Uncertainty and Risk in Natural Hazards

Jonathan Rougier

Department of Mathematics
University of Bristol, UK

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Natural hazards

A general framework for natural hazards

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Not knowing these components, or limitations in doing the calculations, leads to *epistemic uncertainty*.

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5. The loss might then be the total area of damage in the region, with respect to some threshold, say $1\,\text{m}/\text{s}^2$,
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Probability of exceedence (PE) curves

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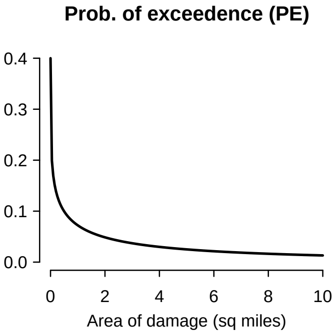

The probability distribution function of $\tilde{\ell}$ is

$$F_{\tilde{\ell}}(v) = \Pr\{\tilde{\ell} \le v\} = \sum_{\omega \in \Omega} 1[\ell_\omega \le v] p_\omega .$$

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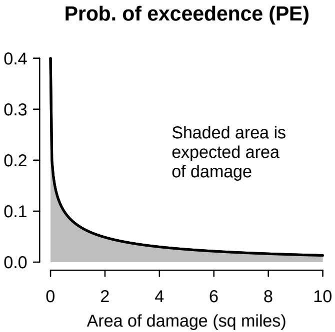

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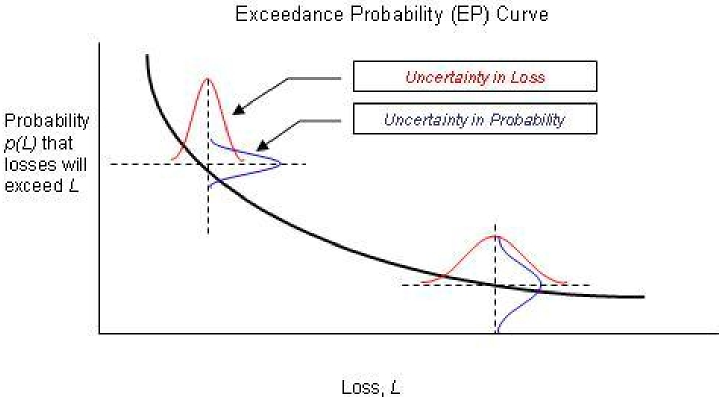

A catalogue of epistemic uncertainties

In order of increasing challenge:

1. Not being able to enumerate Ω and p_ω;
2. Not knowing the loss operator;
3. Not knowing the form and parameters of the aleatory process;
4. Not knowing which future will prevail (scenarios);
5. Not knowing the footprint function.

Two crucial tools:

- Adding a 'margin for error' (ubiquitous);
- Sensitivity analysis (very under-used).

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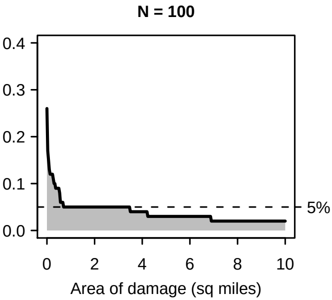

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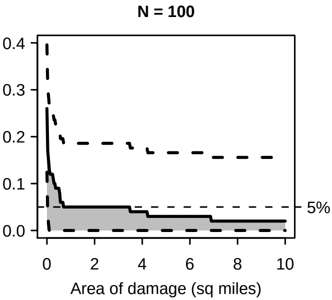

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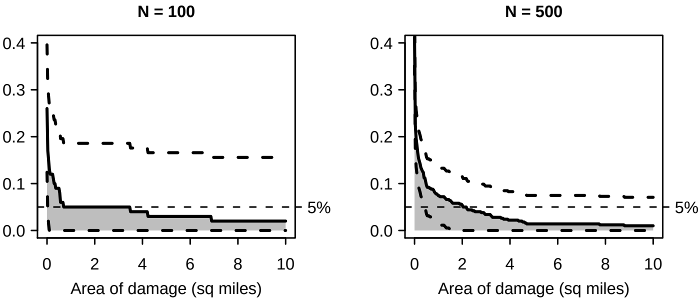

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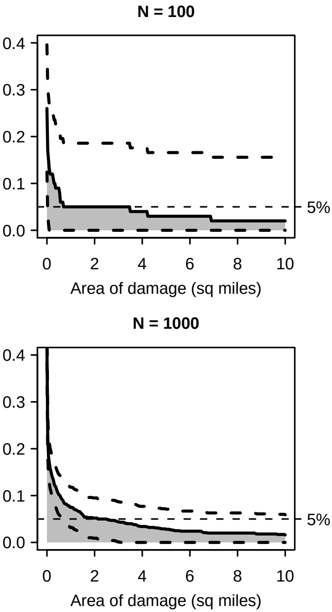

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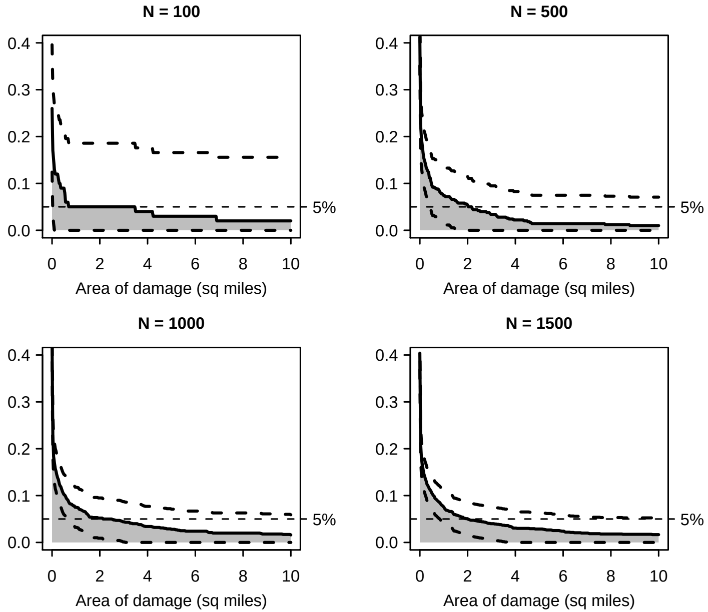

Uncertainty about the loss operator can be integrated out

Previously we had

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But it is straightforward to generalise to

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- This changes the shape of the PE curve *but it does not make the PE curve uncertain.*

Multiple future scenarios

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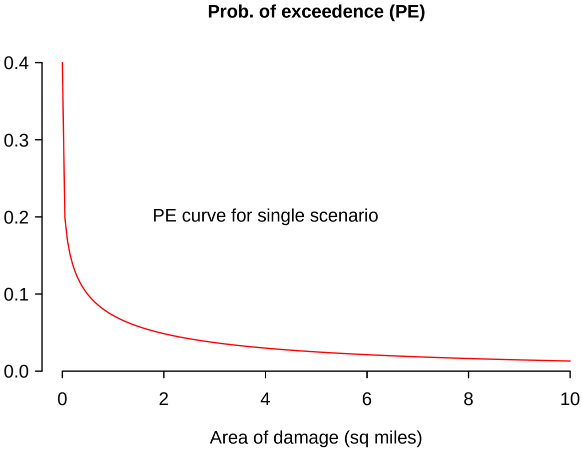

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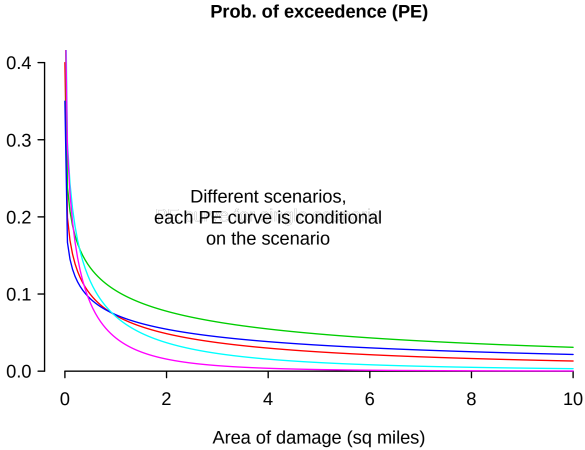

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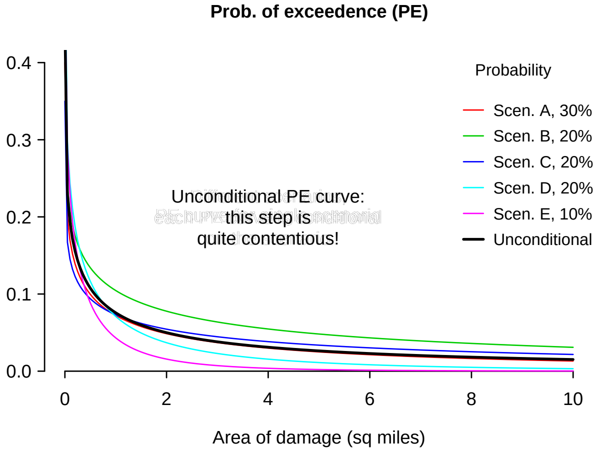

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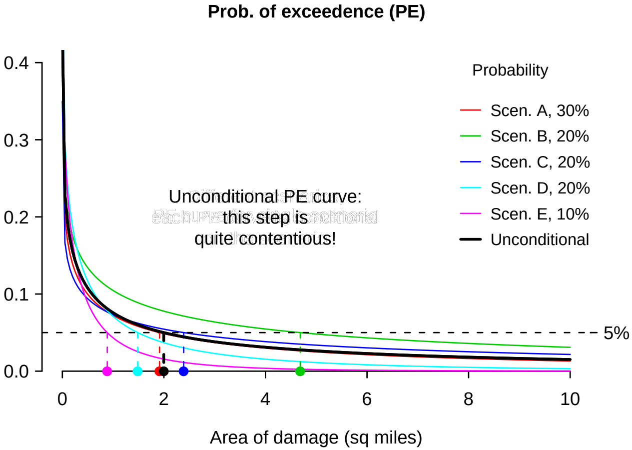

Other sources of epistemic uncertainty

Not knowing the form and parameters of the aleatory process.

This is standard statistics: challenging, but not obscure.

Not knowing the footprint function.

This is indeed hard, but progress is being made in the statistical field of Computer Experiments, notably in the MUCM project.

Summary

‘Aleatory’ uncertainty

The inherent uncertainty in a natural hazard. A PE curve is a summary of aleatory uncertainty as represented in terms of loss.

‘Epistemic’ uncertainty

‘Other’ uncertainty, notably that which arises from our incomplete knowledge. Things to remember:

1. Limited sampling introduces uncertainty, and can be represented as confidence bands on the PE curve, e.g. when estimating quantiles.
2. Some epistemic uncertainties can be integrated out. The PE curve does not become ‘uncertain’, it becomes *conditional*.
3. Others are more challenging, notably multiple future scenarios, and accounting for footprint function limitations.