TCC Topics in Algebraic Geometry: Assignment #3.

Problem 3 (for 26th November).

As usual, let k be an algebraically closed field, $\mathbb{P}^1 = \mathbb{P}^1_k$, $\mathbb{A}^1 = \mathbb{A}^1_k$.

(1) Prove the result stated in the course that $\operatorname{Aut}\mathbb{P}^1 \cong \mathrm{PGL}_2(k)$. Thus, every isomorphism of varieties $\mathbb{P}^1 \to \mathbb{P}^1$ is of the form $t \mapsto \frac{at+b}{ct+d}$ for a unique $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{PGL}_2(k) = \mathrm{GL}_2(k)/k^\times$.

(2) Determine $\operatorname{Aut}\mathbb{A}^1$ and $\operatorname{Aut}(\mathbb{A}^1 \setminus \{0\})$.

(3) Find $\operatorname{Aut}\mathbb{G}_m$, the group of isomorphisms $\mathbb{G}_m \to \mathbb{G}_m$ as an algebraic group.

Please hand in your solution by emailing it to **tccalggeom@gmail.com** by 26th November.