MATH11007 SHEET 15: THE DIRECTIONAL DERIVATIVE.

Set on Tuesday, February 22: Qs 1, 2 and 5.

(1) In what direction(s) does the directional derivative of $f(x, y) = xy$ at the point $(2, 0)$ take the value -1?

(2) You are given that the directional derivative of a function $f(x, y)$ at a point (a, b) in the direction of the vector $(1, 2)$ is 2, and in the direction of the vector $(-1, 1)$ is 3. Find the partial derivatives of f at (a, b).

(3) Find the directional derivative of $f(x, y) = \sqrt{xy}$ at the point $P(2, 8)$ in the direction of the point $Q(5, 4)$.

(4) Find the maximum rate of change of $f(x, y, z) = \ln \left|xy^2z^3\right|$ at the point $(1, -2, -3)$. In what direction doest it occur?

(5) For each of the following, find a normal vector to the given curve (or surface) at the given point, and the equation of the tangent line (or plane) at that point:
   (a) $x^2y^2 = 1$ at $(2, -1/2)$.
   (b) $x^2 + 2y^2 + 3z^2 = 6$ at $(1, -1, 1)$.
   (c) $z = p^4 - x^2 - y^2$ at $(1, -1, p^2)$.

(6) You are standing on a very steep hillside; the height of the ground at a point (x, y) is given by $f(x, y)$. If you head due North, you go uphill at a slope of $60°$; if you head due East, you go uphill at $30°$. In which direction is the slope greatest? What is the value of the greatest slope?

(7) Let $z = f(x, y) = x^2y$, and let $\mathbf{u} = (u, v)$ be a unit vector. Write a formula for $\mathrm{D}_{\mathbf{u}}f(1, -1)$ in terms of u and v. Find u and v such that the derivative is zero.

References

1. Frank Ayres, Jr. and Elliott Mendelson, *Schaum's Outline of Calculus, Fourth Edition* Chapters 48 and 52, Mc-Graw–Hill, 1999.