Galois Representations (Lecture 3)

Exercise. Let $X = \operatorname{Spec} \mathbb{Q}(i)$ $(= \operatorname{Spec} \mathbb{Q}[x]/(x^2+1))$, viewed as a scheme $/\mathbb{Q}$.

(1) Explain why $H^i_{\text{ét}}(X_{\bar{\mathbb{Q}}}, \mathbb{Q}_l) = 0$ for $i > 0$.
(2) Describe $X(\bar{\mathbb{Q}})$ and the action of $G_{\mathbb{Q}}$ on this set.
(3) Prove that
$$L(H^0(X), s) = \zeta(s)L(\chi, s)$$
where χ is the unique non-trivial Dirichlet character $(\mathbb{Z}/4\mathbb{Z})^\times \to \mathbb{C}^\times$.