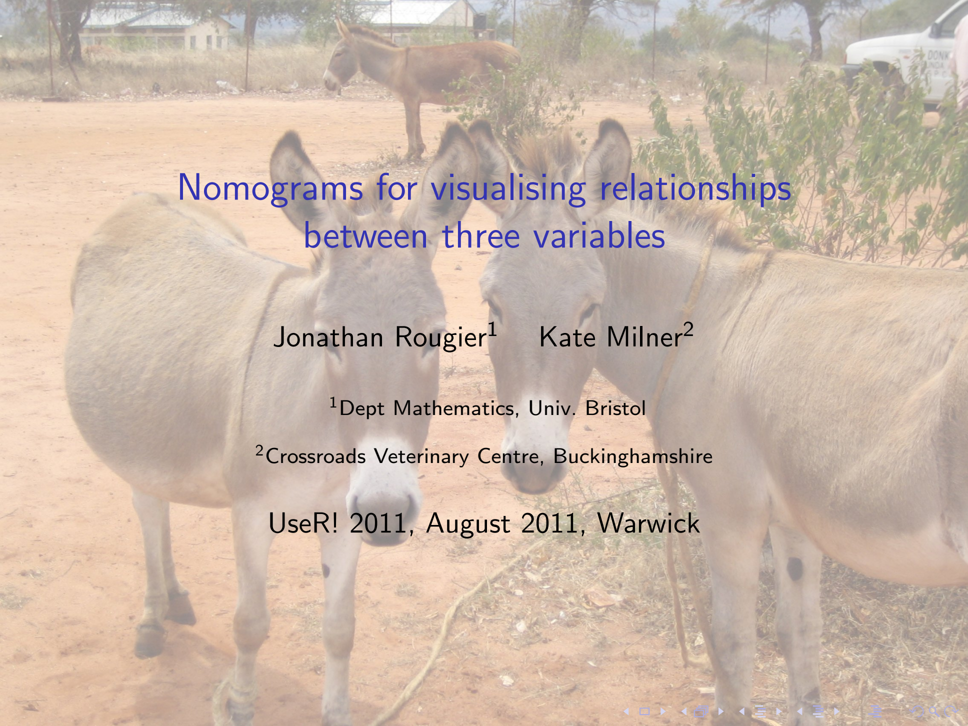

Nomograms for visualising relationships between three variables

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[1]Dept Mathematics, Univ. Bristol

[2]Crossroads Veterinary Centre, Buckinghamshire

UseR! 2011, August 2011, Warwick

Background

A donkey drawn by my housemate Caroline (in the pub).

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This donkey is not enjoying being weighed.

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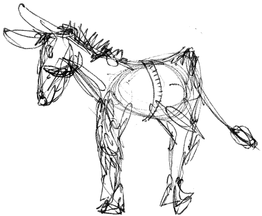

A happy baby donkey being measured.

Usual practice

The standard practice is to fit a relationship

$$\log(\texttt{Weight}) = a + b\log(\texttt{HeartGirth}) + c\log(\texttt{Height})$$

to adult donkeys in good condition, and possibly other relationships for juveniles and donkeys in poor condition. What value can we statisticians add?

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1. Explicit inclusion of factors for `Age`, `Gender`, and `BCS` (Body Condition Score);
2. Box-Cox assessment of the appropriate transformation of the lefthand side (`boxcox` in the MASS package);
3. Initial model to include interactions, then stepwise reduction to maximise AIC (`stepAIC` in the MASS package).

Building the statistical model

Box-Cox plot for transformations of the response favours square root

Building the statistical model

Backwards stepwise deletion removes all interaction terms :) and Gender completely

```
Stepwise Model Path
Analysis of Deviance Table

Initial Model:
sqrt(Weight) ~ BCSis + Gender + Age + log(HeartGirth) + log(Height) +
    log(HeartGirth):log(Height) + BCSis:log(HeartGirth) + Gender:log(HeartGirth) +
    Age:log(HeartGirth) + BCSis:log(Height) + Gender:log(Height) +
    Age:log(Height)

Final Model:
sqrt(Weight) ~ BCSis + Age + log(HeartGirth) + log(Height)


                           Step Df   Deviance Resid. Df Resid. Dev        AIC
1                                                   504   78.14041  -972.7873
2          - Age:log(HeartGirth)  5 0.37630656       509   78.51672  -980.1883
3        - BCSis:log(HeartGirth)  4 0.49082973       513   79.00755  -984.8168
4            - BCSis:log(Height)  4 0.41453445       517   79.42208  -989.9858
5              - Age:log(Height)  5 0.91895494       522   80.34104  -993.7620
6           - Gender:log(Height)  2 0.13986420       524   80.48090  -996.8210
7 - log(HeartGirth):log(Height)  1 0.00927524       525   80.49018  -998.7587
8       - Gender:log(HeartGirth)  2 0.31844543       527   80.80862 -1000.6226
9                       - Gender  2 0.06633122       529   80.87496 -1004.1787
```

Building the statistical model

Resulting model has additive adjustments for BCS and Age

```
Call:
lm(formula = sqrt(Weight) ~ BCSis + Ageis + log(HeartGirth) +
    log(Height), data = donk, subset = subset)

Residuals:
      Min        1Q    Median        3Q       Max
-1.016797 -0.275575 -0.005298  0.255089  1.519246

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)     -58.89411    2.42162 -24.320  < 2e-16 ***
BCSis1.5         -0.49820    0.17939  -2.777  0.00568 **
BCSis2           -0.24978    0.08253  -3.026  0.00260 **
BCSis3.5          0.37485    0.05833   6.426 2.91e-10 ***
BCSis4            0.57031    0.11024   5.173 3.27e-07 ***
Ageis<2yo        -0.35353    0.07676  -4.605 5.16e-06 ***
Ageis5-10yo       0.19782    0.06255   3.162  0.00165 **
Ageis>10yo        0.27681    0.05070   5.459 7.35e-08 ***
log(HeartGirth)  10.22732    0.50604  20.211  < 2e-16 ***
log(Height)       4.84926    0.60029   8.078 4.45e-15 ***
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.392 on 531 degrees of freedom
Multiple R-squared: 0.8724, Adjusted R-squared: 0.8703
F-statistic: 403.5 on 9 and 531 DF,  p-value: < 2.2e-16
```

Nomogram for our donkeys

Our statistical estimate of `Weight` is

$$\texttt{Weight} = \Big(-58.9^{\dagger} + 10.2 \log \texttt{HeartGirth} + 4.8 \log \texttt{Height} \Big)^2$$

where † indicates adjustments to be made for `BCS` and `Age`. *How do we turn this into something that can be used in the field?*

- Most statisticians would immediately think of a contour plot, which would work for any relationship of the form $f(u, v) = w$. This requires two straight lines and an interpolation.

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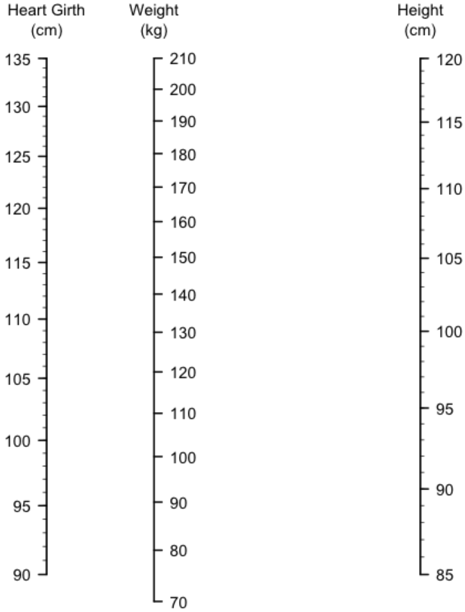
- Most statisticians would immediately think of a contour plot, which would work for any relationship of the form $f(u, v) = w$. This requires two straight lines and an interpolation.
- For a large subset of such relationships, though, we can construct a nomogram, which needs one straight line and no interpolation.

Nomogram for our donkeys



Additive corrections:

```
BCS: 1.5,    -11kg
     2,       -6kg
     3.5,    +10kg
     4,      +16kg

Age: <2yo,     -7kg
     5-10yo, +5kg
     >10yo,   +7kg
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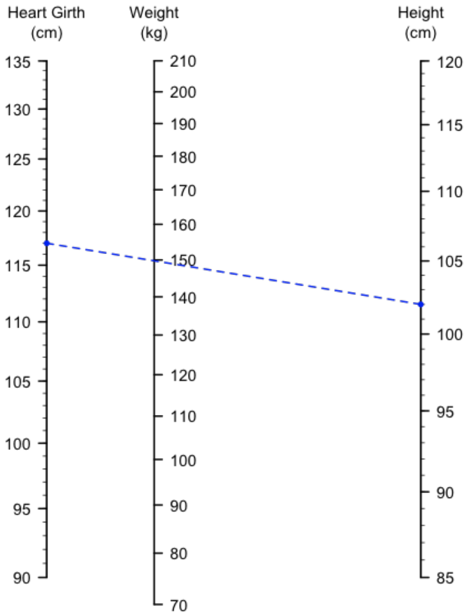

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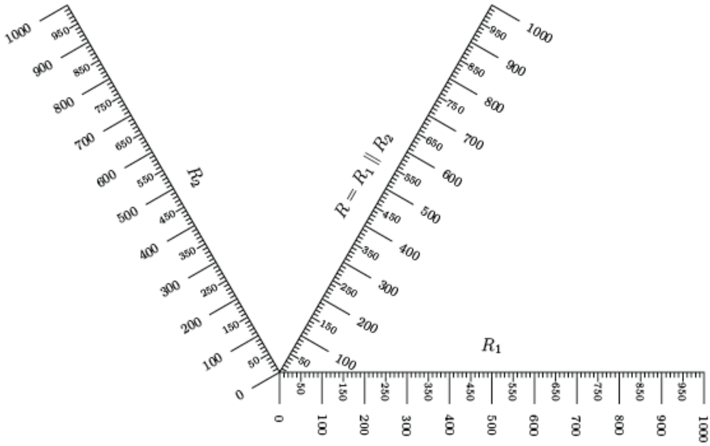
A healthy (BCS 2.5 or 3) 2-5yo donkey with a `HeartGirth` of 117cm and a `Height` of 102cm has a predicted weight of about 150kg.

Digression on nomograms

Nomograms are visual tools for representing the relationship between three or more variables, in such a way that the value of one variable can be inferred from the values of the others by drawing a straight line.

- $f_1(u) + f_2(v) = f_3(w)$ gives a parallel scale-nomogram, like ours;
- We could also have used an N chart, used for $f_1(u)/f_2(v) = f_3(w)$;
- Proportional nomograms can handle more than three variables, e.g. in two stages using a pivot;
- An entire theory based around determinants allows the construction of nomograms for much more general relationships; typically these are curved scale nomograms.

Digression on nomograms



Equivalent Resistance of Two Resistors in Parallel

$$1/R = 1/R_1 + 1/R_2$$

All figures from Ron Doerfler, 2009, The Lost Art of Nomography, *The UMAP Journal*, **30**(4), pp. 457-493.

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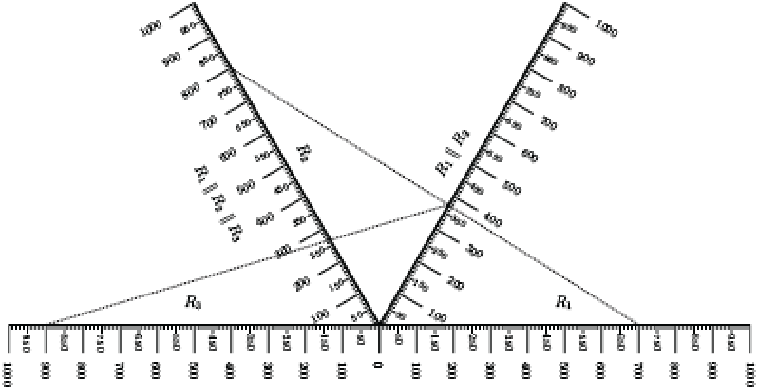

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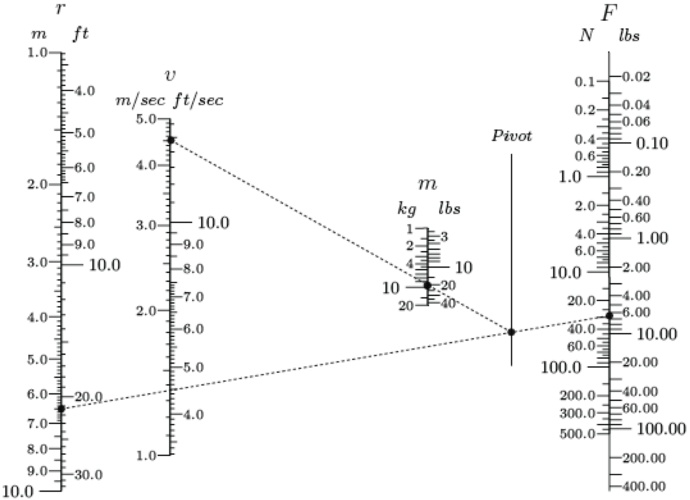

Centripetal Force: $F = mv^2/r$

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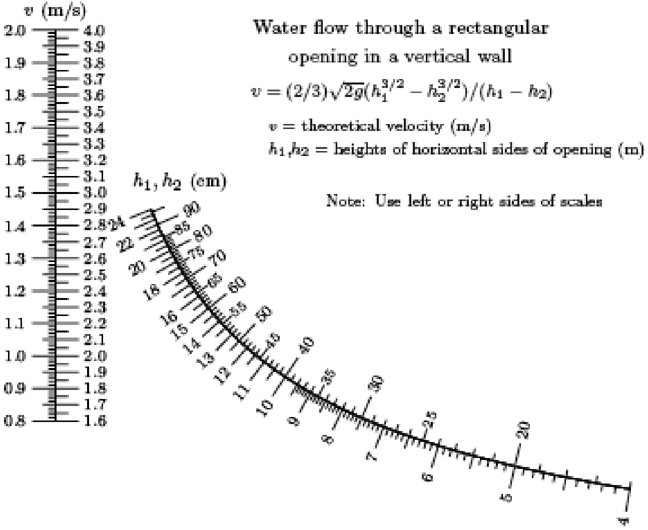

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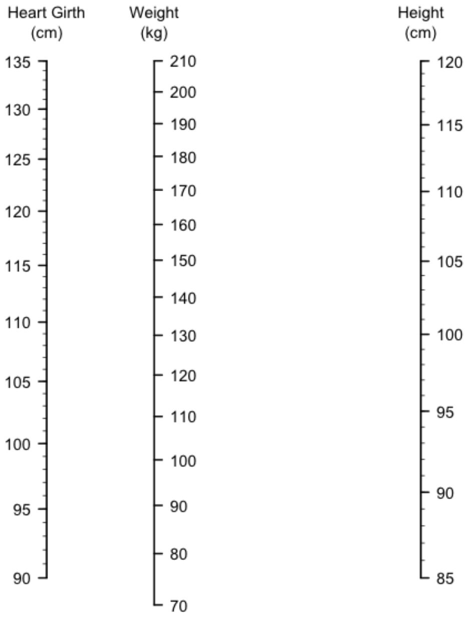
Back to the donkeys!



What is the effect of replacing sqrt(`Weight`) with log(`Weight`), which would be the more usual transformation?

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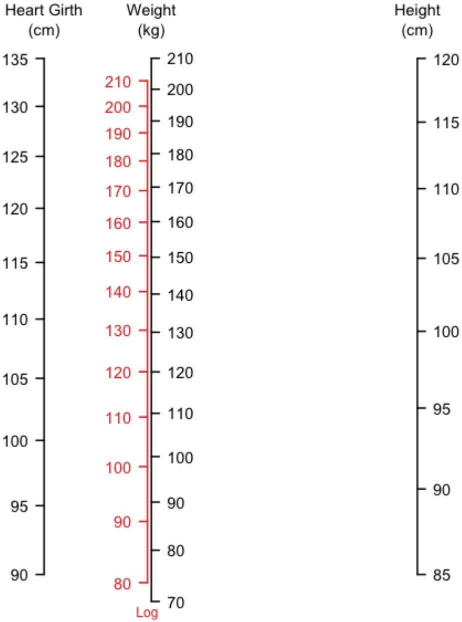

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Gives slightly higher weights (∼5kg) for small and large donkeys. This difference is smaller than the residual standard deviation, which is 10kg.

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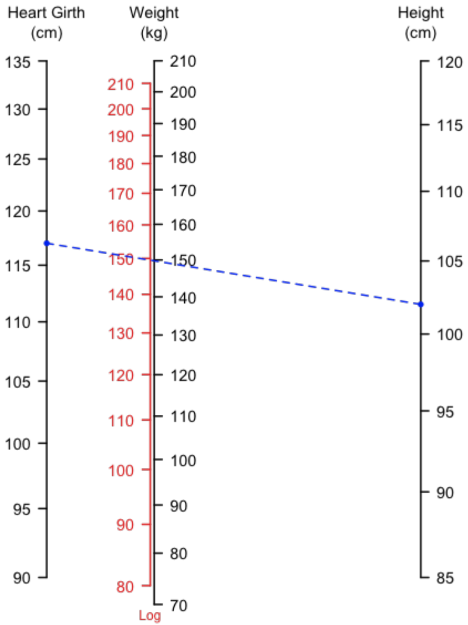

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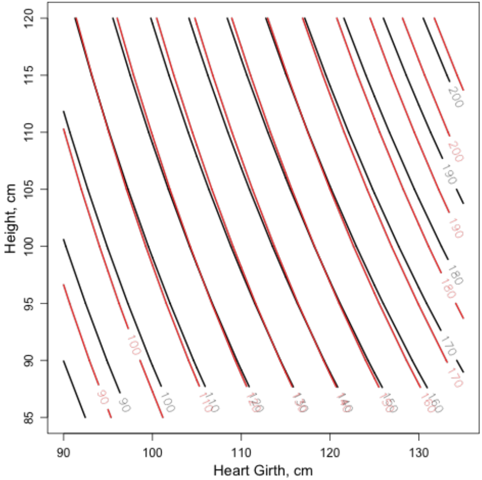
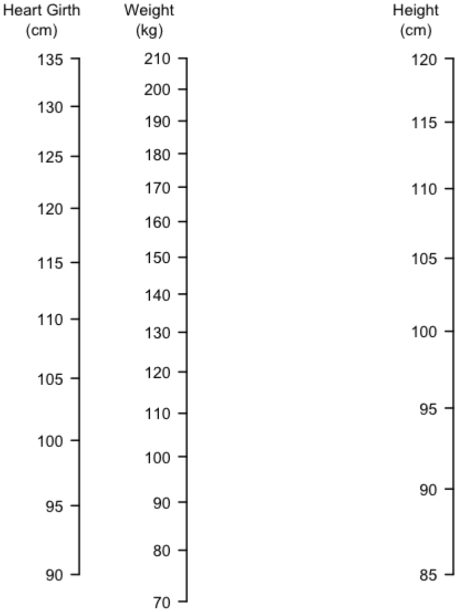
Things are a lot less clear if we try to visualise this using a contour plot.



Different relationships on one plot



`Height` and `Length` seem to be interchangeable; so could estimate `Weight` with either.

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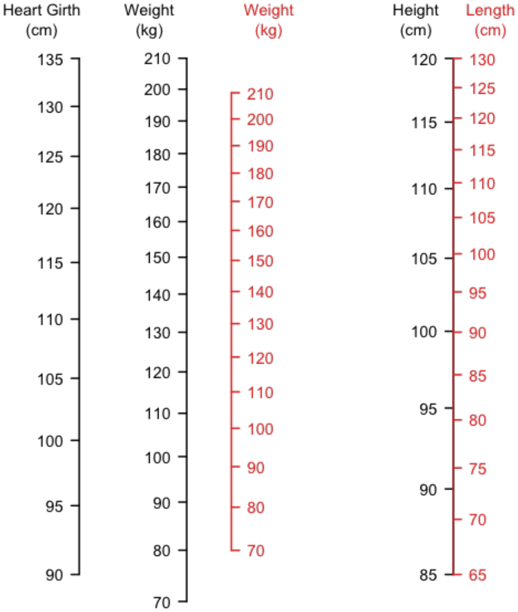

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Estimate using Length can be added to existing nomogram, to give vets the choice of which measurement to make.

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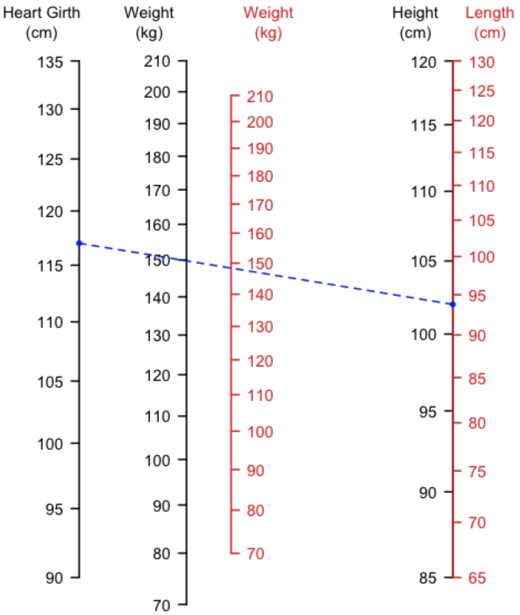

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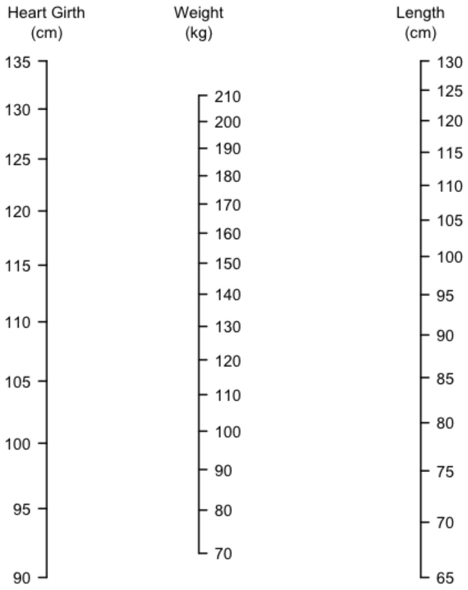
Different types of donkey



Different types of donkey can be displayed on the same plot. Here are our Kenyan donkeys, shown with a `Length` covariate.

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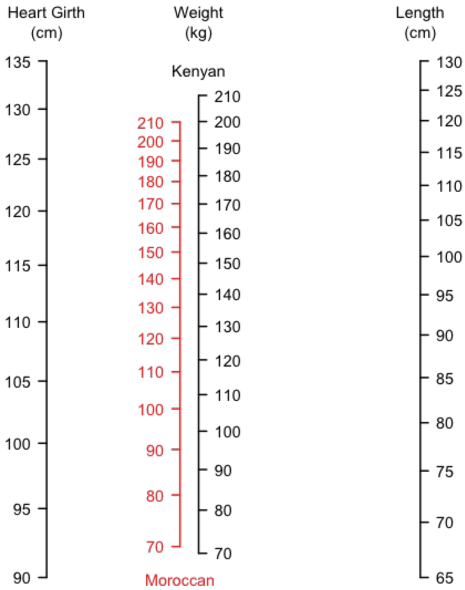

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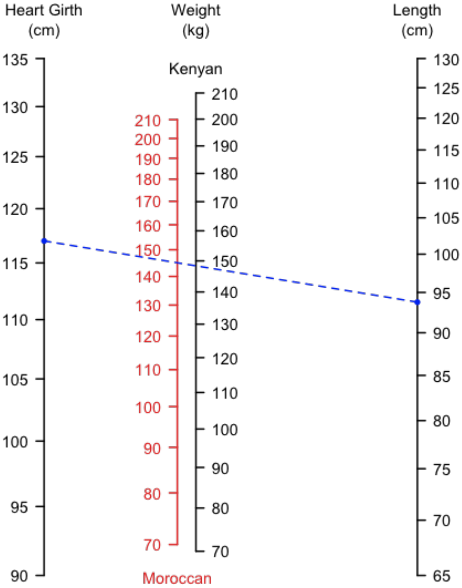

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Summary

Visualisation is an important part of both data analysis and statistical communication.

- For relating three variables, contour plots will always work, but where they are available, nomograms might be clearer and simpler to use.
- Our donkey nomogram will be used by practicing vets in Kenya, but it has also been a useful tool for us in model choice and model comparison.
- Nomograms are also available for some relationships between four or more variables.
- One catch: Contour plots can be overlaid on a field showing predictive uncertainties. Unfortunately it is not as easy to visualise predictive uncertainty with a nomogram.

Resources

Ron Doerfler, 2009, The Lost Art of Nomography, *The UMAP Journal*, **30**(4), pp. 457-493.
http://myreckonings.com/wordpress/wp-content/uploads/JournalArticle/The_Lost_Art_of_Nomography.pdf

Ron Doerfler, Creating Nomograms with the *PyNomo* Software, Version 1.1 for PyNomo Release 0.2.2.
http://www.myreckonings.com/pynomo/CreatingNomogramsWithPynomo.pdf

Leif Roschier, 2009, http://www.pynomo.org/