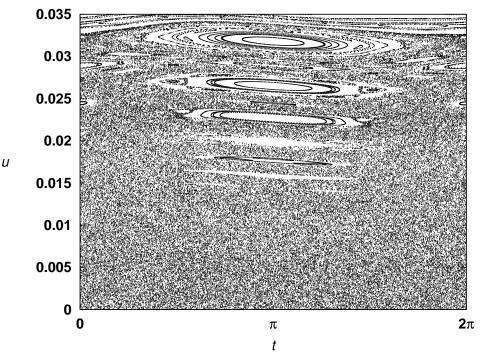
## Applied Dynamical Systems Problem Sheet 5

- 1. [2013 exam] The Fermi-Ulam model consists of a particle colliding alternately with a fixed wall at x = 0 and an oscillating wall at  $x = L - a \cos t$ , where a is a small positive parameter. Both walls are assumed to have infinite mass.
  - (a) If the time of the *n*th collision with the moving wall is denoted by  $t_n$  and the velocity immediately before that collision has magnitude  $v_n$ , the dynamics may be approximated by

$$u_{n+1} = |u_n + \epsilon \sin t_n|$$
  
 $t_{n+1} = t_n + u_{n+1}^{-1}$ 

State assumptions that may be used to derive these equations, and give expressions for  $u_n$  and  $\epsilon$  in terms of the previously defined quantities.

- (b) Show that the above map is area preserving, and analyse the stability of fixed points where  $t_n$  (considered modulo  $2\pi$ ) is zero or  $\pi$ , and hence explain the location and number of the vertical sequence of elliptic islands.
- (c) Explain how the portrait shown in the figure would change as  $\epsilon$  is decreased toward zero.



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