Theory of Inference: Homework 2

1. Study the statement and proof of the Bayes Rule theorem. Now state and prove the theorem in the special case where the choice of action does not affect the client's beliefs about X.

2. A volcano can be either inactive or active; it is active with probability θ. If it is inactive its eruption rate is zero. If it is active, its eruption rate has a Gamma distribution with shape parameter α and rate parameter β. Let λ be its eruption rate. Show that

$$\Pr(\lambda \dot{\leq} v) = 1 - \theta + F(v; \alpha, \beta) \cdot \theta$$

for $v \geq 0$, and zero otherwise, where F is the distribution function of the Gamma distribution. Hint: introduce the random quantity $A \in \{0, 1\}$, where $A = 1$ exactly when the volcano is active, and use the Law of Total Probability.

3. Here is an exam-style revision question on the first strand of lectures.

   (a) The Fundamental Theorem of Prevision (FTP) is an if-and-only-if theorem. Pick one branch (either if or only-if), state it, and prove it. [5 marks]

   (b) Outline a model for data, distinguishing between random quantities, observables, and observations. Give an example of how an observable differs from the random quantity it measures, owing to limitations in the instrument. [5 marks]

   (c) State and prove the Muddy Table theorem. Illustrate it with a diagram. [5 marks]

   (d) 'Bayesian conditionalisation' is the name given to a model for learning in which new information is incorporated into beliefs through conditioning. What are the attractive features of this model? Why does it *not* describe the typical practice of statistical inference? [10 marks]

Please hand in your answers for marking next Tue (3 Mar), at the lecture or by 5pm in the box outside my office door. I will return them in Thu's lecture.

Jonathan Rougier
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