

Package ‘SymTS’

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Type Package

Title Symmetric Tempered Stable Distributions

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Description Contains methods for simulation and for evaluating the pdf, cdf, and quantile functions for symmetric stable, symmetric classical tempered stable, and symmetric power tempered stable distributions.

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SymTS-package

*Symmetric Tempered Stable Distributions***Description**

Contains methods for simulation and for evaluating the pdf, cdf, and quantile functions for symmetric stable, symmetric classical tempered stable, and symmetric power tempered stable distributions.

Details

The DESCRIPTION file:

```
Package:      SymTS
Type:        Package
Title:       Symmetric Tempered Stable Distributions
Version:     1.0-2
Date:       2023-01-14
Author:      Michael Grabchak <mgrabcha@uncc.edu> and Lijuan Cao <lcao2@uncc.edu>
Maintainer:  Michael Grabchak <mgrabcha@uncc.edu>
Description: Contains methods for simulation and for evaluating the pdf, cdf, and quantile functions for symmetric stable, symmetric classical tempered stable, and symmetric power tempered stable distributions.
License:     GPL (>= 3)
```

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References

- M. Grabchak (2016). Tempered Stable Distributions: Stochastic Models for Multiscale Processes. Springer, Cham.
- S. T. Rachev, Y. S. Kim, M. L. Bianchi, and F. J. Fabozzi (2011). Financial Models with Levy Processes and Volatility Clustering. Wiley, Chichester.
- J. Rosinski (2007). Tempering stable processes. Stochastic Processes and Their Applications, 117(6):677-707.
- G. Samorodnitsky and M. Taqqu (1994). Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance. Chapman & Hall, Boca Raton.

dCTS

PDF of CTS Distribution

Description

Evaluates the pdf for the symmetric classical tempered stable distribution. When $\alpha=0$ this is the symmetric variance gamma distribution.

Usage

```
dCTS(x, alpha, c = 1, ell = 1, mu = 0)
```

Arguments

x	Vector of points.
alpha	Number in $[0,2)$
c	Parameter $c > 0$
ell	Parameter $\text{ell} > 0$
mu	Location parameter, any real number

Details

The integration is preformed using the QAWF method in the GSL library for C. For this distribution the Rosinski measure $R(dx) = c \cdot \delta_{\text{ell}}(dx) + c \cdot \delta_{-\text{ell}}(dx)$, where δ is the delta function. The Levy measure is $M(dx) = c \cdot \text{ell}^{\alpha} \cdot e^{-x/\text{ell}} \cdot x^{-1-\alpha} dx$. The characteristic function is, for α not equal 0,1:

$$f(t) = \exp(2 \cdot c \cdot \gamma(-\alpha) \cdot (1 + \text{ell}^2 t^2)^{\alpha/2} \cdot (\cos(\alpha \cdot \text{atan}(\text{ell} \cdot t)) - 1)) \cdot e^{i \cdot t \cdot \mu},$$

for $\alpha = 1$ it is

$$f(t) = (1 + \text{ell}^2 t^2)^c \cdot \exp(-2 \cdot c \cdot \text{ell} \cdot t \cdot \text{atan}(\text{ell} \cdot t)) \cdot e^{i \cdot t \cdot \mu},$$

and for $\alpha=0$ it is

$$f(t) = (1 + t^2 \text{ell}^2)^{-c} \cdot e^{i \cdot t \cdot \mu}.$$

Note

When $\alpha=0$ and $c \leq .5$, the pdf is unbounded. It is infinite at μ and the method returns Inf in that case. This does not affect pCTS, qCTS, or rCTS.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2016). Tempered Stable Distributions: Stochastic Models for Multiscale Processes. Springer, Cham.

Examples

```
x = (-10:10)/10
dCTS(x, .5)
```

dPowTS

PDF of PowTS Distribution

Description

Evaluates the pdf for the symmetric power tempered stable distribution.

Usage

```
dPowTS(x, alpha, c = 1, ell = 1, mu = 0)
```

Arguments

x	Vector of points
alpha	Number in [0,2)
c	Parameter $c > 0$
ell	Parameter $\text{ell} > 0$
mu	Location parameter, any real number

Details

The integration is performed using the QAWF method in the GSL library for C. For this distribution the Rosinski measure $R(dx) = c * (\alpha + \text{ell} + 1) * (\alpha + \text{ell}) * (1 + |x|)^{-2 - \alpha - \text{ell}}(dx)$.

Note

We do not allow for the case $\alpha=0$ and $c \leq .5 * (1 + \text{ell})$, as, in this case, the pdf is unbounded. This does not affect pPowTS, qPowTS, or rPowTS.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2016). Tempered Stable Distributions: Stochastic Models for Multiscale Processes. Springer, Cham.

Examples

```
x = (-10:10)/10
dPowTS(x, .5)
```

dSaS

PDF of Symmetric Stable Distribution

Description

Evaluates the pdf for the symmetric alpha stable distribution. For alpha=1 this is the Cauchy distribution.

Usage

```
dSaS(x, alpha, c = 1, mu = 0)
```

Arguments

x	Vector of points.
alpha	Index of stability; Number in (0,2)
c	Scale parameter, c>0
mu	Location parameter, any real number

Details

The integration is performed using the QAWF method in the GSL library for C. The characteristic function is

$$f(t) = e^{-(c |t|^\alpha)} * e^{i*t*\mu}.$$

Author(s)

Michael Grabchak and Lijuan Cao

References

G. Samorodnitsky and M. Taqqu (1994). Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance. Chapman & Hall, Boca Raton.

Examples

```
x = (-10:10)/10
dSaS(x, .5)
```

pCTS

CDF of CTS Distribution

Description

Evaluates the cdf for the symmetric classical tempered stable distribution. When $\alpha=0$ this is the symmetric variance gamma distribution.

Usage

```
pCTS(x, alpha, c = 1, ell = 1, mu = 0)
```

Arguments

x	Vector of probabilities.
alpha	Number in [0,2)
c	Parameter $c > 0$
ell	Parameter $\text{ell} > 0$
mu	Location parameter, any real number

Details

For details about this distribution see the the description of dCTS.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2016). Tempered Stable Distributions: Stochastic Models for Multiscale Processes. Springer, Cham.

Examples

```
x = (-10:10)/10
pCTS(x, .5)
```

pPowTS

PDF of PowTS Distribution

Description

Evaluates the cdf for the symmetric power tempered stable distribution.

Usage

```
pPowTS(x, alpha, c = 1, ell = 1, mu = 0)
```

Arguments

x	Vector of probabilities.
alpha	Number in [0,2)
c	Parameter $c > 0$
ell	Parameter $ell > 0$
mu	Location parameter, any real number

Details

The integration is performed using the QAWF method in the GSL library for C. For this distribution the Rosinski measure $R(dx) = c * (\alpha + ell + 1) * (\alpha + ell) * (1 + |x|)^{-2 - \alpha - ell} dx$.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2016). Tempered Stable Distributions: Stochastic Models for Multiscale Processes. Springer, Cham.

Examples

```
x = (-10:10)/10  
pPowTS(x, .5)
```

pSaS

CDF of Symmetric Stable Distribution

Description

Evaluates the cdf for the symmetric alpha stable distribution. For alpha=1 this is the Cauchy distribution.

Usage

```
pSaS(x, alpha, c = 1, mu = 0)
```

Arguments

x	Vector of probabilities.
alpha	Index of stability; Number in (0,2)
c	Scale parameter, c>0
mu	Location parameter, any real number

Details

The integration is performed using the QAWF method in the GSL library for C. The characteristic function is

$$f(t) = e^{(-c |t|^{\alpha})} * e^{(i*t*\mu)}.$$

Author(s)

Michael Grabchak and Lijuan Cao

References

G. Samorodnitsky and M. Taqqu (1994). Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance. Chapman & Hall, Boca Raton.

Examples

```
x = (-10:10)/10  
pSaS(x, .5)
```

qCTS

Quantile Function of CTS Distribution

Description

Evaluates the quantile function for the symmetric classical tempered stable distribution. When $\alpha=0$ this is the symmetric variance gamma distribution.

Usage

```
qCTS(x, alpha, c = 1, ell = 1, mu = 0)
```

Arguments

x	Vector of quantiles.
alpha	Number in $[0,2)$
c	Parameter $c > 0$
ell	Parameter $ell > 0$
mu	Location parameter, any real number

Details

For details about this distribution see the the description of dCTS.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2016). Tempered Stable Distributions: Stochastic Models for Multiscale Processes. Springer, Cham.

Examples

```
x = (1:9)/10  
qCTS(x, .5)
```

qPowTS

Quantile Function of PowTS Distribution

Description

Evaluates the quantile function for the symmetric power tempered stable distribution.

Usage

```
qPowTS(x, alpha, c = 1, ell = 1, mu = 0)
```

Arguments

x	Vector of quantiles.
alpha	Number in [0,2)
c	Parameter $c > 0$
ell	Parameter $ell > 0$
mu	Location parameter, any real number

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2016). Tempered Stable Distributions: Stochastic Models for Multiscale Processes. Springer, Cham.

Examples

```
x = (1:9)/10
qPowTS(x, .5)
```

qSaS

Quantile Function of Symmetric Stable Distribution

Description

Evaluates the quantile function for the symmetric alpha stable distribution. For $\alpha=1$ this is the Cauchy distribution.

Usage

```
qSaS(x, alpha, c = 1, mu = 0)
```

Arguments

x	Vector of points.
alpha	Index of stability; Number in (0,2)
c	Scale parameter, $c > 0$
mu	Location parameter, any real number

Details

The characteristic function is

$$f(t) = e^{(-c |t|^\alpha)} * e^{(i*t*\mu)}.$$

Author(s)

Michael Grabchak and Lijuan Cao

References

G. Samorodnitsky and M. Taqqu (1994). Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance. Chapman & Hall, Boca Raton.

Examples

```
x = (1:9)/10
qSaS(x, .5)
```

rCTS

Simulation from CTS Distribution

Description

Simulates from the symmetric classical tempered stable distribution. When $\alpha=0$ this is the symmetric variance gamma distribution. The simulation is performed by numerically evaluating the quantile function.

Usage

```
rCTS(r, alpha, c = 1, ell = 1, mu = 0)
```

Arguments

r	Number of observations.
alpha	Number in [0,2)
c	Parameter $c > 0$
ell	Parameter $\text{ell} > 0$
mu	Location parameter, any real number

Details

For details about this distribution see the the description of dCTS.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2016). Tempered Stable Distributions: Stochastic Models for Multiscale Processes. Springer, Cham.

Examples

```
rCTS(10, .5)
```

 rPowTS

Simulation from PowTS Distribution

Description

Simulates from the symmetric power tempered stable distribution. The simulation is performed by numerically evaluating the quantile function.

Usage

```
rPowTS(r, alpha, c = 1, ell = 1, mu = 0)
```

Arguments

r	Number of observations.
alpha	Number in [0,2)
c	Parameter $c > 0$
ell	Parameter $ell > 0$
mu	Location parameter, any real number

Details

For this distribution the Rosinski measure $R(dx) = c*(\alpha+ell+1)*(\alpha+ell)*(1+|x|)^{-2-\alpha-ell}(dx)$.

Author(s)

Michael Grabchak and Lijuan Cao

References

M. Grabchak (2016). Tempered Stable Distributions: Stochastic Models for Multiscale Processes. Springer, Cham.

Examples

```
pPowTS(10, .5)
```

 rSaS

Simulation from Symmetric Stable Distribution

Description

Simulates from the symmetric alpha stable distribution. When alpha=1 this is the Cauchy distribution. The simulation is performed using a well-known approach. See for instance Proposition 1.7.1 in Samorodnitsky and Taqu (1994).

Usage

```
rSaS(r, alpha, c = 1, mu = 0)
```

Arguments

r	Number of observations.
alpha	Index of stability; Number in (0,2)
c	Scale parameter, c>0
mu	Location parameter, any real number

Details

The characteristic function is

$$f(t) = e^{(-c |t|^\alpha) * e^{i * t * \mu}}.$$

Author(s)

Michael Grabchak and Lijuan Cao

References

G. Samorodnitsky and M. Taqu (1994). Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance. Chapman & Hall, Boca Raton.

Examples

```
rSaS(10, .5)
```

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