This paper contains two sections, Section A and Section B. Answer each section in a separate answer book.

Section A contains ten short questions worth 4 marks each. All TEN answers will be used for assessment. This section is worth 40% of the marks for the paper.

Section B contains six longer questions worth 15 marks each. A candidate’s FOUR best answers will be used for assessment. This section is worth 60% of the marks for the paper.

Calculators of the approved type are permitted in this examination.

Do not turn over until instructed.
Section A: Short Questions

A1. (a) (2 marks)
Expand \((3x - 2y)^3\).

(b) (2 marks)
Simplify
\[27 \left(\frac{4a^8}{12a^6}\right)^3 + 9a^0.\]

A2. Differentiate. (You do not need to simplify.)

(a) (2 marks)
\[f(x) = e^{3x^2 + 4x}\]

(b) (2 marks)
\[f(x) = \frac{7}{x^4} + \sqrt{x^3}\]

A3. (a) (2 marks)
Rearrange the following into an expression that does not involve improper algebraic fractions:
\[
\frac{4x^3 - 2x^2 + 5x + 1}{x^2 - 1}
\]

(b) (2 marks)
Find the equation of the circle with centre \(C(-4, 0)\) and radius \(r = 5\).

A4. Find \(x\).

(a) (2 marks)
\[\log_3 \sqrt{81} = x\]

(b) (2 marks)
\[\log_x \frac{1}{625} = 4\]
A5. (4 marks)
Let \( f(x) = \frac{1}{4}(x - 3) \), \( g(x) = x^5 \), and \( h(x) = \frac{1}{x} \).
Give algebraic expressions for \( f^{-1}(x) \), \( (g \circ h)(x) \), \( (f \circ g \circ h)(x) \), and \( g(x^3) \).

A6. (4 marks)
Evaluate the sum \( S_{30} \) of the first 30 terms of the following geometric series, giving your answer correct to three decimal places:
\[
-2 + \frac{4}{5} - \frac{131}{50} + \ldots
\]

A7. (4 marks)
Solve the following equation for \( 0^\circ \leq x < 360^\circ \), giving your answer correct to two decimal places:
\[
\cos 2x = -0.4
\]

A8. (4 marks)
Prove the following identity:
\[
\frac{\tan x(\cos^2 x + \sin^2 x)}{2 \cos x} = \frac{\sin^2 x}{\cos x \sin 2x}
\]

A9. (4 marks)
Find \( y' \) in terms of \( x \) and \( y \) if
\[
12x + 3x^2y - 5y^2 = 0.
\]

A10. Let \( z_1 = 4 - 3i \) and \( z_2 = 6 - 5i \).

(a) (1 mark)
Find real numbers \( a \) and \( b \) such that \( z_1 - z_2 = a + bi \).

(b) (3 marks)
Find real numbers \( c \) and \( d \) such that \( \frac{z_1}{z_2} = c + di \).

Continued...
Section B: Longer Questions

B1. (a) (4 marks)
Express the following as a single fraction and simplify:

\[
\frac{(x + 5)^2}{x^2(x^2 - 25)} \div \frac{x^2 - 9}{8x^2(x - 5)(x + 3)}
\]

(b) (5 marks)
Solve the quadratic equation by completing the square:

\[6x^2 + 6x - 72 = 0\]

(NB: In this question you will not get any marks for using the quadratic formula.)

(c) (6 marks)
Solve the following inequality:

\[(6x + 2)(x - 4) + 10 - x^2 < (3x - 1)^2 - (2x + 4)^2\]

B2. (a) (5 marks)
Differentiate. (You do not need to simplify.)

\[f(x) = \ln \frac{4x^2 + 5}{x} + \log_7 \sqrt{x}\]

(b) (10 marks)
Find the x- and y-coordinates of the maximum point and the minimum point of the curve

\[f(x) = -\frac{1}{4}(5x^3 - 60x + 4)\].
B3. (a) \textbf{(7 marks)}
Evaluate whether the straight line \( L \) cuts, touches or misses the circle \( C \).

\[
L: \quad 2x + y = 4 \\
C: \quad x^2 + y^2 - 4x + 5y - 9 = 0
\]

(b) \textbf{(8 marks)}
Find the \( x \)- and \( y \)-coordinates of the points where the following function \( f \) cuts the \( x \)-axis:

\[
f(x) = 12x^3 - 13x^2 + 1
\]

B4. (a) \textbf{(5 marks)}
Find the sum of the following arithmetic series:

\[
110 + 106 + 102 + \ldots + (-90)
\]

(b) In 1990, there were 300 mobile phone subscribers in Smallville. Since then the number of subscribers has been increasing exponentially. In 2003, there were already 4000.

i. \textbf{(5 marks)}
How many mobile phone subscribers were in Smallville in 1997?

ii. \textbf{(5 marks)}
In which year will 16137 people have a mobile phone in Smallville?

B5. (a) Differentiate. (You do not need to simplify.)

i. \textbf{(2 marks)}
\[
f(x) = \tan 2x
\]

ii. \textbf{(3 marks)}
\[
f(x) = 3 \sin^7 5x
\]

(b) \textbf{(10 marks)}
Find the \( x \)-coordinates of the points of inflexion of the curve

\[
f(x) = \sin x - \cos x
\]

for \( 0 \leq x < 2\pi \), giving your answers correct to two decimal places.

Continued...
B6.  (a) **(4 marks)**
Find the modulus $r$ and the principal argument $\theta$ (in degrees) of $z = -8 + 9i$, giving your answers correct to two decimal places.

(b) **(5 marks)**
Solve the following equation:
\[
\frac{1}{6 - x} - \frac{1}{6 + x} = \frac{x^2 + 5}{36 - x^2}
\]

(c) **(6 marks)**
Find the equation of the tangent $t$ to the curve
\[
3x^2 + y^2 = 4
\]
at the point $(-1, 1)$ on the curve.

End of examination.