TCC course: Nonlinear Schrödinger Equations

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The study of nonlinear Schrödinger equations is currently a very active area of research. It has many applications, for instance Bose Einstein condensation, dispersive waves and optical waves. In this course we will give an introduction to the topic with the aim to introduce some of the basic ideas used in the study of time dependent nonlinear Schrödinger equations.

The course will have three parts. In the first part we introduce several types of nonlinear Schrödinger equations and and discuss some of the physical motivation for them. We will introduce as well the basic conservations laws. In the second part we will discuss well-posedness of the initial value problem, i.e., under which conditions a unique solution exists and if this solutions depends continuously on variations of the initial conditions. This is based on tools from harmonic analysis, like Strichartz estimates, which we will introduce. In the third part we discuss semiclassical methods which can be used to construct approximate solutions to nonlinear Schrödinger equations.

A Introduction

- Some classes of nonlinear Schrödinger equations, their background and applications.
- Conservation laws and Symmetries.

B Well-posedness.

- some background on Banach and metric spaces, contraction mapping theorem and well-posedness for nonlinear ODE's
- Dispersion and Strichartz estimates for the linear Schödinger equation.
- Local well-posedness in different function spaces
- Conservation laws and global existence.
- breakdown of existence and blowup
- nonlinear Schrödinger equations on the torus.

C Semi-classical approximations

- WKB analysis in the linear case
- Extension to the weakly nonlinear case
- Coherent states

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Literature:

- F. Linares and G. Ponce, *Introduction to Nonlinear Dispersive equations*, Springer 2009
- T. Tao, *Nonlinear Dispersive Equations: Local and Global Analysis*, AMS 2006
- T Cazenave, *Semilinear Schrödinger Equations*, AMS 2003
- R. Carles, *Semi-Classical Analysis for Nonlinear Schrödinger Equations*, World Scientific 2008
- C Sulem and P-L. Sulem, *The Nonlinear Schrödinger Equation*, Springer 1999