Projects Sebastian Müller

Level 3

- **Classical perturbation theory in astronomy** (10 or 20 cp)

  Many systems in nature are non-integrable. Roughly speaking this means that there are not enough conserved quantities to find an analytical solution for the equations of motion. However for systems that are close to being integrable it is possible to find approximate solutions. The technique used to get these approximate solutions (classical perturbation theory) is based on Hamiltonian mechanics. Classical perturbation theory has many applications in astronomy:

  - The motion of *planets* in the solar system becomes non-integrable if one takes into account not only the gravitational attraction by the Sun, but also the influence of other planets. This influence can be quite important; for instance Neptune was found due to its influence on the orbit of Uranus.
  - The shape of the *rings of Saturn* can be obtained using perturbation theory and some results on integrable systems.
  - While passing Saturn, the Voyager probe collected data about *two moons of Saturn that have very close orbits.* To understand why they do not collide one has to take into account their gravitational interaction.

  In this project one of these problems should be studied in detail, along with the techniques of perturbation theory. The project will involve reading and analytical and/or numerical work.

  Prerequisites: Mechanics 2/23 or equivalent

- **Chaos and integrability** (10 or 20 cp)

  Mechanical systems can be integrable, close to integrable (see above) or far away from being integrable. Systems that are far away from being integrable are also called chaotic. They show various interesting features different from integrable systems. In particular the motion of particles in these systems strongly depends on the initial conditions; if one slightly changes the initial position or momentum of a particle its trajectory may look completely different. In this project these different types of behaviour should be investigated for a family of systems, e.g.:

  - *Billiards:* On a billiard table, a ball moves on a straight line and is reflected at the boundary. While billiard players prefer rectangular tables, billiards of a different shape have been studied by mathematicians and physicists as examples for various different effects. Depending on the shape of the boundary billiards can be integrable (e.g. if one considers a circular boundary), close to integrable (e.g. if one slightly deforms a circle) or chaotic (e.g. if one strongly deforms a circle into a so-called cardioid curve).
  - *Pendulums*, e.g. the double pendulum

  One way of distinguishing between different kinds of behaviour is to draw so-called *Poincaré sections.* In the example of billiards one would follow a trajectory, and for each bounce from the boundary write down the position s on the boundary and the angle θ enclosed between the trajectory and the boundary. One then draws a two-dimensional coordinate system and marks all arising combinations of s and θ by a dot. Similar plots can be made for pendulums. The results then look strikingly different for different dynamics: for almost integrable systems they show an intricate structure related to the periodic orbits of the system; for chaotic systems the dots are distributed uniformly over the plot.

  This project will start with reading and numerically producing Poincaré plots for one family of systems such as billiards or pendulums. Then one can go further into the theory and e.g. study which impact deformations of a billiard have on the periodic orbits. Alternatively it is possible to put a stronger emphasis on the programming side.

  Prerequisites: Mechanics 2/23 or equivalent

Level M

- more extended versions of the level 3 projects
- **Quantum chaos**

  In quantum chaos one considers systems whose classical motion is chaotic, i.e., depends sensitively on the initial conditions. One then investigates the quantum mechanical properties of these systems, for example their energy levels. It turns out that their are deep connections between the classical and quantum mechanical behaviour. Possible projects on this topic are:

  - *Maps* (20 cp)

    An important result in quantum chaos is that the energy levels of a chaotic system are determined by its classical periodic orbits (Gutzwiller trace formula). In this project this relationship should be explored for simple model systems called maps. Here the state of a system at a time t is represented by a vector $\mathbf{u}_t$ and the state $\mathbf{u}_{t+1}$ at a later time $t+1$ is given as a function of $\mathbf{u}_t$. The project would involve reading about the Gutzwiller trace formula for maps, finding the periodic orbits of a chaotic map and then using these orbits to numerically determine some of its energy levels.
  - *Self-crossings* (20 cp or more)

    In quantum chaos self-crossings of classical trajectories play an important role. They determine the statistical behaviour of energy levels, which is essentially the same for all chaotic systems. In this project one would consider a chaotic billiard (see the explanation above) and numerically determine classical trajectories inside this billiard. One would then determine the points where these trajectories cross themselves and investigate the statistics of these crossings. I.e., how many crossings are there, and how likely is it to have a crossing with a given angle ϵ? In addition to this numerical work, the project would involve reading parts of the literature to understand the impact these self-crossings have on the quantum mechanical behaviour of chaotic systems.

  Prerequisites: These projects would be suitable for students who have taken some units on Mathematical Physics; please contact me for details.