# UNIVERSITY OF BRISTOL

# Department of Mathematics

# Postgraduate Opportunities in Applied Mathematics

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# 1 Introduction

The Department of Mathematics is one of the largest departments in the Faculty of Science at the University of Bristol with over 50 full time members of academic staff covering a range of Applied Mathematics, Pure Mathematics and Statistics research. It has an international reputation for research excellence in each of these areas. Due to the recent success and continuing expansion of the department, we will be relocating in 2010 to a new purpose-built £34 million building.

The Applied Mathematics group was awarded the highest attainable 5*A rating in the last UK government's Research Assessment Exercise in 2001, a ranking only matched in Applied Mathematics by Warwick and Cambridge. There are currently 21 full time members of academic staff in Applied Mathematics, supported by 14 Postdoctoral Research Assistants and 25 Postgraduate Students.

The success of the Applied Mathematics group is based on attracting not only the best available staff, but also by recruiting the best possible Ph.D. students; it is often their work which is at the cutting edge of modern research. We therefore welcome applications to study for degrees at the level of Ph.D., M.Sc. by research and M.Res. A new Centre for Complexity Sciences will provide an additional 10-15 postgraduate places from October 2007 on an integrated multidisciplinary M.Res/Ph.D. programme in conjunction with Departments in Engineering Mathematics and Computer Science. The Department also is involved in the M.Sc. by Research programmes in the Science of Natural Hazards (run by the Earth Sciences department)

The research interests in the group are wide and varied and broadly captured by the following themes:

| | |
|---|---|
| Complex fluids | Discrete geometry |
| Dynamical systems | Granular flows |
| Laboratory experiments | Liquid crystals |
| Nanoscience | Material science |
| Quantum chaos | Quantum information |
| Random matrices | Scientific computing |
| Turbulence | Waves |

This booklet is designed to provide detailed information on Postgraduate Study within the Applied Mathematics group. This information can also be found online at:

`http://www.maths.bris.ac.uk/study/admissions_postgrad/`

and by following the links to the Applied Mathematics research themes. For additional information and enquiries, contact:

Dr. Richard Porter,
Applied Postgraduate Admissions Tutor,
School of Mathematics,
University Walk,
Bristol, BS8 1TW, UK.

Tel: +44 (0)117 9287996
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Email: `richard.porter@bris.ac.uk`

# 2 An Overview of Research in Applied Mathematics

Topics for Postgraduate study in the Applied Mathematics group are usually from the areas listed below. Occasionally students come with specific projects in mind. It is not unusual for prospective students to have an interest in one or more of these areas and to seek guidance about specific project topics.

## Dynamical Systems

Most physical problems can be viewed as a dynamical system. Typically this involves studying the solution structure of nonlinear equations, understanding how these solutions may vary as the dynamical system changes and discerning generic properties of the solutions, for example will they exhibit chaotic behaviour. The following people have interests in this area: Dr. Carl Dettmann, Dr. Misha Rudnev, Dr. Holger Waalkens and Prof. Steve Wiggins.

## Quantum Chaos, Random Matrix Theory and Number Theory

Quantum mechanics, the theory of matter on small scales, plays a centrally important role in many of the most important areas of science and technology (e.g. lasers, mesoscopic and nanoscopic systems). However, few quantum systems can be solved analytically. For the rest, methods of approximation are required. Among these, asymptotic methods based on classical (Newtonian) mechanics are of increasing importance, especially in mesoscopic and nanoscopic systems, which lie at the boundary between the classical and quantum worlds. Within classical mechanics there is a broad spectrum of qualitatively different dynamics, ranging from integrable (completely regular) to strongly chaotic (highly irregular). Quantum Chaos is the area of research concerned with how this fact manifests itself in quantum mechanics. It is an exciting and rapidly developing field, encompassing the mathematical analysis of new quantum phenomena and a wide variety of applications in many areas of science and technology (e.g. in nanoscale systems and microlasers). There are deep connections with Random Matrix Theory – the study of the statistical distribution of the eigenvalues of matrices picked at random from some suitably defined ensemble – ergodic theory, and several areas of number theory, such as the theory of the Riemann zeta function and other related objects. Many fundamental developments in the subject have followed from work carried out here in Bristol. The following people have interests in this area: Dr. Carl Dettmann, Prof. Jon Keating, Dr. Francesco Mezzadri, Dr. Jonathan Robbins, Dr. Nina Snaith, Dr. Martin Sieber. There is a close relationship with the Dynamical Systems and Quantum Information groups, and with the group in Physics led by Professor Sir Michael Berry FRS. There is also a close connection with the newly-formed number theory group in Pure Mathematics.

Some examples of research in number theory, random matrix theory, and quantum chaos can be found online:

`http://www.maths.bris.ac.uk/research/applied/themes/random_matrix/`

## Fluid dynamics

Fluid dynamics describes a tremendous variety of phenomena from the large scale (e.g. weather and ocean systems on Earth and other planets or stars) through medium scales (e.g. the flow around grand prix racing cars) to very small scale (e.g. microdroplets for drug delivery). Our main aim is to understand how the nonlinear character of the hydrodynamic equations leads to the wealth of flow properties observed. These can range from the formation of novel flow structures, in particular those covering many length scales, to fully turbulent flows. We are also pushing beyond the boundaries of classical hydrodynamics by studying polymeric fluids, granular media and flows on the nanoscale. This places our group at a junction between mathematics, physics, chemistry, engineering, and geophysics. Our in-house fluid dynamics laboratory keeps us closely connected to the "real world", and presents us with ever new theoretical challenges. The following people have interests in this area: Dr. Lorena Barba, Dr. Robert Deegan, Dr. Andrew Hogg, Prof. Rich Kerswell, Prof. Howell Peregrine, Dr. Richard Porter, Prof. Jens Eggers and Dr. Maria Zaturska

Traditional research into fluid dynamics at Bristol involves free surface flows, especially water waves (Prof. Howell Peregrine, Dr. Richard Porter), and turbulence and transition to turbulence (Prof. Howell Peregrine, Prof. Rich Kerswell). More recently, it expanded to include granular media and particle laden flows and complex fluids (Dr. Andrew Hogg, Prof. Rich Kerswell, Dr. Robert Deegan, Prof. Jens Eggers), and vortical flows (Dr. Lorena Barba, Prof. Rich Kerswell).

Further information about fluid dynamics group is provided by the web site

`http://www.maths.bris.ac.uk/research/applied/themes/fluid_dynamics/`

## Quantum Computation and Quantum Information Theory

In the past five years the new subjects of quantum computation and quantum information theory have emerged which both offer the potential for immense practical computing power and also suggest deep links between the well-established disciplines of quantum theory and information theory and computation. On the one hand computer chips will soon be so small that we will have to grapple with the fact that electrons inside the processing elements become "smeared out", for example they can tunnel out of the wires: Heisenberg's Uncertainty Principle seems to be at odds with the desire for reliable computation. On the other hand it has been realised very recently that one might be able to take advantage of intrinsically quantum features to build quite new types of computers – quantum computers. We are only just beginning to understand what quantum information is and what quantum computers can do. We have close links with the physics and computer science departments and our group is interested in all aspects of Quantum Information Theory (foundations, nonlocality, entanglement, Quantum Shannon theory, Quantum computational models, and Quantum algorithms), and Experiment (Quantum Key Distribution, Single photon sources, Nonlocality experiments).

The following people have interests in this area: Prof. Noah Linden and Dr. Andreas Winter. Quantum computation and quantum information theory group may advertise additional PhD and other positions on the web:

`http://www.maths.bris.ac.uk/QCIG/positions.html`

## Numerical Methods

The mathematical modeling of nonlinear phenomena sometimes leads to differential equations that are too difficult to solve by known analytical methods. In such cases, numerical methods can provide much insight into the properties of the solution set. Two threads of research are represented in the group; one where the use of numerical techniques is motivated by particular applications, and another where the focus is on the theoretical study of the effectiveness of the numerical methods themselves. The following people have interests in this area: Dr. Lorena Barba, Prof. Rich Kerswell, Prof. Jens Eggers, Dr. Yves Tourigny.

## Harmonic Analysis, Geometric Combinatorics, Distance Set Theory

The Erdos/Falconer distance conjectures ask, in a variety of settings, for the smallest number of distances determined by subsets of the Euclidean space, sufficiently large with respect to an appropriate measure. These problems can be studied from both the analytic and combinatorial standpoint and have their analogues in the finite field setting. For example, there are links between Fourier estimates related to the Falconer distance problem and Freiman's theorem in additive number theory, as well as the distribution of lattice points in convex domains. Similarly, there is a connection between Fourier estimates of measures supported on convex curves and estimates on the number of solutions of certain diophantine equations. They are linked to fundamental open questions in harmonic analysis such as the restriction/Kakeya conjecture and have intriguing manifestations in additive and geometric number theory. The following people have interest in the area: Dr. Misha Rudnev.

## PDEs, Variational Problems and Applications

Recent years have seen intense interaction between mathematics and materials science, including solid mechanics and liquid crystals. This has borne much fruit – in solid mechanics, the explanation of intriguing material behavior (e.g., the shape memory effects) by mathematical models that relate behavior to microstructure; in liquid crystals, models relating static and dynamic properties to the existence and regularity of harmonic maps, possibly with defects, between topologically nontrivial spaces. This successful interaction has in turn raised a number of questions, many of which are of interest simultaneously in mathematics, in the physical sciences and in engineering. The relevant mathematical areas are primarily, but not exclusively, calculus of variations, partial differential equations, functional and real analysis and topology. Specific research problems include the following. Solid mechanics: microstructure evolution in solids, homogenization of (polycrystalline) materials with degenerate energy, computation of quasiconvex hulls and morphology formation in biological tissues as a result of stresses induced by growth. Liquid crystals: topological classification, energy bounds, for nematics in polyhedral geometries with natural, e.g. tangent, boundary conditions; number of smooth solutions, regularity of weak solutions; switching mechanisms, applications to bistable display technology. The following people have interests in this area: Dr. Isaac Chenchiah, Dr. Jonathan Robbins.

## Nanomathematics

It has been stated that nanotechnology and nanoscience is in the process of giving rise to the second industrial revolution. The hope and promise is that nanotechnology and nanoscience will lead to the realization of many broad goals of society; such as an improved understanding of nature and an ability to control it, new manufacturing processes that increase productivity and are environmentally friendly, breakthroughs in healthcare and health treatments that are cheap and widely available, solutions to issues surrounding sustainable development, and, in general, extending the limits of human potential. Solely for nanotechnology and quantum information the University is building a new four story million building consisting of research and teaching laboratories, offices and seminar and discussion rooms, that is purpose designed to facilitate interdisciplinary collaboration. The building is specifically designed to enable collaboration and interchange of ideas between workers in Bristol and with collaborators in the UK, Europe, and the US. The University of Bristol currently has a critical mass of workers resulting in a vibrant and stimulating multidisciplinary research environment in nanotechnology and nanoscience. Workers across the University including in the departments of physics, mathematics, computer science, electrical engineering, biology, and chemistry are all currently contributing to this effort. Within the Department of Mathematics we work within the following research themes which are fundamental for many aspects of nanotechnology:

- *Transport in nanostructures* – the driving force that makes many nanosystems "work".
- *Interaction of deterministic and stochastic dynamics across many differing time and length scales* – a central issue that has received little attention which is important for understanding, e.g., the control and manipulation of proteins and DNA.
- *Bridging length and time scales* – possibly the central problem of nanoscience involving a huge number of new modelling issues and considerations, e.g., important issues are the merging of continuum and molecular descriptions, and the merging of quantum and classical descriptions.
- *Self-assembly* – a key mechanism for manufacturing and fabrication at the nanoscale and an area where new modelling and predictive tools may lead to new technological breakthroughs.
- *Coherence and decoherence: the quantum/classical interface*
- *Quantum information theory*
- *Nanofluidics* – important for a variety of processing and fabrication techniques at the nanoscale.
- *Data visualization and software development* – the modern partner of theory and modelling, but requiring new approaches for dealing with massive data sets.

Each of these themes demands new techniques and approaches for theory, modelling, and simulation and our interdisciplinary program built around the unification of these ideas and approaches will lead to a new *nanomathematics*. Applied mathematics researchers involved are Prof. Jens Eggers, Prof. Jon Keating, Prof. Noah Linden, Prof Stephen Wiggins, Dr. Carl Dettmann, Dr. Martin Sieber, Dr. Holger Waalkens, and Prof. Andreas Winter.

## Complexity

The Mathematics Department is part of a major initiative to create a new Doctoral Training Centre, the Bristol Centre for Complexity Sciences. The Centre has been set up thanks to a successful £4 million bid to the EPSRC and the theoretical hub of the programme is a collaboration between the Applied Mathematics and Statistics groups in the Mathematics department together with the departments of Engineering Mathematics and Computer Science.

With the first students starting in October 2007, the Centre will be recruiting 10-15 students a year to an integrated multidisciplinary research and training environment, linking mathematics, statistics and computer science with application areas in engineering, life and molecular sciences.

Within the Applied Mathematics group this initiative is led by Prof. Noah Linden together with Prof. Jens Eggers, Prof. Jon Keating, Prof. Rich Kerswell, Prof. Stephen Wiggins, Prof. Andreas Winter, and Dr Robert Deegan, Dr. Carl Dettmann, Dr. Martin Sieber and Dr. Holger Waalkens.

For further information see: `http://bccs.bris.ac.uk/`

# 3 Academic Staff in Applied Mathematics

Below is a list of academic staff and a summary of their main research interests.

Dr. L. A. Barba: fluid mechanics and computational methods in applied science.
Dr. I. V. Chenchiah: applications of calculus of variations to solid mechanics, microstructures in solids.
Dr. R. R. Deegan: experimental nonlinear dynamics, specifically in fracture, dielectric breakdown, hydrodynamics of contact lines and non-newtonian fluids.
Dr. C. P. Dettmann: classical dynamical systems including periodic orbit theory; symbolic dynamics; Lyapunov exponents and decay of correlations, with applications to nonequilibrium statistical mechanics of fluids and turbulence.
Prof. J. Eggers: hydrodynamics, complex fluids, and statistical mechanics; in particular phenomena involving many length scales and pattern formation.
Dr. A. J. Hogg: geophysical and environmental fluid dynamics; two phase flows, including suspensions, sedimentation and erosion; granular flows; experimental fluid dynamics.
Prof. J. P. Keating: quantum chaos, random matrix theory, and number theory.
Prof. R. R. Kerswell: geophysical and astrophysical fluid dynamics: magnetohydrodynamics, rotating fluid mechanics, hydrodynamic stability theory, numerical methods for partial differential equations.
Prof. N. Linden: quantum information theory and quantum computation; global/topological aspects of classical and quantum mechanics.
Dr. F. Mezzadri: applications of random matrix theory to quantum chaos, statistical mechanics and number theory.
Prof. D. H. Peregrine: fluid mechanics, especially free-surface flows: water waves, including propagation, breaking, impact on structures and problems related to coastal hydrodynamics; two-phase flow.
Dr. R. Porter: linearised wave theory in surface waves, acoustics, and elasticity; the interaction of waves with structures.
Dr. J. M. Robbins: liquid crystals and harmonic maps; quantum chaos; topology of integrable systems; the spin-statistics connection.
Dr. M. Rudnev: mathematical mechanics, hamiltonian chaos, discrete geometry, geometric measure theory.
Dr. M. Sieber: applications of random matrix theory to number theoretical functions such as the Riemann zeta function.
Dr. N. Snaith: quantum chaos, in particular the connection between random matrix theory and number theoretical functions such as the Riemann zeta function.
Dr. Y. J. M. Tourigny: Numerical analysis with a particular focus on the approximation of functions and partial differential equations; computational aspects of number theory, particularly continued fractions and their generalisations, and lattice basis reduction.
Dr. H. Waalkens: hamiltonian systems with focus on applications in atomic and molecular physics and chemistry, the topology and geometry of integrable and non-integrable systems, frequency analysis, semiclassical quantum mechanics.
Prof. S. Wiggins: dynamical systems and applications.
Prof. A. Winter: quantum information and quantum computation; discrete mathematics.

# 4 Postgraduate Environment and Accommodation

The University, which has about 12,000 undergraduates and 2,500 postgraduates, is near the centre of the City of Bristol. Its buildings are of very varied design and most are close together. The School of Mathematics is centrally located in a building opened in 1969, which is now which has an attractive garden area nearby. Due to the recent success and growth of the department, some of the groups are located away from the main building, although a new Mathematics building has been commissioned for 2010.

Bristol itself is a thriving modern city of some 500,000 people, and is effectively the capital of England's West Country, with a long history that can be traced back a thousand years. Indeed it was the most important city in England outside London until the early 19th century, due mainly to its historic sea port and docks about which the city has grown. Bristol now has a large commercial business community based around finance in the centre of the city and many scientific and industrial business based on aerospace and telecommunications. There are also many cultural events and activities that take place in and around Bristol. For more details, see

`http://visitbristol.co.uk/`

It is usual for students to find accommodation within about 2km of the University, mostly in large houses of the late nineteenth century which have been subdivided into bed-sitting rooms and flats. It is the students' responsibility to find their own accommodation, but the University's accommodation service provides assistance. There are also a limited number of places for postgraduates at favourable rates if they are willing to be 'in charge' of student houses, or to be in similar positions of responsibility within halls of residence. Extra help is given to students from abroad by the Overseas Students Office. More details are sent to students when they are accepted for courses or with the details of registration before they arrive. Further useful information is sent to overseas students.

# 5 Facilities, Postgraduate Training and Experience

### Computing

Research students are accommodated according to their research groups and interests. All postgraduates are given desks and computers often in shared offices and located close to people within their research groups. Students are given basic training by the Computer Support Officers in how the computing network is set up and each student is given a web site. The Department is equipped with a range of computational facilities and software. Apart from having PC's on desks which are connected on a campus-wide network, there are servers and dedicated workstations running Linux grouped together to form a Linux Farm. Currently the department also has access to a Beowulf Cluster of 160 processors which is maintained by one of the three Computer Officers.

From 2007, the Department will also have access to a new High-Performance Computing (HPC) facility, cluster of 2024-CPU IBM computers which will be one of the fastest supercomputers in the UK.

Postgraduates are encouraged to use mathematical packages such as Matlab, Maple and LaTeXas well as become well-versed in the Unix and Linux Operating systems. The University runs courses in Computer-related topics which Postgraduates are free to attend.

### Other facilities

The Department of Mathematics has a fluid dynamics laboratory used by staff, postdoctoral research assistants and postgraduates. There are a number of experimental facilities for studying environmental and geophysical flows including a rotating table in addition to facilities for observing and making detailed measurements of natural phenomena such as sparks and drop impacts.

There are good Library facilities for Applied Mathematics publications based mainly within the Engineering building of the University and the Physics Library. The Library services also provide online subscription to many of the leading journals.

### Learning

Within each sub-group of the Applied Mathematics group there are seminar series attended by Staff, Research Assistants and Postgraduate students. These attract world class speakers leading in research from around the UK and beyond, as well as providing a platform for postgraduates to talk about their research within their group. When relevant, Postgraduate students are encouraged to attend undergraduate lecture courses.

Students are encouraged and supported financially to attend meetings and conferences relevant to their work, this includes overseas travel.

The department is currently involved in several initiatives to provide an enhanced level of Postgraduate training in Mathematics. A successful bid to the UK's EPSRC funding body will lead to the introduction of Postgraduate taught courses from October 2007, in conjunction with Mathematics departments in Warwick, Oxford, Imperial and Bath. The delivery of the courses will involve interactive audio-video technology so that lectures can be "virtually attended" by students sitting in their host institution. Postgraduate students will be enroled on several courses in their first year that are deemed appropriate for them by their Supervisors and by the Director of Graduate Studies. These courses will be given by leading experts across a broad range of research areas and are aimed at providing Postgraduate students an advanced level of tuition of a broad range of Applied Mathematics topics.

# 6 Collaborations and Research Groups

The Applied Mathematics group has many strong research links with the Pure and Statistics groups and in particular with other departments and faculties within the University. This is one of its strengths. Within Applied Mathematics, there are opportunities to interact with groups in Engineering Mathematics, Physics, Earth Sciences, Civil and Aerospace Engineering, Physical and Theoretical Chemistry and Computer Science and Biology.

The Centre for Environmental and Geophysical Flows brings together interested staff from the Departments of Mathematics, Geology, and Geography. The objective of this Centre is to identify and develop connections between geophysical and industrially relevant problems in order to further the understanding of fluid motions within the environment. For example, an understanding of the flow of a hot viscous liquid over a cold surface is relevant to the hazard analysis of both magma flows from volcanoes and hypothetical containment breaches in nuclear reactors. The property of buoyancy reversal of a coignimbrite volcanic eruption cloud is also observed in accidental releases of highly toxic hydrogen fluoride into the atmosphere. Models to describe the dispersal of pollutants in water or in porous rock can be applied to modelling the movement of traffic on a motorway. The existing seminar series run by the Centre provides an excellent complement to the Department's activities and is being followed up with coordinated computer and laboratory support.

The Laboratory for Advanced Computations in the Mathematical Sciences is a facility based around the 160-node Beowulf cluster (soon to be upgraded to the new High-Performance Computing facility), and provides computing resources to the department of mathematics and any University researcher who may need to run heavy-duty computational experiments. These are vital for exploring complex systems such as climate modelling, turbulence and nuclear explosions, for example.

Many staff members have research connections with industrial and government organisations. In particular the mathematical physics group have close research links with Hewlett-Packard Laboratories, both in Bristol and in Palo Alto.

# 7 Postgraduate Degrees: Ph.D. and M.Sc. by research

The normal entry requirement is a good honours degree (1st class or upper second class) in Mathematics or a related subject from a British University; equivalent qualifications from overseas institutions are accepted. The funded period of study is three and a half years for a Ph.D., and two years for an M.Sc. by research. The UK Research Councils currently award research studentships for maximum three years and a half for British and EU students who have been residents in UK for 3 years prior to application.

Each student has an adviser with whom he or she works. In many cases the adviser is chosen before the student arrives. This is necessary for certain studentships. In other cases, the adviser is chosen in the first few weeks of study. Second advisers are also appointed because we believe that their support is valuable in improving the quality of the student's studies. It can open up new areas and techniques to the student, and also helps in integrating students into the Department.

Research degrees are awarded on the recommendation of the examiners of the student's final dissertation; this includes an oral examination.

# 8 Studentships and Funding: British and EU students

The studentships described in this section are for students who are normally resident in Great Britain.

Funding for studentships is sought from industrial firms but the only regular source of studentships is from UK Research Councils. Details of the terms of their studentships may be obtained from:

Engineering and Physical Sciences Research Council (EPSRC)
Polaris House
North Star Avenue
Swindon, SN2 1ET, UK

Or web: `http://www.epsrc.ac.uk`

or from Natural Environment Research Council (NERC) – same address as above, with postcode SN2 1EU, and web `nerc.ac.uk` or from Biotechnology and Biological Sciences Research Council (BBSRC), with postcode SN2 1UH and web `bbsrc.ac.uk`.

## Research studentships

Application for research council studentships is through the Department. They are available for study leading to a Ph.D. They are awarded for a maximum period of $3\frac{1}{2}$ years to students with First Class and Upper Second Class Honours degrees or equivalent. Studentships for Applied Mathematics

1. **EPSRC Doctoral Training Awards**

   The department applies on behalf of specific candidates (British and EU students who have been residents in the UK for 3 years prior to application) for a studentship from a 'pool' of studentships set aside for mathematics by the EPSRC. These studentships are awarded competitively, that is to say to the best qualified students nationally. Preference may be given to candidates who are taking up projects in certain 'earmarked areas' or changing university. In the recent past, candidates with first-class degrees and some good upper seconds have been awarded studentships. The closing date for us to send applications to EPSRC is in early July, and we need around two weeks to process the forms. The awards are announced in mid-August.

2. **CASE studentships**

   CASE is an acronym for Cooperative Awards in Science and Engineering. These research studentships are awarded by research councils jointly to a university department and an industrial organization. There is a named adviser at the university and named individual in the outside organization; together they are responsible for supervising work in a specific area. There may be some choice of topic within that area. The work is done largely at the university and the student is expected to spend a number of weeks each year at the outside institution (necessary expenses are paid). These studentships have the advantage of providing direct industrial experience for the student and also of being more remunerative, the outside institution also pays the student at least £3,000 per annum. Although the student is committed to work on a topic within a specific area, there is no further obligation after the period of the studentship.

3. **Complexity**

   The new Bristol Centre for Complexity Sciences will be recruiting 10-15 students a year starting in October 2007. The scheme is being run between the Applied Mathematics and Statistics groups together with the departments of Engineering Mathematics and Computer Science. The initiative is designed to appeal to students wishing to take part in multidisciplinary research. The programme for the first year involves taught courses across the different research areas during which time students are registered for an M.Res degree. Specific Ph.D. projects are started in the second year. The degrees are fully funded for four years.

4. **Other Studentships from research councils**

   Departments receive quotas of studentships which they may award to suitably qualified candidates before the end of July. The Mathematics Department may receive one or two of these studentships a year. Also studentships may be awarded as part of research grants (Project Studentships). These are for work on specific projects and connected to specific supervisors.

### Value of studentships

Research studentships are £12,300 plus fees. In future sessions Research Studentships are expected to increase in value. No tax is payable on this income, nor is there any abatement of the allowance on account of income from other sources.

### University Postgraduate Research Scholarships

Students from the UK or EU countries, showing special promise may be considered for a Bristol University scholarship: application is by the Department of Mathematics on behalf of the student, the awards are made on a competitive basis. The financial support from a University Scholarship is similar to that from a research council studentship, see below.

These awards will provide a maintenance stipend of £5,000 plus £500 towards travel/project expenses and cover the home tuition fees per annum for a period of up to three years. The estimated annual living cost in Bristol currently is £9000. The difference will be covered by the Department of Mathematics £2000 and the Faculty of Science £2000. They are renewed annually, subject to satisfactory academic progress. The awards are available to new entrants on full time PhD research degrees only. Neither existing PhD students nor part time students will be eligible to apply.

## 9 Studentships and Funding: Overseas Students

### Overseas Research Scholarships

For students from other countries outside the EU, Overseas Research Fellowship (ORS) awards are tenable at Bristol University. The ORS Awards Scheme provides partial funding of tuition fees to overseas postgraduate students of outstanding merit and research potential. There is intense competition for these awards both for the limited number of applications the University is permitted to make and at the national level. Application is again through the University. Under this scheme, if a candidate is successful, the Unviersity also awards a scholarship award to top up both living expenses and to the full amount.

Further details on ORS awards and a (more comprehensive) leaflet listing possible sources of funding for overseas students is available from:

Student Funding Office
University of Bristol
Senate House (Ground Floor)
Tyndall Ave

Bristol BS8 1TH
England

Also see: http://www.prospects.csu.ac.uk/student/

### Dorothy Hodgkin Postgraduate Awards

The University is offering 3 Dorothy Hodgkin Postgraduate Awards (DHPAs) for overseas students from China, India, Hong Kong, Russia and other countries from the developing world wishing to study for Ph.D degree at the University of Bristol. A full list of eligible countries can be found at: http://www.bristol.ac.uk/studentfunding/overseas_pg/dhpa.html

The scholarship is awarded on the basis of merit, and not an individual's financial circumstances, and is aimed to encourage top quality students from overseas to study for PhDs in the UK. Students will need to complete an application form which can be downloaded from the Dorothy Hodgkin web site, listed above. They will also need to submit transcripts (where applicable) and 2 references, one from someone with knowledge of their previous degree and one from their future supervisor at Bristol. Under this scheme, if successful, both living expenses and the fees will be covered.

## 10 Application and entry procedure

Prospective postgraduate students can obtain all the application materials online at

http://www.maths.bris.ac.uk/study/admissions_postgrad/apply/

Material can be also requested by contacting

Postgraduate Co-ordinator
School of Mathematics,
University Walk,
Bristol, BS8 1TW.

Tel: +44 (0)117 9288664
Fax: +44 (0) 117 9287999
Email: pgsec-maths@bris.ac.uk

The completed application, along with the reference letters (see below), which can be sent separately or together with the rest of the application in sealed envelopes, should be sent to the person above. Return of the completed form constitutes a formal application, but in no way commits the applicant. In this respect it may be likened to a formal enquiry.

For all other inquiries, please contact

Dr. Richard Porter,
Applied Postgraduate Admissions Tutor,
School of Mathematics,
University Walk,
Bristol, BS8 1TW, UK.

Tel: +44 (0)117 9287996
Fax: +44 (0)117 9287999
Email: richard.porter@bris.ac.uk

Along with the application form, reports from two referees nominated by the applicant are requested. Applicants in the United Kingdom, are usually invited to visit Bristol to meet members of the Applied Mathematics group, once satisfactory reports are received. Travel expenses are reimbursed at a young person's rail card rate. A decision on whether to accept the applicant will usually be made shortly after the interview. Acceptance may be conditional on obtaining a certain class of degree.

Once a student accepts an offer of a place for postgraduate study, further formalities of admission are handled by the office of the University's Faculty of Science. Evidence must be provided to the Faculty that the student has financial support, both for the fees and for subsistence. For most British students this is satisfied by the award of a studentship. The latest figures for the fees for full-time postgraduate study are given below. You can find complete information on `http://www.bristol.ac.uk/prospectus/`

British and E.C. students: £3,235 (2007-08)
Other overseas students: £12,100 (2007-08)

The fees normally change each year in line with inflation. We estimate that students require at least £9000 per annum to live in Bristol.

Overseas students whose native language is not English are required to give evidence of their fluency in English. There are various ways in which this may be done, e.g. in many countries there are offices of the British Council where a test such as the IELTS may be taken leading to a report for the University. The University's normal minimum requirement is 6.5 on IELTs and 600 on TOEFL (old style) and 250 on TOEFL (computerised). However, the faculty of Science offers a degree of flexibility for postgraduate students in Mathematics and will accept an IELTS score of 6.0 or a TOEFL score of 577 (233 computerised).

## Closing Dates

There is no fixed closing date for application for the Ph.D. or M.Sc. study. However, we strongly urge prospective candidates **to apply as early as possible** (preferably by April, since we will begin allocating studentships and making offers in March – April). Studentships are often conditionally allocated well before final degree examinations.

The closing date for University of Bristol Postgraduate Research Studentships is early **May**.

The closing date for ORS and Dorothy Hodgkin awards is Mid **February**, but in practice applications need to be made to the department even earlier.

# 11 Ph.D. Research Project Descriptions

The project descriptions in this booklet were compiled in October 2006 as guide for prospective students seeking admission in 2007/2008.

## 11.1 Lorena Barba

My research areas are fluid mechanics and computational methods in applied science. In particular, I work with vortex particle methods and their applications for the computation of unsteady viscous flows. I have strong interests in fundamental problems of fluid dynamics, in particular high-Reynolds number flows with concentrated areas of vorticity and vortex dynamics in general.

In addition, I am interested in the development of the meshless paradigm for computational methods. The vortex particle method is a meshless method; there are also a variety of new methods being developed that do not rely on the construction of a mesh in the computational domain. This approach holds great promise to allow for computations of highly complex, unsteady flows, flows with moving boundaries, material problems with discontinuities (such as cracks), multi-scale computations, and many other extremely challenging problems.

Some research topics are given below:

**Axisymmetrization (or not!) of perturbed vortices, and emergence of vortex tripoles.**

When a strong vortex is subject to a perturbation, the first fundamental question that arises is whether the vortex will return to an axisymmetric shape (i.e., its stability). One can also be interested in the time scale of the relaxation process. For more details visit:

`http://www.maths.bris.ac.uk/~aelab/`

## 11.2 Isaac Chenchiah

I am interested in mathematical problems in solid mechanics and materials science, particularly those motivated by microstructure and phase transitions.

Recent years have seen intense interaction between mathematics, solid mechanics and materials science. This has borne much fruit, chief among which has been the explanation of intriguing material behaviour (e.g., the shape memory effect) by mathematical models that relate behaviour to microstructure[1]. This successful interaction has in turn raised a number of questions many of which are of interest simultaneously in mathematics, in the physical sciences and in engineering.

The relevant mathematical areas are primarily, but not exclusively, calculus of variations, partial differential equations (PDE), functional analysis and real analysis.

Two of many possible projects are outlined below. These would be suitable for students with a good background in analysis (a specific background in calculus of variations or PDE is not a prerequisite). A genuine interest in solid mechanics is essential. Questions are welcome at `Isaac.Chenchiah@bris.ac.uk`.

### PDE, geometry and polycrystalline materials

PDE with geometric constraints are fascinating but poorly understood objects. Moreover qualitative features of their solutions are of relevance to the behaviour of materials. Of particular interest are polycrystalline materials, where the presence or absence of certain behaviour (e.g., the shape memory effect, rigid-perfect plasticity etc.) is related to the existence or non-existence of non-trivial solutions to equations of the form

$$Lu(x) \in R(x)\mathcal{S}, \quad x \in \Omega \subset \mathbb{R}^n, \quad u \in W^{1,\infty}(\Omega, \mathbb{R}^n)$$

where $L$, a first-order linear partial differential operator, $R\colon \Omega \to SO(n)$, a piece-wise constant map and $\mathcal{S} \subset \mathbb{R}^{n\times n}$ are dictated by mechanics/physics. (See [1] for more on this problem in the context of shape memory materials and [3] for a similar problem in rigid-perfect plasticity.)

The goal of the project is to understand better the nature of solutions to such PDE – and thus the behaviour of the related polycrystalline materials.

### Methods to compute quasiconvex hulls

The existence of microstructures in materials is related to the relevant energy-density functions not being quasiconvex. (Quasiconvexity is a weakening of convexity that is of relevance in the calculus of variations). Though the notion of quasiconvexity is more than 50 years old it remains poorly understood.

In this project we focus on a problem of much relevance to engineering, namely, that of computing (analytically, not computationally) quasiconvex envelopes of energy-density functions that arise in mechanics. The currently available methods are very limited in their power and thus only a few quasiconvex envelopes have been computed. (For a flavor of the

[1] For some optical micrographs of microstructures in solids c.f., e.g., `http://www.aem.umn.edu/images/images3/microstructure1sml.jpg` and `http://www.aem.umn.edu/images/images3/struct-nmg6sml.jpg`.

methods c.f., e.g., [2, Chap3] where the quasiconvex envelope of an function that arises in the study of nematic elastomers is computed.)

The goal of this project would be develop more powerful methods to compute quasiconvex envelopes.


1. Kaushik Bhattacharya and Robert V. Kohn, *Elastic energy minimization and the recoverable strains of polycrystalline shape-memory materials*, Arch. Ration. Mech. Anal. **139** (1997), 99–180, `http://www.springerlink.com/openurl.asp?genre=article&eissn=1432-0673&volume=139&issue=2&spage=99`.

2. Georg Dolzmann, *Variational methods for crystalline microstructure—analysis and computation*, Lect. Notes Math. 1803, Springer-Verlag, (2003), `http://www.springerlink.com/openurl.asp?genre=issue& eissn=1617-9692& volume=1803`.

3. Robert V. Kohn and Thomas D. Little, *Some model problems of polycrystal plasticity with deficient basic crystals*, SIAM Journal on Applied Mathematics **59** (1998), no. 1, 172–197, `http://epubs.siam.org/sam-bin/dbq/article/32001`.

## 11.3 Robert Deegan

**Holes in Shear Thickening Fluid** (with R. Kerswell)

The excitation of Faraday waves on the surface of a vertically-vibrated layer of a Newtonian fluid like water is well known and now well studied. However, it has recently been discovered experimentally (Merkt et al. 2004) that non-Newtonian fluids can behave surprisingly different under vibration. Under certain conditions, deep holes which persist indefinitely can be generated in the surface. So far, only the fact that the fluid needs to be 'shear thickening' seems clear; otherwise, little is understood. This project (joint with R. Kerswell) has both an experimental and theoretical component, the exact mix of which can be tailored to the interests/tastes of the student. The objective will be to understand how and why the holes can exist with the overall hope to reveal important properties of shear-thickening fluids.

F. S. Merkt, R. D. Deegan, D. I. Goldman, E. C. Rericha & H. L. Swinney, *Persistent Holes in a Fluid*, Phys. Rev. Lett. 92, 184501 (2004).

**Spark Bifurcations**

Spark discharges are widely used in technological applications ranging from electrochemical discharge machining to pollution control to fabrication of microparticles, and are responsible for maintaining the Earth's electrical potential in their largest incarnation as lightning. A spark is the last step in a multi-step process that begins with formation of an ionised channel, known as a streamer, and culminates with a bright discharge. A familiar characteristic of sparks is their tree-like appearance: the crooked backbone with many branches. It is generally acknowledged that this structure is not well understood. Recent theory and simulations suggest that the branching arises from a hydrodynamic instability of the streamer front (Arrayas 2002 & 2005). The purpose of this project is to discover the origin of spark branching through experiment.

M. Arrayas, M. A. Fontelos, & J. L. Trueba, *Power laws and self-similar behavior in negative ionization fronts*
`http://citebase.eprints.org/cgi-bin/citations?id=oai:arXiv.org:physics/0504005`

M. Arrayas, U. Ebert, & W. Hundsdorfer, *Spontaneous Branching of Anode-Directed Streamers between Planar Electrodes*, Phys. Rev. Lett. 88, 174502 (2002).

**Crack Paths**

The best continuum theory of brittle fracture predicts crack speeds that are 50% greater than observed in the laboratory. This discrepancy is known emperically to arise from an instability that causes cracks to branch and hence slow down. Nonetheless, the physical origin of this instability remains unknown. For this project we will experimentally search for an empirical equation of motion for the crack tip.

## 11.4 Carl Dettmann

Here are some suggestions, but I am also happy to supervise well motivated students who want to work on other aspects of mathematical physics and/or nonlinear dynamics. Most projects involve a combination of numerical and asymptotic calculations (occasionally rigorous), and most involve aspects of dynamical systems theory. Some travel to visit international collaborators (in addition to conferences) could be appropriate. The reading suggestions below can be downloaded from my website `http://www.maths.bris.ac.uk/~macpd/`. They are research articles, hence rather technical; please read initially only to get the general idea, and feel free to ask me questions directly.

- Open dynamical systems, regular and/or chaotic. The regular systems involve some number theory. Read: "Open circular billiards and the Riemann hypothesis," L. A. Bunimovich and C. P. Dettmann, Phys. Rev. Lett. 94 100201 (2005).

- Random dynamical systems, perhaps jointly with Jon Keating. Read: "Stochastic stabilization of cosmological photons." C. P. Dettmann, J. P. Keating, and S. D. Prado, J. Phys. A.: Math. Gen.37 L377-L383 (2004).

- Coupled map lattices. Read: "Stable synchronised states of coupled Tchebyscheff maps," C. P. Dettmann, Physica D 172 88-102 (2002).

- Dynamical approaches to nonequilibrium statistical mechanics. Read: "The Lorentz gas as a paradigm for nonequilibrium stationary states," C. P. Dettmann, pp 315-365 in Hard ball systems and the Lorentz gas (edited by D. Szasz), Encyclopaedia of Mathematical Sciences Vol 101 (Springer, 2000).

## 11.5 Jens Eggers

### Formation and stability of Taylor cones

When a drop of dielectric fluid is placed in an electric field, it first extends along field lines. With slowly increasing field strength the resulting series of stationary shapes becomes unstable at a critical field, and evolves dynamically toward a new shape with sharply tipped cones at its end. This shape is strongly reminiscent of a stationary solution originally found by G. I. Taylor, representing a balance of surface tension and electric forces. Often, these cones are unstable at the tip, and a tiny jet is ejected from it. These so-called "electric jets" represent one of the most important methods to inject tiny droplets into a gas phase. The object of this thesis is to focus on the dynamics of the formation of the Taylor cone. Preliminary numerical calculations indicate that the dynamical structure that develops after the drop becomes unstable is a cone with an angle different from Taylor's prediction. Equally unexplored and perhaps related is the mechanism leading to the instability of the tip.

A fascinating aspect of the Taylor cone solution is that a variety of very similar structures are observed for example when a drop is placed in a strong shear flow, in the so-called "selective withdrawal" experiment, or in viscous drops running down an inclined plane. The first stage always consists of the formation of a singular, tip-shaped structure on the free surface. However, upon increasing the driving, this structure typically turns unstable to eject a jet. A second aim of this thesis is thus to identify common features in these diverse situations, and to draw conclusions about possible universal mechanisms of instability.

### Drop impact

When a drop of water impacts on a smooth surface, it first flattens out to a thin "pancake" structure, which then retracts and rebounds from the surface. For all applications whose object is to deposit material on the surface, this typical behavior is of course extremely undesirable. The object of this thesis is to quantitatively describe the phenomenon of impact by developing simplified model equations for both phases of spreading and retraction. All previous descriptions were based on global balances, without paying attention to the actual drop shapes. However, there are a number of characteristic features of drop impact that need to be explained more quantitatively.

1. The above-mentioned "pancake" is in reality far from uniform, but ultimately an extremely thin film forms, bordered by a thick rim.

2. In retraction, low-viscosity liquids form a series of sharp steps, somewhat reminiscent of hydraulic jumps. If the liquid freezes during recoil owing to heat transmitted to the substrate (for example in the case of liquid solder), some of these features are strikingly preserved.

Most importantly, the actual droplet shapes as well as the flow field come into play when calculating the viscous dissipation during impact, which can no longer be neglected for higher viscosities. An even more dramatic effect is produced by the presence of tiny amounts of polymer additives: while the initial spreading is virtually unaffected, the recoil is strongly suppressed and the liquid is deposited even when its base viscosity is very small. While this effect is subject of several recent papers and patents for the agricultural industry (think of pesticides being deposited on plant leaves), it is not understood. Again, only the flow field will tell us about the stretching of the high molecular weight polymer, and where it eventually ends up. Further interesting and poorly explored subjects include the influence of solid surface properties and of the contact line motion.

The student would join a group of researchers actively investigating related topics.

## 11.6 Andrew Hogg

Projects are available in a range of two-phase flows, including suspended sediment transport, dense granular flows and rapid avalanches. Particular attention is given to environmentally relevant processes and phenomena and research projects may employ analytical, numerical and experimental techniques. There follows some focused projects, but many other topics could be investigated as well.

#### Avalanches and rapid granular flows

Avalanches of snow and rock pose significant natural hazards. Their motion down slopes is driven by gravitational acceleration and controlled by the interactions between the constituent particles. Currently there is no universally accepted mathematical model for these flows and there are many possibilities for exciting research projects that develop the fundamental descriptions of the motion and apply them to environmental phenomena.

One topic that has been the focus of recent research in Bristol is avalanche defence barriers. Inhabited regions may be protected from destructive snow avalanches by constructing solid obstacles in the flow path to deflect, retard or arrest the oncoming motion. Although such barriers are widely deployed, there is only limited understanding of the nature of this interaction. These particulate flows exhibit characteristics that are both fluid- and solid-like. There is a need to develop new mathematical models of this type of motion and to learn about the physical processes by performing new laboratory experiments. Interesting Ph.D. projects could tackle various aspects of this phenomena.

Additionally granular flows exhibit many important characteristics that are as yet only poorly understood, but which could be studied as part of a Ph.D. project. These include the way in which grains of different sizes and densities become segregated by the motion; and the way in which these avalanche flows slow and come to rest when they flow down slopes of sufficiently weak gradients.

#### Suspended sediment transport

River currents and coastal waves may pick up sedimentary particles if their velocity is sufficiently high. Thereafter the particles are carried along in suspension with their submerged weight being supported by the action of fluid turbulence. It is important to be able to predict the quantity of sediment that a flow may transport so that accurate assessment may be made of the rates of coastal erosion or the effects of building new engineering structures such as barrages and harbours on the surrounding environment. The current formulae for predicting this erosion are highly inaccurate and estimates may be incorrect by orders of magnitude.

This Ph.D. project will adopt a new approach to modelling suspended sediment transport. It will employ recent ideas from studies of turbulent fluid flows in which coherent eddies and other flow structures have been identified. These structures are potent means for transporting sediment in suspension and this project will examine how models involving these flows may significantly improve our understanding and ability to quantify the amount of particulate material that may be transported

### Volanic ash flows and oceanic turbidity currents: particle-driven flows

Flows of volcanic ash (pyroclastic flows) and oceanic sediment flows (turbidity currents) are driven by the presence of suspended particles. These render the suspension denser than the surrounding fluid and this buoyancy difference drives the dispersion of the particle-laden cloud. The particles, however, continually settle out of the suspension to the underlying boundary thus progressively reducing the excess density of the cloud and the speed of the flow. Eventually all of the particles have settled and the flow is arrested, leaving a distribution of deposited particles. Understanding this type of ow is important in the assessment of the hazard posed by volcanic flows and in understanding the deposits left by ancient sediment flows that may now form important hydrocarbon reservoirs.

A number of Ph.D. projects are possible in this area and could entail mathematical modelling, computation or laboratory experimentation on small-scale flows that share many features with these important environmental phenomena. At a fundamental level there is a need to develop a mathematical description of the motion that captures its key features and is able to predict the deposit that arises. However there are a number of additional features: for example, how do these flows interact with topography such as small hills or width constrictions? How are particles of different sizes and settling velocities transported and deposited? And what effect does an underlying slope have upon this type of motion?

### Mud and debris flows

When a fluid is highly laden with particles or contains a significant fraction of mud, it may exhibit dynamical features that differ strongly from situations where the particulate phase is dilute. Most notably the fluid may exhibit a yield strength that must be overcome for motion to occur. This poses a significant control on the flows: in particular they may arrest or form channelised and braided streams. Recent experiments and field observations have highlighted a number of these features and Ph.D. projects in this area would develop quantitative, mathematical models that yield rational, physical explanation for the observed phenomena.

## 11.7 Jonathan Keating

### Project areas

Projects are available in the following areas.

1. Quantum chaos (the quantum properties of systems in which the classical dynamics is chaotic).

2. Random matrix theory (properties of matrices whose elements are random variables), including applications to complex wave problems, the Riemann zeta function and families of L-functions, and quantum information theory.

3. Semiclassical asymptotics (asymptotics of wave theories in the limit as the wavelength tends to zero; eg quantum asymptotics in the limit of small de Broglie wavelength).

4. Cosmological chaos.

Students interested in any of these areas can contact Professor Keating `j.p.keating@bristol.ac.uk` for related references.

There are close links with research interests of Dr C. P. Dettmann, Dr J. Marklof, Dr F. Mezzadri, Dr N. C. Snaith, Dr M. Sieber.

## 11.8 Rich Kerswell

I am interested in supervising projects in the following areas.

a) transition to turbulence in shear flows (Newtonian and non-Newtonian)

b) nonlinear dynamics of fluid flows generally (bifurcations, stability, generating nonlinear solutions to the Navier Stokes equations, dns)

c) variational calculus applied to turbulence

d) the dynamics of granular media

e) magnetic field generation in fluids (dynamo action)

A specific project (with R.D. Deegan and A.J. Hogg) this year concerns the modelling of drug delivery into the brain. Neurosurgeons at Frenchay hospital in Bristol are currently developing a technique called "convection-enhanced delivery" using extremely fine catheters and very low infusion rates to deliver drugs directly into the brain. This technique offers extremely exciting possibilities for treating patients with brain tumours, Alzheimers disease, multiple sclerosis, Parkinson's disease, Huntington's disease, motor neurone disease, spinal cord injuries and a vast number of other neurological conditions. The key issue is being able to predict (and control) where the drug infuses for given choices of the injection point, catheter size, drug and flow rate. The project will consist of building a hierachy of mathematical (and possibly laboratory) models to simulate and ultimately inform this medical procedure which will be in collaboration with neurosurgeons at Frenchay Hospital.

## 11.9 Noah Linden

My research is in the new field of Quantum Information which is one of the most exciting and dynamics areas science and technology. It is an interdisciplinary subject where mathematicians, physicists and computer scientists have made major contributions. Deep links have been forged between the previously unrelated disciplines of quantum physics and computer science/information theory. On the one hand there have been insights into fundamental issues in physics. On the other, totally new methods of computation, communication and information processing have emerged. Quantum information is concerned both with the fundamental science of quantum systems and with how one can use quantum resources to perform computational and other information processing tasks.

Despite a major international research effort, we are still at a very early stage in developing our understanding of this field. Many deep questions remain about the nature of quantum information and the possible scope of quantum systems for information processing tasks. Quantum information theory is also forcing us to ask new questions in fundamental physics.

My current research in is a number of different theoretical aspects of quantum information and quantum computation. These include the theory of entanglement and non-locality, quantum communication and foundational questions in quantum computation [such as what gives quantum computation its power].

My recent papers can be found via `http://www.maths.bris.ac.uk/~manl/` These give an idea about what issues particularly interest me at present, and areas in which I would be interested in taking on PhD students.

## 11.10 Francesco Mezzadri

### Asymptotic properties of spectra and eigenfunctions of quantum chaotic systems

The central problem is to understand how the chaotic nature of the underlying motion of classical systems affects the corresponding quantum mechanics. The main theoretical questions concern the behaviour of the eigenfunctions and eigenvalues in the semiclassical limit, that is as Planck's constant tends to zero. In particular, it is believed that in the semiclassical limit the energy levels of generic chaotic systems are correlated like the eigenvalues of large random Hermitian matrices. This is known as the Random Matrix Theory conjecture. The models used to study these problems will be quantum systems whose time evolution is discrete; these are known as quantum maps.

### Random matrix theory and critical phenomena in spin models

Recent studies have shown that random matrix theory can give new insight into the understanding of phase transitions in lattice systems. The problems to address include the behaviour of entanglement in the ground state of quantum spin chains, the study of universality of critical exponents and of the scaling hypothesis in two dimensional spin models. Thermodynamics variables in proximity of critical temperatures obey power laws whose exponents seem to depend only on the dimensionality and symmetries of the system; the scaling hypothesis, instead, asserts that the thermodynamic properties of a macroscopic system depend only on few relevant variables that characterize its behaviour on a particular time or length scale.

### Correlations at the edge of the spectrum of non-Hermitian random matrices

The spectral correlations at the edge of the spectrum of random matrices with real spectra is well understood by the theory developed in the early '90s by Tracy and Widom. Their theory, however, does not apply to matrix ensembles with complex spectra. The main problem consists in computing the asymptotics of the kernel of the measure. Such kernel is usually a sum of orthogonal polynomials defined in the complex plane. The main goal is to develop suitable asymptotics techniques to compute such kernels. This would give us the tools to determine the correlations at the edge of the spectrum of ensembles of matrices with complex and quaternionic elements.

### Distributions of zeros of derivatives of characteristic polynomials of random matrices from the classical compact groups

The behaviour of the zeros of the Riemann zeta function and other L-functions is affected by the properties of the zeros of their derivatives. The distributions of the zeros of derivatives of characteristic polynomials of random matrices are good models for the distributions of the zeros of derivatives of L-functions. Up to now the only distribution known is when the matrices are from the unitary group equipped with Haar measure, and only in certain asymptotic regimes. The purpose of this project is to improve previous results by computing higher order terms in the asymptotic expansion and extend them to the other classical compact groups.

## 11.11 Richard Porter

I am interested in the use of analytical techniques to solve equations which are derived from studying the interaction of waves with structures in the context of water waves, acoustics, electromagnetic and optics, quantum theory and elasticity.

### Waves in ice sheets and elastic plates

When the sea freezes over in polar regions during the winter months, a 1-2m layer of sea ice is formed containing a series of irregularities including cracks, ridges, open leads, etc. Due to the large aspect ratio of the ice sheets that are formed, flexural waves are able to propagate within the ice, which can be modelled as a thin elastic sheet. The way in which the waves behave and interact with irregularities is of interest as a remote sensing device to monitor ice-thickness, for example, as well as assisting the understanding of ice break-up. This area of work has much in common with understanding how waves interact with irregularities on elastic plates that occur in airframe structures and turbomachinery, and there is scope to consider a variety of problems in any one of these areas.

### Methods for focussing and capturing wave energy

The hydrodynamic theory of wave energy converters was developed over 25 years ago. Some remarkable results were discovered which often allows one to easily assess the theoretical potential for success of any particular wave energy converter design. In practice, engineering design makes it hard to realise the theoretical potential and so energy converter efficiency is often traded in for robustness. This project is broadly aimed at considering alternative ways of increasing wave energy efficiency, not by necessarily considering converter design but by using various structures to capture and focus energy. To do this, a variety of analytical and numerical tools will be developed which will hopefully lead to some powerful new methods and results.

### Large array scattering

There are many physical applications where one wishes to assess the diffraction of waves by a ‘large’ but finite number of objects placed in a medium which supports wave propagation. For instance, waves on the ocean interacting with a large number of vertical supporting columns is of practical interest in the proposed design of offshore runways in Japan, or in the ribs of an airframe structure in which the air supports acoustic waves and the panels between the ribs support flexural waves. Typically, when calculating the solutions to these problems, one can readily develop analytic techniques to solve for scattering by a finite number of scatterers and for an infinite periodic array of scatterers. However, the problems become much more complicated when one considers large, but finite, numbers of scatterers, or randomly-placed scatterers. Very recently, some progress has been made on developing analytical techniques for solving such problems, but there is plenty of scope for to develop further techniques and applying them to physically important problems.

## 11.12 Jonathan Robbins

### Liquid crystals in confined geometries

Liquid crystals provide a variety of mathematical problems of both theoretical and practical interest. Our research concerns liquid crystals in polyhedral geometries, and combines elements of topology, partial differential equations, calculus of variations and harmonic maps. It is also motivated by our collaboration with researchers in the Digital Media Department at Hewlett-Packard Laboratories, Bristol.

Nematic liquid crystals are composed of rod-like molecules which are orientationally ordered but spatially disordered. The preferred orientation of the rods may be described by a unit-vector field (or, more precisely, a director field) which is a local minimiser of a certain energy functional. The associated Euler-Lagrange equations are nonlinear elliptic PDE's. Solutions are examples of harmonic maps between Riemannian manifolds.

In polyhedral geometries, liquid crystals often satisfy (approximately) tangential boundary conditions; on the faces, the director lies in the plane of the face, but is otherwise unconstrained. The boundary conditions produce a rich family of topologically distinct configurations. Our programme involves classifying these topologies and calculating, both analytically and numerically, the minimising configurations and their energies. Applications include bistable displays, in which the pixel geometry supports multiple energetically stable solutions which are topologically and optically distinct.

Topics for PhD research include (i) existence and regularity of local minimisers of given homotopy type, (ii) dynamics of configurations under applied fields and noise, including the creation and dynamics of surface and edge defects. The research would suit students with a broad range of interests in mathematics and mathematical physics. There is ample scope for numerical computations, too.

### Quantization and topology of integrable systems: periodic toda chain, singularities, and higher Maslov classes

Background: Singularities of finite-dimensional integrable systems are where the gradients of the constants of the motion become linearly dependent, and the foliation into tori degenerates (they are higher-dimensional generalisations of fixed points in 1-freedom systems). There is a body of work on the global topology of finite-dimensional integrable systems in which these singularities play a central role. There is also renewed interest in the semiclassical quantization of more-than-one-dimensional integrable systems, including quantization to higher orders in $\hbar$ and topological signatures such as monodromy. The periodic Toda chain is a well-known, non-separable, finite-dimensional integrable system. Gutzwiller initiated the study of the quantum problem in the early 1980s, and there have been important recent development, in particular concerning the quantum-integrability of the Toda chain (existence of observables commuting with the quantized Hamiltonian).

Recently I've been interested in a) the general relationship between integrable (codimension-one) singularities and the Maslov index which appears in the semiclassical (EBK) quantization conditions, and b) specifically the singularities of the periodic Toda chain, which turn out to be related in a nice way to eigenvalue degeneracies of the Lax matrix.

Research problems: a) Higher-order semiclassical quantization of the Toda chain. Recent work of Littlejohn et al and Colin de Verdiere gives a general geometrical framework for calculating higher-order $\hbar$ terms in the EBK quantisation scheme, but specific multidimensional systems have yet to be explored. The $n$-particle Toda chain is a good candidate for explicit calculations and for investigating the asymptotic behaviour of the semiclassical series for the energy levels. b) Investigate higher-dimensional Maslov classes (the next after the "Maslov index" requires at least 3 degrees of freedom), which should be related to higher-codimension singularities. Calculate higher Maslov classes for candidate integrable systems, and investigate their role on semiclassical quantisation.

### Configurations of points in $R^3$

Atiyah has constructed a certain natural map from $C_n$, the configuration space of $n$ distinct and distinguishable particles in three-dimensional Euclidean space, to the flag manifold $U(n)/T(n)$ (here $U(n)$ is the unitary group, and $T(n)$ its maximal torus), which is equivariant with respect to permutations. Atiyah has conjectured that the map is continuous. The conjecture has been proved for $n = 1$ (trivially), 2 (easily), 3 (analytically) and 4 (by computer algebra), but for $n > 4$ remains open. One motivation for the problem arises from a nonrelativistic derivation of the spin-statistics relation for point particles in three dimensions.

A sharper formulation may be given in terms of conjectured properties of a certain $n$-body potential energy function, $V_n$, which is very interesting in its own right. $V_n$ is invariant under translations, rotations, and dilations, and is constructed purely from Euclidean geometry. For $n = 3$ it is related to a certain many-body quantum potential due to Calogero and Marchioro for which some exact eigenstates can be calculated. Recent studies of minimum-energy configuration reveal striking similarities to, amongst other problems, minimum-volume sphere-packings and low-energy configurations of skyrmions.

Topics for PhD research include i) classical and (possibly) quantum dynamics of $V_n$, and ii) an algebraic study of its properties. Another direction iii) involves a generalisation due to Atiyah and Bielawski in which $U(n)$ is replaced by another classical Lie group $G(n)$, and $C_n$ is replaced by the tensor product of $\mathbb{R}r^3$ with the associated Lie algebra. One would like to generalise the related quantum theory, and find analogues of the spin-statistics connection, in this generalised Lie group setting.

## 11.13 Misha Rudnev

The main theme of my today's research is the interaction of harmonic analysis, geometric combinatorics, geometric measure theory and analytic number theory, in particular various manifestations of the notion of curvature therein. There is a number of fundamental questions arising at the crossroads of these areas of mathematics. These questions have lead to conjectures which today are wide open and mysteriously intertwined. Around these conjectures, there is a constant supply of exciting new problems which appeal to a variety of mathematical taste and expertise.

### Some problems in geometric combinatorics and measure theory

My recent efforts in geometric combinatorics and measure theory are centered around the Erdös/Falconer distance conjectures which ask, in a variety of settings, for the smallest number of distances determined by subsets of the Euclidean space. Namely, let $E \subset \mathbb{R}^d$. Define its *distance set* as

$$\Delta(E) = \{\|x - y\|, \ \ x, y \in E\}.$$

How big does $E$ have to be so that its distance set is conspicuous? What I mean by that precisely is – suppose $E$ is a discrete set of $|E| \gg 1$ elements and let $q \gg 1$ be a large real. What is the minimum cardinality of $E$ to guarantee that the number of elements $|\Delta(E)|$ in the distance set is at least $q$? Paul Erdös conjectured in 1946 that $|\Delta(E)| \geq q$, whenever $|E|$ is asymptotically greater than $q^{\frac{d}{2}}$. More precisely, for any $\varepsilon > 0$ there exists a constant $C_\varepsilon$ (depending also on the dimension $d$, but not on $E$) such that

$$|\Delta(E)| \geq q, \quad \text{whenever} \quad |E| \geq C_\varepsilon q^{\frac{d}{2}+\varepsilon}.$$

This conjecture is nowhere near resolution. As of today, the best result in dimension 2 (which in fact may be the most difficult dimension: there is some not totally unfounded hope that in $d \geq 4$ the matters may become easier) is due to N. Katz and G. Tardos, saying that $|\Delta(E)| \geq C|E|^{\approx .86}$ for some $C$. The result is obtained by methods of combinatorial geometry and rests on a repeated application of the Szemerédi-Trotter incidence theorem. The latter theorem says that if one has $m$ lines and $n$ points in the plane, the total number of incidences $I$, i.e. pairs $(p, l)$ such that a point $p$ lies on the line $l$, is bounded from above as follows:

$$I \leq 5[m + n + (mn)^{\frac{2}{3}}].$$

The continuous analogue of the Erdös distance conjecture is known as the Falconer distance problem. Suppose now that $E$ is a compact Borel set in $\mathbb{R}^d$. It is conjectured that $\Delta(E)$ has *positive Lebesgue measure*, whenever the Hausdorff dimension of $E$ exceeds $\frac{d}{2}$. The fact that there is the same $\frac{d}{2}$ in both discrete and continuous problems (even if you do not quite know what the Hausdorff dimension is) is certainly not an accident! Falconer himself proved in 1986 that $\Delta(E)$ has positive Lebesgue measure, whenever the Hausdorff dimension of $E$ exceeds $\frac{d+1}{2}$. The Falconer conjecture has been studied by methods of harmonic analysis, and the closest to $\frac{d}{2}$ one could get so far is $\frac{d+1}{3}$, due to T. Wolff and M. B. Erdoğan.

The distance conjectures and related problems can be studied from the analytic, number theoretic and combinatorial viewpoint. For example, there is a link between the Fourier transform estimates related to the Falconer distance problem and the celebrated Freiman theorem in additive number theory. The latter theorem says that if a set of numbers $A \subset \mathbb{Z}$ has a property that $|A+A| \le C|A|$ (i.e. the set of possible pair-wise sums of the elements of $A$ does not exceed $A$ in cardinality by more than a constant factor $C$), then $A$ is contained in an arithmetic progression with at most $k(C)$ generators. (Freiman's theorem has been recently generalised by B. Green and I. Ruzsa to the case of an arbitrary Abelian group instead of $\mathbb{Z}$.)

From the analytical point of view, it would not matter if the distance $\|\cdot\|$ in the definition of the distance set became anisotropic, i.e. instead of the unit sphere $S^{d-1}$ one dealt with the boundary $\partial K$ of some smooth well-curved body $K$. Studying $K$-distances is fruitful for at least one good reason: they provide a rich source of examples and counter-examples. The approach of T. Wolff and M. B. Erdoğan, vindicating the dimensions greater than $\frac{d+1}{3}$ for the Falconer distance problem target best possible point-wise estimates for the "spherical average" $\sigma_\mu(r) = \int_{S^{d-1}} |\widehat{\mu}(r\omega)|^2 d\omega$, where $\mu$ is a Borel measure, supported on the set $E$. (In order that $\Delta(E)$ have positive Lebesgue measure it suffices that the so-called *Mattila integral* $M_\mu = \int_1^\infty \sigma_\mu^2(r)\, r^{d-1} dr$ converge.) Best possible point-wise estimates for the spherical average $\sigma_\mu(r)$ are available in $d = 2$ (where they alone do not suffice to resolve the Falconer conjecture), however in dimensions $d \ge 4$ they are not known, but can in principle turn out to be in a sense more regular than for $d = 2$. In $d \ge 4$ these estimates alone could possibly resolve the Falconer conjecture. This question does not appear to be totally unfathomable within the harmonic analysis approach. Interestingly enough, due to a recent work of myself and A. Iosevich, its positive resolution would entail that there exist no well-curved smooth convex bodies $K$ in higher dimensions, such that for no matter how large $t$, the boundary $\partial K$ blown up $t$ times had more than integer lattice points, for any $\varepsilon > 0$. This is a classical analytic number theory fact if $K$ is a sphere.

There is also a connection between the asymptotic behaviour of Fourier transforms of measures in $\mathbb{R}^d$ and properties of a class of diophantine equations. In the context of the finite field analog of the Erdös/Falconer distance problem, there is a connection between this problem and asymptotic properties of classical number-theoretic Kloosterman sums and its "angles".

Finally, it is not understood, what is the proper niche, occupied by the Erdoös/Falconer distance problem amidst other giant open questions in harmonic analysis, such as first of all the Restriction/Kakeya problem (claiming that the Hausdorff dimension of a set in $\mathbb{R}^d$, containing a unit segment in each direction is $d$). To this effect, it seems very sensible to study the Falconer problem on the sphere $S^{d-1}$ and see how the curvature comes into play. It does not seem unreasonable that the resolution of the Falconer conjecture on $S^2$ is strongly correlated with the Restriction conjecture in $\mathbb{R}^3$, but the precise connections are yet to be discovered.

If you are interested in any of these questions, please do not hesitate to e-mail me at `m.rudnev@bris.ac.uk`. You can look up my recent work on these problems on
`http://www.maths.bris.ac.uk/~maxmr/papers.html`

Here is a list of excellent on-line notes which provide more than the introduction to harmonic analysis and adjacent areas of mathematics (see also the links on these pages).

- Ben Green's notes on Restriction and Kakeya
  `http://www.maths.bris.ac.uk/~mabjg/rkp.html`

- Expository notes by Alex Iosevich
  `http://www.math.missouri.edu/~iosevich/expositorypapers.html`
- Thomas Wolff's Lectures on Harmonic Analysis, edited by Izabella Łaba
  `http://www.math.ubc.ca/~ilaba/wolff`
- Terry Tao's Harmonic Analysis page
  `http://www.math.ucla.edu/~tao/harmonic/`

**Arnold's diffusion**

My mother field is Hamiltonian dynamics. One of the central question in the theory of Hamiltonian systems is the *Arnold diffusion conjecture*, which has recently been partially solved by John Mather. There is a variety of possibilities for a Ph.D. project in this area.

Namely, in 1964 V. I. Arnold published a short note describing an example of a simple classical mechanical system whose phase space trajectories could wander "arbitrarily far" from their point of departure. This gave rise to a conjecture, now known as Arnold diffusion, stating that such a behaviour must be generic in the whole of Hamiltonian mechanics, namely that Hamiltonian systems with three or more degrees of freedom are generically unstable. I.e. on a "typical" energy surface of dimension 5 and higher, any two open sets shall be connected by a trajectory. The existing mathematical approach to the conjecture combines methods of geometric singular perturbation theory, analysis in the large, Hamilton-Jacobi theory and variational techniques. The problem has numerous connections with approximation theory and harmonic analysis. It is clear that in order to achieve a more than $O(\varepsilon)$ progress toward the Arnold diffusion, new methods should be developed.

These new methods may base on understanding of the interplay of the variational and Hamilton-Jacobi approach to the problem. Variational methods enable one (at least in two dimensions) to deal with "cantori", which are fractals, forming a residual set in the phase space of a "typical" perturbed integrable Hamiltonian system. These objects are most likely to be responsible for Arnold diffusion in a system of general position (despite their set has Lebesgue measure zero, its Hausdorff dimension supposedly equals the dimension of the phase space (this is hardly related to the Kakeya conjecture mentioned earlier). Little is understood about the higher-dimensional relatives of the cantori. It would be highly desirable – and very challenging – to develop analytic techniques for dealing with these sets, formally represented by almost everywhere divergent series, at least in dimension two, even within a class of specific examples.

## 11.14 Martin Sieber

In short-wave asymptotics one studies approximate solutions to wave equations that are valid in the limit of small wave lengths. One example is the semiclassical approximation in quantum mechanics. It is an approximation that connects wave properties to the trajectories of Newtonian mechanics. This is analogous to the geometrical optics approximation for electro-magnetic waves, that expresses wave properties in terms of rays.

Semiclassical approximations are one of the major tools in the field of *quantum chaos*. They provide a link between quantum mechanics and classical mechanics, and thus they allow to study how these theories are connected. One of the major questions is: What can one say about a quantum system, if one knows characteristic properties of a classical system, for example that it is chaotic? Currently available research projects are as follows.

### Diffraction in quantum systems

Usual semiclassical approximations do not describe wave phenomena like the diffraction of wave functions on objects that are not smooth on the order of a wave length. These effects require the inclusion of the next-order correction to the leading order approximation for short wave lengths. The corresponding theory is the geometrical theory of diffraction (or in an extended form the uniform theory of diffraction). Although diffraction is a second-order effect, it can play an important rôle in short-wave theories. For example, it can influence the statistical distribution of energy levels (see below). For this reason, there has been recently an increased interest in diffraction effects in quantum systems. One project involves the study of the consequences of adding one and more point-like objects to a quantum system.

### Statistical distributions of energy levels

One of the central topics in quantum chaos is the random matrix hypothesis. It makes a prediction about statistical distributions of energy levels (which are the eigenvalues of the Schrödinger wave equation) in quantum systems whose corresponding classical system is chaotic. It states that these statistical distributions coincide with those of eigenvalues of large random matrices. There is a strong numerical support for this hypothesis, but analytically it has been shown only in certain limiting regimes. Recently there have been new theoretical developments that point to a progress in the semiclassical treatment of statistical distributions. This would lead to a large number of possible applications. In the proposed project these theoretical developments should be pursued in detail.

There are close connections to research interests of Prof. J. Keating, Dr J. Marklof, and Dr J. M. Robbins.

### 11.15 Nina Snaith

My research interests are in quantum chaos and random matrix theory, but in particular in the application of random matrix theory to number theory. Random matrix theory was developed in the context of nuclear physics to predict the statistics of the eigenenergies of complicated nuclear systems. However, in the 1970s, it was shown that the Riemann zeta function (a function much studied by number theorists and subject of the famous Riemann Hypothesis) has zeros that show the same statistics as these nuclear energy levels. So, random matrix theory has become a powerful tool to study number theoretical functions like the Riemann zeta function and other L-functions.

Current projects in this area involve applying random matrix theory to mean values of the Riemann zeta function and other L-functions, to the order of zeros of L-functions at the critical point (in connection with the conjecture of Birch and Swinnerton-Dyer) and in the distribution of prime numbers.

Projects in this area can involve both numerical and analytical work and students will learn techniques that are of use both in mathematics and physics.

## 11.16 Yves Tourigny

### Spectral properties of non-Hermitian Jacobi operators

A Jacobi operator $\mathcal{H}$ acts on sequences, say $\psi$, of general term $\psi_n$ and, for each $\psi$, returns a sequence $\mathcal{H}\psi$ of general term

$$(\mathcal{H}\psi)_n := a_n\psi_{n-1} + b_n\psi_n + c_n\psi_{n+1}$$

where the $a_n$, $b_n$ and $c_n$ are given (fixed) complex numbers. There is a link between this operator (and some of its generalisations) and various methods for summing power series; the project will exploit this link to obtain new results on the asymptotic performance of these series summation methods. The case where the operator is Hermitian, i.e. $a_n$ is the complex-conjugate of $c_{n-1}$ is reasonably well-understood; it corresponds to the case where the Padé method is used to sum series that are the moment generating series of certain measures supported on the real line. The non-Hermitian case brings in many new fascinating aspects. Their study will be approached from two directions; one will make a careful analysis of some special choices of the coefficients $a_n$, $b_n$ and $c_n$; the other will consider the "opposite case" where the coefficients are completely random.

The project is suitable for someone with an interest in computing, mathematical physics and analysis.

### Applications of series summation methods to problems of fluid dynamics

Many problems of fluid dynamics involving a "small parameter" can be solved formally in terms of a series in powers of the parameter, and the terms in the infinite sum can be computed successively by use of a recurrence relation. Some examples where this approach could yield new and/or interesting results are (a) the solution of the Birkhoff-Rott equation modelling the Kelvin-Helmhotz instability of a vortex sheet between two fluids and (b) the solution of simplified forms of the Navier-Stokes equations in particular domains with a high level of symmetry. The project will consider the problem of making optimal use of the first few terms of the series computed in such examples; the concepts of *algebraic approximant* and its multidimensional version will play a central part.

The project should appeal to someone with an interest in computing, programming and fluid dynamics.

## 11.17 Holger Waalkens

I am interested in various fields of dynamical system theory and semiclassical quantum mechanics and their application to molecular and atomic systems. Some possible Ph.D. projects with me are listed below. Part of them involve collaborations with Prof. Stephen Wiggins. I am also happy to discuss suggestions from Ph.D. applicants for other projects.

### Transition State Theory

Recent developments in dynamical system theory allow to describe various types of "reaction-dynamics" like isomerization or dissociation of molecules and ionization of atoms in external fields in a coherent picture. The basic ingredients for this geometric theory referred to as *transition state theory* are the stable and unstable manifolds of a so-called *normally hyperbolic invariant manifold* (NHIM). The NHIM can be proven to exist near a certain type of saddle equilibrium point which is characteristic for reaction-type systems. The manifolds can be computed from a normal form approach. A possible PhD projects could be the application of this theory to a particular system. It is desirable to extend this at first purely classical theory to include quantum effects like tunnelling. This can be done in a semiclassical setting which could be the subject of another PhD project. Furthermore it is desirable to extend the theory in such a way that a weak coupling to a so-called heat bath with infinitely many degrees of freedom can be described. This would allow to describe reactions of molecules like isomerization in solvents and could be the subject of another PhD project.

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### Quantum Monodromy

For an integrable system there exist as many constants of the motion as degrees of freedom. The map from phase space to the constants of the motion is the *energy momentum map*. If the pre-image of a regular value of the energy momentum map is compact it follows from the Liouville-Arnold theorem that the pre-image is a torus and there exist action angle variables. In the context of integrable systems monodromy refers to obstructions to the *global* existence of action angle variables in a connected component or *phase* of regular values of the energy momentum map. If the connected component is not simply connected the actions are in general not globally defined. Classical monodromy carries over to quantum mechanics via the EBK quantization of tori where it leads to a topological defect in the lattice of the joint eigenvalue spectrum know as *quantum monodromy*. A famous example is the (quantum) spherical pendulum. In a PhD projects more general potentials (than the gravitational potential) on the sphere are to be discussed. This leads to new variants of (quantum) monodromy which have applications to rotational-vibrational spectra of triatomic molecules.

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**Instantaneous Frequencies from Wavelet Transforms**

For an integrable system trajectories are generically confined to invariant Liouville tori. The motion on a torus is characterized by $n$ frequencies where n is the number of degrees of freedom. The Kolmogorov-Arnold-Moser (KAM) theorem states that if an integrable system is slightly perturbed most of the tori survive the perturbation. For a system with 3 or more degrees of freedom a trajectory with an initial condition which is not on an invariant torus may wander through the mesh of invariant tori. This is known as *Arnold diffusion* (not to be confused with the notion of the *Arnold mechanism* which describes a very special mechanism leading to *Arnold diffusion*). J. Laskar introduced the numerical procedure of *frequency map analysis* to describe Arnold diffusion in terms of time varying frequencies. This procedure only applies to near-integrable systems. Within a PhD project Laskar's approach is to be modified using wavelets which will allow one to define instantaneous frequencies for motions further away from the integrable limit. The method is to be applied to questions of IVR (intra-molecular vibrational energy redistribution) in molecular systems.

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## 11.18 Steve Wiggins

I am always interested in talking with potential postgraduate students whose ambition is to use mathematics and computation to pioneer a new area of knowledge where mathematical thinking holds the promise of prediction and understanding. Consequently, I do not hand out what I would necessarily call a "project". Rather, I work with students to develop an understanding of the important issues in a given area and how they can be developed and understood through mathematical analysis and/or computation. The Ph.D. research must contain a substantial new mathematical and/or computational result that is of recognized value, but I also expect the student to develop a clear vision for their own future research program.

Several areas of current interest to me are described below.

**Lagrangian transport and mixing in geophysical flows: a dynamical systems approach**

Over the past 10 years the analogy between the global, geometrical study of nonlinear dynamical systems and Lagrangian transport and mixing studies in fluid mechanics has been used to obtain a deeper understanding of Lagrangian transport issues in a variety of fluid flows. However, the vast majority of this work has been in the context of two-dimensional, time-periodic fluid flows. This is due to the fact that through time periodicity the study of the equations for fluid particle trajectories can be reduced to the study of a two-dimensional area-preserving Poincaré map, and once the problem has been cast in this setting a variety of well-known techniques and ideas from dynamical systems theory can be applied for the purpose of studying fluid transport and mixing issues. For example, KAM tori represent barriers to fluid transport and mixing, chaotic dynamics should act to enhance mixing, and invariant manifolds, such as the stable and unstable manifolds of hyperbolic periodic points, are manifested as "organized structures" in the fluid flow.

There is a remarkable similarity between the mathematical framework of dynamical systems theory and the experimental and observational framework of modern oceanography. On the one hand, quasi-Lagrangian current following floats and drifters, as well as remote sensing data, show numerous localized, coherent motions ranging from major currents like the Gulf Stream to mesoscale phenomena such as rings, and associated vortex structures, down to a variety of submesoscale vortical motions. On the other hand, the theoretical tools of dynamical systems address the role that localized structures play in governing the motion over extended regions of space. Furthermore, application of dynamical systems theory relies on the existence of certain geometrical structures in the flow, not on a specific analytical form for the velocity field. Consequently, it is an ideal tool for "mining" the large data sets that result from remote sensing or large scale, high-resolution numerical simulations.

In the past five years there have been significant advances in dynamical systems theory to the point where the framework can now be utilized in the context of "real" problems. However, this approach leads to a new idea of a "dynamical system" in this context. Rather than being defined by a set of equations, our dynamical systems are defined by "data sets", which can only be known for a finite amount of time. This brings up some fundamentally new mathematical problems that must be addressed in order to apply notions such as hyperbolic trajectories, stable and unstable manifolds, and lobe dynamics since standard dynamical systems theory develops these notions as "infinite time phenomena".

The proposed Ph.D. projects will be involved with continuing the development of the theoretical framework for a geometrical theory of dynamical systems that are defined for a finite time interval. At the same time, computational algorithms will be developed and implemented in software. Along with the theory we will also be interested in pursuing "real world" applications of these tools such as for velocity fields in coastal zones derived from high frequency radar arrays and other velocity fields derived from large scale computation and remote sensing. A laboratory component to the program will provide yet another source of velocity fields derived from data. As oceanography enters a "data rich" era with the availability of barely manageable amounts of data from satellites and radar arrays I feel that the framework of dynamical systems theory is now poised to lead to numerous breakthroughs in the analysis and interpretation of this data.

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**Specific Projects: Below I list several possible research topics.**

- Development of three-dimensional lobe dynamics.

- A study of the coupling of unresolved small scale dynamics to large scale "coherent structures" through dynamical systems ideas.
- Development of a dynamical systems theory for dynamical systems defined as "data sets". In particular, developing a finite time understanding of infinite time phenomena.
- A rigorous treatment of transport barriers: KAM and Nekhoroshev theory for aperiodically time-dependent systems.
- Issues associated with no-slip boundaries: distinguished non-hyperbolic saddle like trajectories and their stable and unstable manifolds invariant manifolds.
- Application of dynamical ideas to the study of transport in coastal zones using velocity fields obtained from high frequency radar arrays.
- Application of dynamical ideas to the study of transport using general circulation models.

**Global Geometry and Dynamics in Hamiltonian Systems with Three or More Degrees-of-Freedom**

It has long been known that resonances play an important role in phase space transport in systems with three or more degrees of freedom. One of the main reasons for this is that in this case resonances are not isolated by energy conservation. A consequence of this is that resonances form a dense "web" throughout the phase space. Important unanswered questions are: what is the mechanism(s) causing trajectories to remain near a resonance, what causes them to "drift" along a given resonance, and what causes them to transfer to another resonance? There are virtually no theoretical results that provide answers to these questions (although, lately, certain building blocks which will probably play an important role in answering these questions have begun to be developed). Therefore numerical methods, and clever numerical experiments, are absolutely crucial for the development of intuition and understanding of these issues, the importance of which, for phase space transport in molecules, has long been recognized in the chemistry community, see, e.g., Martens et al. [1987].

We outline some particular issues. Consider a three-degree-of-freedom system. The phase space is six dimensional and the energy surface is five dimensional. Some numerical experiments (which could be considered somewhat crude, and not altogether conclusive) indicate that some trajectories tend to be confined near multiplicity one resonant surfaces, and drift slowly along them. At the intersection of two multiplicity one resonant surfaces the motion becomes more complex. The trajectory may drift through the intersection and remain on the same resonant surface, or it may make a transition to the other multiplicity one resonant surface. How and why such motions occur is not at all understood, but high dimensional invariant manifolds (of various dimensions) lie at the heart of all of these questions.

In particular, a multiplicity one resonant surface under certain conditions can be shown to correspond to a codimension two normally hyperbolic invariant manifold having codimension one stable and unstable manifolds (Wiggins [1992]). These stable and unstable manifolds act as higher dimensional separatrices which tend to confine trajectories near the multiplicity one resonance surface. Within these codimension one stable and unstable manifolds are codimension two stable and unstable manifolds of lower dimensional “whiskered tori”. These can possibly be used to construct “transition chains” which lead to drift along the multiplicity one resonance surface. How this picture breaks down at the intersection of two multiplicity one resonances is not at all clear, and is a central question for the understanding of global phase space transport.

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**Specific Projects: Below I list several possible research areas.**

- A study of the “Arnold diffusion problem” in a generic three or more degree-of-freedom Hamiltonian system.
- A study of issues associated with normally hyperbolic invariant manifolds and their foliations in analytic Hamiltonian systems.
- Introduce ideas of symplectic topology into classical Hamiltonian perturbation theory.

### The Geometry and Dynamics of Chemical Reactions

Over the past 15 years advances in lasers and molecular beams have led to the development of experimental techniques where real time dynamical data related to molecular interactions and dynamics can be obtained, which has resulted in the area of research known as “femtochemistry”. In some ways the experimental situation in chemistry today bears a strong resemblance to the experimental situation in fluid mechanics in the late 70’s. At that time new laser based experimental techniques were developed that resulted in global, real time dynamical data from fluid flows. Such data played a key role in motivating the dynamical systems analyses of fluid flows. In fact, one could make the case that these experimental advances were the reason that fluid mechanics became (probably) the main application area for dynamical systems theory throughout the 80’s. Today we are in an analogous situation for theoretical chemistry vis- a‘-vis the relation of experimental advances in fluid mechanics and dynamical systems theory in the late 70’s. Consequently, a focused, interdisciplinary research effort in dynamical systems and theoretical chemistry could play an important role in the interpretation of this new experimental data, as well as the further development of the field of theoretical chemistry. From the point of view of Hamiltonian mechanics, problems from theoretical chemistry could be the inspiration for many new results in much the same way that celestial mechanics played this role in the last two centuries.

Broadly speaking, our research will be concerned with the nonlinear dynamical foundations of reaction dynamics. The fact that this is still an area ripe for fundamental research is clearly brought out in the 1998 Faraday Discussion of chemical reaction theory (Faraday Discuss., 1998, 110). In the summarizing remarks of Professor Donald Truhlar (Faraday

Discuss., 1998, 110, 521–535) it was pointed out that our understanding of dynamical bottlenecks and reaction coordinates is still very much incomplete, with a central theme of the meeting being how to find them in complex systems.

The mathematical approach and techniques of dynamical systems theory are ideally suited for these questions in reaction dynamics since they are intrinsically global and geometrical in nature. For example, questions related to the evolution of molecules from reactants to products, and questions related to intramolecular and intermolecular energy transfer are formulated as phase space transport problems whose answers depend on the geometry and dynamics associated with invariant manifolds in phase space. We want to emphasize that the phase space setting (the arena of dynamical systems theory) is central to out research and differs significantly from the configuration space setting that is more prevalent (and possibly more intuitive) in the chemistry community. Indeed, some of our recent work has shown that certain of the concepts and ideas can only be realized rigorously in phase space (Wiggins et al. [2001]).

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**Specific Projects: Below I list several research areas.**

- A study of phase space structure in the rovibrational Hamiltonian, with applications to a specific molecule, such as formaldehyde.
- A study of the notion of "reaction path" and "reaction path Hamiltonian" from the point of view of normal form theory.
- A study of the geometry of energy partitioning in systems that are far from integrable.
- A study of the mechanism(s), and implications, of "stable chaos" in molecular dynamics.
- A study of transition state theory with fluctuating barriers.
- A study of the phase space structure associated with dissociation dynamics near surfaces.

In addition, there are numerous other projects in this general area that I would be happy to discuss with a potential postgraduate student.

### Time-Frequency Analysis of the Dynamics of Molecules

The concept of "resonance" is a fundamental paradigm in dynamics. It arises in a multitude of applications, e.g., structural mechanics, celestial mechanics, molecular dynamics, fluid mechanics. Regardless of the specifics of the physical setting, resonance is always an important dynamical phenomenon. Very generally speaking, it tends to result in some type of "cooperative" or "directed" behavior between different dynamical degrees-of-freedom. For example, in the study of the dynamics of molecules resonances play an important role in energy transfer amongst different vibrational modes and, consequently, provides a context for theoretical chemists to interpret classical trajectory calculations.

Much of the intuition for the understanding and description of resonance phenomena comes from a narrow mathematical background. In particular, much of our intuition about resonances and their importance comes from studies of low dimensional dynamical systems (e.g., forced one degree-of-freedom oscillators) and perturbations of integrable Hamiltonian systems, where a special system of coordinates (action-angle variables) play a central role. This is not a sufficiently broad and general mathematical framework for the wealth of new applications in today's age of rapid scientific and technological development.

This project is concerned with developing new methods to study the dynamics associated with resonances in Hamiltonian systems with many degrees-of-freedom that are far from integrable, and are independent of the particular coordinate system in which the dynamical system is expressed.

There has been some previous work along these lines. The frequency map analysis method has been used as a numerical tool to study the global dynamics of systems of three degrees of freedom and more. By the computation of the fundamental frequencies associated to trajectories, this method allows one to determine the existence of quasiperiodic and chaotic zones in the phase space of a given dynamical system. The method was introduced by Laskar [1993] to study the chaotic dynamics of the Solar System, and since then it has been widely used in celestial mechanics and galactic dynamics. Also, frequency analysis has been applied to Hamiltonian systems arising from the classical study of polyatomic molecules, see Losada et al. [1997], Martens et al. [1987], von Milczewski et al. [1997].

Laskar's method gives good accuracy for the case of quasiperiodic trajectories, in which the frequencies remain constant in time. However, when the trajectory is chaotic, we expect that the values of the fundamental frequencies will vary greatly with respect to time. Still, the same approach has been used to compute the frequency of chaotic signals in short intervals of time assuming that the trajectory is close to quasiperiodic in that interval. The only theoretical justification of the Fourier based frequency map analysis method is in the setting of near integrable Hamiltonian systems expressed in action-angle coordinates (Laskar [1999]).

The insights given by this method in the limited situations where it can be justified indicate that there is a need for a similar method that can be applied to much more general systems, and that is what we will develop and apply to systems that are strongly nonlinear and far from integrable. We will provide a theoretical justification for them that is independent of the particular coordinates in which they are expressed.

The overall aim is to develop a methodology, based on both theory and computation, for understanding the role of resonances in strongly nonlinear, far from integrable, high dimensional dynamical systems. Specific aims and objectives are as follows.

1. Develop the ability to determine whether a generic trajectory at a given level of excitation is chaotic.

2. Develop a quantification of the nature of regions of quasiperiodicity, and determine how they influence long-time trappings of trajectories.

3. Develop an understanding of how resonance junctions play a key role in these trappings.

4. Determine if there are generic structures that organize the transport in phase space, like the "Arnold Web", in near integrable systems.

5. Determine the speed of evolution along these structures.

The project will also involve heavy use of the Laboratory for Advanced Computation in the Mathematical Sciences at the University of Bristol (`http://lacms.maths.bris.ac.uk`). The laboratory provides us with a unique capability for carrying out complex simulations. The laboratory is home to a 160 node Beowulf type system. Each processor has a speed of 1 GHz, with 1 Gb of memory. The entire cluster has a storage capacity of about one terabyte.

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### Chaotic Stirring and Mixing in Microfluidic Devices

Microfluidics is the recently coined term to describe the study of fluid flows in devices having dimensions ranging from millimeters to micrometers. The volumes of fluid involved range from nanoliters ($10^{-a}$ liters) to picoliters ($10^{-12}$ liters). The applications of such fluid flows have literally exploded on the past few years. Some examples are blood cell separation equipment, biochemical assays, chemical synthesis, genetic analysis, drug screening, electrochromatography, surface micromachining, laser ablation, and mechanical micromilling.

Microfluidics is the core technology for microbiological analysis systems, or the so-called "lab-on-a-chip", or micrototal analysis systems (m TAS) (Polson and Hayes [2001], Beebe et al. [2000]). The demand for biomedical microdevices is increasing at a very rapid rate. The market for such devices was $400,000,000 in the year 2000, and it is expected to increase by a factor of five by the year 2005 (Stone and Kim [2001]).

Because of the length scales associated with the flow of fluids in microchannels the flow is characterized by a low Reynolds number (Re), and is therefore laminar (smooth in space, as opposed to turbulent). Hence, there is a need to understand transport, stirring, and mixing, in low Reynolds number, laminar flows. From the point of view of fluid mechanics, many aspects of this subject are well-developed and quite mature. However, the problems posed by applications at the microscale introduce some new difficulties. The channel lengths and flow speeds in "micromixers" are not sufficient to allow for the mixing effects of molecular diffusion. In this case, it is critical to understand the details of particle paths in the fluid flow over relatively short time scales (compared to the molecular diffusive time scale). This is precisely the subject area of what has been broadly termed "chaotic advection" (Ottino [1989], Wiggins [1992]), and it is significant that in recent review articles on microfluidics (Whitesides and Stroock [2001], Stone and Kim [2001]) it has been asserted that chaotic advection should play a crucial role in future designs of "micromixers" for these very reasons.

This project is concerned with developing an understanding of chaotic advection and stirring on the microscale so as to provide concrete design principles for the development of complex microfluidic devices and the control of fluid flows on microscales.

Several different types of micromixers have been produced by microfabrication techniques. These micromixers fall into three categories: nozzle arrays, static mixers, and active mixers.

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**Generation of Chaotic Mixing in Micromixers by Electro-Osmosis**

An electric field can be used to drive a fluid through micromixer using the electro-osmotic effect (EOF). Roughly speaking, this is carried out by applying a layer of charge to the surfaces of a flow channel. This results in a layer of oppositely charged ions near the surfaces. When an electric field is applied the charged layer moves, thus "dragging" the neutrally charged fluid along with it. An important area of current interest if concerned with the understanding of how to apply specific patterns of charges on the surfaces of the flow channel specific forces and torques so as to precisely control the motion of the fluid. In this project we will study these issues, as well as examine more complicated applications of charges, and applied electric fields, for the purpose of producing chaotic advection within the flow channel.

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**The Dynamics of Atomic Force Microscopes**

One of the most revolutionary devices developed in the last century was the atomic force microscope (AFM) of Binnig, Quate, and Gerber [1986]. This is the device that provides images of atoms and molecules. Roughly speaking, an AFM works by scanning a fine ceramic or semiconductor tip over a surface in much the same way a phonograph needle (in the days before CDs) scanned the surface of a record. The tip is positioned at the end of a cantilever beam and is repelled or attracted to the surface. The magnitude of deflection of the cantilever provides knowledge about the topography of the surface. There are many different "operating modes" for an AFM. For example, these may involve the tip, cantilever base, or sample being subjected to some form of excitation. This project will involve theoretical and computational studies for different modes of operations for AFMs.

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**Dynamical Properties of Carbon Nanotubes**

Carbon nanotubes were discovered in 1991 by Sumio Iijima of NEC Corporation.

A carbon nanotube is a seamless, cylinder with a very large aspect ratio. A nanotube is described by its diameter and the chirality of the atomic rows of carbon atoms relative to the cylinder axis. The lengths are on the micrometer to millimeter scale, while the diameters are typically much smaller (around a nanometer). Numerous important technological applications have been proposed for carbon nanotubes. For example, they could be key components in new generations of transistors and diodes, new generations of batteries, or as materials strengtheners. Theses are described in the recent Scientific American article by Steve Mirsky available at `http://www.pa.msu.edu/cmp/csc/nanotube.html`

According to Sohlberg et al. [1998], “simulations of the behavior of nanocomponents have revealed one common theme, the rigidity of a component is the key to its proper performance”. This project is concerned with investigating the mechanical properties of carbon nanotubes from the point of view of continuum mechanics, which will build on the ideas of Sohlberg et al. [1998]. As a preliminary step, the student must learn to ”build” carbon nanotubes on the computer using molecular dynamics (see, e.g., Hinsen [2000]. We also will examine in more detail the role played by nonlinear resonances in the stability of carbon nanotubes, as well as the role played by deterministic chaos in the organization of carbon atoms into nanotubes, first noted in Sumpter and Noid [1995].

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**Transport in Ratchets and Molecular Motors**

The study of ratchets has blossomed in the past few years, being motivated by the desire to understand the way in which molecular motors in biology function. The Scientific American article by Alan Hall provides an excellent introduction to molecular motors. It can be obtained at `http://www.sciam.com/exhibit/1999/092099molecularmotor/index.html`

While the equations of motion of a ratchet are relatively simple to write down (See the references below), the role that different mechanisms (e.g., dissipation, deterministic forcing, noise, parameter variations) in producing “directed motion” is only now starting to be understood. This issues surrounding local and global bifurcations are also intriguing. This project will explore these mechanisms and issues.

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**Collective Coordinates, Invariant Manifolds, and Synchronization**

Recent work has shown some intriguing relations between collective coordinates, invariant manifolds, and synchronization in dynamical systems. This project is concerned with studying these relations with an eye towards applications in certain molecular dynamics and protein folding problems.

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## 11.19 Andreas Winter

The following PhD projects are exemplary but not exhaustive for what I could supervise. I am an information theorist with a background in probability theory and quantum mechanics and an interest in combinatorics; these are the approximate boundaries of my expertise.

### Information theory

Maurer and Ahlswede/Csiszar have shown how to treat cryptographic key distillation in an information theoretically optimal way, from statistical assumptions, in some restricted models. Many questions remain: for example, what is the ultimate key yield? Or, following Wolf and Gisin: is there "bound information" in the sense of genuine correlation which nevertheless does not allow extraction of secret key? In recent work with Japanese colleagues I found rates for other cryptographic tasks, bit commitment and coin tossing, again from statistical assumptions. These are optimal in the case of bit commitment. For coin tossing this is still open, and for the important primitive of oblivious transfer there does not even exist a lower bound.

### Quantum information theory

In a joint work with Devetak, I characterized the amount of secret key obtainable from a quantum state, in a restricted model. Like in the classical model, the ultimate key rate is still open. Also, there is a connection to entanglement distillation, which allows in principle to characterize the optimal distillation rates but the formulas are "non-computable" as they involve infinite dimensional optimizations. A better understanding, or at least easily computed bounds would be extremely interesting. Noah Linden offers a project on quantum entanglement which focuses on multi-party systems. Also these pose optimal rate problems about which almost nothing is known, but which could be tractable by the recently developed methods. In general however, it is felt that new ideas are needed.

### Coding theory

Even though I am not an expert, I would be able to supervise an able student. A question about "small" codes which I find particularly intriguing is the so-called Hadamard conjecture: it says that there exists an n-by-n orthogonal matrix with all entries +1 or -1 if and only if n=2 or n is divisible by 4. This boils down to constructing such 1 orthogonal matrices (Hadamard matrices) for all orders 4k. The problem is equivalent to constructing binary codes of length 4k on 4k bits with Hamming distance 2k.