Introduction to Statistics

Problem Sheet 2

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1. The number of vehicles which pass a given point on a motorway over a 5-minute period is a Poisson random variable with mean 75 under normal conditions; if an accident has occurred a short distance ahead, then it is Poisson with mean 15.

   Develop an optimal test of the null hypothesis of normal conditions against the alternative of an accident ahead, with a false alarm rate of no more than 10^{-3}. You may use the following facts: if X_λ denotes a random variable with a Poisson(λ) distribution, then, $\mathbb{P}(X_{75} \leq 48) \leq 10^{-3}$ and $\mathbb{P}(X_{15} \geq 30) \leq 10^{-3}$.

2. The number of requests received by a certain website over a 1-minute period has a Gaussian distribution with mean 100 and variance 100 under normal conditions. When it is subjected to a botnet attack, the mean goes up to 500 but the variance remains unchanged. The numbers of requests received over disjoint time intervals are mutually independent.

   (a) Specify the distribution, mean and variance of the number of requests received over a 5-minute period under normal conditions, and also when the website is subjected to a botnet attack.

   (b) Develop an optimal test with a false alarm probability of 10^{-4} for detecting a botnet attack within 5 minutes of its inception. You may use the fact that, if $Z \sim N(0,1)$, i.e., Z is a standard normal random variable, then $\mathbb{P}(Z > 2.75) \approx 10^{-4}$.

3. A tyre company claims its tyres have a mean useful lifetime of 42,000 miles. A consumer association bought one of the company's tyres from each of 10 randomly chosen outlets and tested them on a test rig that simulated normal road conditions. The observed lifetimes (in thousands of miles) were

   $$42 \quad 36 \quad 46 \quad 43 \quad 41 \quad 35 \quad 43 \quad 45 \quad 40 \quad 39.$$

   Thinking carefully about the context, what is an appropriate alternative hypothesis H_1 to use in testing the manufacturer's claim?

   Use the data to test whether or not there is sufficient evidence to reject the manufacturer's claim, using a test procedure with significance level $\alpha = 0.05$.

4. A certain manufacturer produces packets of biscuits with a nominal weight of $200g$. You may assume that it is known from experience that the standard deviation of the weight of the packets is $4g$. To carry out a control check on the actual weight of the packets produced, an employee weighs 25 packets selected at random from a day's production and finds that the average weight of the sample is $\overline{x} = 202.275g$.

   Let μ denote the actual mean weight of $200g$ packets produced by the manufacturer. Test the null hypothesis $H_0 : \mu = 200$ against the alternative $H_1 : \mu \neq 200$, using a test procedure with significance level $\alpha = 0.01$.