

GALOIS REPRESENTATIONS — EXERCISES TO LECTURE 1

**Exercise 1.** Let  $E/\mathbb{Q}_7$  be the elliptic curve  $y^2 + y = x^3 - x^2$ . Let  $K/\mathbb{Q}_7$  be some horrible extension with residue degree 11 and ramification degree 76. Find  $\#\tilde{E}(\mathbb{F}_K)$ , where  $\mathbb{F}_K$  is the residue field of  $K$ .

**Exercise 2.** Let  $E/\mathbb{Q}_7$  be the elliptic curve  $y^2 = x^3 + 7^2$ . Identify its  $\ell$ -adic representation  $\rho_E : \text{Gal}(\overline{\mathbb{Q}}_7/\mathbb{Q}_7) \rightarrow \text{GL}_2(\overline{\mathbb{Q}}_\ell)$  on  $H_{\text{ét}}^1(E, \mathbb{Q}_\ell) \otimes_{\mathbb{Q}_\ell} \overline{\mathbb{Q}}_\ell$  for  $\ell \neq 7$ .  
Hint: compute  $\#\tilde{E}(\mathbb{F}_K)$  over two different cubic ramified extensions  $K/\mathbb{Q}_7$ .