Statistical mechanics problem sheet 1

Unstarred problems follow quickly from definitions and equations in the lectures; single starred problems require more effort; double starred questions are designed as a challenge. The exam will contain questions similar to the first two categories (not those requiring internet access...). To obtain credit points, either get 40 on the exam; or 30 on the exam and submit reasonable attempts at half the homework assignments.

1. Classify the following systems as isolated, closed or open, and give a brief explanation of your answer:
   (a) The solar system
   (b) A tin of baked beans
   (c) A balloon

2. Classify the following quantities as state variables or otherwise, and for the state variables, state whether they are intensive or extensive or neither.
   (a) The mass density
   (b) The average energy of a molecule.
   (c) The maximum energy of a molecule.
   (d) The logarithm of the number of molecules.

3. Give examples of
   (a) A system in a steady state but not in thermodynamic equilibrium
   (b) A system in thermal equilibrium but not in mechanical equilibrium
   (c) A system in mechanical equilibrium but not in thermal equilibrium
   (d) $**$ A system not in equilibrium that absorbs heat from a cold system and imparts heat to a hot system [Hint: the cold and hot systems may be of different types]

4. $*$ Prove the following useful result for partial derivatives in a space with three variables and a single relation, say $f(x, y, z) = 0$:

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

5. An ideal gas moves quasistatically from a state (p_1, V_1) to a state (p_2, V_2), with temperatures $T_1 = T_2$. Compute the work done on the system for
   (a) An isobaric followed by an isochoric process.
   (b) An isochoric followed by an isobaric process.
   (c) An isothermal process.

   Explain what bearing these results have on whether work is a state variable.

6. Which of these are exact differentials?

   (a) $xdx + y^2dy$

   (b) $ydx + x^2dy$

   (c) $(y - z)dx + (x - z)dy + (y - x)dz$

7. State the second law of thermodynamics, and give an example of an irreversible process, and $**$ a perpetual motion machine of the second kind [hint: search the internet].

8. $*$ The corona of the sun is at a few million degrees, outside and much hotter than the photosphere (visible surface). Does this violate the second law, and how can it be explained? [Hint: search the internet]

9. Compute the heat capacities at constant volume and at constant pressure for air at standard temperature and pressure, assuming an $f = 5$ ideal gas and a molecular mass of $28u$. Express your answer in units of $JK^{-1}kg^{-1}$ and compare it with $C_p/M = 4.184kJK^{-1}kg^{-1}$ for water.

10. For a system involving chemical reactions, such as

$$2H_2 + O_2 \Leftrightarrow 2H_2O$$

    we can talk either about chemical potentials for conserved quantities (eg H and O atoms) or of the non-conserved molecules (H_2 etc.). Show that in the latter case, the chemical potentials satisfy a relation with the same coefficients as the chemical reaction, ie

$$2\mu_{H_2} + \mu_{O_2} = 2\mu_{H_2O}$$

11. $*$ A monatomic gas which obeys van der Waal's equation has energy

$$E = \frac{3}{2}NkT - \frac{aN}{V}$$

    where a is a constant. It undergoes adiabatic free expansion from a state (V_1, T_1) to a state (V_2, T_2). Express T_2 in terms of the other quantities.