1. Simplex tableaus (there may be typos, please report!)

   (a) Initial tableau, the columns labeled by $(z, x_1, \ldots, x_5, \text{Val})$ (for *value*).

$$T0 = \begin{bmatrix} 0 & -1 & 2 & 1 & 0 & 0 & 4 \\ 0 & 3 & 2 & 0 & 1 & 0 & 14 \\ 0 & \mathbf{1} & -1 & 0 & 0 & 1 & 3 \\ 1 & -3 & -2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Pivot $T0_{32}$:

$$T1 := \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 7 \\ 0 & 0 & \mathbf{1} & 0 & \frac{1}{5} & \frac{-3}{5} & 1 \\ 0 & 1 & -1 & 0 & 0 & 1 & 3 \\ 1 & 0 & -5 & 0 & 0 & 3 & 9 \end{bmatrix}$$

Pivot $T1_{23}$:

$$T2 = \begin{bmatrix} 0 & 0 & 0 & 1 & \frac{-1}{5} & \frac{8}{5} & 6 \\ 0 & 0 & 1 & 0 & \frac{1}{5} & \frac{-3}{5} & 1 \\ 0 & 1 & 0 & 0 & \frac{1}{5} & \frac{2}{5} & 4 \\ 1 & 0 & 0 & 0 & 1 & 0 & 14 \end{bmatrix}$$

Optimal value $z = 14$, achieved by $x_4 = 0$, $x_1 = 4 - .4x_5$, $x_2 = 1 + .6x_5$, $x_3 = 6 - 1.6x_5$, $0 \le x_5 \le 3.75$: multiple solutions.

As this is the manufacturing problem, the shadow prices are the reduced costs of the slack variables (in the final tableau, bottom row), i.e. $\boldsymbol{y} = (0, 1, 0)$.

   (b) Initial tableau, the columns labeled by $(z, x_1, \ldots, x_5, \text{Val})$.

$$T0 = \begin{bmatrix} 0 & 3 & 1 & 0 & 1 & 0 & 18 \\ 0 & 0 & \mathbf{2} & 1 & 0 & 1 & 7 \\ 1 & -2 & 8 & 5 & 0 & 0 & 0 \end{bmatrix}$$

Pivot $T0_{23}$:

$$T1 = \begin{bmatrix} 0 & 3 & 0 & \frac{-1}{2} & 1 & \frac{-1}{2} & \frac{29}{2} \\ 0 & 0 & 1 & \mathbf{\frac{1}{2}} & 0 & \frac{1}{2} & \frac{7}{2} \\ 1 & -2 & 0 & 1 & 0 & -4 & -28 \end{bmatrix}$$

Pivot $T2_{24}$:

$$T2 = \begin{bmatrix} 0 & 3 & 1 & 0 & 1 & 0 & 18 \\ 0 & 0 & 2 & 1 & 0 & 1 & 7 \\ 1 & -2 & -2 & 0 & 0 & -5 & -35 \end{bmatrix}$$

Optimal value -35, with a BOS $x_4 = 18$, $x_3 = 7$, $x_1 = x_2 = x_5 = 0$. Note: pivoting $T0_{24}$ in the original tableau would have completed the procedure in one step, as x_3 had a reduced cost 8, with a minimum ratio 3.5, while x_4 had a minimum ratio 7 with reduced cost 5, i.e. 7 units of x_4 reduce the cost by more than 3.5 units of x_3.

As this is the manufacturing problem, (if we change $\boldsymbol{c}$ to $-\boldsymbol{c}$) the shadow prices are minus the reduced costs of the slack variables (in the final tableau, bottom row), i.e. $\boldsymbol{y} = (0, 5)$.

(c) Let's do two-phase method. We shall need excess variables x_4, x_6 to the first and third inequality and slack x_5 to the second one. The matrix of the system of equations, without the objective, is

$$\begin{bmatrix} 1 & -2 & 0 & -1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 3 \\ 0 & -1 & 2 & 0 & 0 & -1 & 2 \end{bmatrix}$$

To complete it in order to have all the columns of the 3×3 identity matrix therein we shall need two artificial variables, so on Phase I we get

$$T0 = \begin{bmatrix} \tilde{z} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & \\ 0 & 1 & -2 & 0 & -1 & 0 & 0 & \mathbf{1} & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & -1 & 2 & 0 & 0 & -1 & 0 & \mathbf{1} & 2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

with the temporary objective Min $\tilde{z} = x_7 + x_8$ – the sum of the artificial variables. Eliminating the variables x_7, x_8 from the objective row yields

$$T0' = \begin{bmatrix} 0 & 1 & -2 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & -1 & \mathbf{2} & 0 & 0 & -1 & 0 & 1 & 2 \\ 1 & 1 & -3 & 2 & -1 & 0 & -1 & 0 & 0 & 3 \end{bmatrix}$$

Note that if you wanted to use the short tableau, you would have simply have

$$T0'' = \begin{bmatrix} & x_1 & x_2 & x_3 & x_4 & x_6 & \\ x_7 & 1 & -2 & 0 & -1 & 0 & 1 \\ x_5 & 1 & 1 & 1 & 0 & 0 & 3 \\ x_8 & 0 & -1 & \mathbf{2} & 0 & -1 & 2 \\ \tilde{z} & 1 & -3 & 2 & -1 & -1 & 3 \end{bmatrix}$$

Namely the last row would be just the sum of the first and third rows.
In any case, there are two artificial variables to get rid of – two steps will be needed. First, pivot the bold **2**. Let us proceed with the long tableau and make remarks about the short ones.

$$T1 = \begin{bmatrix} 0 & \mathbf{1} & -2 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 3/2 & 0 & 0 & 1 & 1/2 & 0 & -1/2 & 2 \\ 0 & 0 & -1/2 & 1 & 0 & 0 & -1/2 & 0 & 1/2 & 1 \\ 1 & 1 & -2 & 0 & -1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Pivoting the bold **1** now:

$$T2 = \begin{bmatrix} 0 & 1 & -2 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 7/2 & 0 & 1 & 1 & 1/2 & -1 & -1/2 & 1 \\ 0 & 0 & -1/2 & 1 & 0 & 0 & -1/2 & 0 & 1/2 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

Phase I is done, we have a BFS $x_1 = x_5 = x_3 = 1$, check that it is indeed feasible.
Note that the short tableau here – just throw away the unit matrix columns – would be

$$T2' = \begin{bmatrix} & x_2 & x_4 & x_6 & x_7 & x_8 & \\ x_1 & -2 & -1 & 0 & 1 & 0 & 1 \\ x_5 & 7/2 & 1 & 1/2 & -1 & -1/2 & 1 \\ x_3 & -1/2 & 0 & -1/2 & 0 & 1/2 & 1 \\ \tilde{z} & 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

Now Phase II begins: replace the objective row with $z = 4x_1 - .5x_2 + 8x_3$, remove the two columns corresponding to the artificial variables:

$$T3 = \begin{bmatrix} 0 & \mathbf{1} & -2 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 7/2 & 0 & 1 & 1 & 1/2 & 1 \\ 0 & 0 & -1/2 & \mathbf{1} & 0 & 0 & -1/2 & 1 \\ 1 & -4 & 1/2 & -8 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Eliminate the basic variables $x_{1,3}$ from the objective row:

$$T4 = \begin{bmatrix} 0 & 1 & -2 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 7/2 & 0 & 1 & 1 & \mathbf{1/2} & 1 \\ 0 & 0 & -1/2 & 1 & 0 & 0 & -1/2 & 1 \\ 1 & 0 & -11.5 & 0 & -4 & 0 & -4 & 12 \end{bmatrix}$$

Note that the short tableau here would be

$$T4' = \begin{bmatrix} & x_2 & x_4 & x_6 & \\ x_1 & -2 & -1 & 0 & 1 \\ x_5 & 7/2 & 1 & \mathbf{1/2} & 1 \\ x_3 & -1/2 & 0 & -1/2 & 1 \\ z & -11.5 & -4 & -4 & 12 \end{bmatrix}$$

It would be simply obtained from $T2'$, having removed the artificial variables, by adding 4 times the x_1 row and 8 times the x_3 row – the basic variables in the objective, and in addition adding .5 to the x_2-entry, a there is $-.5x_2$ in the objective.
Let us do some intelligent pivoting: pivoting the element **1/2** in the long tableau will increase the objective by 8, which is the best we can get, so:

$$T5 = \begin{bmatrix} 0 & 1 & -2 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 7 & 0 & 2 & 2 & 1 & 2 \\ 0 & 0 & 3 & 1 & 1 & 1 & 0 & 2 \\ 1 & 0 & 16.5 & 0 & 4 & 8 & 0 & 20 \end{bmatrix}$$

This is the final tableau $x_1 = 1$, $x_3 = 2$, $x_6 = 2$ is optimal.

If we want dual simplex, we should not drop the artificial columns after Phase I has been finished. In the tableau $T0$ the 3×3 identity matrix is given by the columns x_5, x_7, x_8. So, the difference is: when passing from Phase I to Phase II, do *not* erase the free variables' columns, but always keep them free. So, let us return to the final tableau $T2$ from Phase I and redo Phase II, keeping track of the artificial variables' columns:

$$T2 = \begin{bmatrix} 0 & 1 & -2 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 7/2 & 0 & 1 & 1 & 1/2 & -1 & -1/2 & 1 \\ 0 & 0 & -1/2 & 1 & 0 & 0 & -1/2 & 0 & 1/2 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

Replace the objective row with $z = 4x_1 - .5x_2 + 8x_3$:

$$T3' = \begin{bmatrix} 0 & \mathbf{1} & -2 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 7/2 & 0 & 1 & 1 & 1/2 & -1 & -1/2 & 1 \\ 0 & 0 & -1/2 & \mathbf{1} & 0 & 0 & -1/2 & 0 & 1/2 & 1 \\ 1 & -4 & 1/2 & -8 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Eliminate the basic variables $x_{1,3}$ from the objective row:

$$T4'' = \begin{bmatrix} 0 & 1 & -2 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 7/2 & 0 & 1 & 1 & \mathbf{1/2} & -1 & -1/2 & 1 \\ 0 & 0 & -1/2 & 1 & 0 & 0 & -1/2 & 0 & 1/2 & 1 \\ 1 & 0 & -11.5 & 0 & -4 & 0 & -4 & 4 & 4 & 12 \end{bmatrix}$$

Equivalently, to the short tableau $T4'$ we would add two more columns x_7, x_8 from $T2'$, and recall how the objective row has been computed:

$$T4''' = \begin{bmatrix} & x_2 & x_4 & x_6 & x_7 & x_8 & \\ x_1 & -2 & -1 & 0 & 1 & 0 & 1 \\ x_5 & 7/2 & 1 & \mathbf{1/2} & -1 & -1/2 & 1 \\ x_3 & -1/2 & 0 & -1/2 & 0 & 1/2 & 1 \\ z & -11.5 & -4 & -4 & 4 & 4 & 12 \end{bmatrix}$$

Never consider the artificial variables' rows for pivoting any more! They are zero, and we keep track of what is happening to them only to obtain finally the shadow prices of the constraints.

So, the pivot gives us

$$T5' = \begin{bmatrix} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & \\ 0 & 1 & -2 & 0 & -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 7 & 0 & 2 & 2 & 1 & -1 & -1 & 2 \\ 0 & 0 & 3 & 1 & 1 & 1 & 0 & -1 & 0 & 2 \\ 1 & 0 & 16.5 & 0 & 4 & 8 & 0 & -4 & 0 & 20 \end{bmatrix}$$

This tableau is final, in spite of the -4 in the bottom row, as we are not going to make the artificial variable x_7 basic, i.e., nonzero. The short tableau is clearly obtained by erasing the unit matrix columns.

The shadow prices of the constraints are now the entries in the bottom row in those columns where we had the 3×3 identity matrix in the initial tableau $T0$, i.e. the columns x_5, x_7, x_8. The shadow prices then are $8, -4, 0$ of the second, first, and third constraints, respectively. Indeed, the variable x_5 was added to the second constraint, the variable x_7 to the second one, and x_8 to the last one.

Observe that the optimal solution $x_1 = 1, x_3 = 2, x_2 = 0$ of the original system of inequalities satisfies the first two of them tightly and the third one with slack. So complementary slackness tells us that the shadow price of the third constraint should be zero, as it is.

(d) Two-phase method. Initial tableau, the columns labeled by $(\tilde{z}, x_1, \ldots, x_8, \text{Val})$ with artificial variables x_5, x_7.

$$T0 = \begin{bmatrix} 0 & 1 & -4 & 2 & -1 & \mathbf{1} & 0 & 0 & 0 & 12 \\ 0 & 2 & 1 & 1 & 0 & 0 & -1 & \mathbf{1} & 0 & 10 \\ 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & 7 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 \end{bmatrix}$$

Let us do the short tableaus: start out with the free variables only, without the objective so far.

$$T0' = \begin{bmatrix} & x_1 & x_2 & x_3 & x_4 & x_6 & \\ x_5 & 1 & -4 & 2 & -1 & 0 & 12 \\ x_7 & 2 & 1 & 1 & 0 & -1 & 10 \\ x_8 & 1 & -1 & 1 & 0 & 0 & 7 \end{bmatrix}$$

Now, the Phase I objective is $\tilde{z} = x_5 + x_7$, so the short tableau row now is just the sum of the x_5 and x_7 rows in the above tableau $T0'$.

$$T0'' = \begin{bmatrix} & x_1 & x_2 & x_3 & x_4 & x_6 & \\ x_5 & 1 & -4 & \mathbf{2} & -1 & 0 & 12 \\ x_7 & 2 & 1 & 1 & 0 & -1 & 10 \\ x_8 & 1 & -1 & 1 & 0 & 0 & 7 \\ \tilde{z} & 3 & -3 & 3 & -1 & -1 & 22 \end{bmatrix}$$

Pivot **2**:

$$T1 = \begin{bmatrix} & x_1 & x_2 & x_5 & x_4 & x_6 & \\ x_3 & 1/2 & -2 & 1/2 & -1/2 & 0 & 6 \\ x_7 & 3/2 & 3 & -1/2 & 1/2 & -1 & 4 \\ x_8 & 1/2 & \mathbf{1} & -1/2 & 1/2 & 0 & 1 \\ \tilde{z} & 3/2 & 3 & -3/2 & 1/2 & -1 & 4 \end{bmatrix}$$

Pivot **1**:

$$T2 = \left[\begin{array}{cccccccc} & x_1 & x_8 & x_5 & x_4 & x_6 & \\ x_3 & 3/2 & 2 & -1/2 & 1/2 & 0 & 8 \\ x_7 & 0 & -3 & 1 & -1 & -1 & 1 \\ x_2 & 1/2 & 1 & -1/2 & 1/2 & 0 & 1 \\ \tilde{z} & 0 & -3 & 0 & -1 & -1 & 1 \end{array}\right]$$

Alas, this is the final tableau, showing that $\tilde{z} = 1$ cannot be reduced any further (we still have the artificial variable $x_7 = 1$, so one cannot have all artificial variables zero, the original problem is unfeasible. Which means, the dual is unbounded.

(e) Two-phase method. Initial tableau, the columns labeled by $(\tilde{z}, x_1, \ldots, x_7, \text{Val})$ with an artificial variable x_4.

$$T0 = \left[\begin{array}{cccccccc} 0 & 1 & 0 & 1 & \mathbf{1} & 0 & 0 & 5 \\ 0 & -1 & -4 & 2 & 0 & 1 & 0 & 6 \\ 0 & -1 & -3 & 3 & 0 & 0 & 1 & 7 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{array}\right]$$

Let's do the short tableau again: the basic variables are x_4, x_5, x_6, the free ones x_1, x_2, x_3, and the objective row $\tilde{z} = x_4$ in the short tableau simply copies the x_4 row.
Eliminate the artificial variable x_4 from the objective row:

$$T1 = \left[\begin{array}{ccccc} & x_1 & x_2 & x_3 & \\ x_4 & \mathbf{1} & 0 & 1 & 5 \\ x_5 & -1 & -4 & 2 & 6 \\ x_6 & -1 & -3 & 3 & 7 \\ \tilde{z} & 1 & 0 & 1 & 5 \end{array}\right]$$

Pivot **1**

$$T2 = \left[\begin{array}{ccccc} & x_4 & x_2 & x_3 & \\ x_1 & 1 & 0 & 1 & 5 \\ x_5 & 1 & -4 & 3 & 11 \\ x_6 & 1 & -3 & 4 & 12 \\ \tilde{z} & -1 & 0 & 0 & 0 \end{array}\right]$$

This is the final Phase I tableau, with $\tilde{z} = x_4 = 0$, giving a BFS $x_1 = 5$, $x_6 = 11$, $x_7 = 12$ to the original problem. Let us now substitute the proper objective $z = x_1 + x_2 + x_3$. This means, we shall put down the x_1 row as the objective row, and then subtract -1 from the entries in the x_2 and x_3 columns.
As for the artificial variable x_4, we do not need it any more, but let us still keep it for the purposes of the dual simplex method to get the shadow prices. Just remember never to try to bring it to the basis, whatever its reduced cost.

$$T2 = \left[\begin{array}{ccccc} & x_4 & x_2 & x_3 & \\ x_1 & 1 & 0 & 1 & 5 \\ x_5 & 1 & -4 & 3 & 11 \\ x_6 & 1 & -3 & 4 & 12 \\ z & 1 & -1 & 0 & 5 \end{array}\right]$$

This is the final tableau, as the variable $x_2 \geq 0$ can be brought into the feasible solution in unlimited quantities, increasing the objective *ad infinitum*, for instance $x_1 = 5$, $x_5 = 11 + 4x_2$, $x_6 = 12 + 3x_2$, $x_3 = 0$, while $z = 5 + x_2$. So, the problem is unbounded. As for the shadow prices, unboundedness of the primal implies that the dual is unfeasible.

2. The task is to do two pivot in the basic columns.

- First introduce the excess variable x_4 and slack variable x_5 to get equations from the constraints:

$$x_1 + 2x_2 + x_3 - x_4 = 3, \quad x_1 + x_2 + 2x_3 + x_5 = 2,$$

  Given the basis (x_1, x_2), solve these equations with $x_3 = x_4 = x_5 = 0$, getting $x_1 = x_2 = 1$.
  Note that the the second constraint says $x_1 + x_2 + 2x_3 \leq 2$, and the objective is to maximize the left-hand side, so the optimal value cannot exceed 2.

- Now, from the constraints express the basic variables x_1 and x_2 in terms of free variables $x_3, \ldots, x_5$. Subtract the second constraint equation from the first one. Get $x_2 = 1 + x_3 + x_4 + x_5$. Substitute for x_2 into the second equation, get $x_1 = 1 - 3x_3 - x_4 - 2x_5$. Besides $z = x_1 + x_2 + 2x_3 = 2 - x_5$. Hence the tableau sought after is

$$\begin{bmatrix} 0 & 1 & 0 & 3 & 1 & 2 & 1 \\ 0 & 0 & 1 & -1 & -1 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

  The columns are marked as $(z, x_1, \ldots, x_5, Val)$, the rows as (x_1, x_2, z).

3. If $\boldsymbol{b}$ is a linear combination of fewer than m columns of A, then denoting $\mathbf{a}^1, \ldots, \mathbf{a}^n \in \mathbb{R}^m$ the columns of the matrix A, one has $x_\alpha \mathbf{a}^\alpha + x_\beta \mathbf{a}^\beta + \ldots = \boldsymbol{b}$, where all $x_\alpha, x_\beta, \ldots$ are all nonzero, and their number is less than m. If all the coefficients $x_\alpha, x_\beta, \ldots$ turn out to be positive, they constitute a BFS with fewer than m positive components.

   Geometrically, this corresponds to $\boldsymbol{b}$ lying in "lower-dimensional sub-cones" of the cone

$$\mathcal{C}_A = \{\boldsymbol{y} \in \mathbb{R}^m : \boldsymbol{y} = x_1\mathbf{a}^1 + \ldots + x_n\mathbf{a}^n,\, x \in \mathbb{R}^n_+\},$$

   which are spanned by fewer than m columns of A.

   If such a degenerate BFS has $r < m$ positive components, the tableau, corresponding to such a BFS will have $m-r$ zeroes in the value column, as the example tableau in the problem, where $r = 1$. Hence when one does a simplex method step, it may happen that passing to an adjacent solution does not improve the objective, as the latter's improvement equals reduced cost times ratio, and the ratio will be zero.

   To pass to the example

	x_3	x_4	
x_1	1	1	0
x_2	2	1	2
z	1	1	5

   for a minimisation problem, we cannot at first conclude whether this tableau is optimal or not, as we have positive reduced costs. However, the first equation, in fact, says, $x_1 = -x_3 - x_4$, which as $\boldsymbol{x} \geq 0$ necessitates $x_3 = x_4 = 0$, and therefore this tableau is optimal, as the constraints force the whole feasible set to be a single point $x_1 = 0, x_2 = 2, x_4 = x_5 = 0$. This is the end of the story with the above example. What I had in mind, however, was something a bit more complicated.

   Let us modify it by by adding a free variable x_5 as follows.

	x_3	x_4	x_5	
x_1	**1**	1	-1	0
x_2	2	1	0	2
z	2	1	-1	5

   Graphically, the columns of A are now vectors $(1,0), (0,1), (1,2), (1,1), (-1,0)$ and it happens that $\boldsymbol{b} = (0,2)$ is 2 times the second column. Hence, a BFS $x_2 = 2$, the rest of $\boldsymbol{x}$ being zero. However, $\boldsymbol{b} = (0,2)$ sits in the sector (alias "cone") formed by the third and fifth columns of A as well, and those will provide the optimal basis.

   Analytically, the first equation would mean $x_1 = x_5 - x_3 - x_4$. After the x_5-variable has been added, one cannot tell whether the above BFS is optimal or not: having $x_3 > 0$ is good for the objective, but then x_5 must become positive as well, which is bad for the objective. This is a very simple example, but it shall be patent that the basis $\{x_3, x_5\}$ might be better than just $\{x_2\}$. Let us verify it via a computation.

Pivoting **1** yields

	x_1	x_4	x_5	
x_3	1	1	-1	0
x_2	-2	-1	**2**	2
z	-2	-1	1	5

The value 5 has not changed so far, as there was a zero ratio. But now one can see that bringing x_5 into the basis certainly makes sense, so pivoting **2** yields

	x_1	x_4	x_2	
x_3	0	$1/2$	$1/2$	1
x_5	-1	$-1/2$	$1/2$	1
z	-1	$-1/2$	$-1/2$	4

which is the final tableau for the minimisation problem. Observe, that the basic feasible solution is now non-degenerate, i.e. it has two positive components.