TCC Topics in Algebraic Geometry: Assignment #1.

Problem 1 (for 29th October).

(1) Show that $\mathbb{P}^1 \times \mathbb{P}^1 \not\cong \mathbb{P}^2$. (You may want to use 'weak Bezout'.)

(2) If V is any variety, a rational map $f : V \rightsquigarrow \mathbb{P}^n$ is given by $n+1$ rational functions $f_0, ..., f_n \in k(V)$, not all identically zero on V,

$$V \ni P \quad \longmapsto \quad [f_0(P) : ... : f_n(P)] \in \mathbb{P}^n,$$

and $gf_0, ..., gf_n$ give the same map, for $g \in k(V)^\times$. If, for a point $P \in V$, there is such a g that the gf_i are all defined and not all zero at P, we say that f is regular (or defined) at P, and $f(P)$ is the corresponding value. Use this to show that $\mathbb{P}^n$ is complete, by verifying the valuative criterion.

Please hand in your solution by emailing it to **tccalggeom@gmail.com** by 29th October.