This paper contains seven questions.
The best FIVE answers will be used for assessment.

Calculators of the approved type are permitted in this examination.
1. (a) (3 marks)
Simplify
\[ \frac{22x^{-3} \cdot (6x^4)^2}{15x^{-2} \cdot 33x^7}. \]

(b) Factorise.
   i. (1 mark)
   \[ 4y^2 - 100 \]
   ii. (2 marks)
   \[ 15a^3b^8 - 25a^6b^2 + 45ab^4 \]

(c) (2 marks)
Solve for \( T \) the equation
\[ H = \frac{(T - t)\pi}{275}. \]

(d) (4 marks)
Find the equation of the straight line \( g \) that passes through \((8, 3)\) and is parallel to \( f \):
\[ 4y = -5 + x. \]

(e) (8 marks)
Solve the equation
\[ \frac{20}{x - 2} - \frac{20}{x + 3} = 2. \]

2. (a) Differentiate. (You do not need to simplify.)
   i. (3 marks)
   \[ f(x) = \sqrt{\frac{3x^2 - 1}{x^3 - 4x}} \]
   ii. (3 marks)
   \[ f(x) = 6^{-x} \cdot e^x \]
   iii. (3 marks)
   \[ f(x) = \sqrt[4]{81x^3} + \frac{5}{6x^2} - 9 \]
   iv. (2 marks)
   \[ f(x) = \log_9 x^4 \]

(b) (9 marks)
If \( S = 6xy + 4x^2 \) and \( 2x + y = 8 \), find the maximum value of \( S \) and the values of \( x \) and \( y \) that give this maximum value.
3. (a) (3 marks)
Rearrange the following expression into an expression that does not involve improper algebraic fractions:
\[
\frac{3x^3 - 5x^2 + 2}{x^2 - 1}
\]

(b) (5 marks)
Express \( x^2 + 8x + 7 \) in the form \((x + a)^2 + b\), and hence describe the translation which takes the curve \( y = x^2 \) to \( y = x^2 + 8x + 7 \).

(c) (6 marks)
Solve the following simultaneous equations:
\[
\begin{align*}
\text{I: } 4x - 15y &= 13 \\
\text{II: } 7x + 6y &= 12
\end{align*}
\]

(d) Let \( f(x) = 9 - 4x \) and \( g(x) = x^3 \).
   i. (4 marks)
   Find algebraic expressions for \( f^{-1}(x) \) and \( (f \circ g)(x) \).
   ii. (2 marks)
   Find the value of \( g(f(2)) \).

4. (a) (2 marks)
Evaluate the following with your calculator and give your answer to two decimal places:
\[
\log_6 200
\]

(b) (2 marks)
Find \( x \) when
\[
\log_5 x = 3
\]

(c) (3 marks)
Express the following in terms of \( \log a, \log b \) and \( \log c \):
\[
\log\left(\frac{\sqrt{a \cdot b}}{c^3}\right)
\]

(d) (3 marks)
Express
\[
5 \log x + \frac{1}{7} \cdot [\log a - \log(a - b)]
\]
as a single logarithm.

(e) A rumour spreads exponentially through a college. 150 people have heard it by 9 am, and 300 by 11 am.
   i. (4 marks)
   How many people have heard it by 1.30 pm?
   ii. (6 marks)
   At what time approximately will 2018 people have heard it?

Continued...
5. (a) (4 marks)
Find the sum to infinity of the following geometric series:

\[ 7 + \frac{7}{4} + \frac{7}{16} + \ldots \]

(b) (7 marks)
Starting with \( n = 1 \), write down the first three terms of the following sequence and give a recursive definition for it:

\[ u_n = \frac{1}{5} u(n + 1) \]

(c) (9 marks)
Find the sum of the following arithmetic series:

\[ -6 + (-1) + 4 + \ldots + 239 \]

6. (a) Solve each equation (correct to two decimal places) for \( 0^\circ \leq x < 360^\circ \):

i. (2 marks)
\[ \sin x = -0.3 \]

ii. (2 marks)
\[ \cos \frac{x}{5} = 0.8 \]

iii. (4 marks)
\[ \tan 2x = -4 \]

(b) (8 marks)
Given that \( a = 6 \text{ cm}, b = 8 \text{ cm} \) and \( \gamma = 28^\circ \) find the missing angles and the length of the missing side, giving all answers to two decimal places.

(c) (4 marks)
Find the angle \( \alpha \) (in degrees) subtended at the centre by an arc of length \( b = 7 \text{ cm} \) on a circle of radius \( r = 4 \text{ cm} \), giving your answer to two decimal places.
7. (a) Let \( A(-6, 7) \) and \( B(-3, -1) \) be two points in the plane.

i. (1 mark)
Evaluate \( \overrightarrow{AB} \).

ii. (1 mark)
Rewrite the vector \( \overrightarrow{AB} \) using the unit vectors \( \mathbf{i} \) and \( \mathbf{j} \).

(b) Let \( \mathbf{a} = 4\mathbf{i} + \mathbf{j} - 6\mathbf{k} \) and \( \mathbf{b} = -3\mathbf{i} + 4\mathbf{k} \).

i. (2 marks)
Calculate \( \mathbf{a} - \mathbf{b} \).

ii. (2 marks)
Find the unit vector \( \mathbf{b} \).

iii. (2 marks)
Find a vector \( \mathbf{c} \) that is of magnitude 15 units and is parallel to \( \mathbf{b} \).

iv. (6 marks)
Find the angle \( \alpha \) between the vectors \( \mathbf{a} \) and \( \mathbf{b} \).

(c) (6 marks)
If \( \mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \), \( \mathbf{b} = \begin{pmatrix} -6 \\ -2 \end{pmatrix} \) and \( \mathbf{c} = \begin{pmatrix} -4 \\ 12 \end{pmatrix} \), find which vectors are perpendicular to each other and which vectors are parallel and give brief explanations.