

Theory of Inference, 2014/15

Self-assessments questions on Expectation and Inference

Here are some questions to help to assimilate the material on the first three handouts, which you studied in Teaching Week 13 (the first week of Teaching Block 2). You might find it helpful to go through these questions in small groups. Don't feel that you have to master all of these questions by next Tue (3 Feb), but don't delay too long.

1. Suggest some random quantities and their operational definitions. How would you operationalise sea-level rise in the Bristol channel in 2100, relative to today?
2. Suggest a pair of random quantities that do not have a product realm.
3. What do the symbols \boldsymbol{x} , \boldsymbol{X} , and $\boldsymbol{\mathcal{X}}$ represent? What would be different if they were not in a bold font?
4. What are the axioms of expectation? Illustrate each of these with an example. Explain why 'most likely value' does not satisfy the axioms.
5. Distinguish between a 'coherent' set of expectations and a 'defensible' set.
6. What do we mean by 'statistical inference'? Discuss this in the context of samples and populations.
7. Using Schwartz's inequality, prove that the correlation coefficient, where it exists, is always bounded between -1 and 1 .
8. Define a 'random proposition' and suggest some examples.
9. What is the difference in meaning between ' $A = B$ ' and ' $A \doteq B$ '?
10. Define a 'probability', and explain how it can be interpreted as the fair price of a bet.

11. Express $\Pr(A, B)$ in terms of expectations.
12. State the axioms of probability and show that they are implied by the axioms of expectation.
13. State and prove Markov's inequality, making clear how the axioms of expectation are used in your proof.
14. What does the Fundamental Theorem of Prevision (FTP) state?
15. Express $p(x, y, z)$ in terms of expectations.
16. What is the result of marginalising out Y from $p(x, y)$?
17. The FTP states that (x, y) is a coherent expectation for (X, Y) if and only if (x, y) lies in the convex hull of the realm of (X, Y) . Make sure you understand this statement, and illustrate it with a diagram.
18. Explain, using a diagram, how the value of my expectation of X can constrain the value of my expectation of Y , but not necessarily to a single point.
19. Make sure you understand how the truth of a random proposition $Q := q(\mathbf{X})$ will ensure that $\Pr(\mathbf{X} \doteq \mathbf{x}^{(j)}) = 0$ whenever $\mathbb{1}_{q(\mathbf{x}^{(j)})} = 0$.
20. How can I express my belief that Y is a measurement for X in which the measurement error $Y - X$ has expectation zero and standard deviation σ ?
21. What do the symbols ' Y ', ' y ', and ' y^{obs} ' represent? What would be different if they were in a bold font?
22. Suggest some observations which are non-linear functions of random quantities.
23. Provide a verbal description of the value $E(X | Q)$. Explain how it differs from the value $E(X)$.
24. Explain why $E(X | Q)$ is uniquely defined if and only if $\Pr(Q) > 0$.

25. Define the hypothetical probability $\Pr(A | Q)$ and explain why it is the fair value of a called off bet.
26. Explain, informally, why $E(\cdot | Q)$ satisfies the ‘recursive’ property.
27. State the Law of Iterated Expectation (LIE) and explain how it allows you to provide lower and upper bounds for $E(X)$, based on a belief partition.
28. Suggest an example of ‘drilling down’ that does not involve the weather.
29. What is meant by the ‘support’ of \mathbf{X} ? Explain why functional equalities involving hypothetical expectations and probabilities often need to be qualified by a statement restricting the values of one or more of the arguments.
30. Illustrate your previous explanation using Bayes’s theorem for $p(x | y)$.

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