

Statistical mechanics problem sheet 1

Unstarred problems follow quickly from definitions and equations in the lectures; single starred problems require more effort; double starred questions are designed as a challenge. The exam will contain questions similar to the first two categories (not those requiring internet access...). To obtain credit points, either get 40 on the exam; or 30 on the exam and submit reasonable attempts at half the homework assignments.

1. Classify the following systems as isolated, closed or open, and give a brief explanation of your answer:
 - (a) The solar system
 - (b) A tin of baked beans
 - (c) A balloon
2. Classify the following quantities as state variables or otherwise, and for the state variables, state whether they are intensive or extensive or neither.
 - (a) The mass density
 - (b) The average energy of a molecule.
 - (c) The maximum energy of a molecule.
 - (d) The logarithm of the number of molecules.
3. Give examples of
 - (a) A system in a steady state but not in thermodynamic equilibrium
 - (b) A system in thermal equilibrium but not in mechanical equilibrium
 - (c) A system in mechanical equilibrium but not in thermal equilibrium
 - (d) ** A system not in equilibrium that absorbs heat from a cold system and imparts heat to a hot system [Hint: the cold and hot systems may be of different types]
4. * Prove the following useful result for partial derivatives in a space with three variables and a single relation, say $f(x, y, z) = 0$:

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

5. An ideal gas moves quasistatically from a state (p_1, V_1) to a state (p_2, V_2) , with temperatures $T_1 = T_2$. Compute the work done on the system for
 - (a) An isobaric followed by an isochoric process.
 - (b) An isochoric followed by an isobaric process.
 - (c) An isothermal process.

Explain what bearing these results have on whether work is a state variable.

6. Which of these are exact differentials?

(a) $xdx + y^2dy$

(b) $ydx + x^2dy$

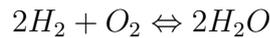
(c) $(y - z)dx + (x - z)dy + (y - x)dz$

7. State the second law of thermodynamics, and give an example of an irreversible process, and ** a perpetual motion machine of the second kind [hint: search the internet].

8. * The corona of the sun is at a few million degrees, outside and much hotter than the photosphere (visible surface). Does this violate the second law, and how can it be explained? [Hint: search the internet]

9. Compute the heat capacities at constant volume and at constant pressure for air at standard temperature and pressure, assuming an $f = 5$ ideal gas and a molecular mass of $28u$. Express your answer in units of $JK^{-1}kg^{-1}$ and compare it with $C_p/M = 4.184kJK^{-1}kg^{-1}$ for water.

10. For a system involving chemical reactions, such as



we can talk either about chemical potentials for conserved quantities (eg H and O atoms) or of the non-conserved molecules (H_2 etc.). Show that in the latter case, the chemical potentials satisfy a relation with the same coefficients as the chemical reaction, ie

$$2\mu_{H_2} + \mu_{O_2} = 2\mu_{H_2O}$$

11. * A monatomic gas which obeys van der Waal's equation has energy

$$E = \frac{3}{2}NkT - \frac{aN}{V}$$

where a is a constant. It undergoes adiabatic free expansion from a state (V_1, T_1) to a state (V_2, T_2) . Express T_2 in terms of the other quantities.