TCC Topics in Algebraic Geometry: Assignment #4.

Problem 4 (for 10th December).

(1) Prove that over $K = \mathbb{R}$, the unit circle group $S^1 : x^2 + y^2 = 1$ is the only non-trivial form of $\mathbb{G}_m$ up to isomorphism (as algebraic groups).

(2) Similarly, over $K = \mathbb{F}_p$, prove that $\mathbb{G}_m$ has a unique non-trivial form. Write it down as an algebraic group (equations + structure morphisms), and determine its number of points over K.

Please hand in your solution by emailing it to **tccalggeom@gmail.com** by 10th December.