Math 595: First half-semester minicourse, Fall 2009

Gromov hyperbolic groups and their boundaries

- **Instructor:** John Mackay
- **Contact:** `jmmackay@illinois.edu`, `http://www.math.uiuc.edu/~jmmackay/`
- **Dates:** Meets August 24 – October 16, 2009.
- **Time:** MWF 12:00 PM - 12:50 PM (Let me know if this time causes a problem.)
- **Location:** Altgeld 443

Description

Geodesic triangles in the hyperbolic plane have the property that every point on a given side is at most distance two from one of the other sides, regardless of how big the triangle is. A Gromov hyperbolic metric space is a metric space with this *thin triangles* property. Amazingly, this simple definition leads to deep mathematics, particularly in relation to group theory. Gromov hyperbolic groups have good algebraic properties (for example, solvable word problem) and are "generic": if you pick a group at random, it will be Gromov hyperbolic.

Since their introduction in the 1980s, Gromov hyperbolic groups have become ubiquitous in mathematics, studied for their own interest and also as a rich source of examples in both algebra and analysis on metric spaces.

The first part of this course will look at Gromov hyperbolic spaces and groups from the inside, working out the basic properties and examples. In the second part of the course, we will look at the asymptotic behavior of Gromov hyperbolic groups and spaces.

Just as hyperbolic space $\mathbb{H}^{n+1}$ has a sphere at infinity S^n which has a conformal structure, a Gromov hyperbolic space or group X has a boundary at infinity $\partial_\infty X$ which carries a canonical topological (in fact "conformal") structure. However, unlike the usual sphere, this boundary will often have fractal like properties. We will explore some of the connections between the geometry of the boundary and the algebraic properties of the group.

Prerequisites

Familiarity with the triangle inequality. Basic graduate classes in topology and real analysis would be helpful.

References

There is no required textbook. Good sources include:

- Bridson, Martin R. and Haefliger, André, *Metric spaces of non-positive curvature*, Springer-Verlag, 1999.
- Ghys, É. and de la Harpe, P. (editors), *Sur les groupes hyperboliques d'après Mikhael Gromov*, Birkhäuser Boston Inc., 1990.
- Gromov, M., *Hyperbolic groups*, pp.75–263, *Essays in group theory*, Springer, 1987.