Theory of Inference: Homework 1

Here are some questions about some of the most important inequalities in the whole of probability and statistics. Before you do these questions, make sure you have looked at the self-assessment questions on chs 1 to 3.

1. Give a complete statement of the Fundamental Theorem of Prevision (FTP) in its 'convex hull' form.

2. Study the proof of Jensen's inequality in ch2 of the notes. When you are sure you understand it, use the same approach to prove Markov's inequality (a diagram is required).

3. Prove Chernoff's inequality

$$\Pr(X \mathbin{\dot{\geq}} a) \leq \inf_{t>0} e^{-at} M_X(t)$$

where M_X is the moment generating function (MGF) of X.

4. Use Chernoff's inequality to provide an upper bound for the tail probability of a Poisson distribution with rate λ. Verify this bound using R.

Please hand in your answers for marking next Tue (24 Feb), at the lecture or by 5pm in the box outside my office door. I will return them in Thu's lecture.

Jonathan Rougier
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