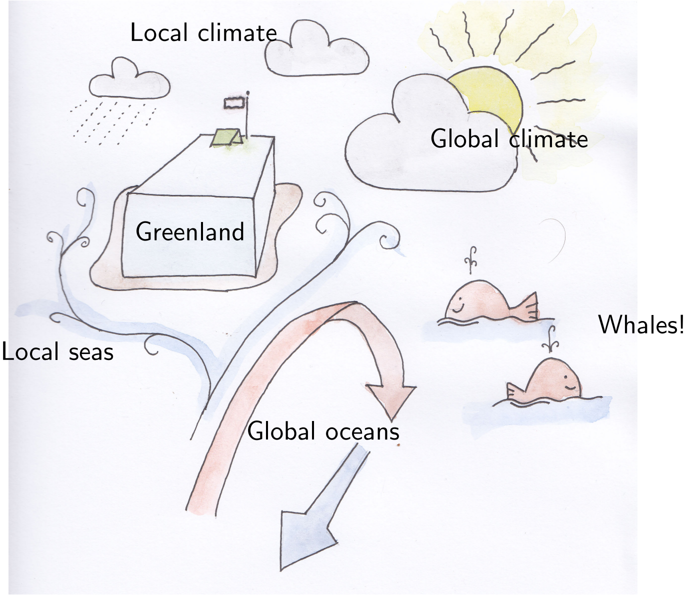
Assessing Model Limitations

Jonathan Rougier

Department of Mathematics
University of Bristol, UK

Bristol, Thu 13 May 2010

Illustration: the Greenland ice-sheet



Two different approaches

The current approach (deterministic model):

$$
\begin{tikzcd}
\text{Observations:} & & y_{t-1} & & y_{t+1} & \\
\text{System:} & \cdots \arrow[r] & x_{t-1} \arrow[u, squiggly] \arrow[r] & x_t \arrow[r] & x_{t+1} \arrow[u, squiggly] \arrow[r] & \cdots \\
\text{Model parameters:} & & & \theta \arrow[ul] \arrow[u] \arrow[ur] & &
\end{tikzcd}
$$

Two different approaches

The current approach (deterministic model):

$$\begin{tikzcd}
\text{Observations:} & & y_{t-1} & & y_{t+1} & \\
\text{System:} & \cdots \arrow[r] & x_{t-1} \arrow[u, squiggly] \arrow[r] & x_t \arrow[r] & x_{t+1} \arrow[u, squiggly] \arrow[r] & \cdots \\
\text{Model parameters:} & & & \theta \arrow[ul] \arrow[u] \arrow[ur] & &
\end{tikzcd}$$

Learning

The initial state x_0 and the model parameters θ are jointly estimated by minimising the sum of squared deviations between the observations and the model output ('maximum likelihood').

Not expected to perform well.

Two different approaches

The *statistical* approach (*stochastic* model):

$$
\begin{tikzcd}
\text{Observations:} & & y_{t-1} & & y_{t+1} & \\
\text{System:} & \cdots \arrow[r, rightsquigarrow] & x_{t-1} \arrow[u, rightsquigarrow] \arrow[r, rightsquigarrow] & x_t \arrow[r, rightsquigarrow] & x_{t+1} \arrow[u, rightsquigarrow] \arrow[r, rightsquigarrow] & \cdots \\
\text{Model parameters:} & & & \theta \arrow[ul, rightsquigarrow] \arrow[u, rightsquigarrow] \arrow[ur, rightsquigarrow] & &
\end{tikzcd}
$$

Two different approaches

The *statistical* approach (*stochastic* model):

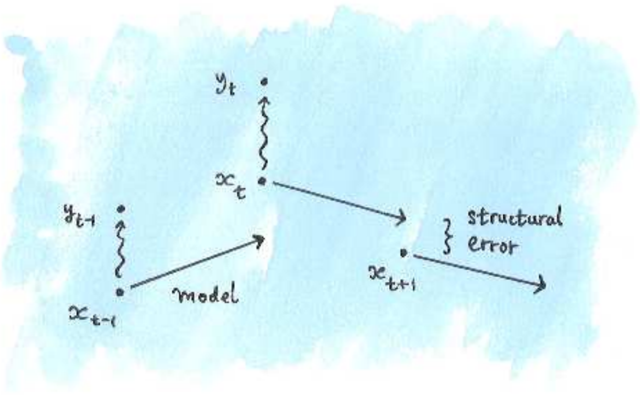
$$
\begin{tikzcd}
\text{Observations:} & & y_{t-1} & & y_{t+1} & \\
\text{System:} & \cdots \arrow[r, squiggly] & x_{t-1} \arrow[u, squiggly] \arrow[r, squiggly] & x_t \arrow[r, squiggly] & x_{t+1} \arrow[u, squiggly] \arrow[r, squiggly] & \cdots \\
\text{Model parameters:} & & & \theta \arrow[ul, squiggly] \arrow[u, squiggly] \arrow[ur, squiggly] & &
\end{tikzcd}
$$

Learning

The joint distribution of the state trajectory $x_0, x_1, \ldots$ and the model parameters θ is updated by the observations $\{y_t\}$. The result is represented as a set of samples of $(x_0, x_1, \ldots, \theta)$.

☎ *Hard to do: call a statistician!*

Structural error on the time-step



Why is it so important to 'noise up' the model?

Statisticians are often grumpy. What's the pay-off?

Why is it so important to 'noise up' the model?

Statisticians are often grumpy. What's the pay-off?

- If we are interested in quantifying uncertainty, then, for our analysis to be defensible, the uncertainty must be sourced correctly.

Why is it so important to 'noise up' the model?

Statisticians are often grumpy. What's the pay-off?

- If we are interested in quantifying uncertainty, then, for our analysis to be defensible, the uncertainty must be sourced correctly.

- In environmental science, the dominant source of uncertainty is structural limitations in the model, and this uncertainty lives in the propagation of the state vector from x_{t-1} to x_t.

Why is it so important to 'noise up' the model?

Statisticians are often grumpy. What's the pay-off?

- If we are interested in quantifying uncertainty, then, for our analysis to be defensible, the uncertainty must be sourced correctly.
- In environmental science, the dominant source of uncertainty is structural limitations in the model, and this uncertainty lives in the propagation of the state vector from x_{t-1} to x_t.
- We cannot simply add on some uncertainty to the solution of a deterministic model, because non-linearities in the model imply that *deterministic and stochastic solutions have fundamentally different characters.*

Illustration: the van der Pol oscillator

Has a 'slow' response x and a 'fast' response $\dot{x}$, related by

$$\ddot{x} + \dot{x} + (\alpha - x^2)x = \sigma x \xi$$

where ξ is white noise. Here $\theta = (\alpha, \sigma)$; we'll take $\alpha = 1$.

Illustration: the van der Pol oscillator

Has a 'slow' response x and a 'fast' response $\dot{x}$, related by

$$\ddot{x} + \dot{x} + (\alpha - x^2)x = \sigma x\, \xi$$

where ξ is white noise. Here $\theta = (\alpha, \sigma)$; we'll take $\alpha = 1$.

As a model for glacial cycles

Deterministic, $\sigma = 0$

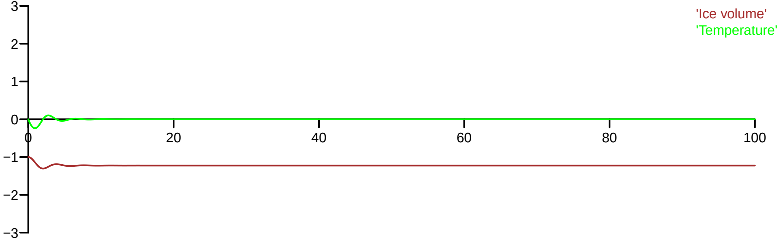


Illustration: the van der Pol oscillator

Has a 'slow' response x and a 'fast' response $\dot{x}$, related by

$$\ddot{x} + \dot{x} + (\alpha - x^2)x = \sigma x \, \xi$$

where ξ is white noise. Here $\theta = (\alpha, \sigma)$; we'll take $\alpha = 1$.

As a model for glacial cycles

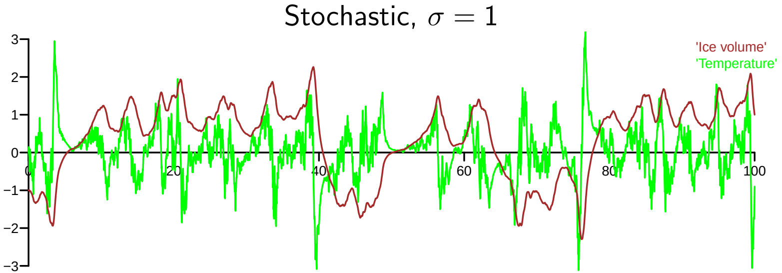


Illustration: the van der Pol oscillator

Has a ‘slow’ response x and a ‘fast’ response $\dot{x}$, related by

$$\ddot{x} + \dot{x} + (\alpha - x^2)x = \sigma x\,\xi$$

where ξ is white noise. Here $\theta = (\alpha, \sigma)$; we’ll take $\alpha = 1$.

As a model for glacial cycles

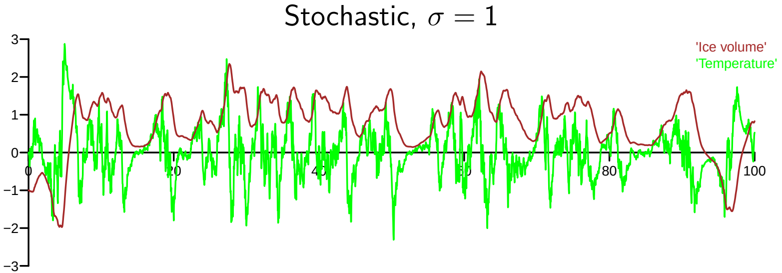


Illustration: the van der Pol oscillator

Has a 'slow' response x and a 'fast' response $\dot{x}$, related by

$$\ddot{x} + \dot{x} + (\alpha - x^2)x = \sigma x \, \xi$$

where ξ is white noise. Here $\theta = (\alpha, \sigma)$; we'll take $\alpha = 1$.

As a model for glacial cycles

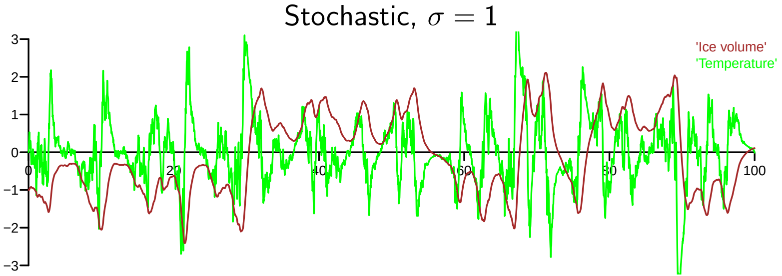


Illustration: the van der Pol oscillator

Has a 'slow' response x and a 'fast' response $\dot{x}$, related by

$$\ddot{x} + \dot{x} + (\alpha - x^2)x = \sigma x \, \xi$$

where ξ is white noise. Here $\theta = (\alpha, \sigma)$; we'll take $\alpha = 1$.

As a model for glacial cycles

Stochastic, $\sigma = 1$

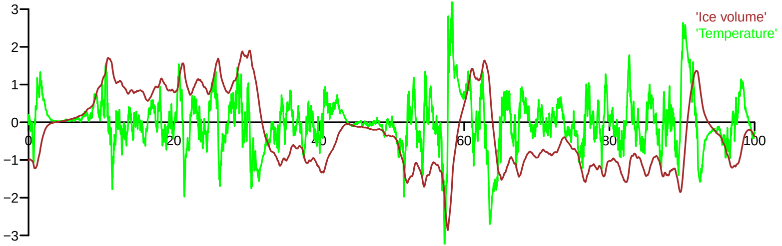


Illustration: the van der Pol oscillator

Has a 'slow' response x and a 'fast' response $\dot{x}$, related by

$$\ddot{x} + \dot{x} + (\alpha - x^2)x = \sigma x \, \xi$$

where ξ is white noise. Here $\theta = (\alpha, \sigma)$; we'll take $\alpha = 1$.

As a model for glacial cycles

Stochastic, $\sigma = 1$

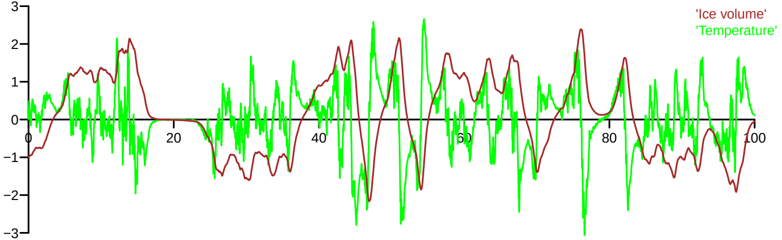


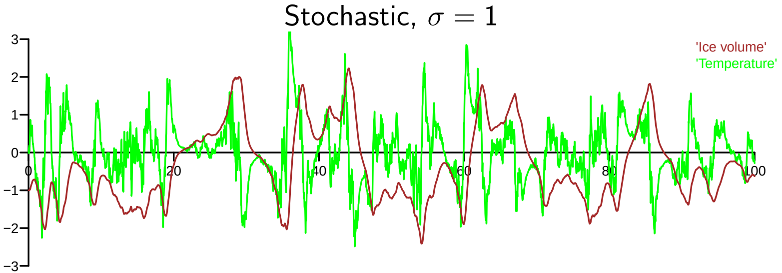
Illustration: the van der Pol oscillator

Has a 'slow' response x and a 'fast' response $\dot{x}$, related by

$$\ddot{x} + \dot{x} + (\alpha - x^2)x = \sigma x \, \xi$$

where ξ is white noise. Here $\theta = (\alpha, \sigma)$; we'll take $\alpha = 1$.

As a model for glacial cycles



The next few steps

Everyone (not just modellers) needs to understand about the limitations of models, and the effect that these limitations have on inference. Here are some steps in the right direction:

1. Increase the measurement uncertainty to incorporate aspects of 'representation uncertainty' and 'structural uncertainty'

The next few steps

Everyone (not just modellers) needs to understand about the limitations of models, and the effect that these limitations have on inference. Here are some steps in the right direction:

1. Increase the measurement uncertainty to incorporate aspects of 'representation uncertainty' and 'structural uncertainty'

2a. Thin the observations where they occur with high density (in space/time), to account for systematic structural error

2b. (Or) Incorporate an auto-regressive process between the model output and the observations

The next few steps

Everyone (not just modellers) needs to understand about the limitations of models, and the effect that these limitations have on inference. Here are some steps in the right direction:

1. Increase the measurement uncertainty to incorporate aspects of 'representation uncertainty' and 'structural uncertainty'

2a. Thin the observations where they occur with high density (in space/time), to account for systematic structural error

2b. (Or) Incorporate an auto-regressive process between the model output and the observations

3. Report ML-based confidence sets for the parameters; use these for hypothesis tests

The next few steps

Everyone (not just modellers) needs to understand about the limitations of models, and the effect that these limitations have on inference. Here are some steps in the right direction:

1. Increase the measurement uncertainty to incorporate aspects of 'representation uncertainty' and 'structural uncertainty'

2a. Thin the observations where they occur with high density (in space/time), to account for systematic structural error

2b. (Or) Incorporate an auto-regressive process between the model output and the observations

3. Report ML-based confidence sets for the parameters; use these for hypothesis tests

And, some time soon,

4. Call in a statistician and noise up your model!