**School of Mathematics** **Spring 2017**

# QUANTUM COMPUTATION

## Exercise sheet 6

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1. **Shor's 9 qubit code.** Imagine we encode the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ using Shor's 9 qubit code, and then an $X$ error occurs on the 8th qubit of the encoded state $|E(\psi)\rangle$.

   (a) Write down the state following the error.

   (b) We now decode the encoded state, starting by applying the bit-flip code decoding algorithm. What are the syndromes returned by the measurements in the algorithm?

   (c) Now imagine that $|E(\psi)\rangle$ is affected by two $X$ errors, on the 7th and 8th qubits. What are the syndromes returned this time? What state does the decoding algorithm output?

   (d) Which patterns of $X$ errors are corrected by Shor's 9 qubit code?

2. **Stabilizers.**

   (a) Show that $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ is stabilized by $\{-X \otimes X, -Z \otimes Z\}$.

   (b) Show that $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ is a stabilizer state and write down its stabilizer.

   (c) List all the stabilizer states of one qubit.

   (d) Prove the claim in the lecture notes that every pair of Pauli matrices on $n$ qubits, i.e. matrices of the form

   $$M = M_1 \otimes M_2 \otimes \cdots \otimes M_n,$$

   where for each $i$, $M_i \in \{I, X, Y, Z\}$, either commutes or anticommutes.