TCC Topics in Algebraic Geometry: Assignment #2.

Problem 2 (for 12th November). Suppose $\operatorname{char} k \neq 2$.

(1) Let C/k be a complete non-singular curve that admits a map $x : C \to \mathbb{P}^1$ of degree 2. Such a curve is called hyperelliptic. Show that C is birational to a curve $y^2 = f(x) \subset \mathbb{A}^2$, with $f \in k[x]$ square-free. [*Hint: Describe* $k(C)$.]

(2) Show that C has genus $g = \lfloor \frac{\deg f - 1}{2} \rfloor$, with regular differentials

$$\Omega_C = \left\langle \frac{dx}{y}, \frac{xdx}{y}, \ldots, \frac{x^{g-1}dx}{y} \right\rangle.$$

[*Hint: If you use Baker, note the two cases* $f(0) \neq 0$ *and* $f(0) = 0$]

(3) Now let C be *any* complete non-singular curve of genus 2. Use $\deg K_C = 2$ and $\dim \mathcal{L}(K_C) = 2$ to prove that C is hyperelliptic.

Please hand in your solution by emailing it to **tccalggeom@gmail.com** by 12th November.