TM and TE Directional Modes of an Optical Microdisk Resonator with a Point Scatterer

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ABSTRACT
Dielectric microresonators are becoming key components for novel opto-electronic devices. Circular microresonators (microdisks) are natural candidates for lasing since some of their modes have extremely high Q-factor (low thresholds). In those modes, which are called whispering gallery modes, light circulates around the circumference of the disk trapped by total internal reflection. Although the microdisk cavities can provide ultra-low threshold lasing, their applicability faces the problem of isotropic light emission which is due to rotational symmetry of the system. In contrast to usual procedure, where a geometric deformation of the microdisk boundary is used to break the symmetry and, as a result, to achieve the output directionality, we propose a scenario inducing rotational symmetry breaking by placing a point scatterer inside the microdisk itself. Numerical results show that both TM and TE highly directional modes are observed in the visible spectrum depending on the position of the scatterer. A simple geometric optic explanation of that dependence is given.
Keywords: two-dimensional circular microresonator, point scatterer, TM/TE directional modes.

1. INTRODUCTION

Dielectric microresonators are becoming key components for novel opto-electronic devices. Circular microresonators (microdisks) are natural candidates for lasing since some of their modes have extremely high Q-factor (low thresholds) [1, 2]. In those modes, which are called whispering gallery modes, light circulates around the circumference of the disk trapped by total internal reflection. Although the microdisk cavities can provide ultra-low threshold lasing, their applicability faces the problem of isotropic light emission which is due to the rotational symmetry of the system.

In order to obtain a directional output one has to break the rotational symmetry, for example, by deforming the boundary of the cavity [3, 4]. This significantly improves the emission directionality but typically spoils the Q-factors. Another approach to breaking the symmetry is to insert an obstacle like a linear defect [5] or a hole [6] into the microdisk. This indeed allows one to obtain resonances with large Q-factors and relatively high directionality.

Following the second approach, we have recently suggested to place a point scatterer inside the microdisk, at some distance away from the centre [7, 8]. We have demonstrated that the presence of the scatterer leads to significant enhancement of the directionality of TM modes in outgoing light in comparison with whispering gallery modes of a circular resonator without scatterer, while preserving their high Q-factors.

In this paper we extend our theory to TE modes and investigate in detail the appearance of both highly directional TM and TE modes depending on the distance of a point scatterer from the disk centre. A simple geometric optic explanation of that dependence is given.

2. THEORY OF 2D MICRODISK CAVITIES

The time-harmonic modes of frequency $\omega = ck$, where k is the wave number and c is the light velocity, of any passive microcavity filled with nonmagnetic dielectric material of permittivity $\varepsilon(\mathbf{r})$ are described by 3D Maxwell's equations. For a microcavity, with the thickness of only a fraction of the mode wavelength, modes themselves can be studied in the 2D approximation with the aid of the effective refractive index $n(x, y) = n(r, \varphi)$ which takes into account the material as well as the thickness of the cavity. In that approximation we omit the coordinate dependence of the EM field in the z direction and, as a result, separate the field into TM ($H_z = 0$) and TE-modes ($E_z = 0$). For TM-modes we have

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \varphi^2} + k^2 n^2(r,\varphi)E_z = 0 , \tag{1}$$

with

$$H_r = -\frac{i}{kr}\frac{\partial E_z}{\partial \varphi} \quad , \qquad H_\varphi = \frac{i}{k}\frac{\partial E_z}{\partial r} , \tag{2}$$

while for TE-modes we introduce a new function $h_z(r,\varphi) \equiv H_z(r,\varphi)/n(r,\varphi)$ and obtain

$$\frac{\partial^2 h_z}{\partial r^2} + \frac{1}{r}\frac{\partial h_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 h_z}{\partial \varphi^2} + k^2 n^2(r,\varphi)h_z + \left[\frac{\nabla^2 n}{n(r,\varphi)} - 2\left(\frac{\nabla n}{n(r,\varphi)}\right)^2\right]h_z = 0 , \tag{3}$$

with

$$E_r = \frac{i}{k\,n(r,\varphi)r}\left[\frac{\partial h_z}{\partial \varphi} + \frac{1}{n(r,\varphi)}\frac{\partial n(r,\varphi)}{\partial \varphi}h_z\right], \quad E_\varphi = -\frac{i}{k\,n(r,\varphi)}\left[\frac{\partial h_z}{\partial r} + \frac{1}{n(r,\varphi)}\frac{\partial n(r,\varphi)}{\partial r}h_z\right]. \tag{4}$$

For a homogeneous microdisk of radius R the effective refractive index $n(r,\varphi) \equiv n$ if $r < R$ and inside the microdisk Eqs. (1) and (3) take the form

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r}\frac{\partial \Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \varphi^2} + k^2 n^2\,\Psi(r,\varphi) = 0 , \tag{5}$$

where either $\Psi = E_z$ or $\Psi = h_z$, depending on the polarization. The refractive index of the environment is fixed to be $n = 1$. Separation of variables and physical conditions in the middle of the disk and at infinity lead to the field function Ψ of the form

$$\Psi = \begin{cases} A_m J_m(knr)e^{im\varphi}, & r < R, \\ B_m H_m(kr)e^{im\varphi}, & r > R, \end{cases} \tag{6}$$

where J_m and H_m are Bessel and Hankel functions of the first kind respectively, and m is the azimuthal modal index. Then, the boundary conditions (continuity of the EM fields) at the point $r = R$ lead to a set of independent transcendental equations for the microdisk resonances

$$J_m(k_{res}nR)H_m'(k_{res}R) - \beta J_m'(k_{res}nR)H_m(k_{res}R) = 0 , \tag{7}$$

where $\beta = n$ for TM-modes and $\beta = 1/n$ for TE-modes. We use the radial modal index q to label different resonances with the same azimuthal index m.

Based on self-adjoint extension theory [9-11], it can be shown that the resonances of a homogenous microdisk of radius R with a point scatterer of strength a located at the distance $d < R$ with corresponding polar coordinates (d, φ_d) are defined by the transcendental equation

$$0 = \frac{i\pi}{2} - \ln\left(\frac{k_{res}n a}{2}\right) - \gamma + \frac{i\pi}{2}\sum_{m=0}^{\infty}\frac{D_m}{C_m}\varepsilon_m\,J_m^2(k_{res}nd) , \tag{8}$$

where $\gamma \approx 0.5772$ is the Euler-Mascheroni constant, $\varepsilon_m = 2$ if $m \neq 0$, $\varepsilon_m = 1$ if $m = 0$, and

$$D_m = H_m(k_{res}nR)\,H_m'(k_{res}R) - \beta H_m'(k_{res}nR)\,H_m(k_{res}R) ,$$
$$C_m = J_m(k_{res}nR)\,H_m'(k_{res}R) - \beta J_m'(k_{res}nR)H_m(k_{res}R) . \tag{9}$$

The field function Ψ is then of the form

$$\Psi = N\begin{cases} -\dfrac{i}{4}H_0(kn|\mathbf{r} - \mathbf{d}|) + \dfrac{i}{4}\sum_{m=0}^{\infty}\dfrac{D_m}{C_m}\varepsilon_m \cos[m(\varphi - \varphi_d)]J_m(knr_<)J_m(knr_>), & r < R, \\[2mm] \dfrac{1}{2\pi kR}\sum_{m=0}^{\infty}\dfrac{1}{C_m}\varepsilon_m \cos[m(\varphi - \varphi_d)]J_m(knd)H_m(kr), & r > R, \end{cases} \tag{10}$$

where N is a normalization factor and $r_<\,(r_>)$ is the smaller (larger) of r and d.

3. FAR-FIELD DIRECTIONALITY

In order to quantify the far-field directionality of the field Ψ, we consider its asymptotic behaviour for $r \to \infty$ which has the form

$$\Psi(\mathbf{r}, k_{res}) = \Psi(r, \varphi, k_{res}) \propto \frac{\exp(ik_{res}r)}{\sqrt{r}}\,f(\varphi) . \tag{11}$$

To characterize the directionality we compute the directivity of the far-field intensity

$$D = 2\pi \left| f_{\max}\left(\varphi_{\max}\right)\right|^2 \Bigg/ \left(\int_0^{2\pi} | f(\varphi)|^2 \, d\varphi \right)^2 . \tag{12}$$

From this definition it follows that $D = 1$ and $D = 2$ for unperturbed resonances (no point scatterer in the disk, i.e. $a = 0$) which have $m = 0$ and $m \neq 0$, respectively.

Now, we study the directivity of both TM and TE perturbed resonant modes in the range $12 < \mathrm{Re}(kR) < 13$, upon varying the coupling parameter a for three microdisks of radius $R = 1\,\mu\mathrm{m}$ with effective refractive indices of $n = 3.00$, $n = 2.44$, and $n = 2.11$, and a point scatterer placed at seven different distances (0.4μm, 0.5μm, 0.6μm, 0.7μm, 0.8μm, 0.9μm, 0.98μm) from the centre of each disk, see Eq. (8). The chosen range corresponds to visible light with frequencies $(5.730 - 6.207) \times 10^{14}$ Hz (green-blue light). We found that highly directional TM-modes are mostly concentrated in the range $12.3 < \mathrm{Re}(kR) < 12.7$, while highly directional TE-modes are concentrated in the range $12.7 < \mathrm{Re}(kR) < 13.0$. The directivities of the modes with the highest directionality for each set of (n,d) are shown in Fig. 1.
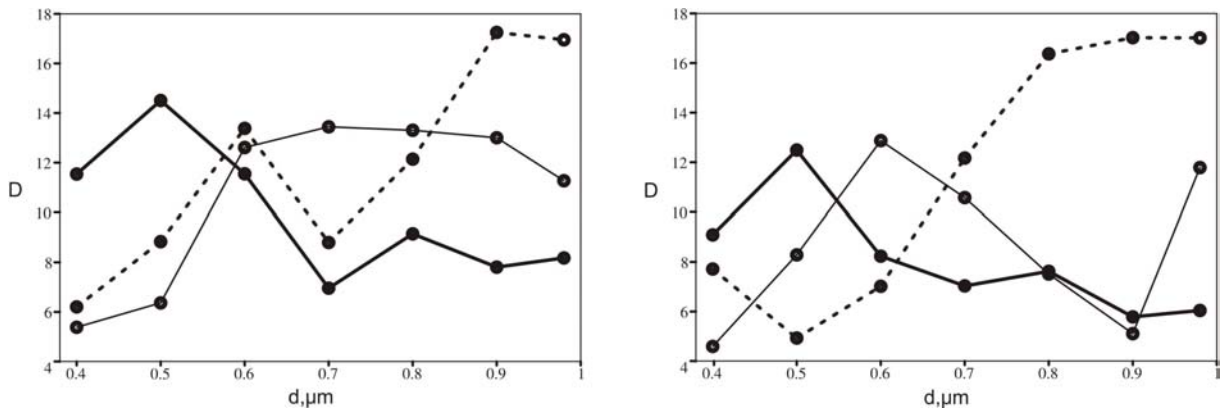

Figure 1. Directivity, D, of the most directional TM-modes in the range $12.3 < \mathrm{Re}(kR) < 12.7$ (left panel) and TE-modes in the range $12.7 < \mathrm{Re}(kR) < 13.0$ (right panel) for each position, d, of a point scatterer in three microdisks of radius $R = 1\,\mu m$ and effective refractive indices of $n = 3.00$ (thick solid line), $n = 2.44$ (thin solid line), $n = 2.11$ (dashed line).

We can see that the directivity of some resonant modes of the microdisk with a scatterer reaches the value $D \approx 18$ which is nine times higher than the directivity of resonant modes with $m \neq 0$ in the microdisk without a scatterer.

Fig.1 indicates that geometric optics seems to give quite a good prediction of the position of a point scatterer that provides the best directionality for resonant modes in a microdisk with a given effective refractive index. If we have a disk of radius $R = 1$ μm and effective refractive index n, a ray coming from infinity will cross the disk axis at the point with the radial coordinate

$$d = \frac{1}{n-1}, \tag{13}$$

if $n > 2$. Inserting $n = 3.00,\ 2.44,\ 2.11$ we find the foci at $d = 0.5,\ 0.7,\ 0.9$ μm, which in most cases coincide with the optimal values for a point scatterer position, see Fig. 1.

4. CONCLUSIONS

In summary, we demonstrated the existence of both highly directional TM and TE-modes in the emission spectrum of a two-dimensional passive microdisk cavity with a point scatterer. A simple geometric optic explanation of the dependence of the output directionality on the position of a point scatterer is given. It would be interesting and potentially very useful to get a more detailed explanation of these results by relating the resonant modes to the ray dynamics in the semiclassical limit.

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