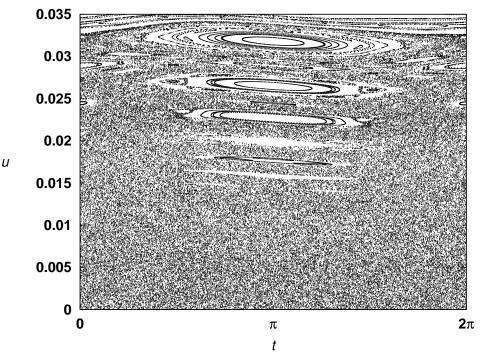# Applied Dynamical Systems Problem Sheet 5

1. [2013 exam] The Fermi-Ulam model consists of a particle colliding alternately with a fixed wall at $x = 0$ and an oscillating wall at $x = L - a \cos t$, where a is a small positive parameter. Both walls are assumed to have infinite mass.

   (a) If the time of the nth collision with the moving wall is denoted by t_n and the velocity immediately before that collision has magnitude v_n, the dynamics may be approximated by

   $$u_{n+1} = |u_n + \epsilon \sin t_n|$$

   $$t_{n+1} = t_n + u_{n+1}^{-1}$$

   State assumptions that may be used to derive these equations, and give expressions for u_n and ϵ in terms of the previously defined quantities.

   (b) Show that the above map is area preserving, and analyse the stability of fixed points where t_n (considered modulo 2π) is zero or π, and hence explain the location and number of the vertical sequence of elliptic islands.

   (c) Explain how the portrait shown in the figure would change as ϵ is decreased toward zero.