Mechanics 1: Week 21 Problems

Marked problems 2, 3

1. Consider the equation:

\[ \ddot{s} = -s, \]

where, for simplicity, we have set \( m = 1 \) (remember from an earlier homework problem that we can “rescale time” so that the mass becomes unity).

(a) Write it as a first order system, or vector field, on the phase plane.
(b) Compute the potential energy and sketch it.
(c) Find all equilibria and classify their stability.
(d) Sketch the phase portrait.
(e) Compute expressions for the trajectories in the phase plane as a function of time (and take \( s(0) = 0 \) for simplicity).

2. Consider the equation:

\[ \ddot{s} = s - s^3, \]

where, for simplicity, we have set \( m = 1 \).

(a) Write it as a first order system, or vector field, on the phase plane.
(b) Compute the potential energy and sketch it.
(c) Find all equilibria and classify their stability.
(d) Sketch the phase portrait.

3. Consider the equation:

\[ \ddot{s} = s - s^2, \]

where, for simplicity, we have set \( m = 1 \).

(a) Write it as a first order system, or vector field, on the phase plane.
(b) Compute the potential energy and sketch it.
(c) Find all equilibria and classify their stability.
(d) Sketch the phase portrait.