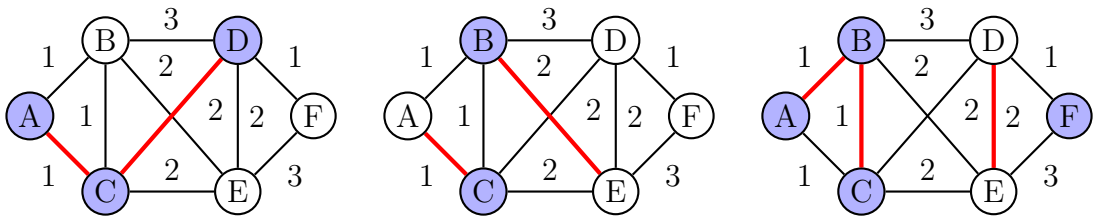
COMS21103: Problems set 8

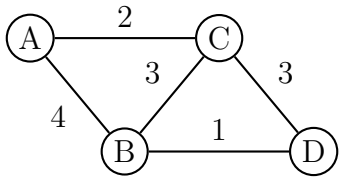
Disjoint sets and minimum spanning trees

1. Assume we have a universe U which consists of the integers between 1 and 8, and we want to store subsets of U in a disjoint-set structure implemented using an array of linked lists with the weighted-union heuristic. Imagine MakeSet(i) has been called for each integer i between 1 and 8 to create 8 disjoint sets containing 1 element each. Describe a sequence of 7 Union operations which makes the largest possible total number of updates to the data of the elements in the data structure.

2. For each following weighted graph G, and subset X of a minimum spanning tree (shown by thick lines), determine whether each subset S (shown by coloured-in vertices) satisfies the cut property.



3. Run Kruskal's and Prim's algorithms on the following graph.



4. Prove the claim made in lecture that Prim's algorithm outputs a tree.

5. Does the minimum spanning tree problem make sense if we allow the input graph to have negative-weight edges? Why or why not?

6. Prove the following claim made in lecture during the discussion of the cut property. Let T be a spanning tree of an undirected graph G, and e be an edge in G. Let p be a path in T between the two endpoints of e. Show that, for any edge e' on the path p, if we replace e' with e in T, the resulting set T' is still a spanning tree.

7. Prove that a spanning tree on a graph with n vertices contains exactly $n - 1$ edges.

8. What happens if we run Kruskal's and Prim's algorithms on a graph which is not connected?