Anomalous fluctuations in one dimensional interacting systems

Joint with
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Márton Balázs

University of Bristol

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The models
- Asymmetric simple exclusion process
- Zero range
- Bricklayers

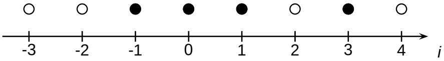
Hydrodynamics
- Characteristics

Tool: the second class particle
- Single
- Many second class particles

Results
- Normal fluctuations
- Abnormal fluctuations

Proof
- Upper bound
- Lower bound
- Microscopic concavity/convexity

Asymmetric simple exclusion



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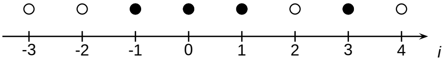

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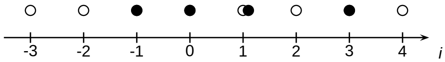

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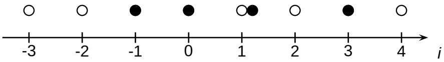

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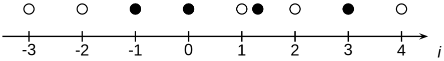

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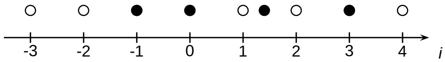

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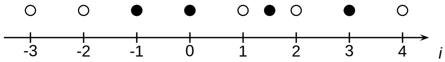

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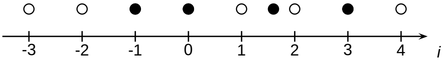

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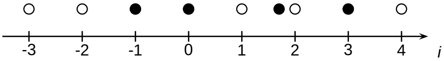

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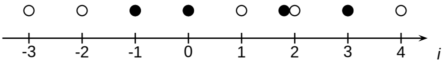

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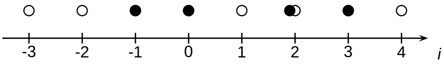

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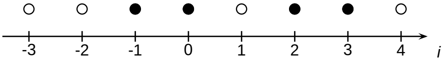

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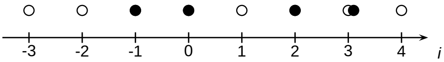

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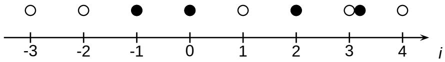

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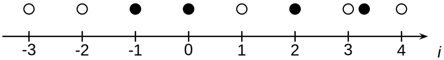

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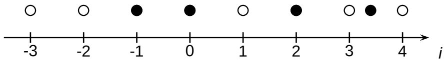

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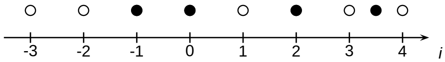

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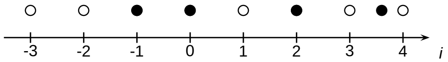

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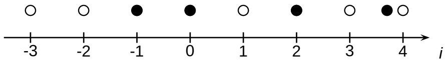

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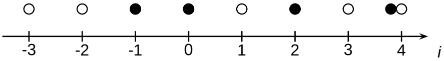

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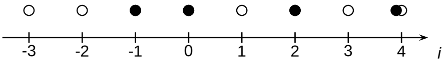

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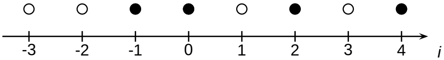

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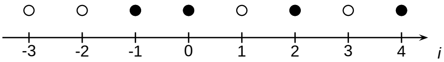

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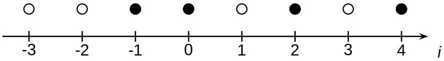

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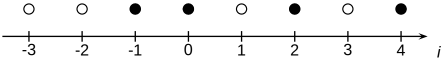

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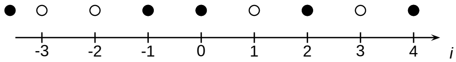

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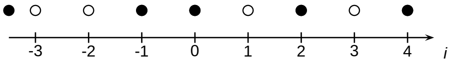

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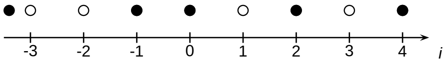

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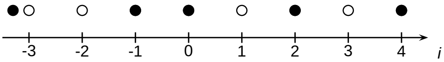

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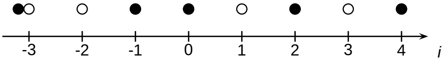

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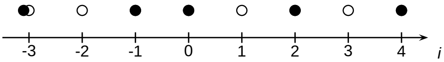

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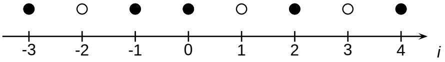

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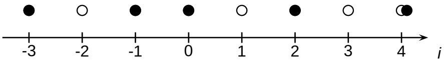

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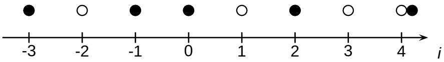

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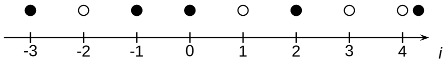

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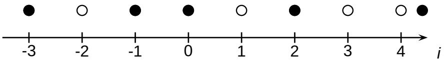

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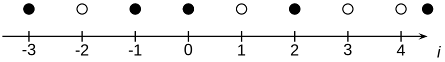

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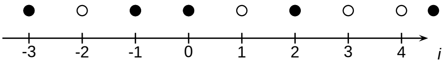

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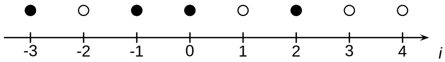

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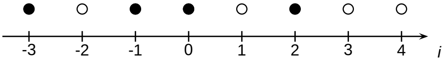

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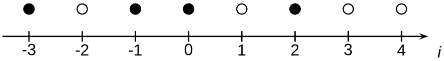

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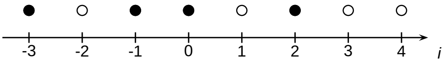

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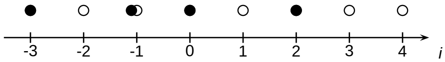

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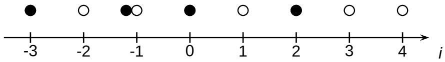

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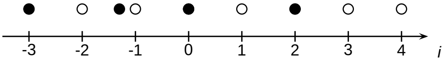

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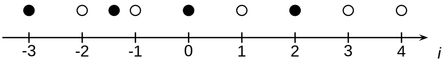

Bernoulli(ϱ) distribution; $\omega_i = 0$ or 1.

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

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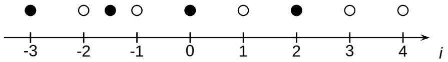

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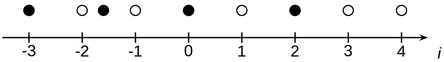

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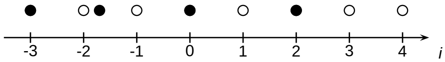

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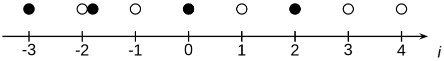

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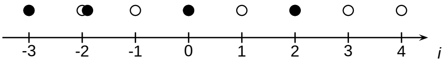

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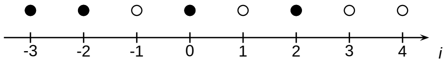

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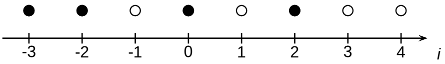

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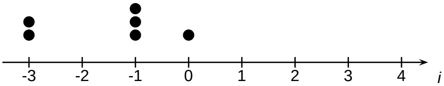
The Bernoulli(ϱ) distribution is time-stationary for any ($0 \le \varrho \le 1$). Any translation-invariant stationary distribution is a mixture of Bernoullis.

The asymmetric zero range process



Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

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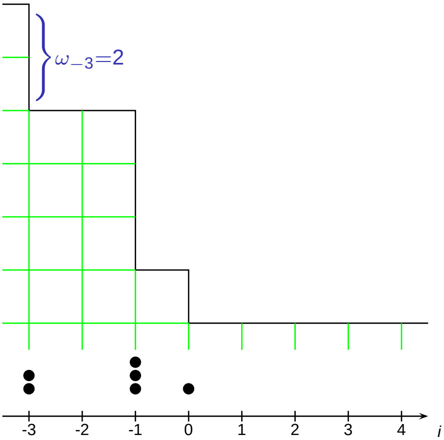

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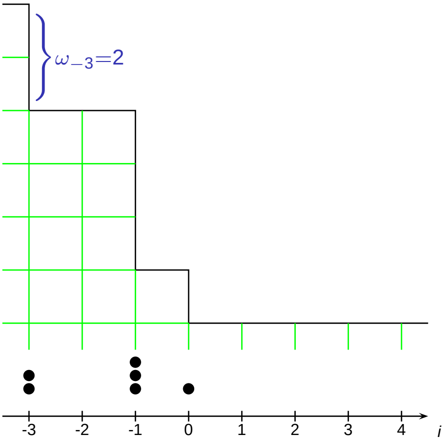

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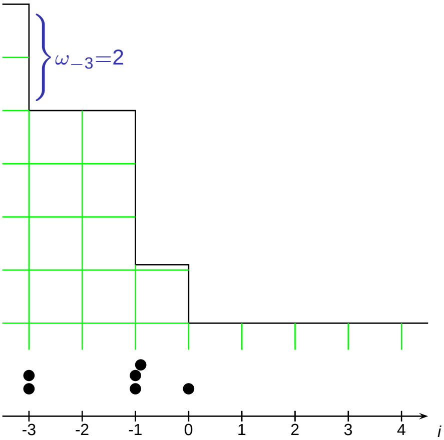

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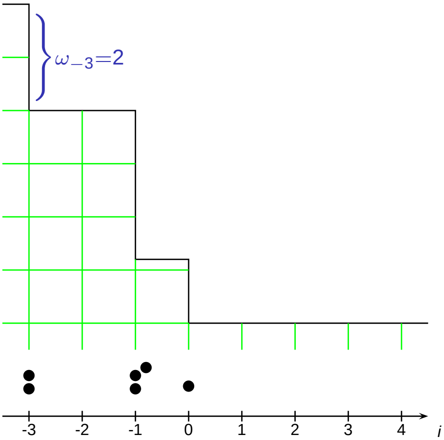

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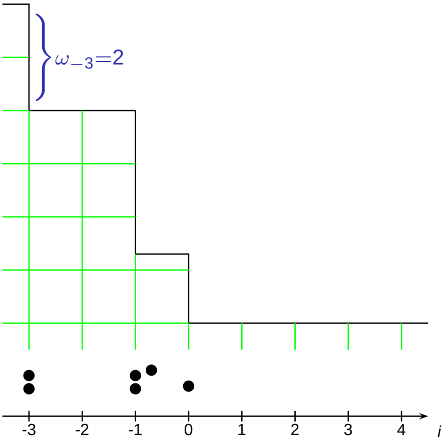

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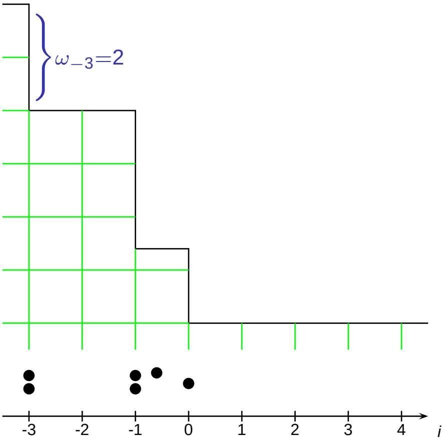

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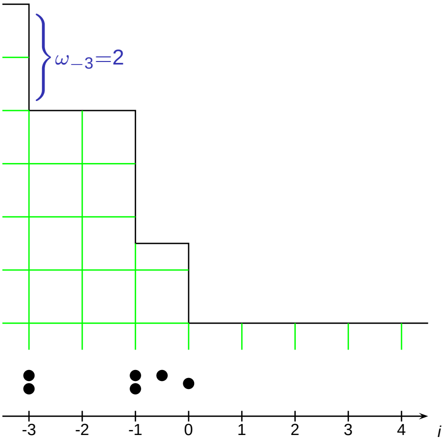

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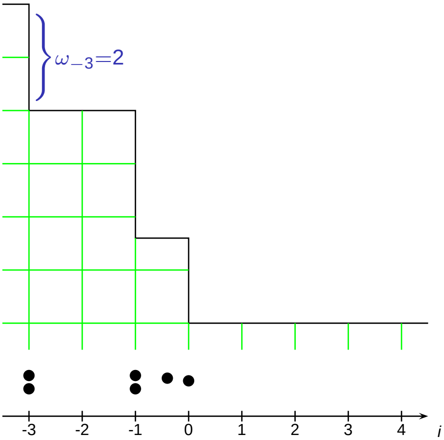

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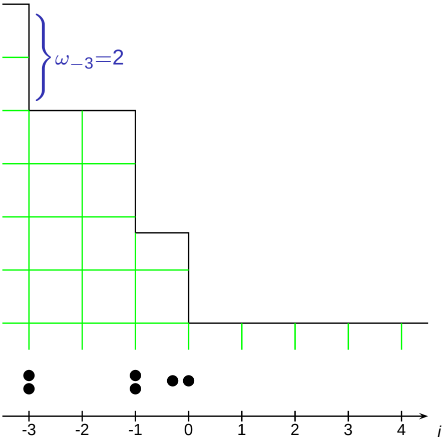

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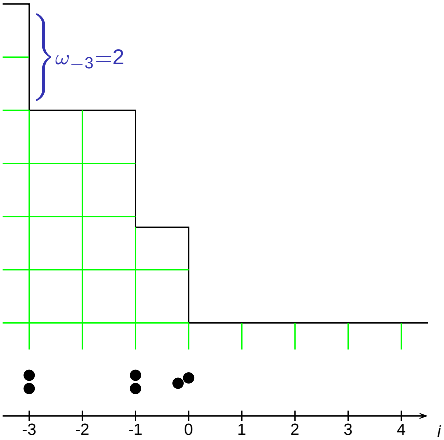

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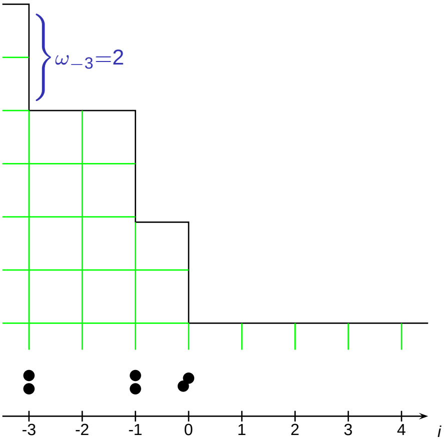

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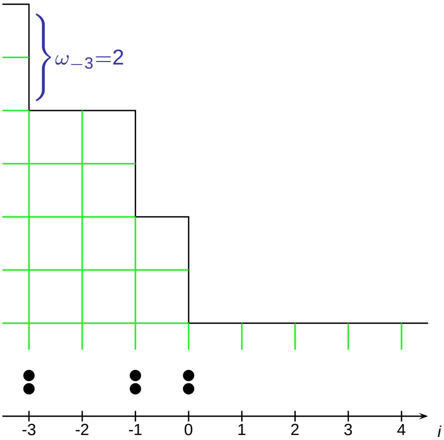

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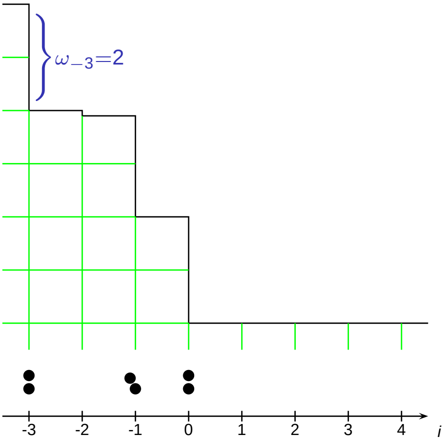

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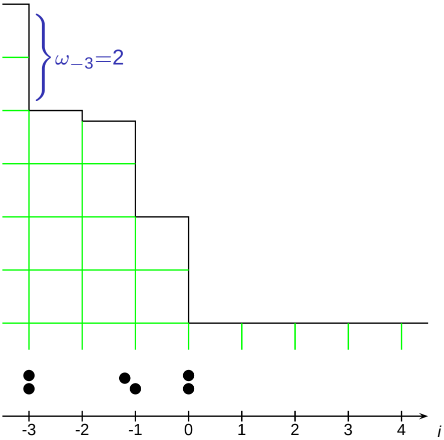

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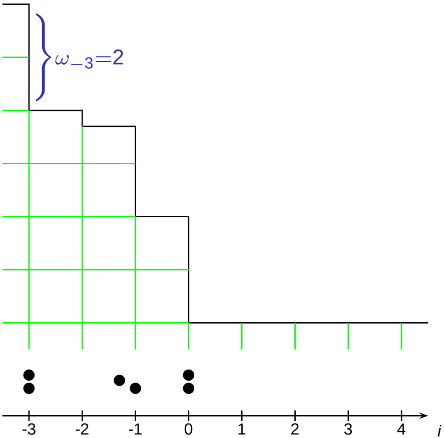

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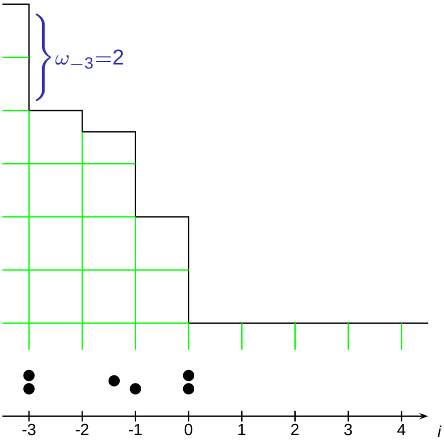

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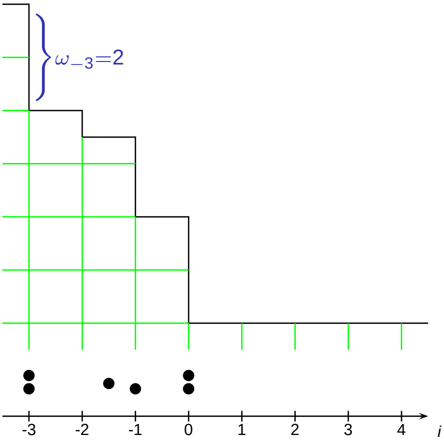

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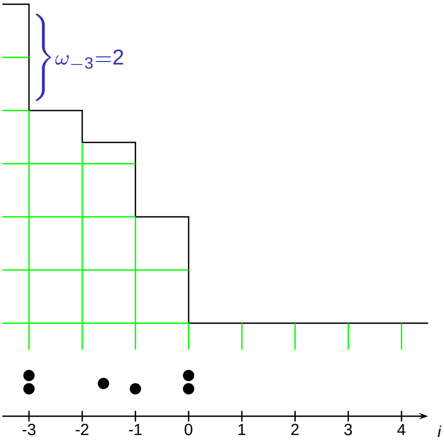

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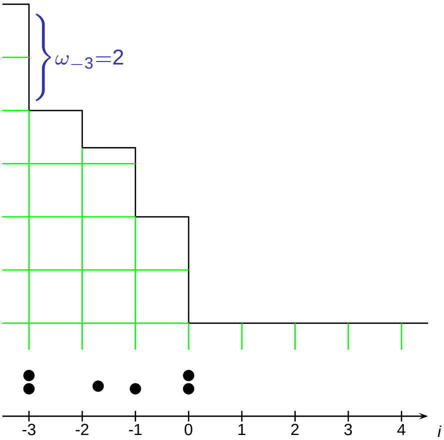

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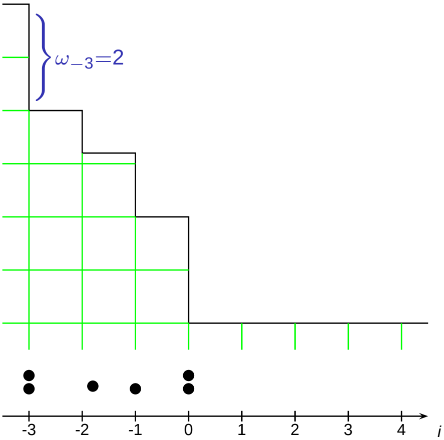

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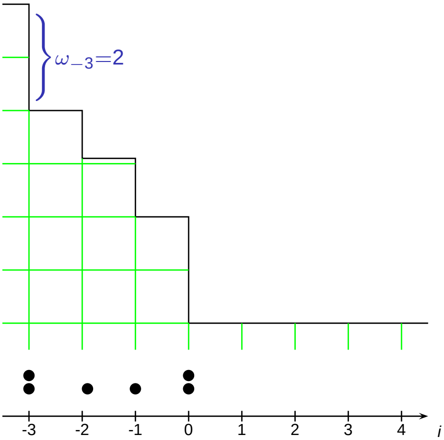

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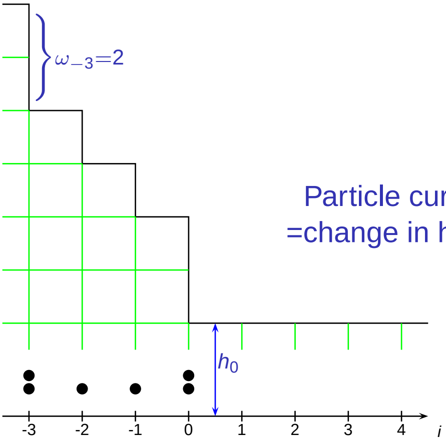

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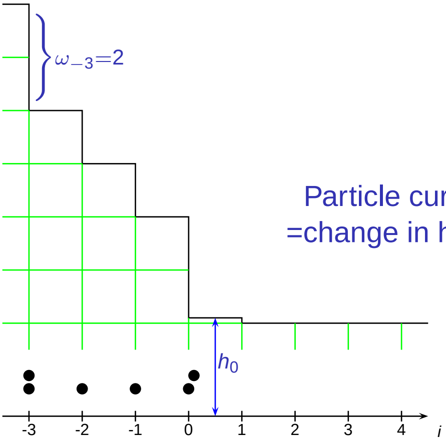

Particle current
=change in height

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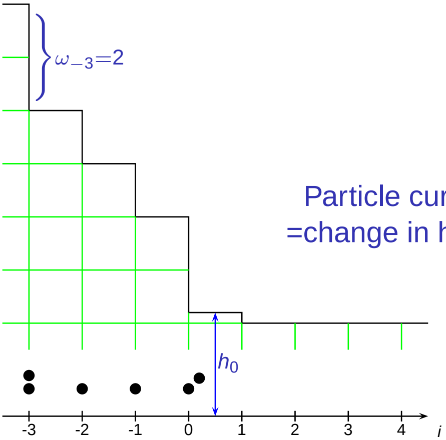

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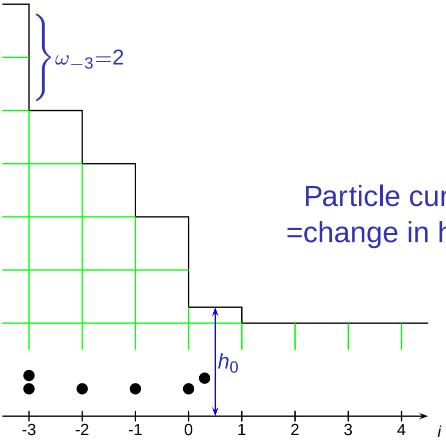

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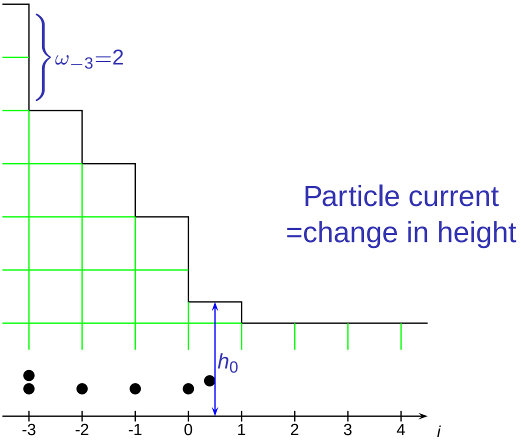

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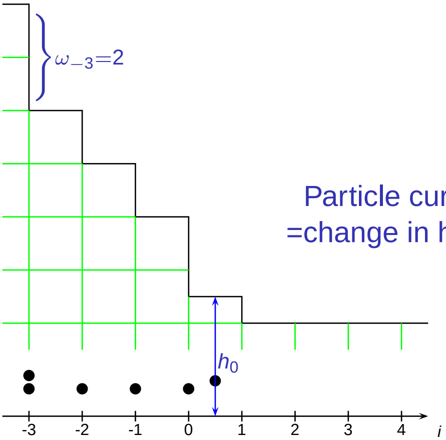

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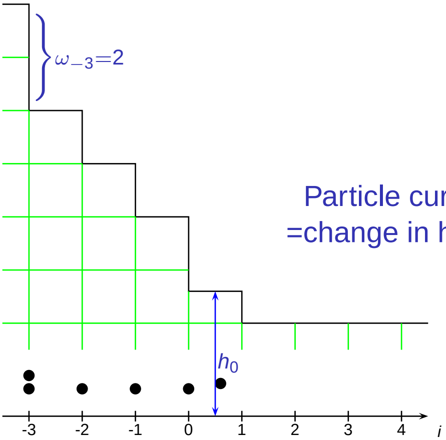

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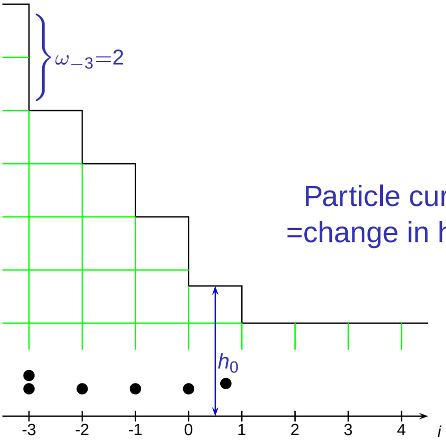

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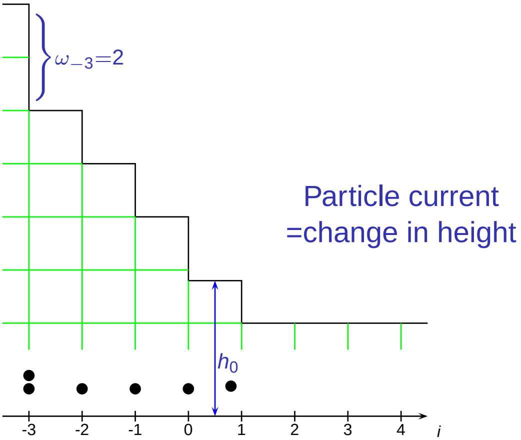

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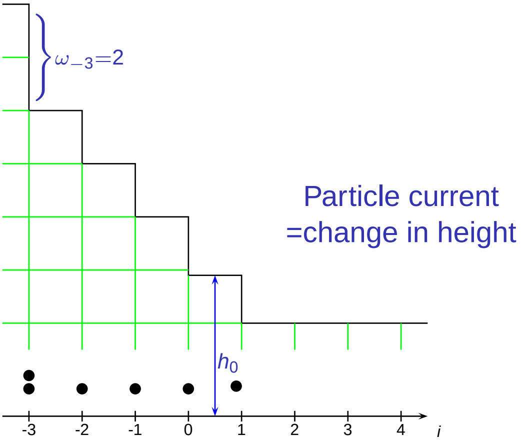

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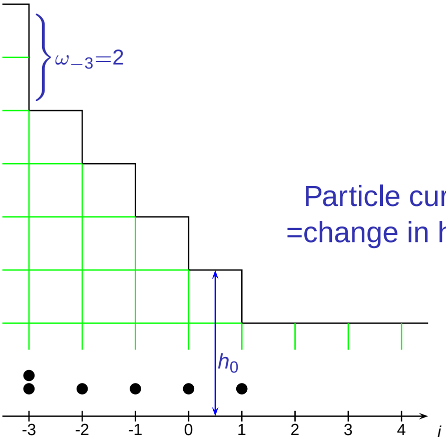

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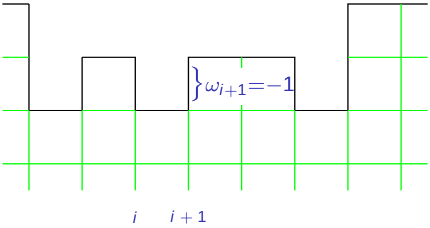
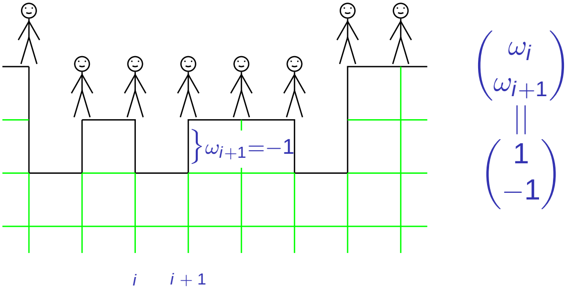
The asymmetric bricklayers process



$$\begin{pmatrix}\omega_i\\ \omega_{i+1}\end{pmatrix} = \begin{pmatrix}1\\ -1\end{pmatrix}$$

Poisson-type distribution; $\omega_i \in \mathbb{Z}$.

The asymmetric bricklayers process

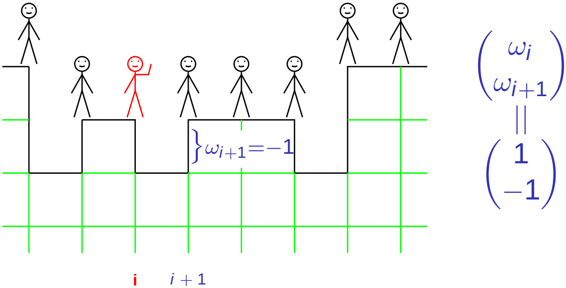


Poisson-type distribution; $\omega_i \in \mathbb{Z}$.

a brick is added with rate $p \cdot [r(\omega_i) + r(-\omega_{i+1})]$
a brick is removed with rate $q \cdot [r(-\omega_i) + r(\omega_{i+1})]$.

$(r(\omega) \cdot r(1-\omega) = 1$; r non-decreasing; $q = 1 - p < p)$.

The asymmetric bricklayers process



Poisson-type distribution; $\omega_i \in \mathbb{Z}$.

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a brick is removed with rate $q \cdot [\mathbf{r}(-\omega_{\mathbf{i}}) + r(\omega_{i+1})]$.

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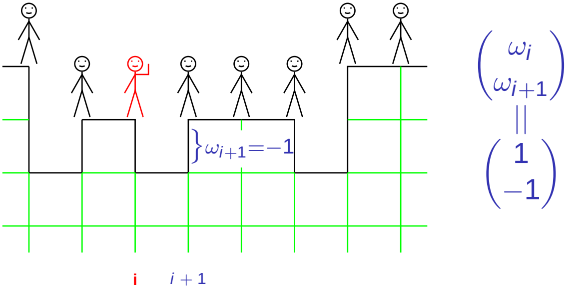
The asymmetric bricklayers process



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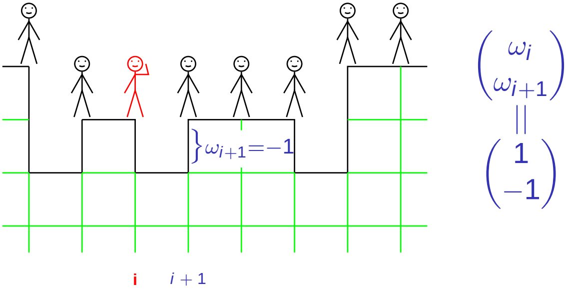

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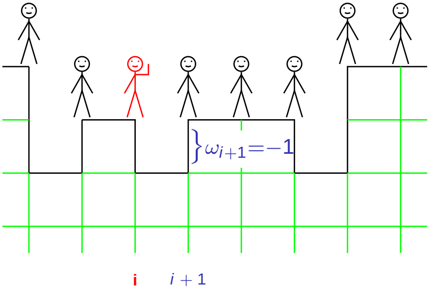

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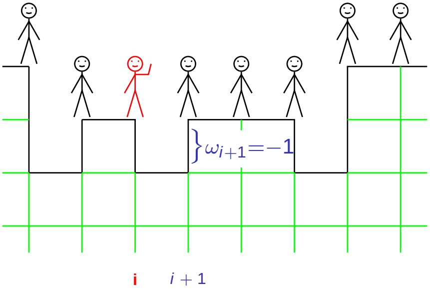

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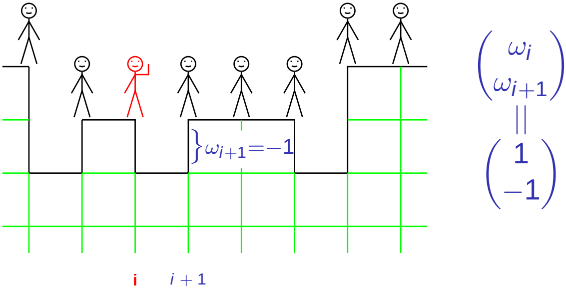

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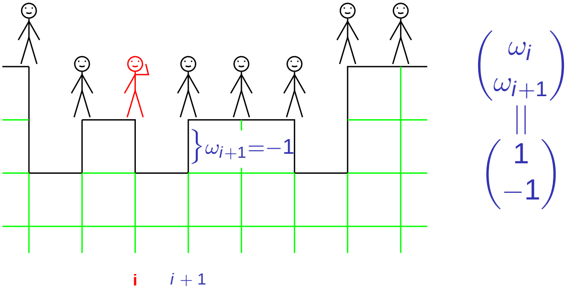

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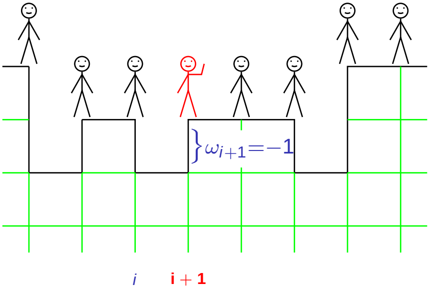

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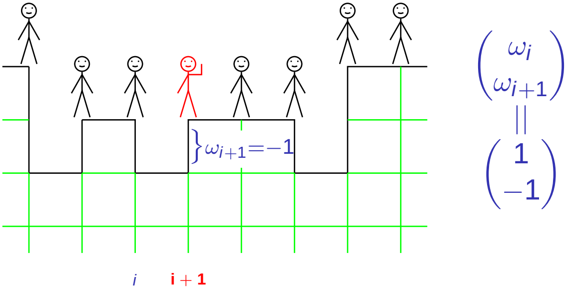

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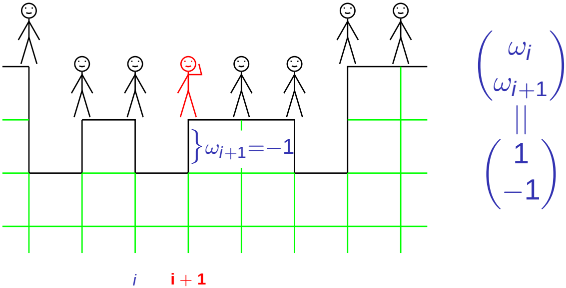

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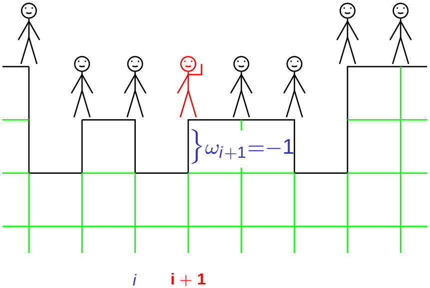

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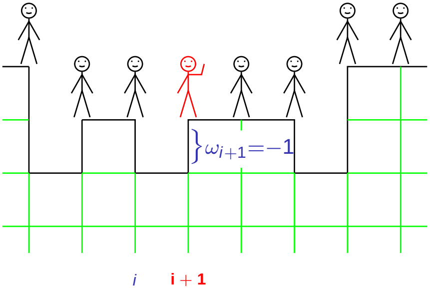

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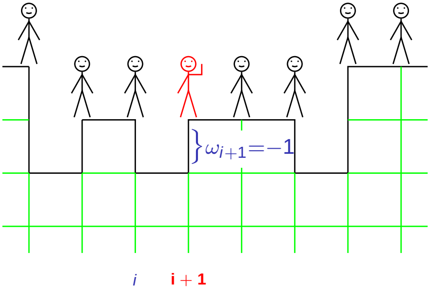

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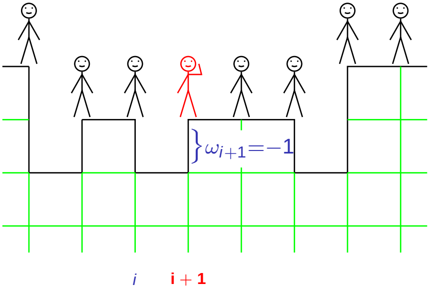

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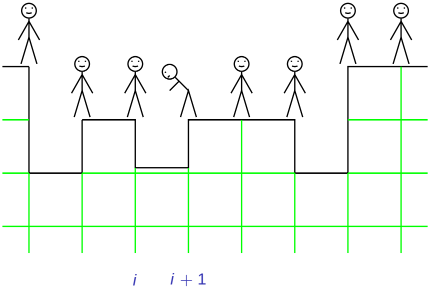

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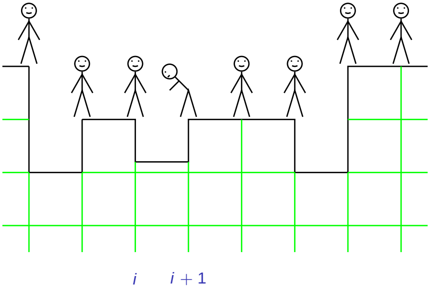

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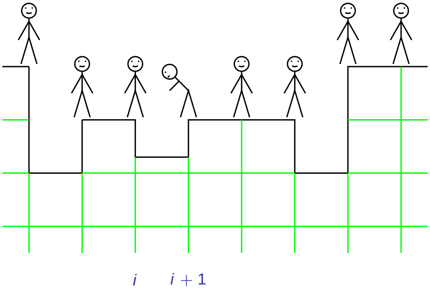

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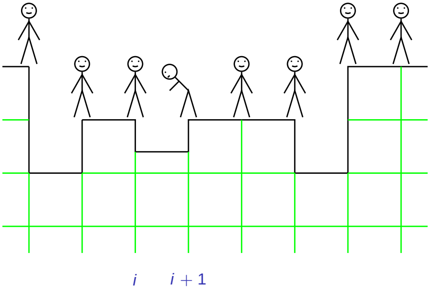

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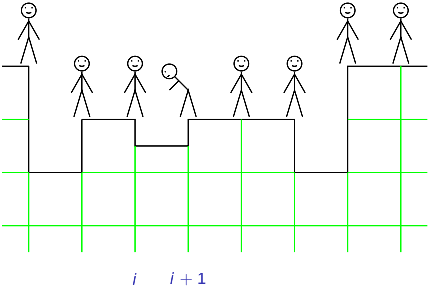

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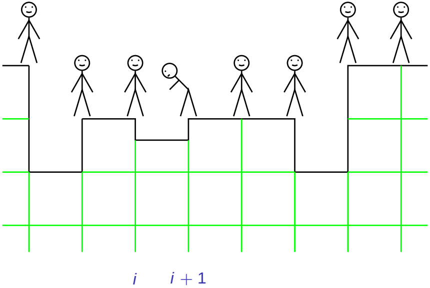

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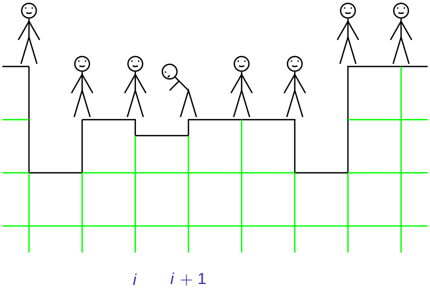

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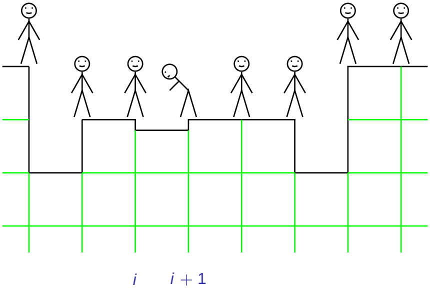

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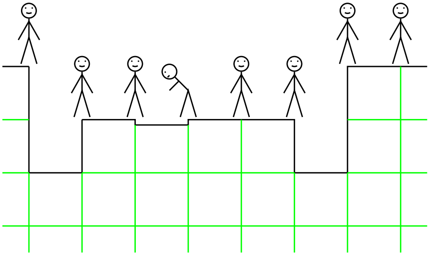

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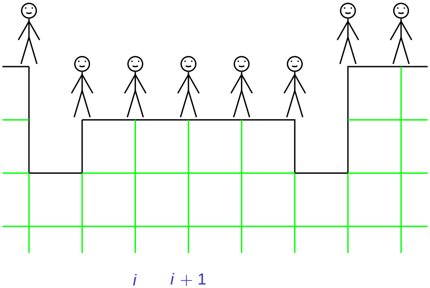

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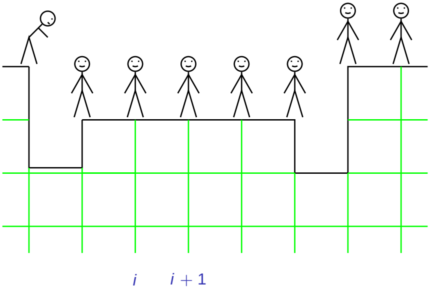

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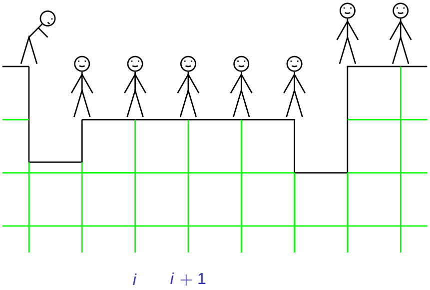

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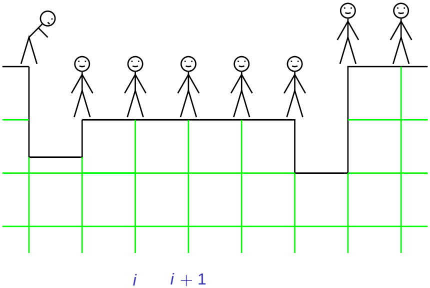

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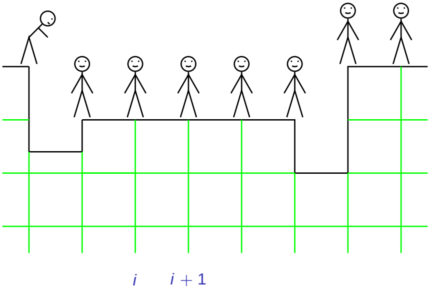

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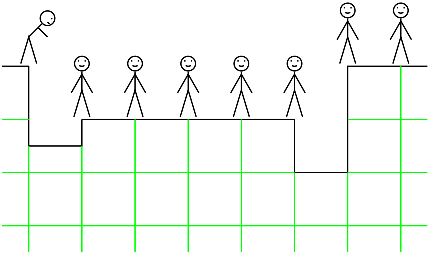

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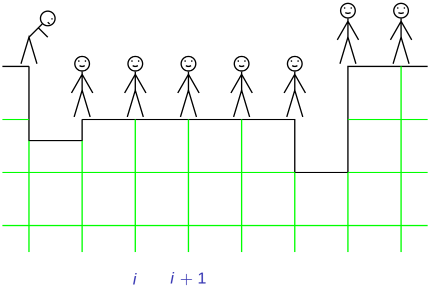

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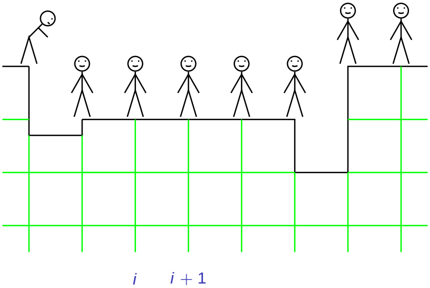

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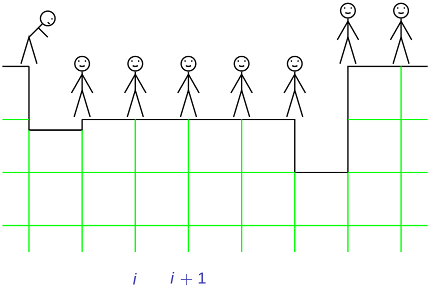

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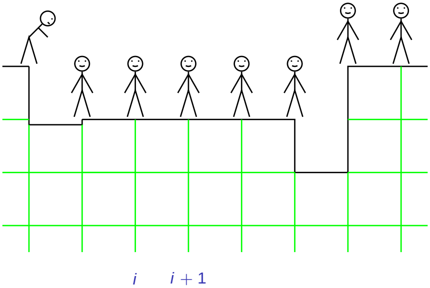

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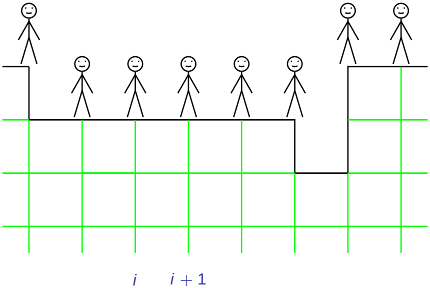

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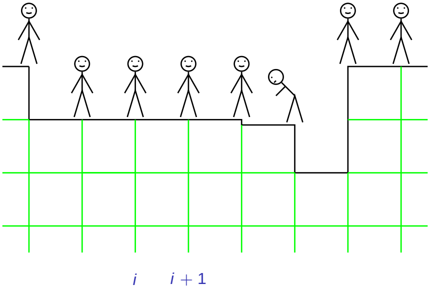

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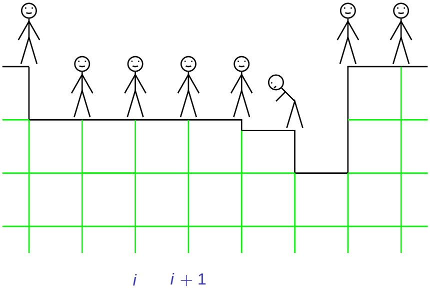

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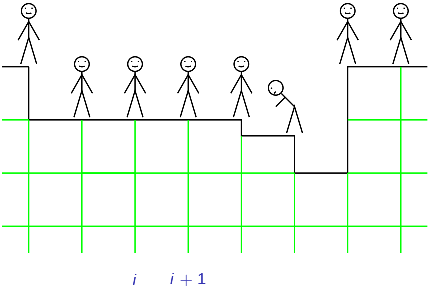

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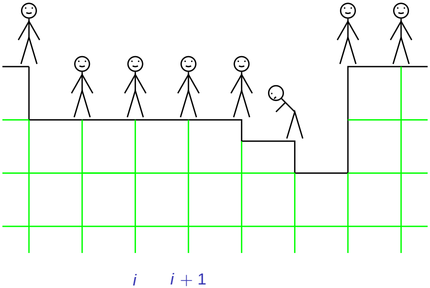

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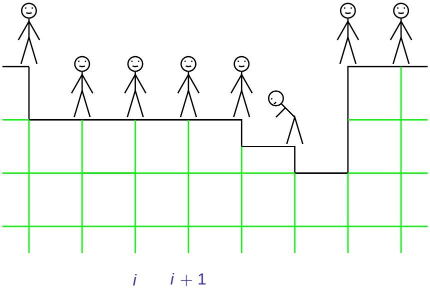

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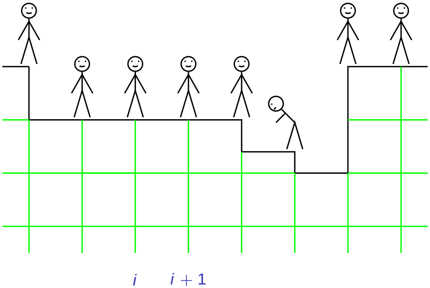

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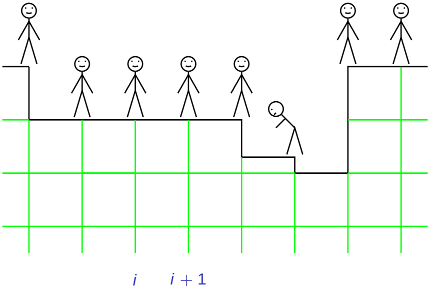

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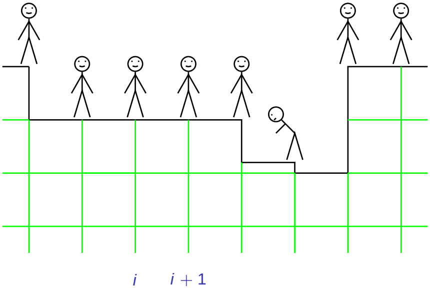

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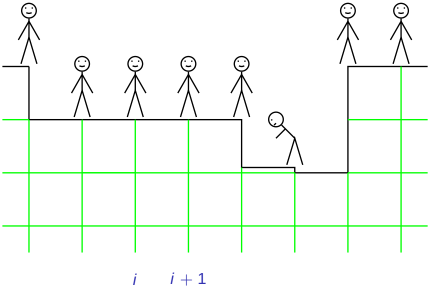

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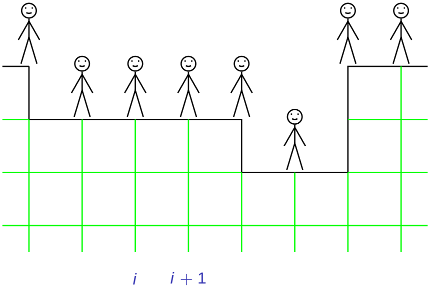

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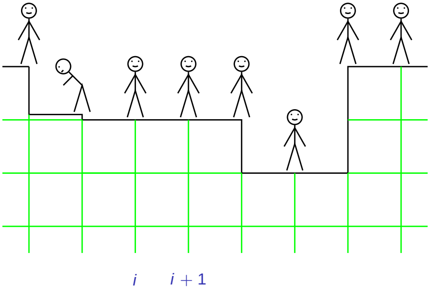

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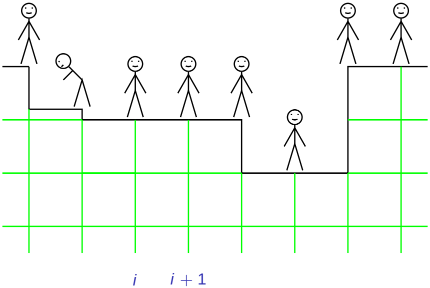

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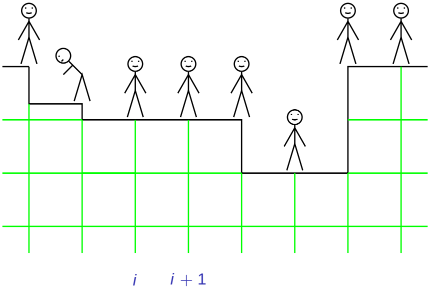

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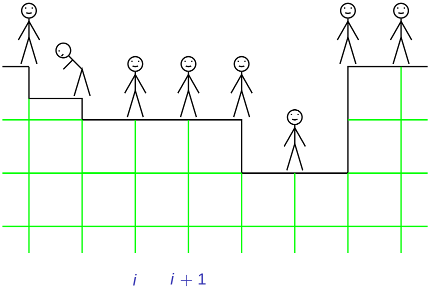

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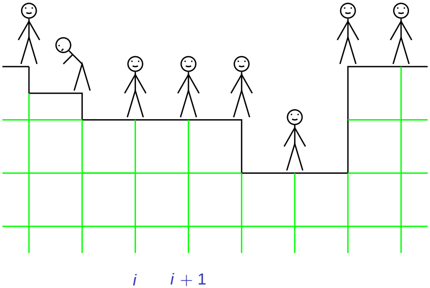

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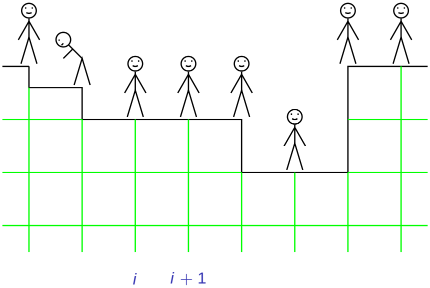

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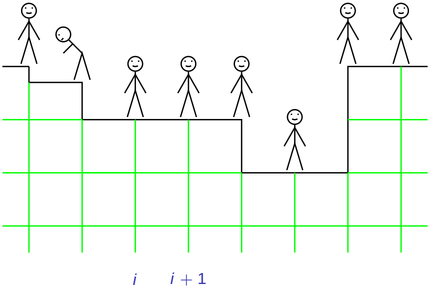

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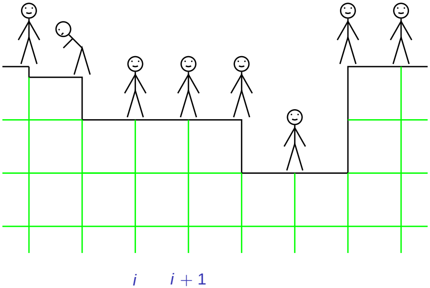

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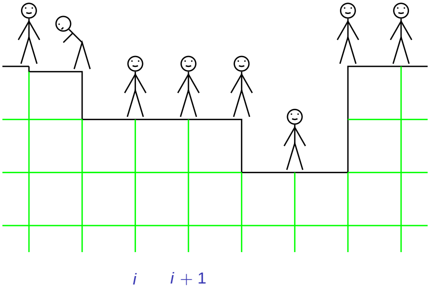

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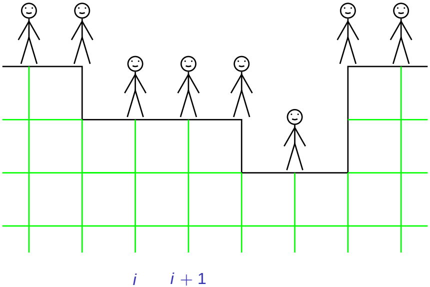

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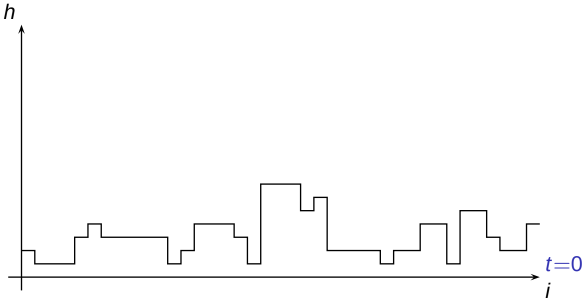

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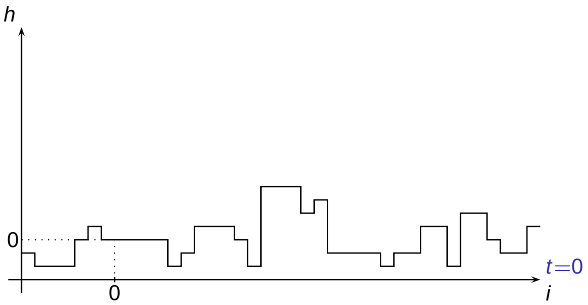

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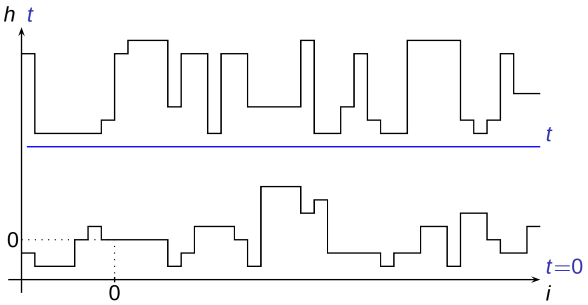

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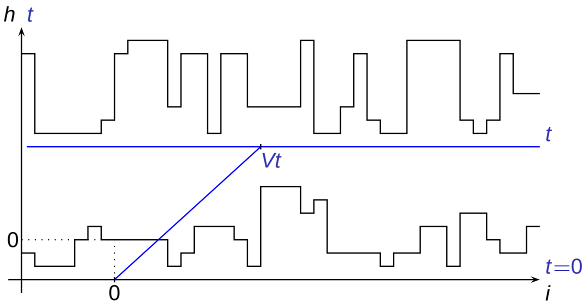

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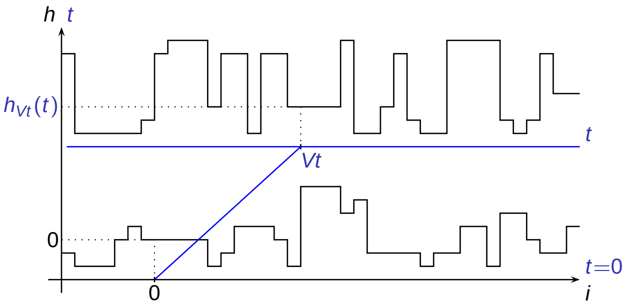

$h_{Vt}(t)$ = height as seen by a moving observer of velocity V.

= net number of particles passing the window $s \mapsto Vs$.

(Remember: particle current=change in height.)

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- The *characteristics* is a path $X(T)$ where $\varrho(T, X(T))$ is constant.

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So, $\dot{X}(T) = H'(\varrho) =: C$ is the *characteristic speed*.

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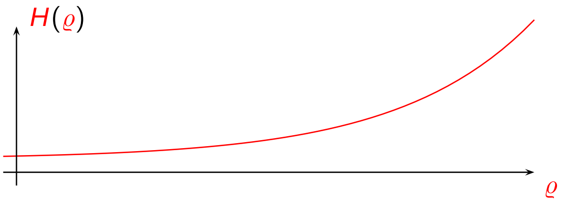

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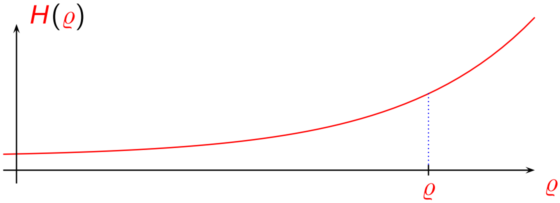

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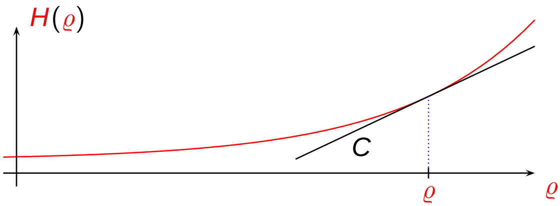

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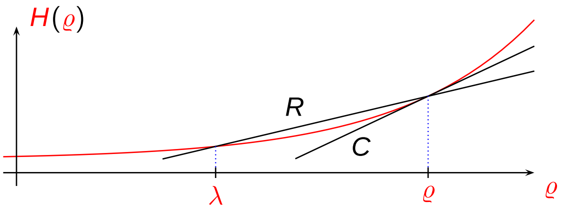

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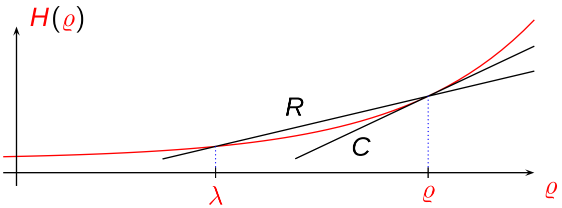

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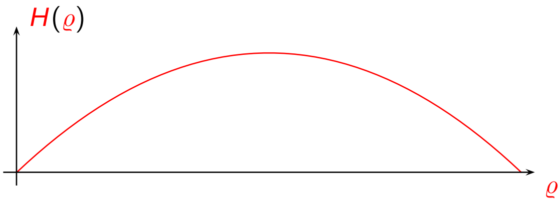

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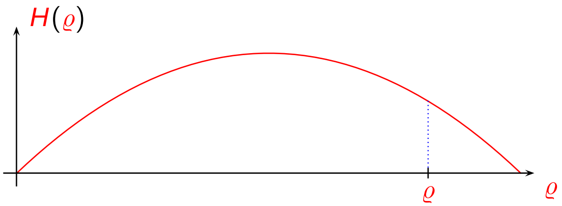

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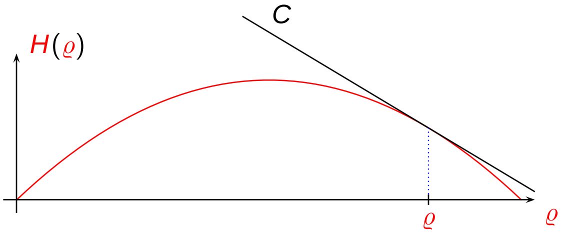

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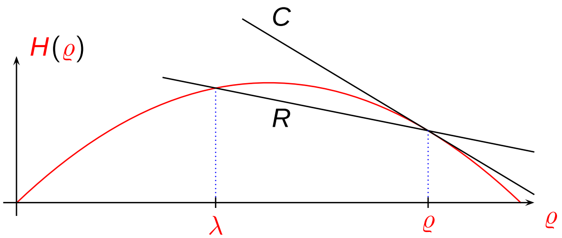

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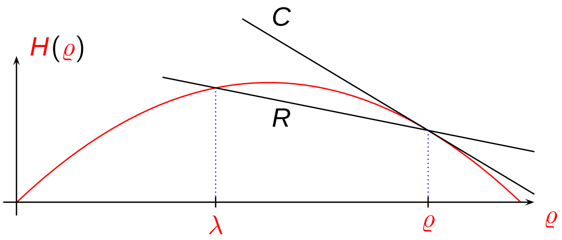

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Tool: the second class particle

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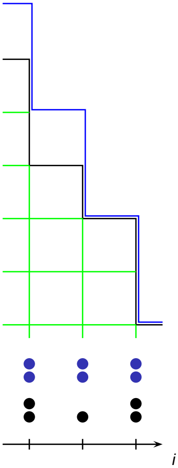

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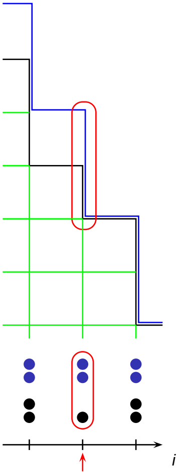

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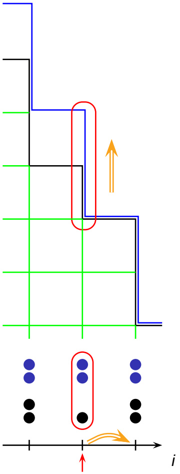

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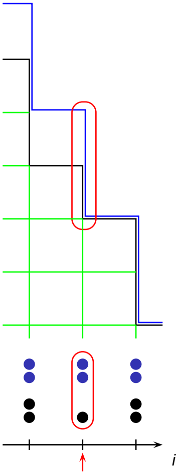

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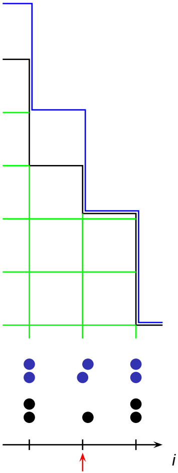

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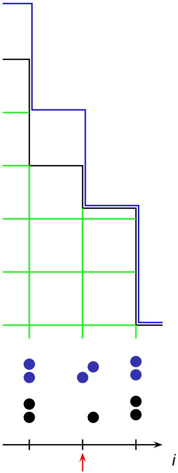

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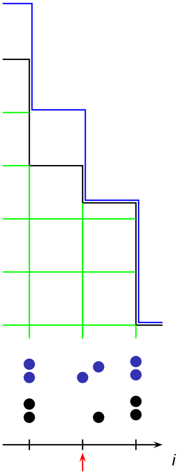

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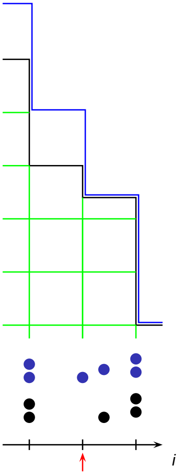

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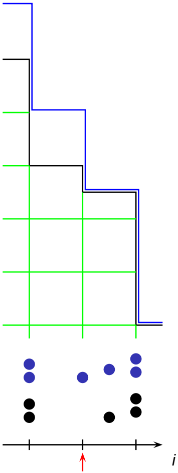

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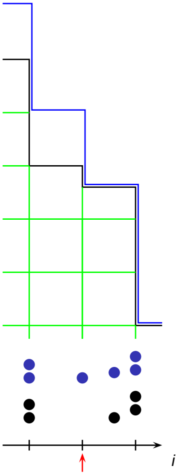

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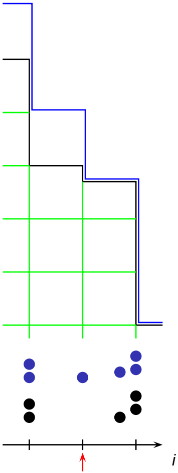

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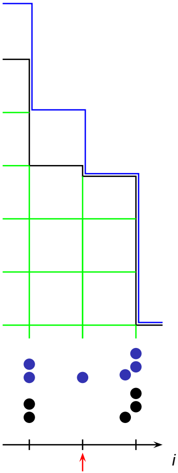

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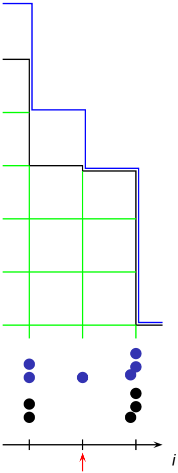

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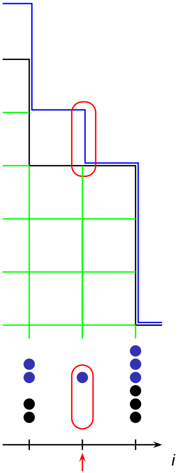

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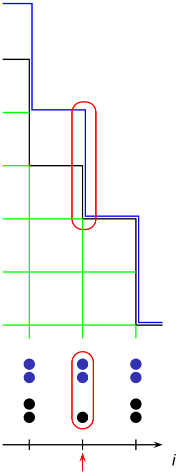

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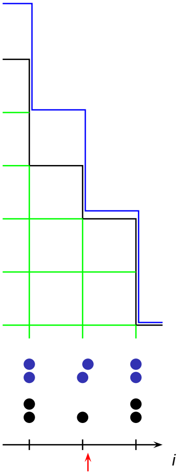

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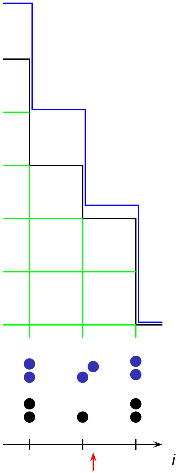

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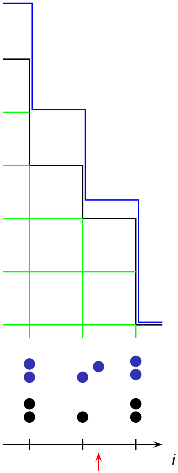

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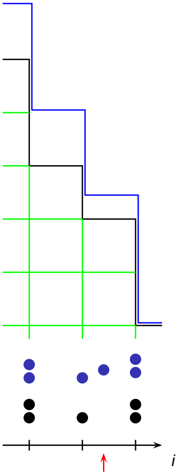

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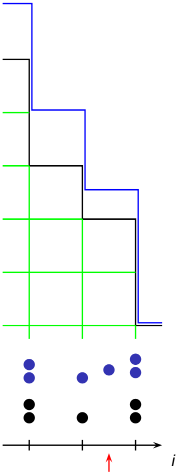

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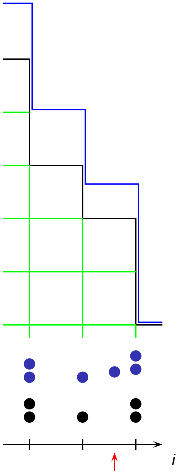

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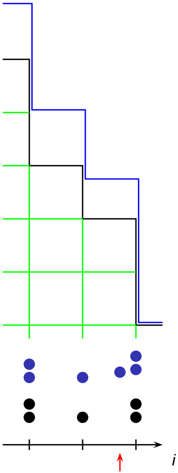

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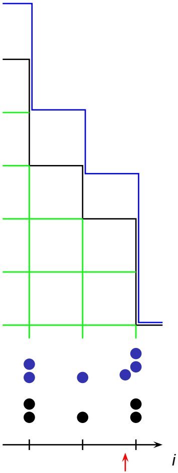

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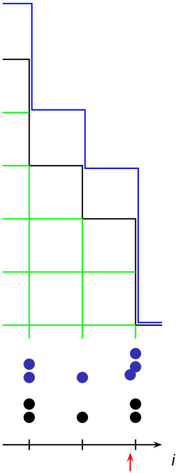

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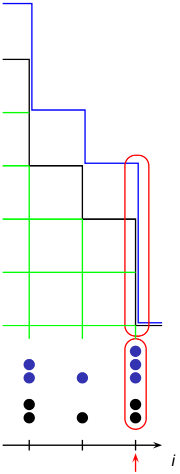

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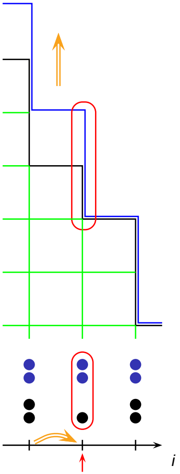

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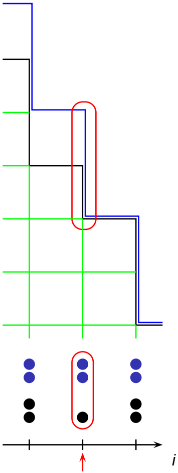

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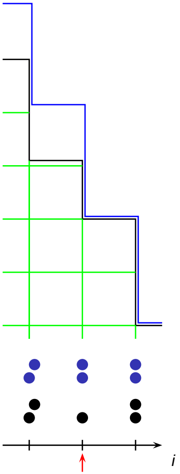

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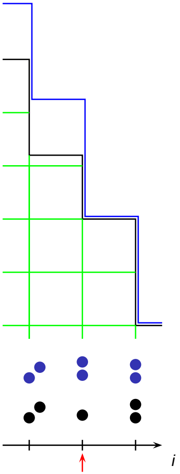

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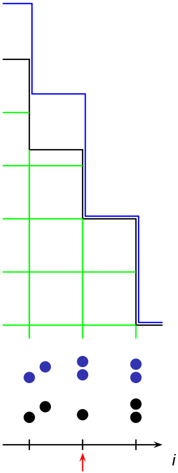

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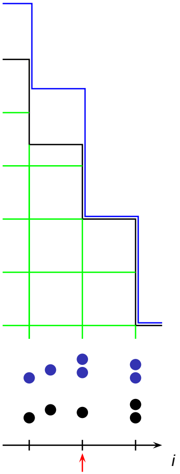

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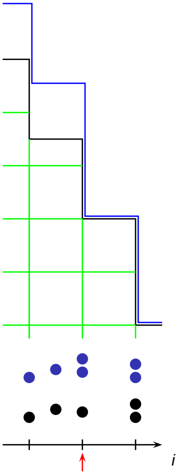

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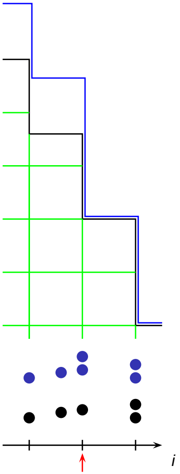

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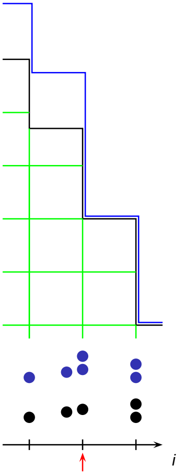

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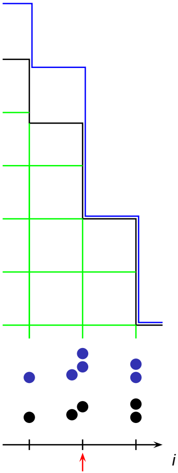

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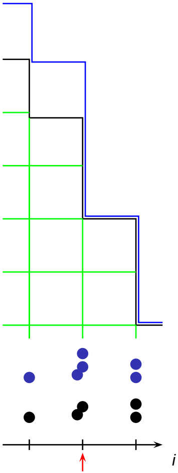

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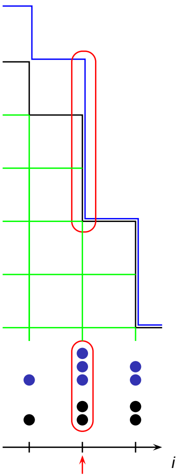

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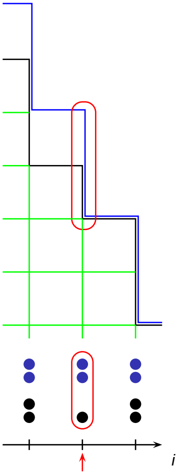

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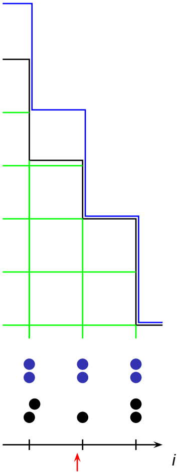

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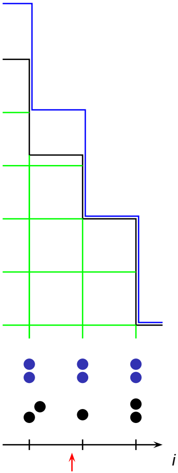

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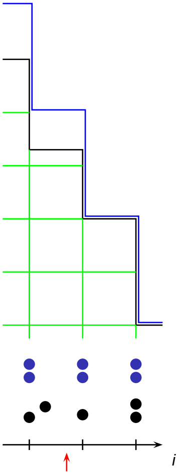

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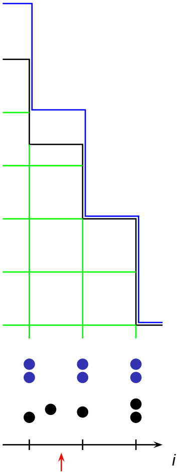
with rate-rate:



Tool: the second class particle

States ω and ω only differ at one site.

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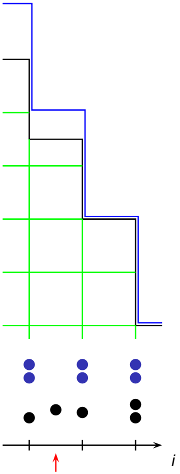

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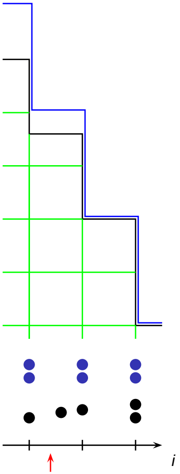

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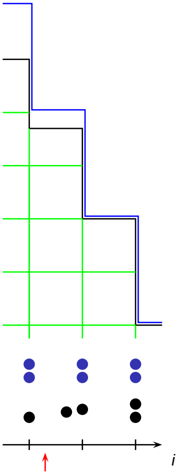

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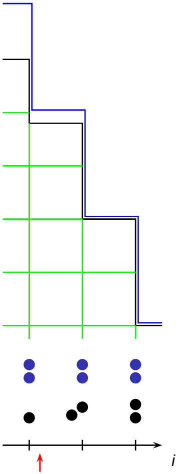

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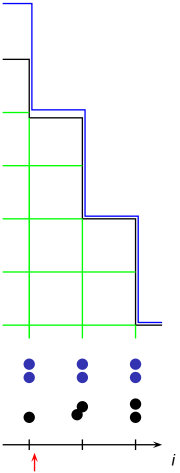

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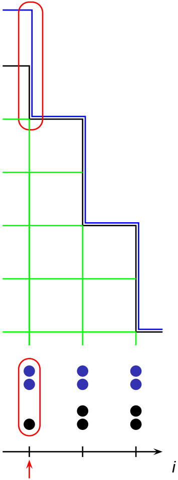

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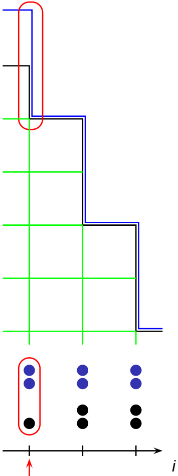

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A single discrepancy↑, the *second class particle*, is conserved.
Its position at time t is $Q(t)$.

Tool: the second class particle

Theorem (B. - Seppäläinen; also ideas from B. Tóth, H. Spohn and M. Prähofer)

Started from (almost) equilibrium,

$$\mathbf{E}(Q(t)) = C \cdot t$$

in the whole family of processes.

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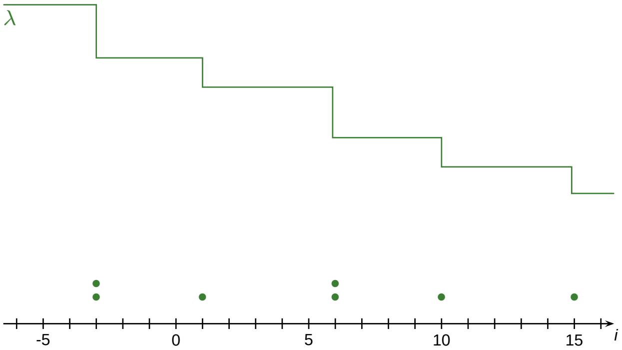

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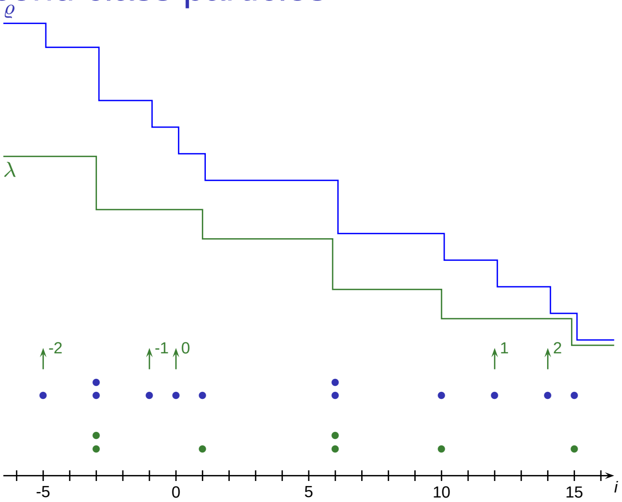

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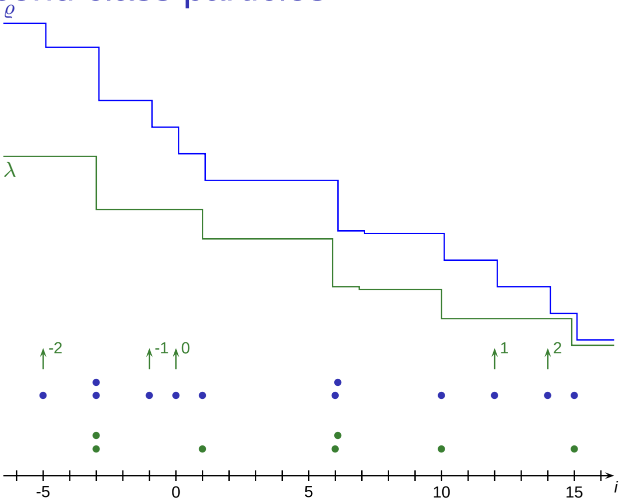

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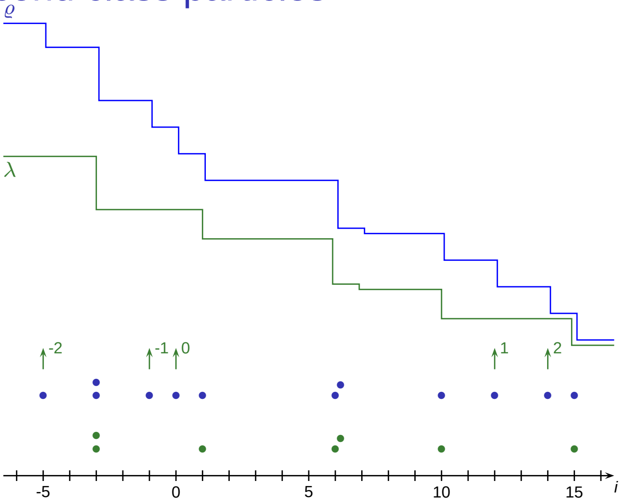

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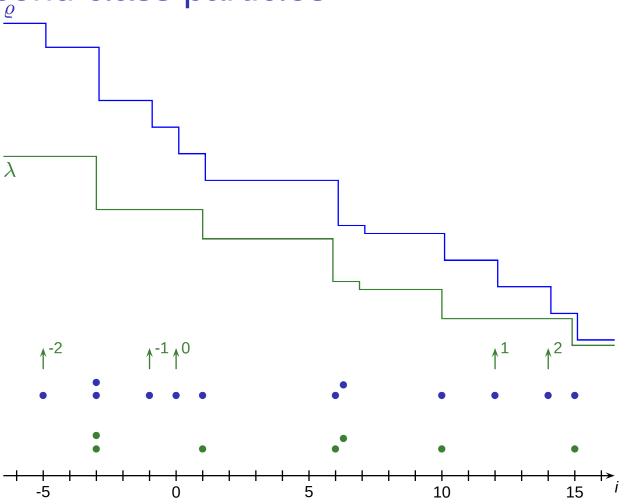

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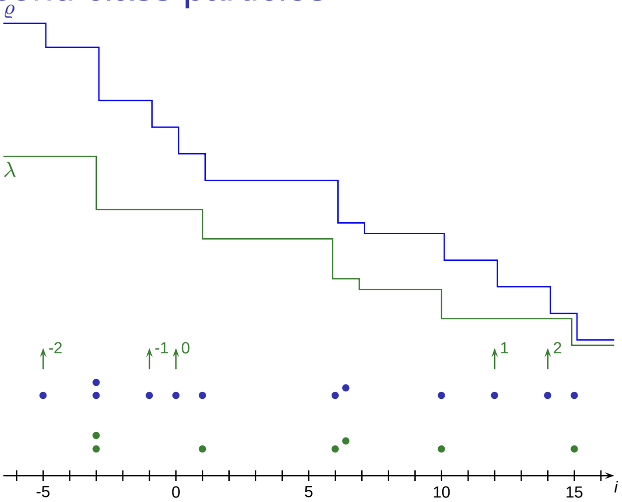

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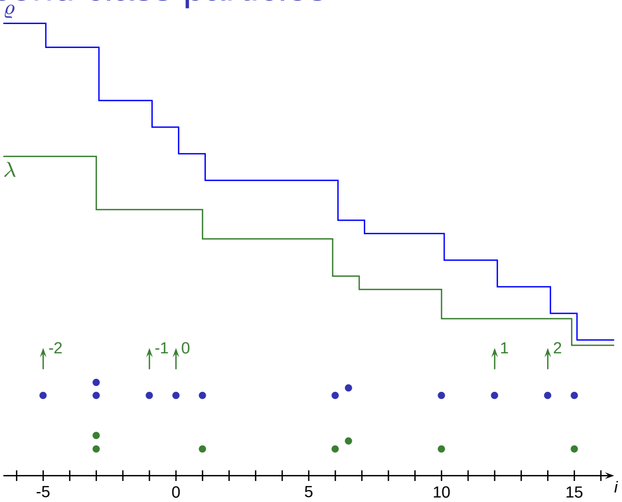

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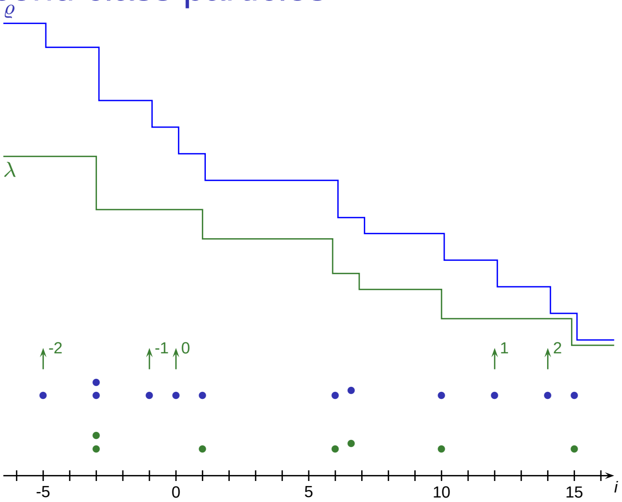

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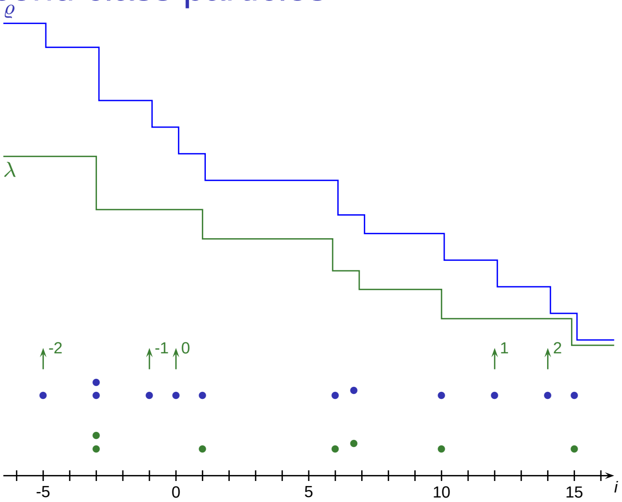

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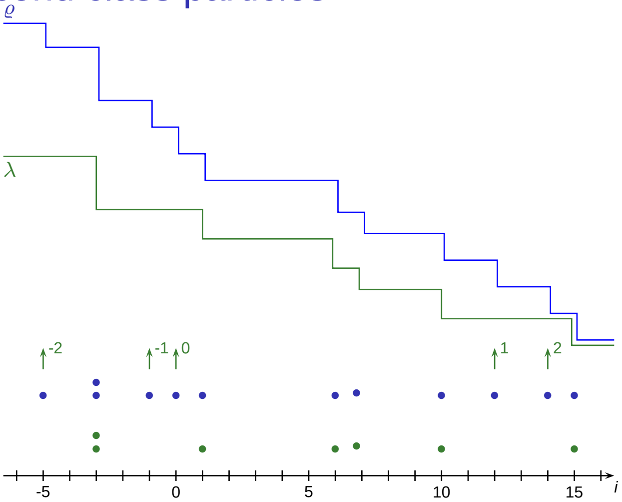

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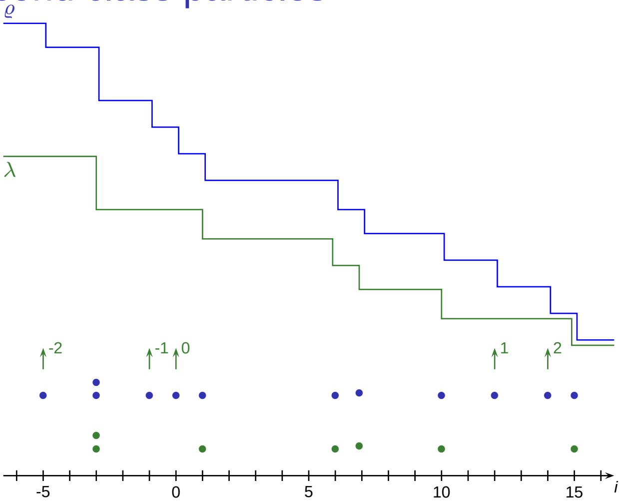

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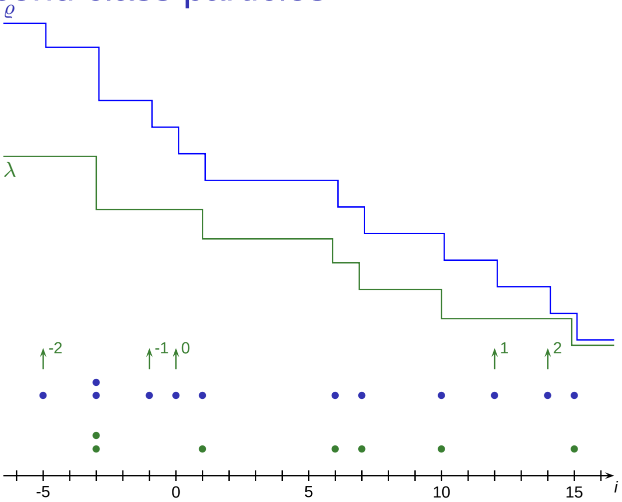

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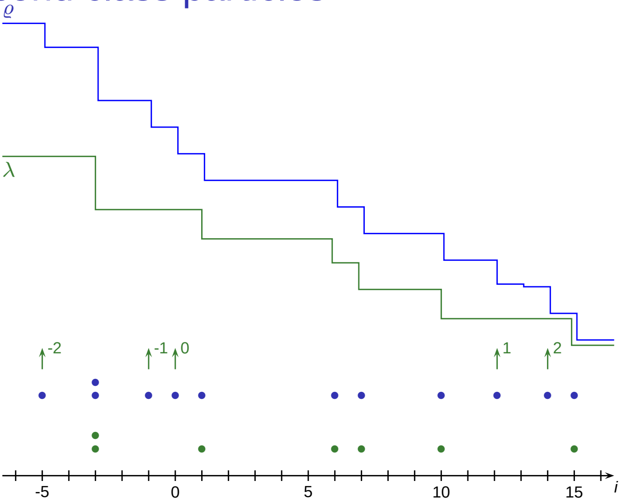

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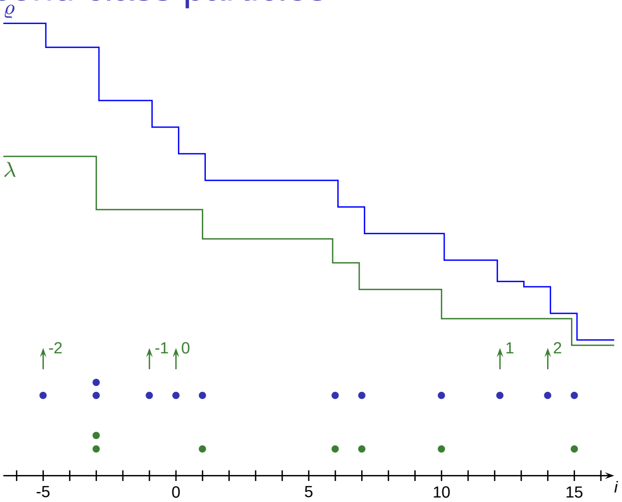

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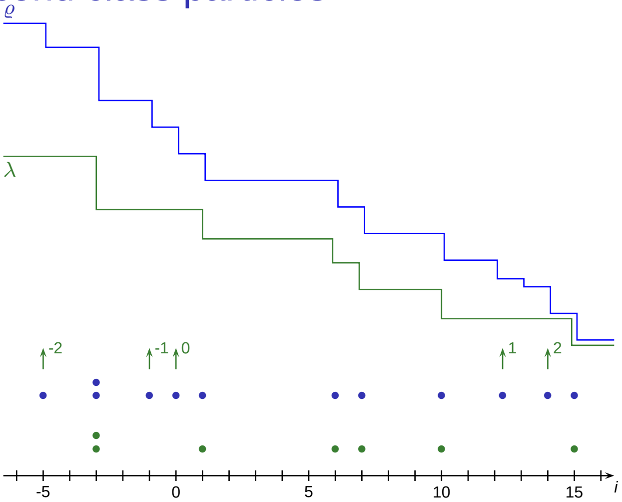

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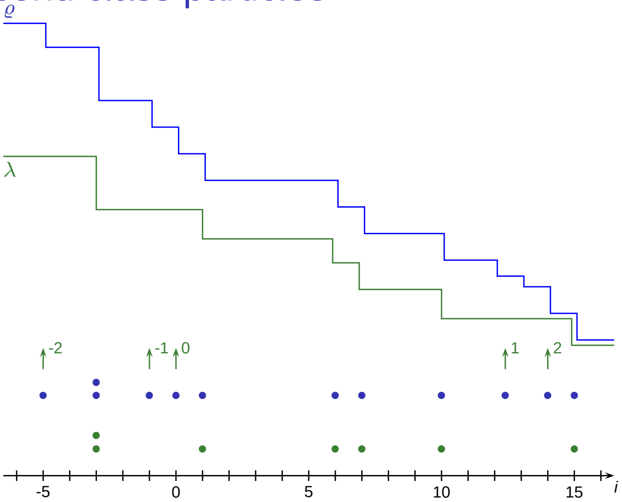

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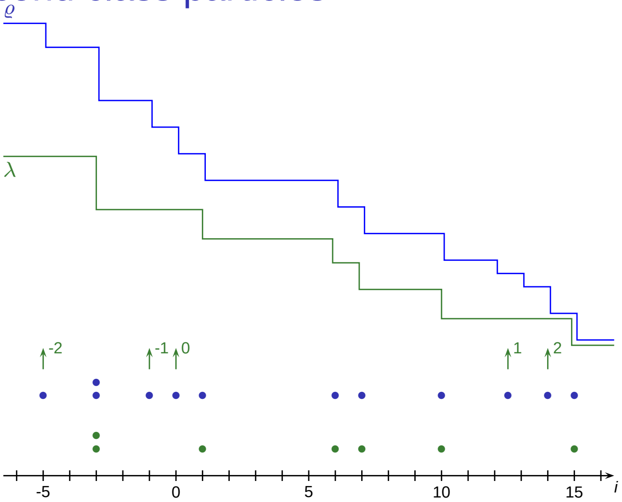

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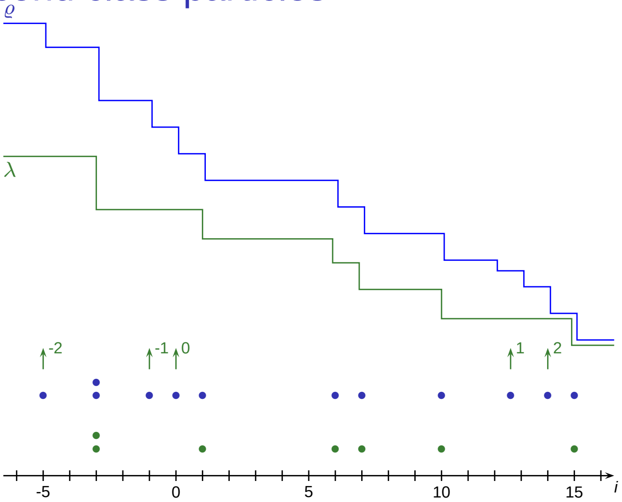

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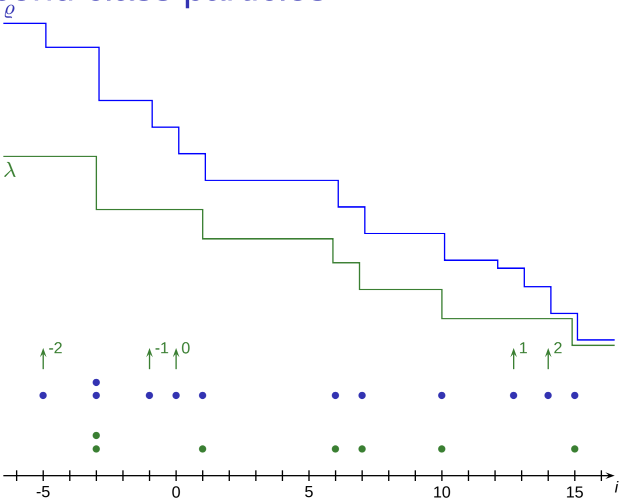

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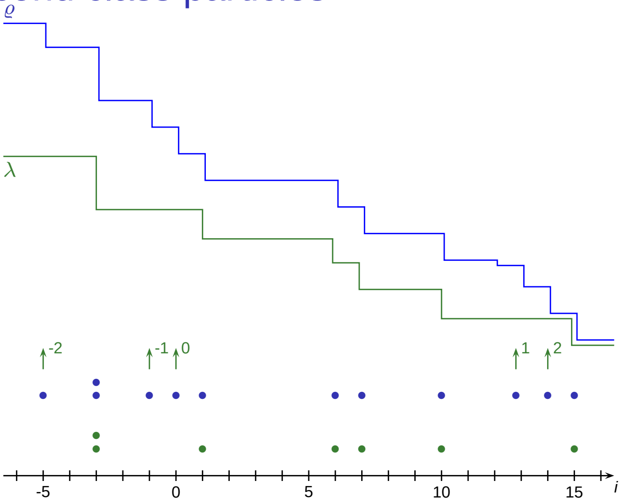

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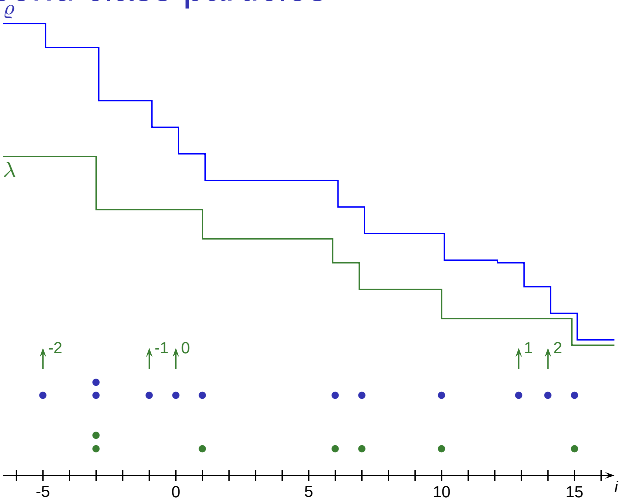

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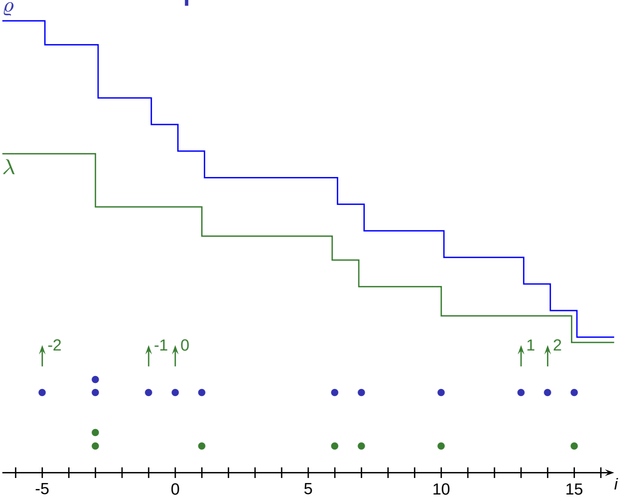

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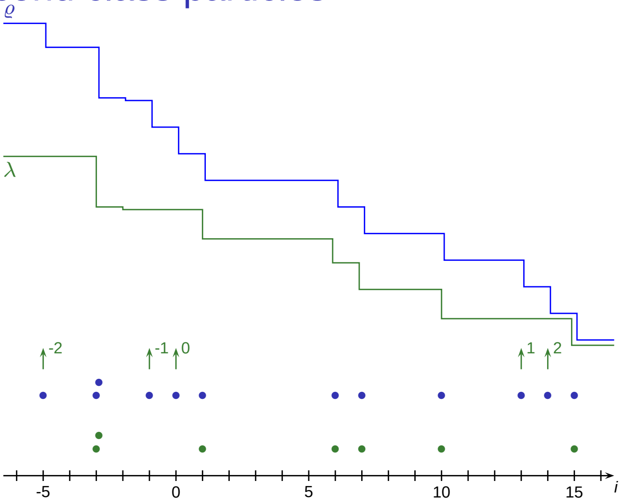

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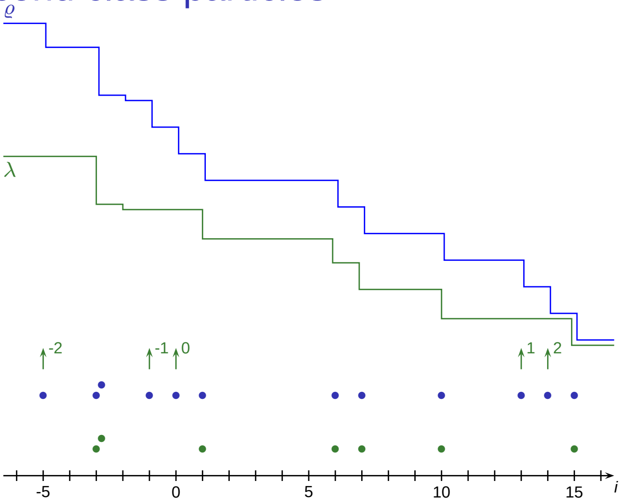

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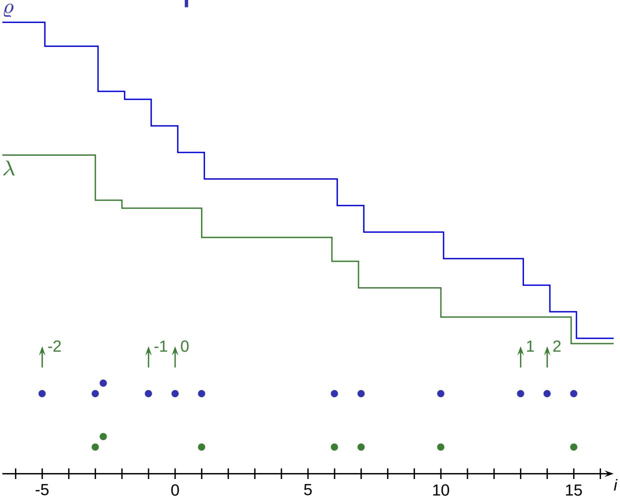

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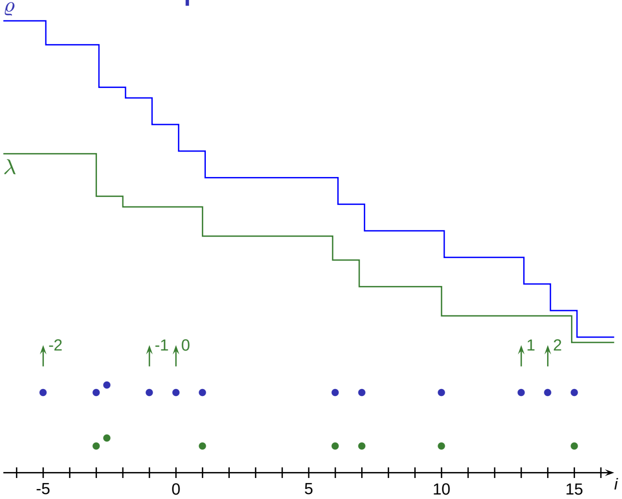

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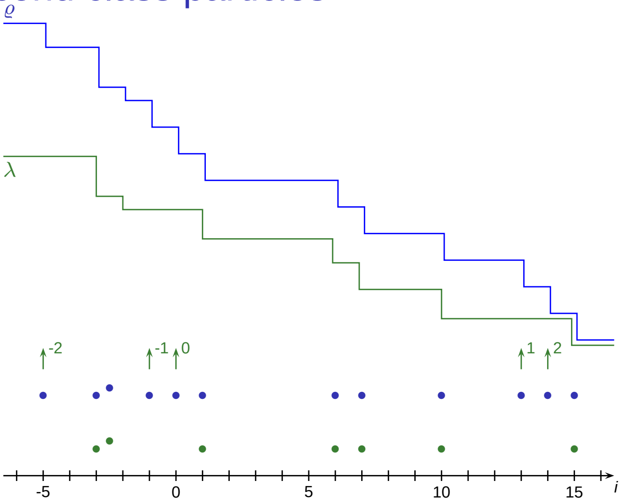

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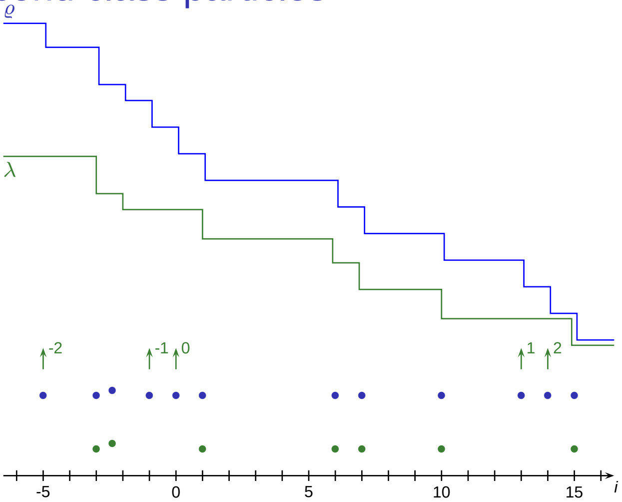

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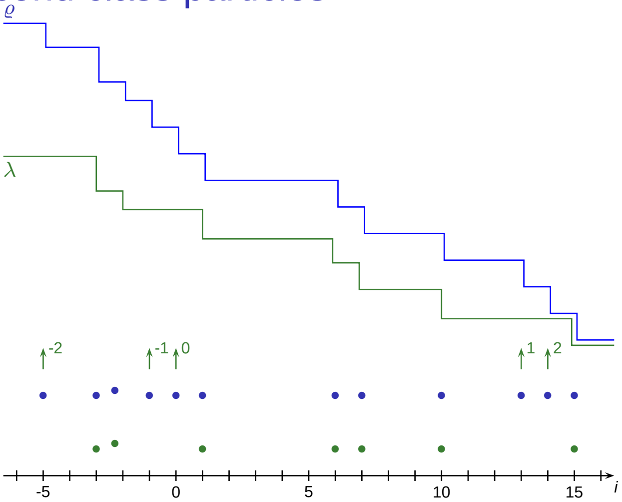

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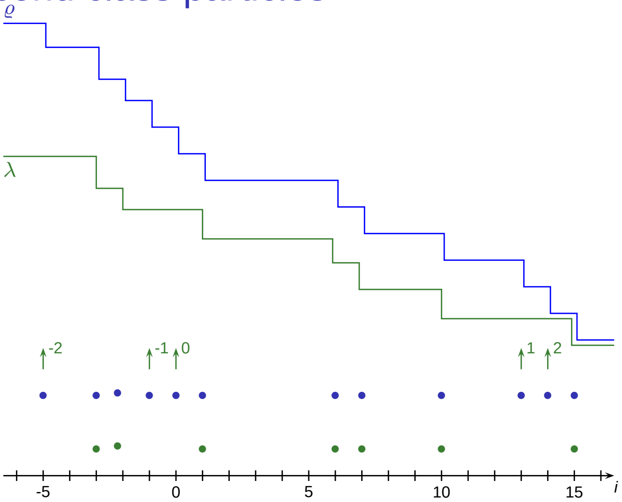

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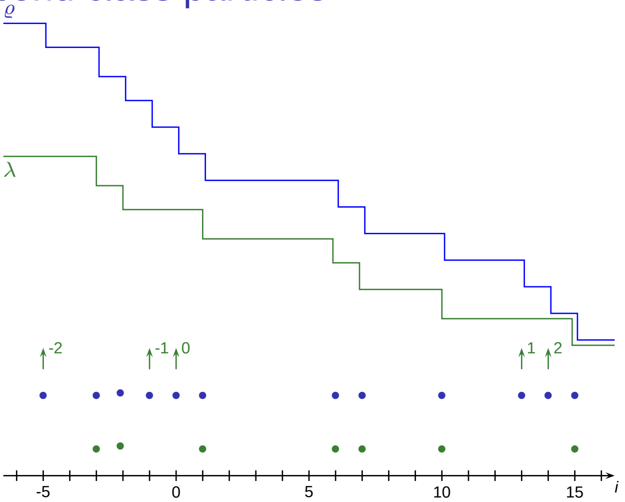

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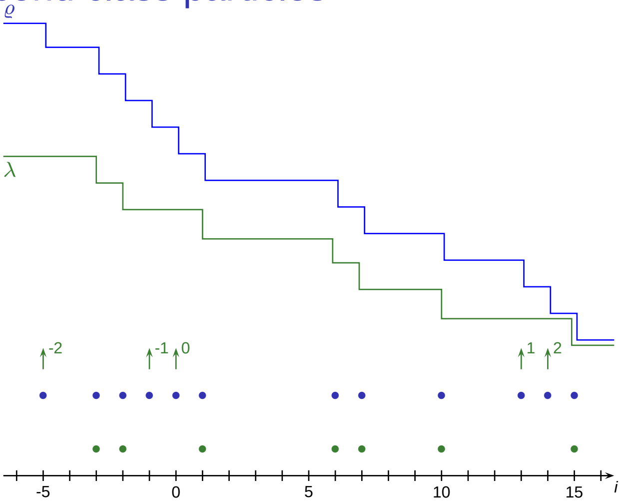

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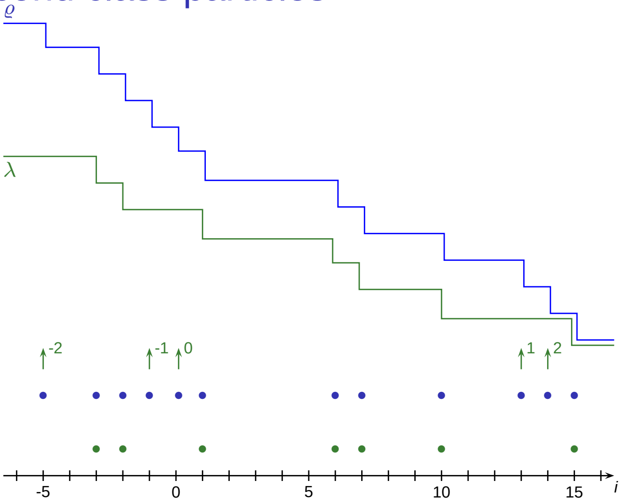

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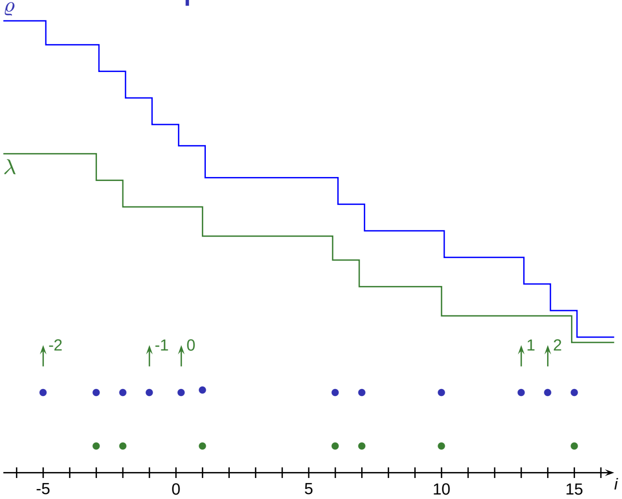

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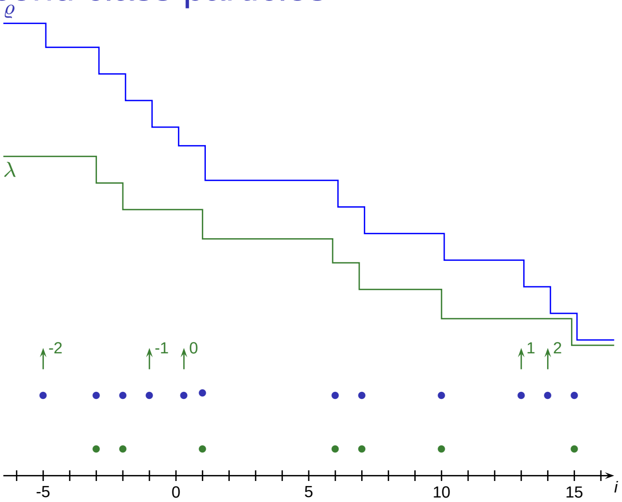

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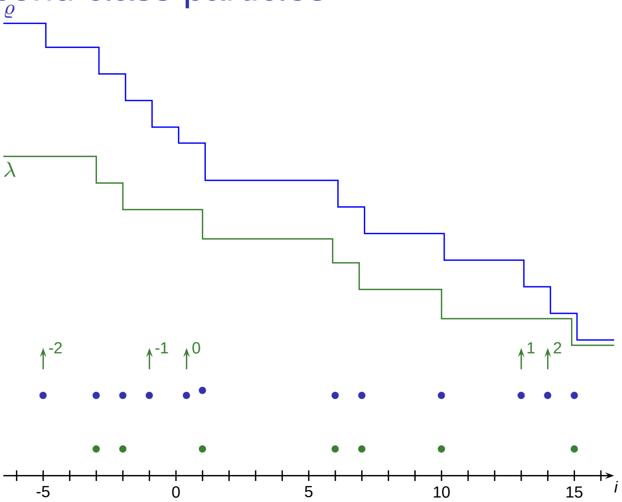

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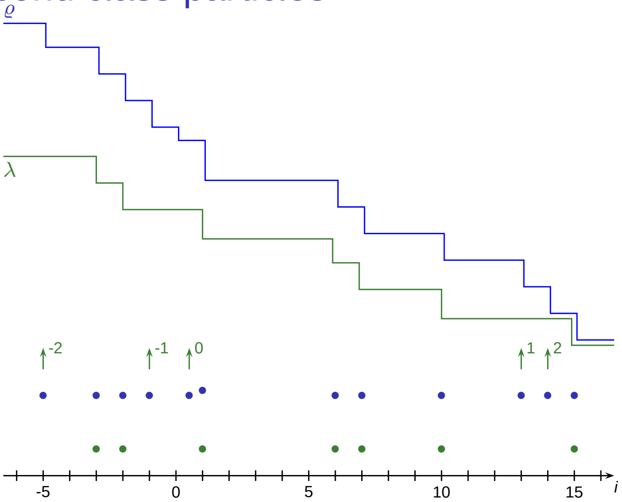

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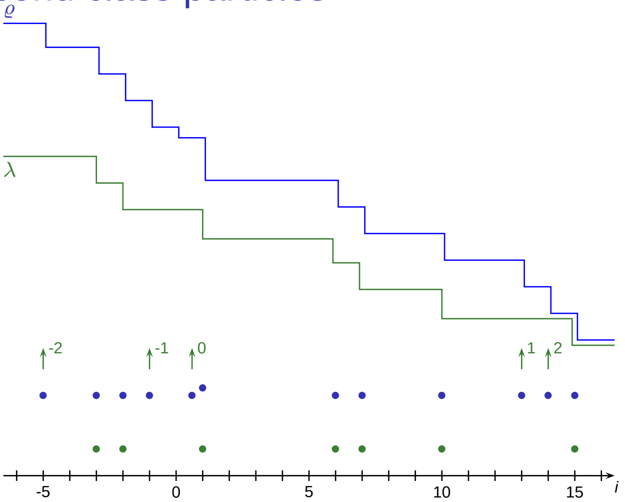

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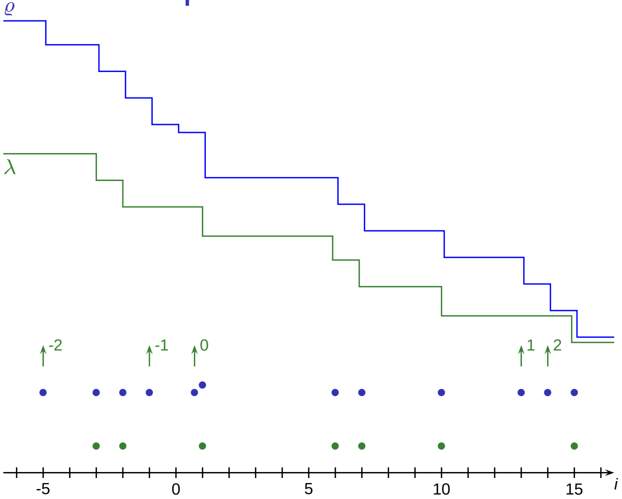

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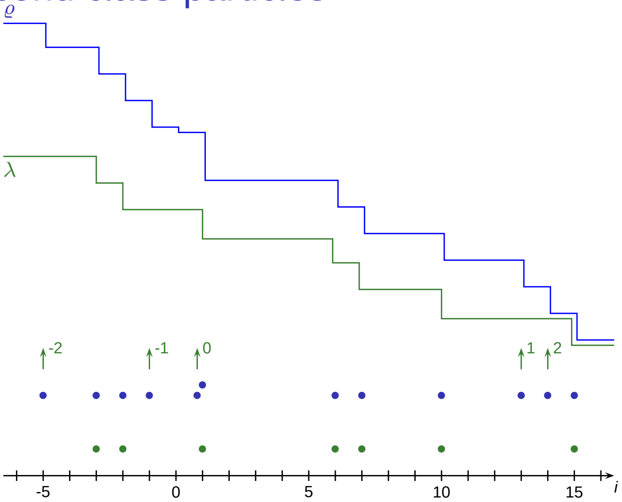

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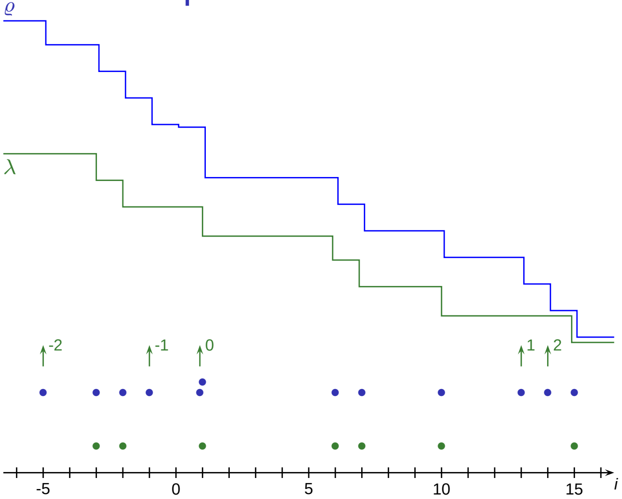

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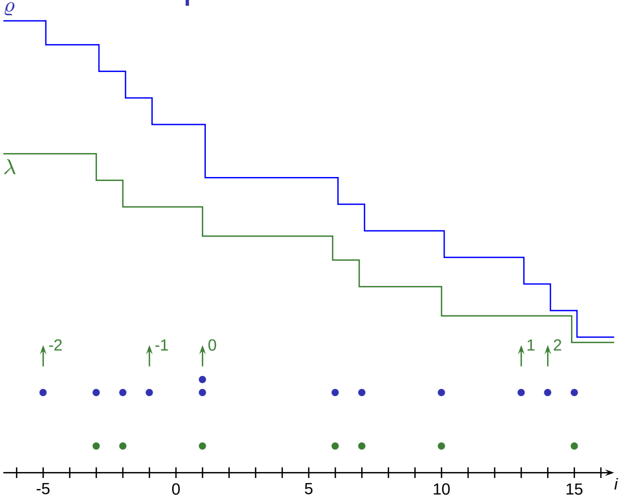

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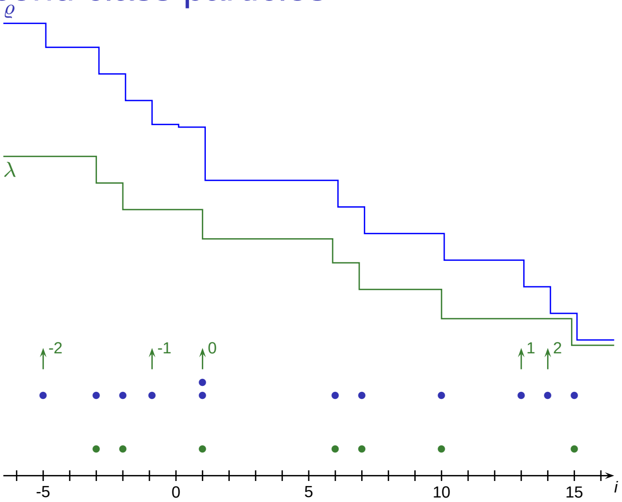

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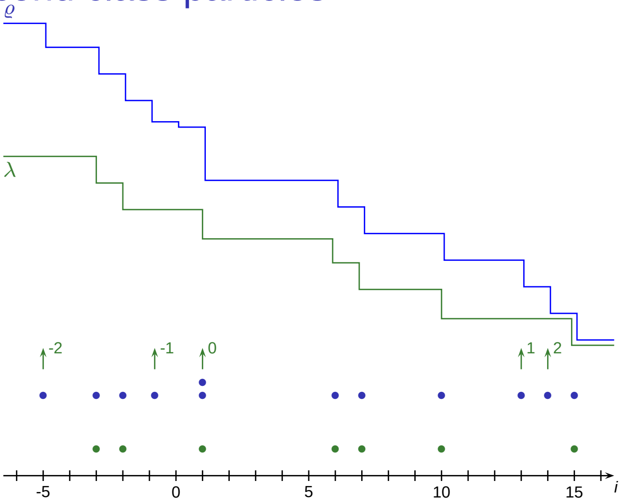

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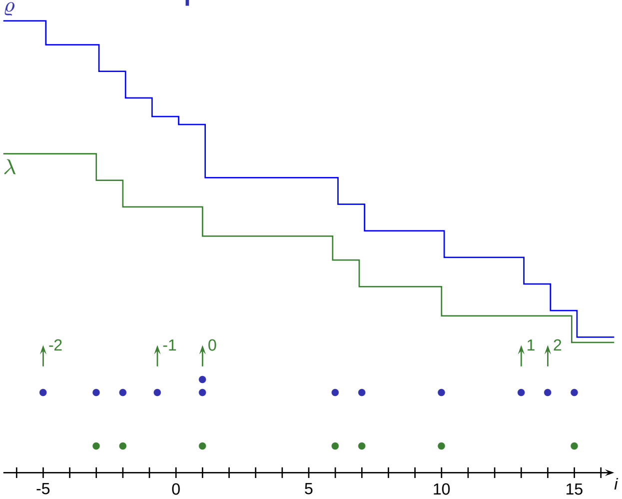

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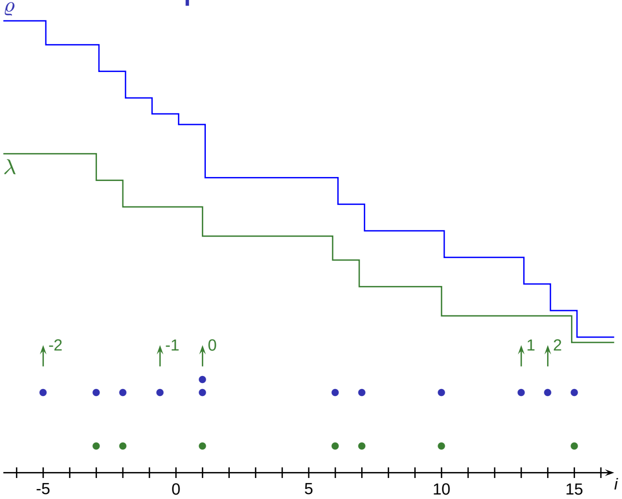

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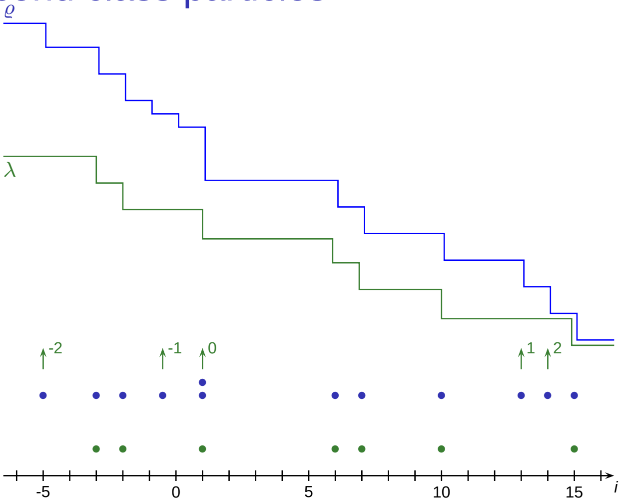

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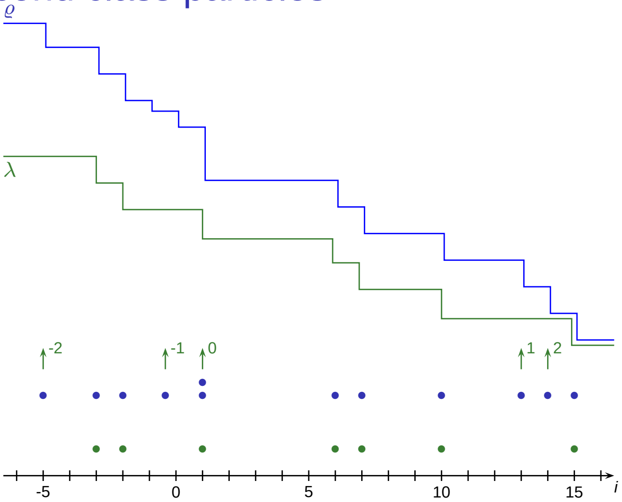

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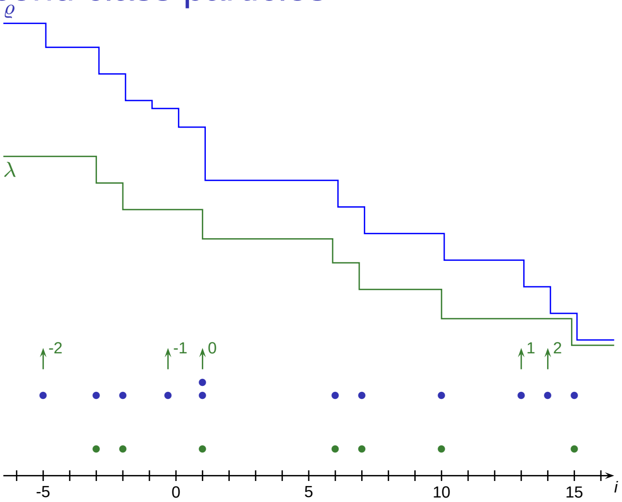

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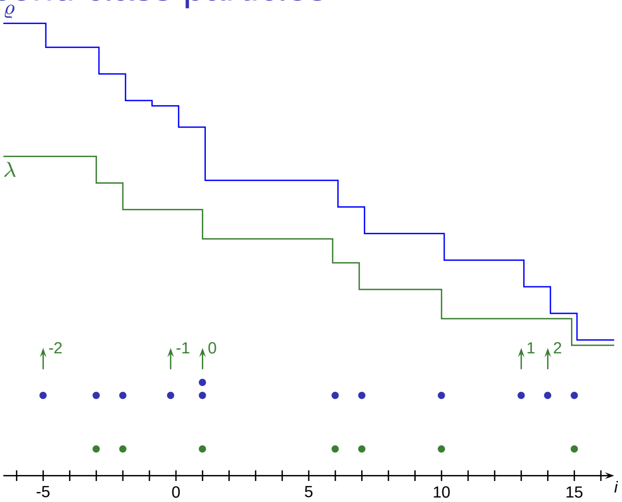

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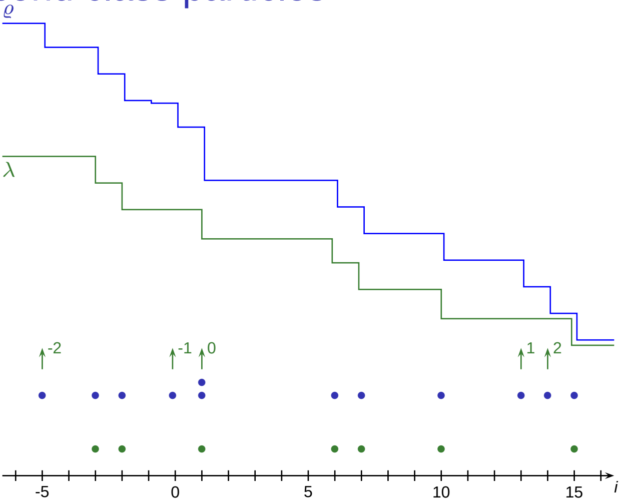

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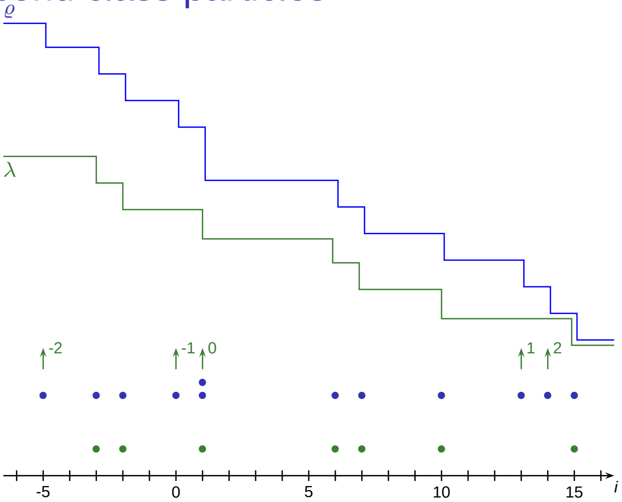

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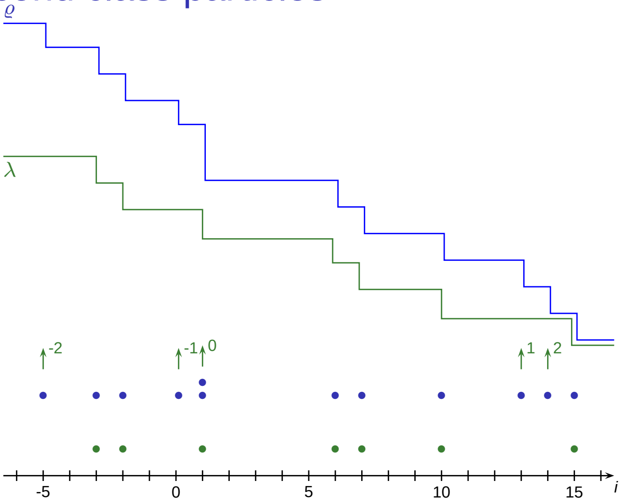

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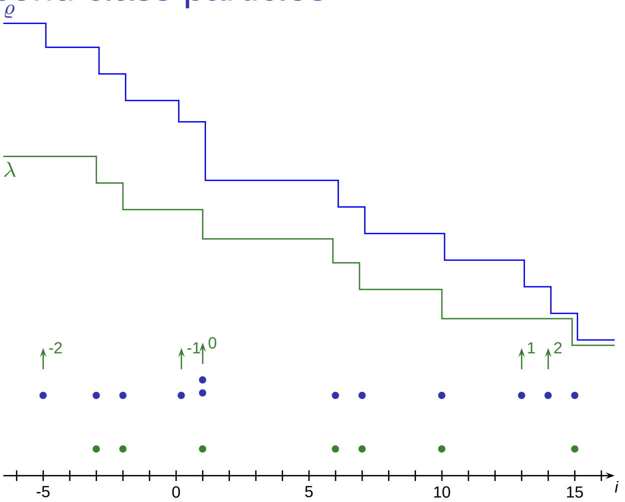

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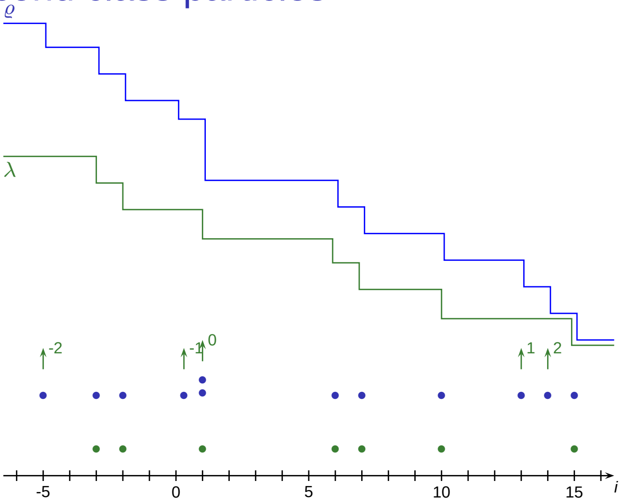

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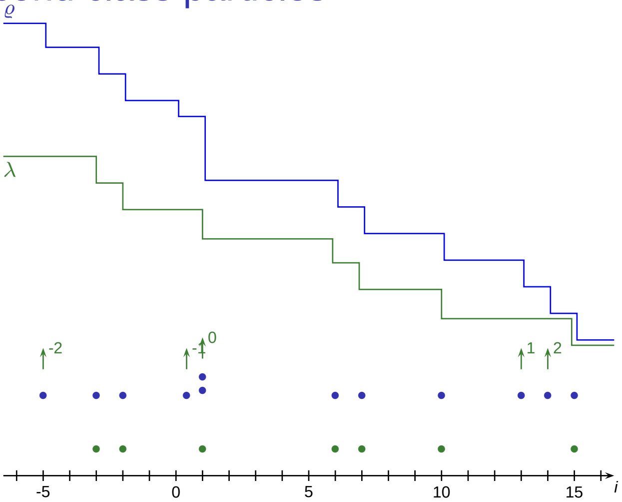

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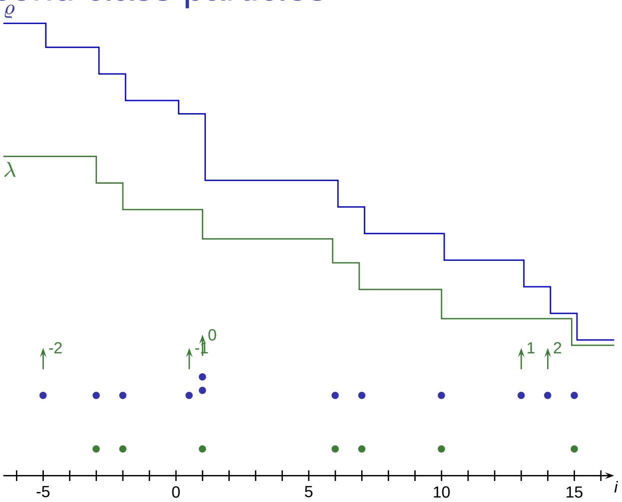

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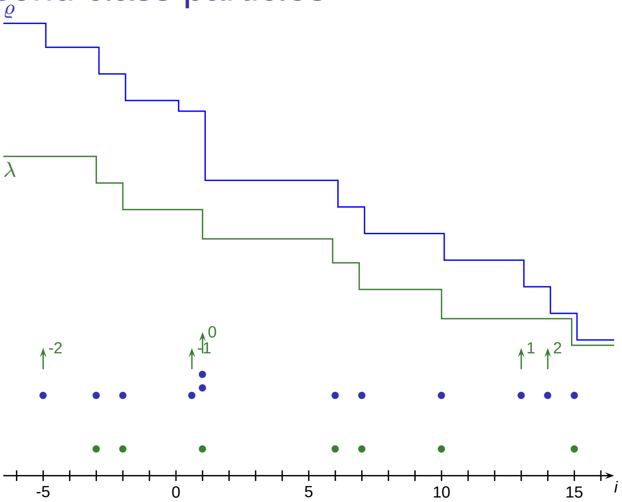

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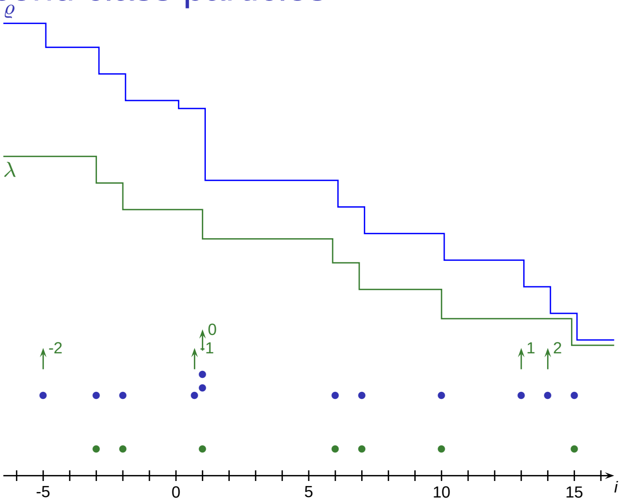

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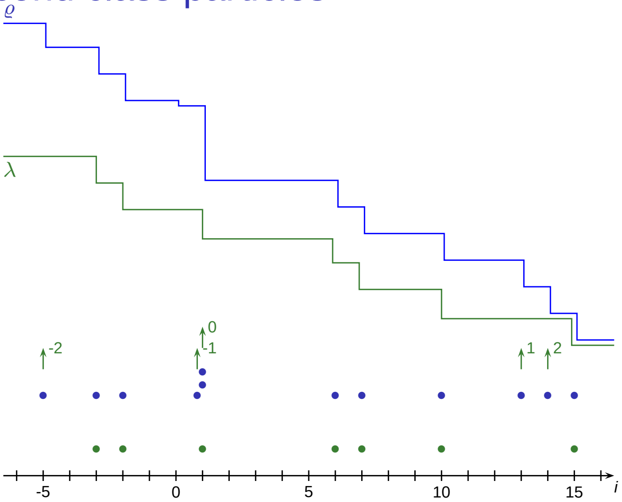

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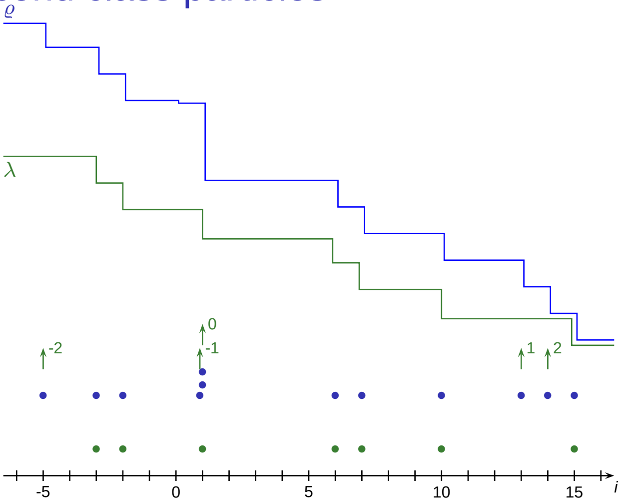

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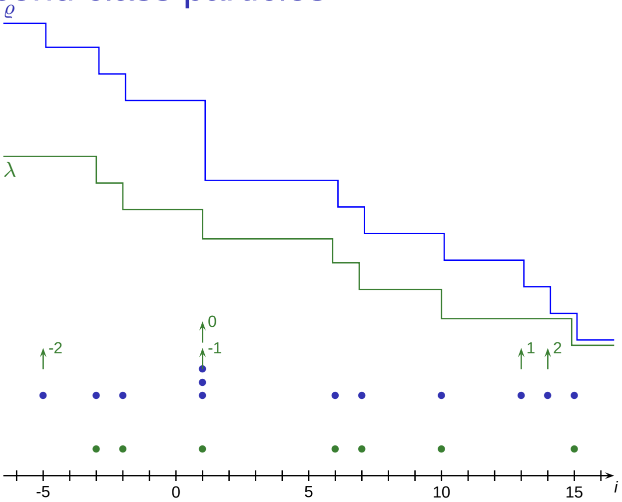

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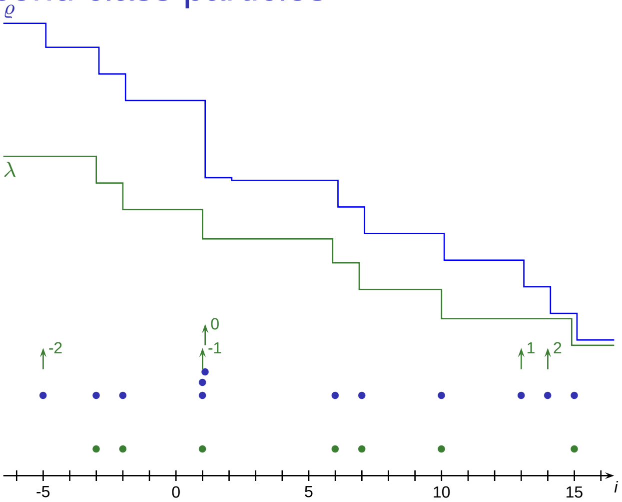

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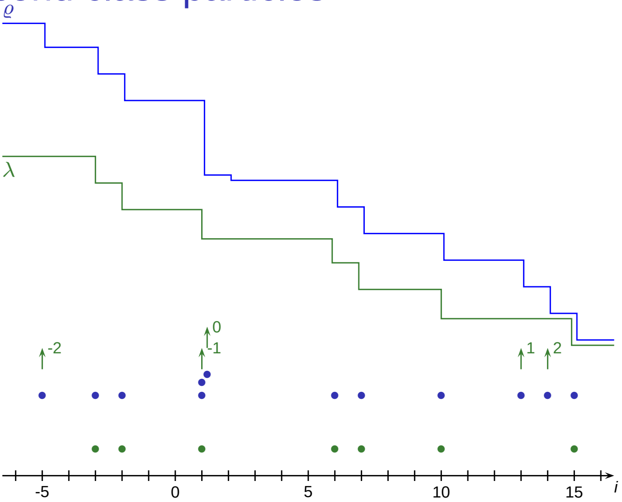

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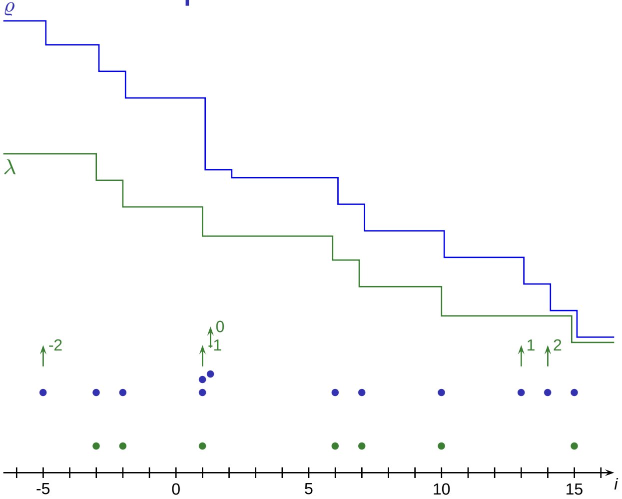

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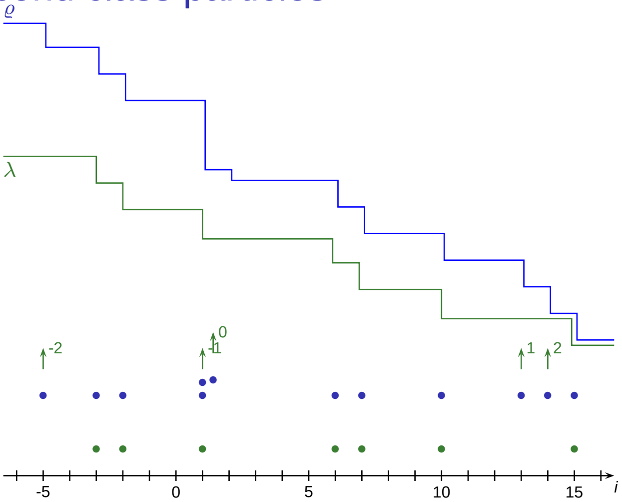

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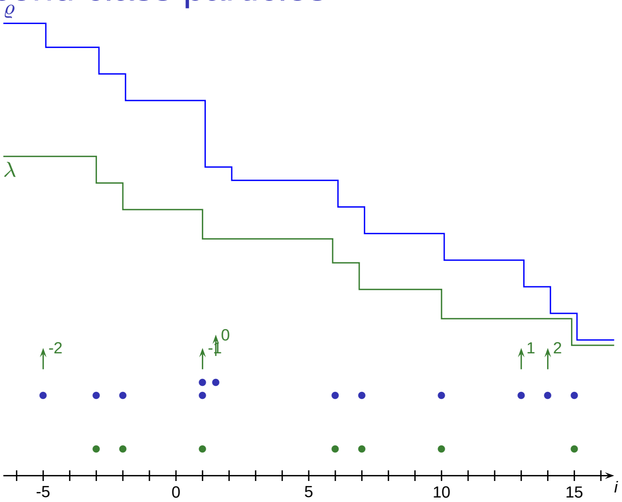

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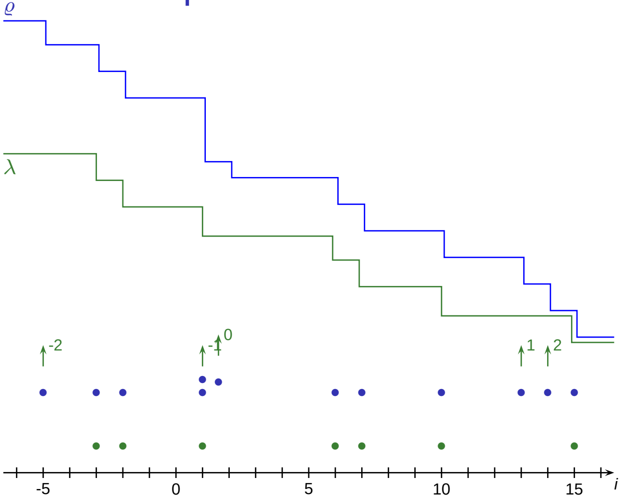

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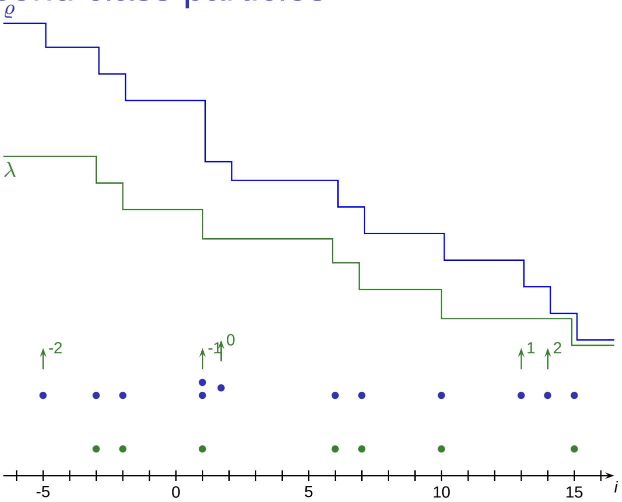

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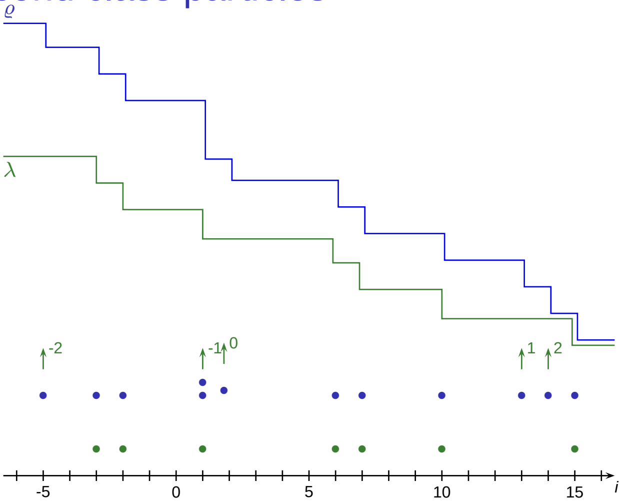

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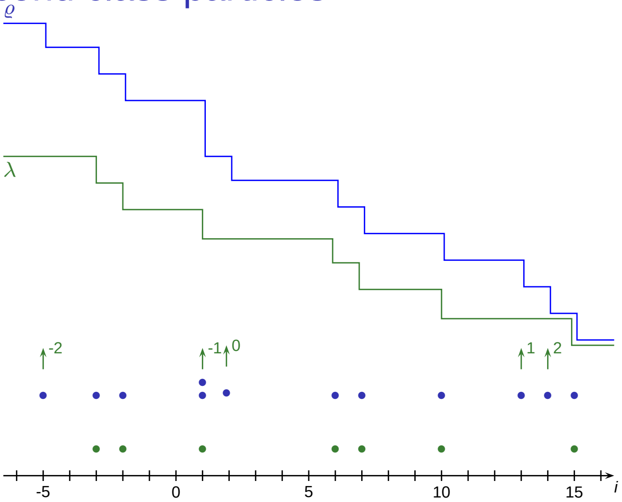

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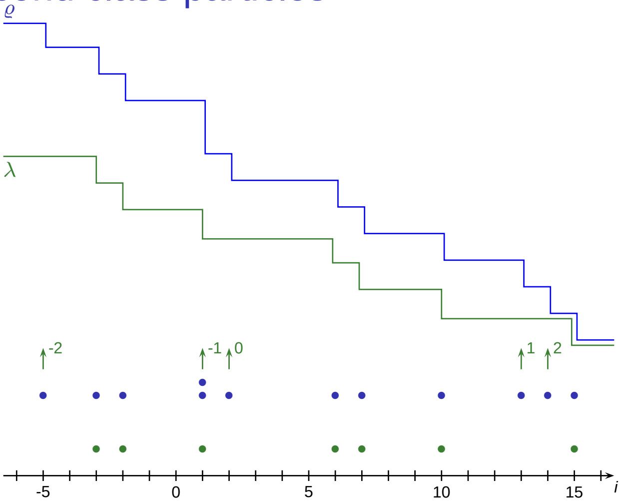

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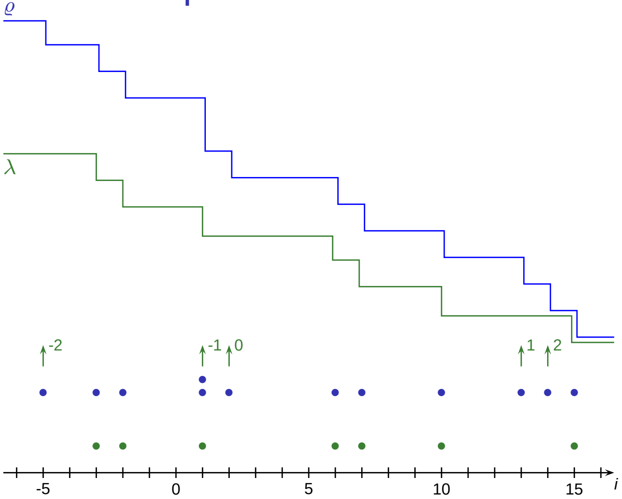

Picture:

The position $X(t)$ of $\uparrow^0$ follows the Rankine-Hugoniot speed R.

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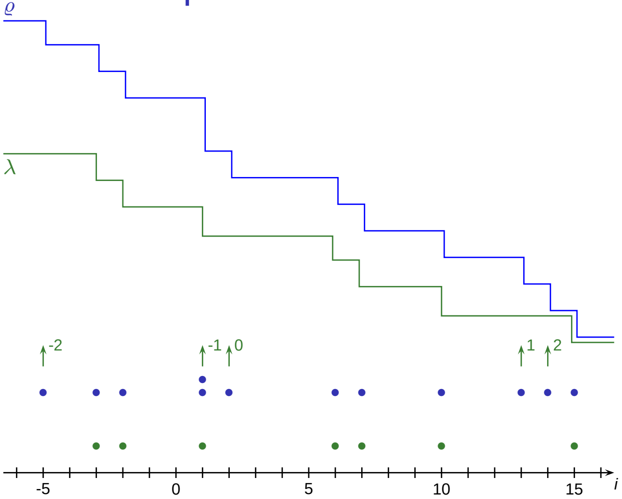

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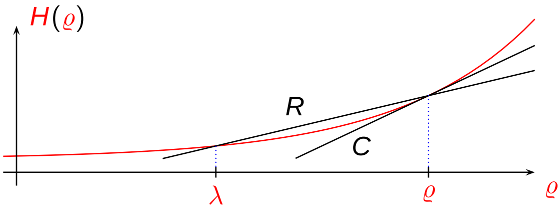
Characteristics (very briefly)

Convex flux (some cases of AZRP, ABLP):



Recall $$C = H'(\varrho) > R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

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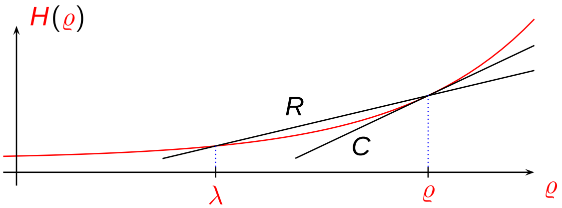

Recall $C = H'(\varrho) > R = \dfrac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$

Do we have $Q(t) \overset{?}{\geq} X(t)$

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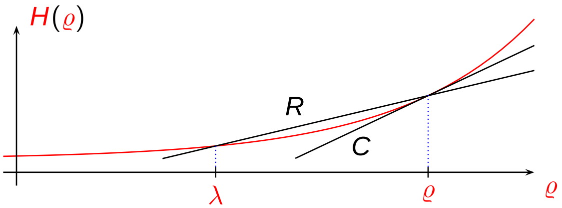

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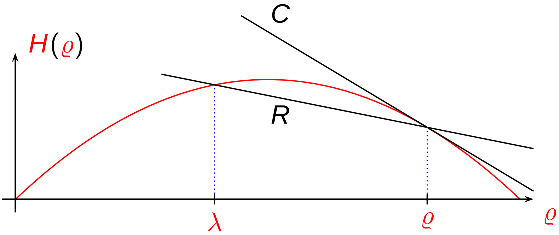

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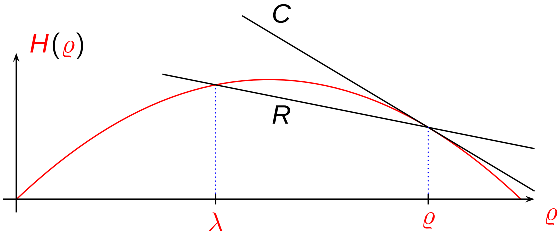

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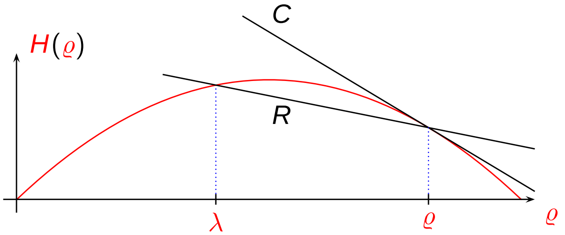

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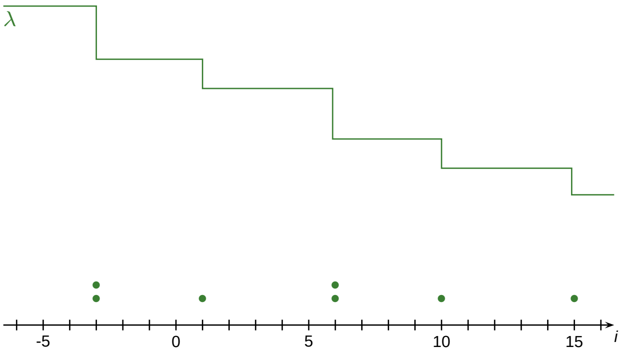

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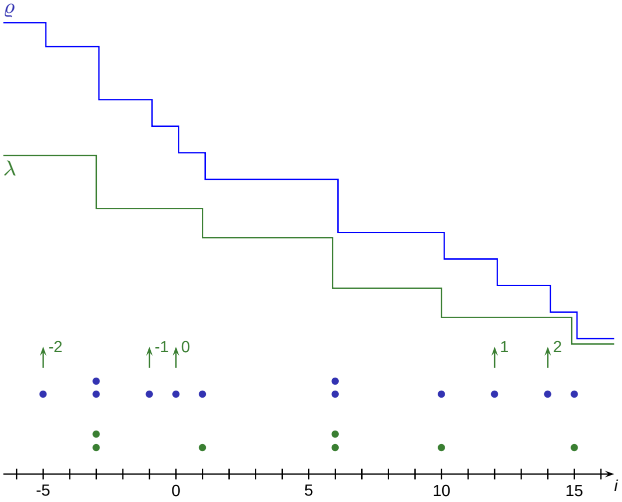

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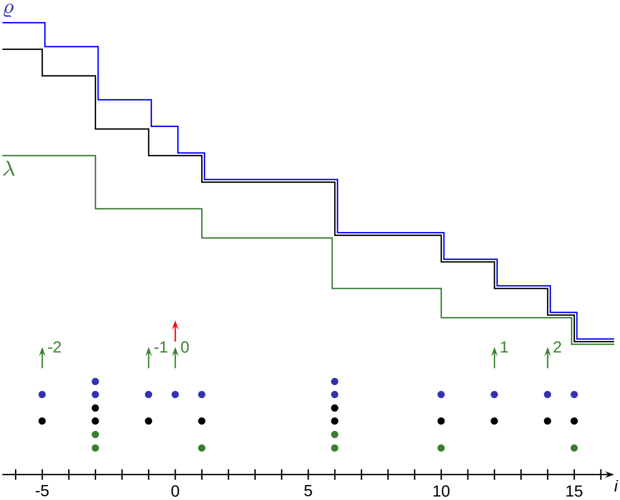

Couple three processes, and $X(t)$ to $Q(t)$.

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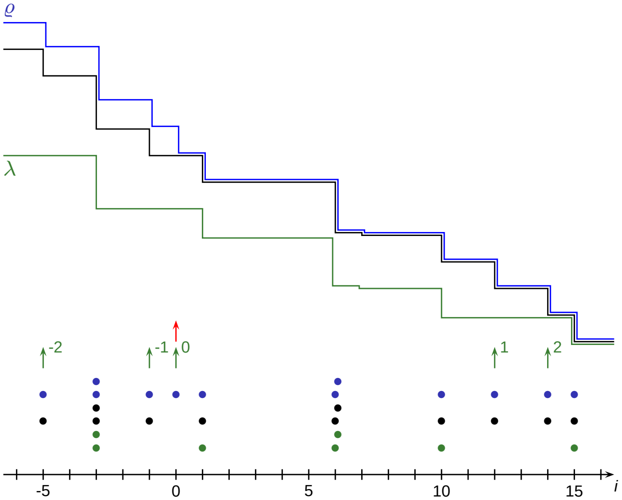

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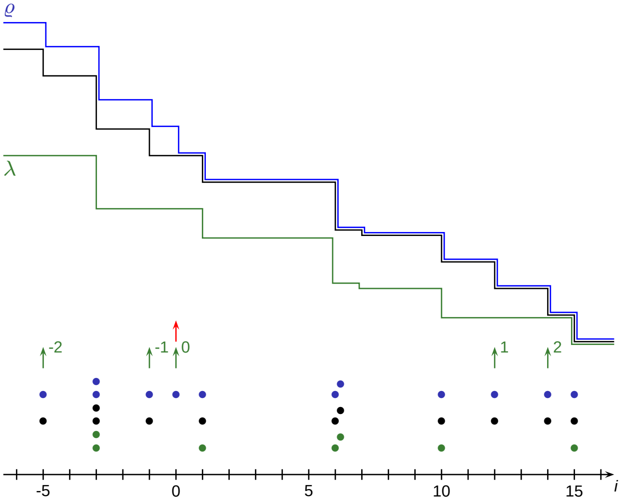

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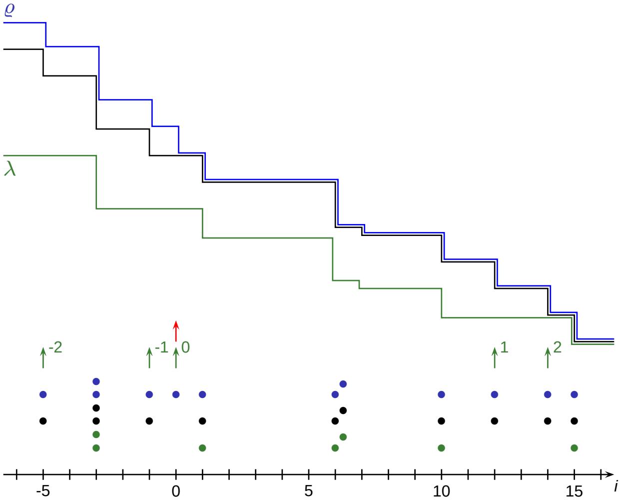

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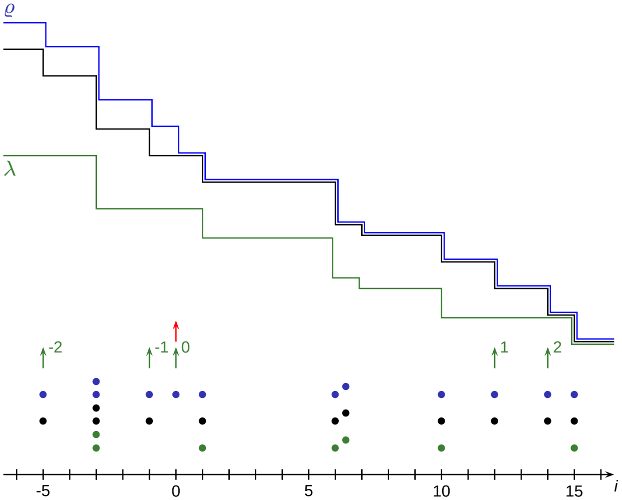

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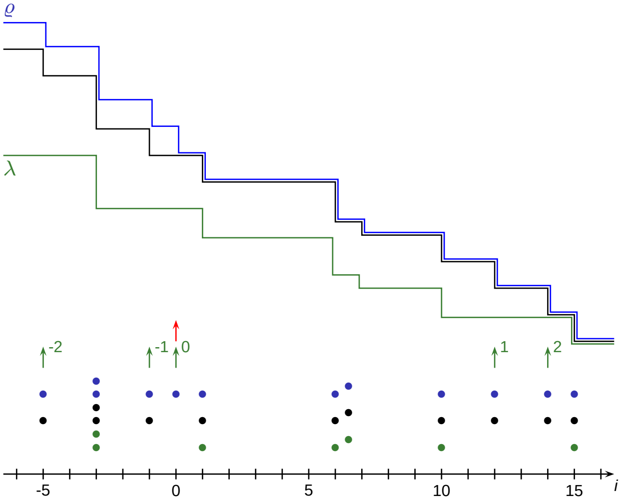

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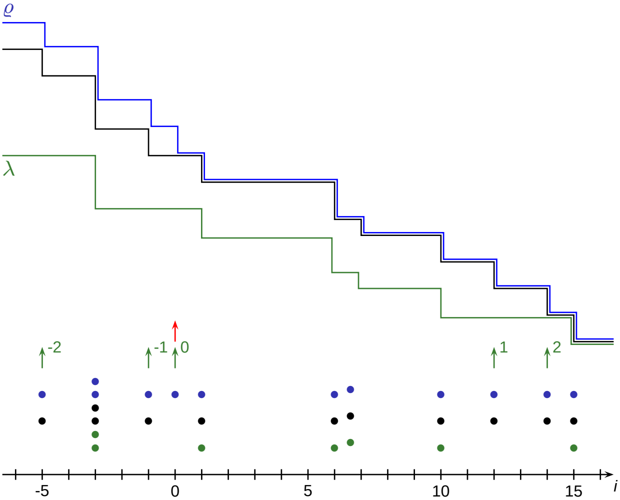

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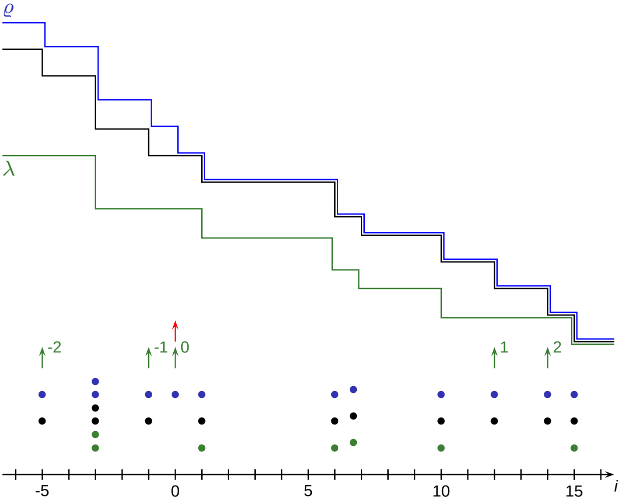

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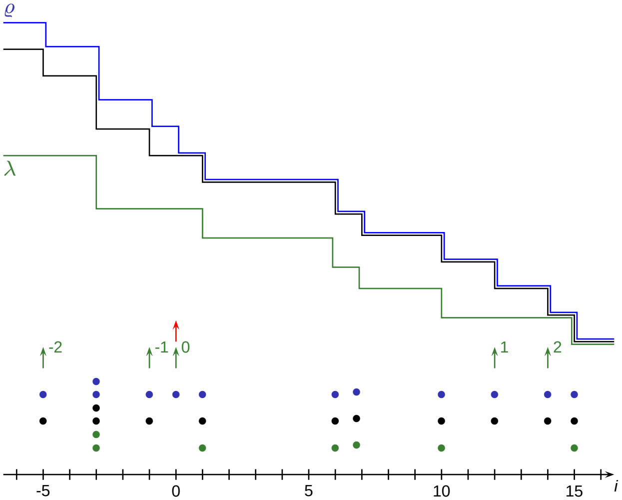

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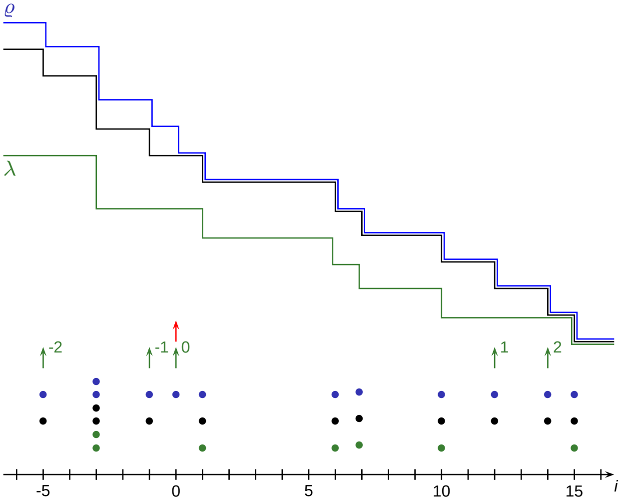

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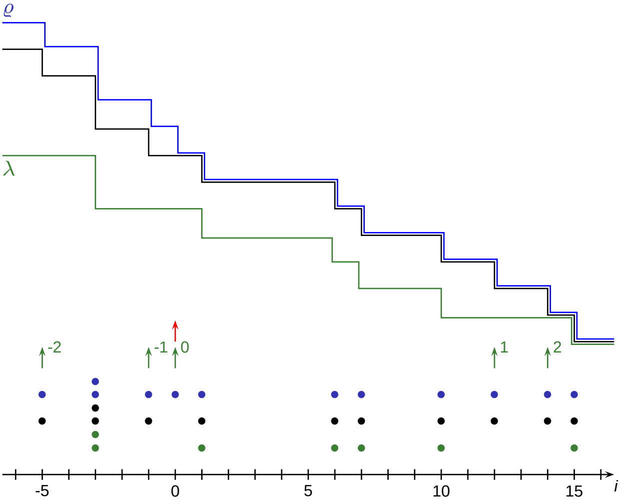

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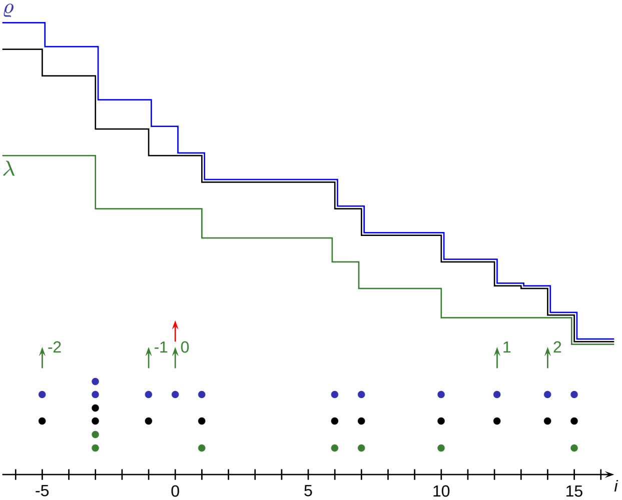

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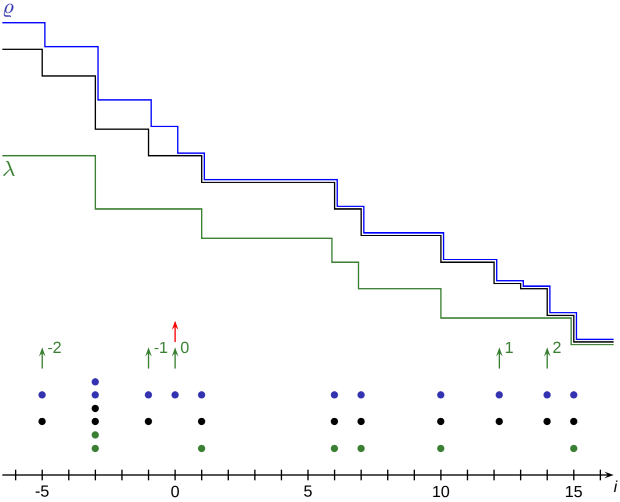

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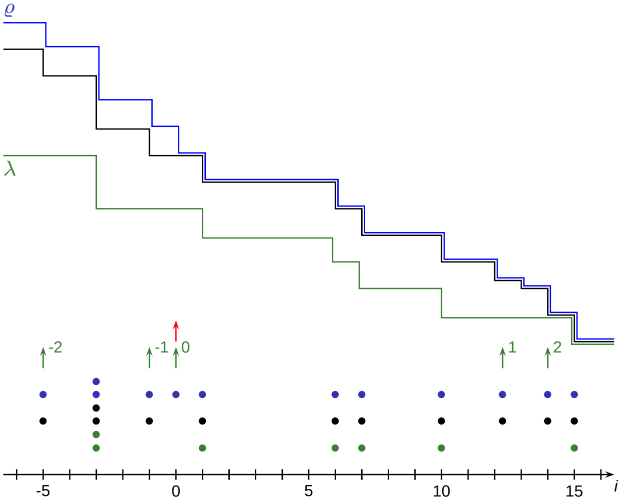

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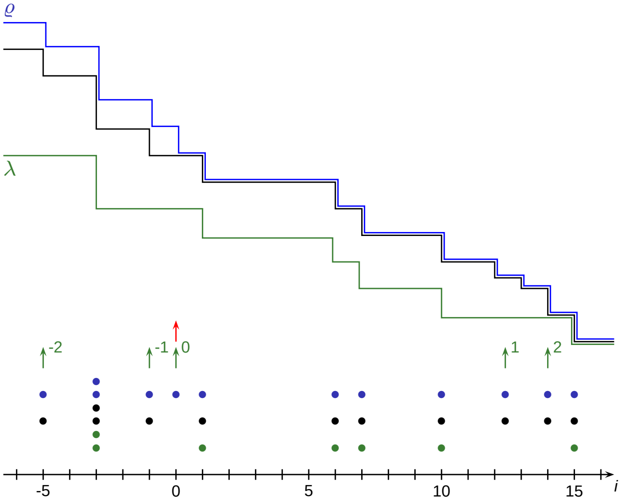

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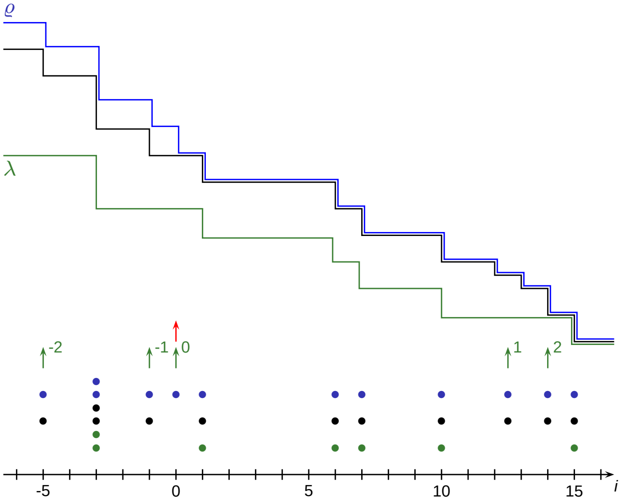

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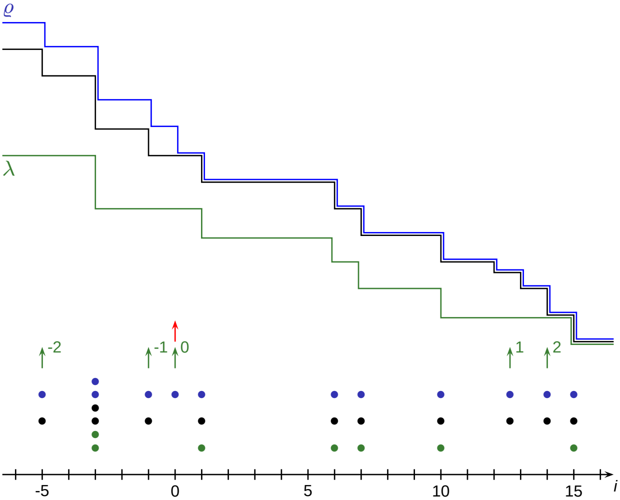

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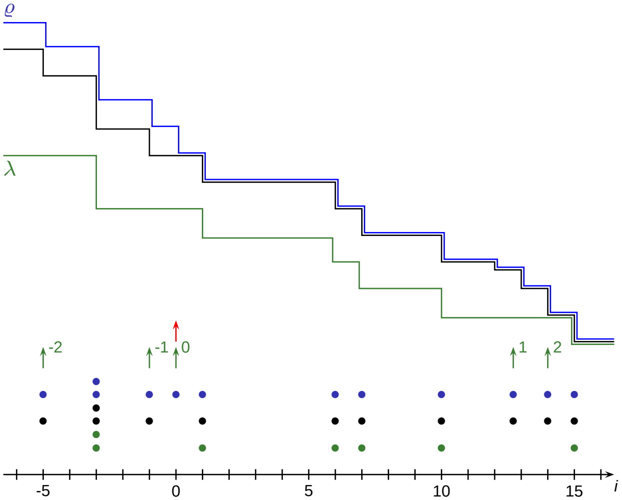

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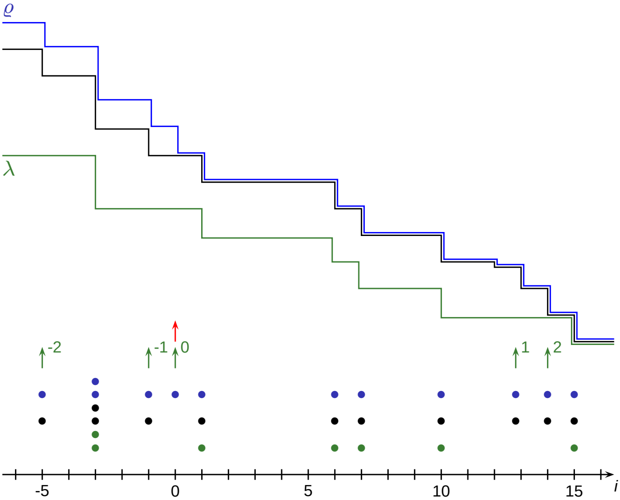

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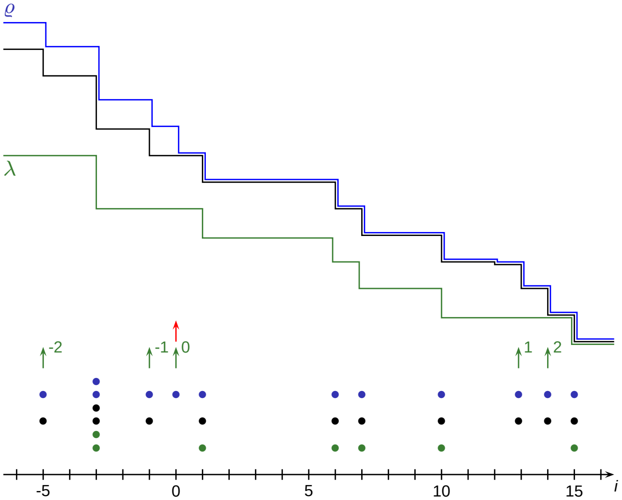

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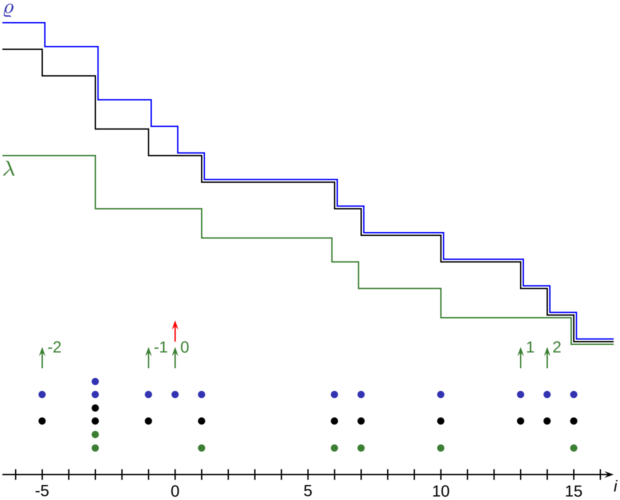

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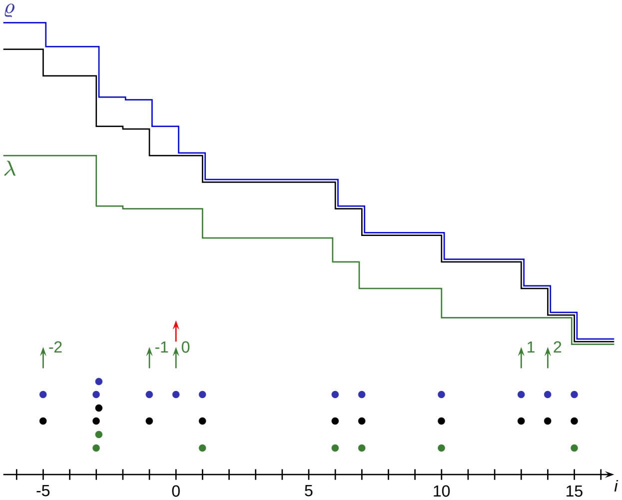

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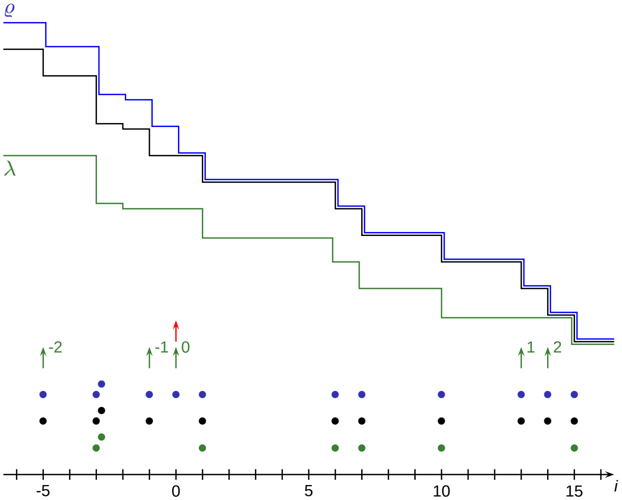

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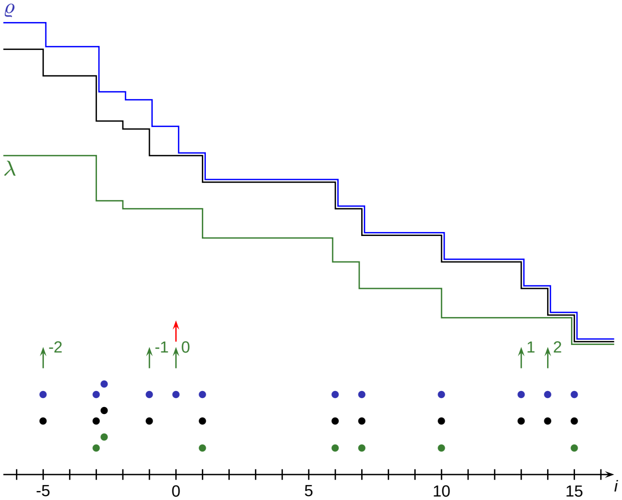

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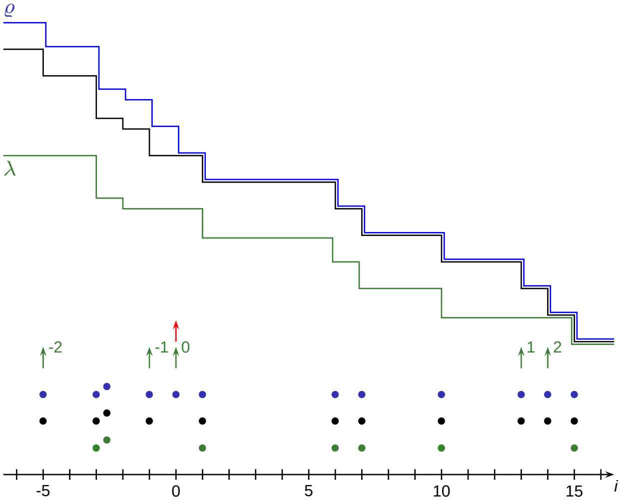

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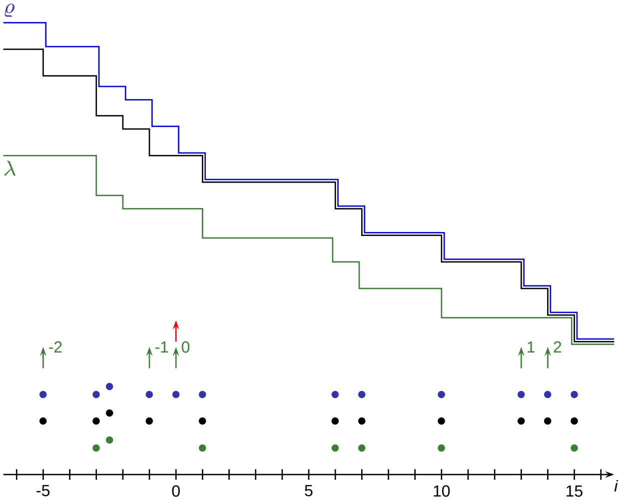

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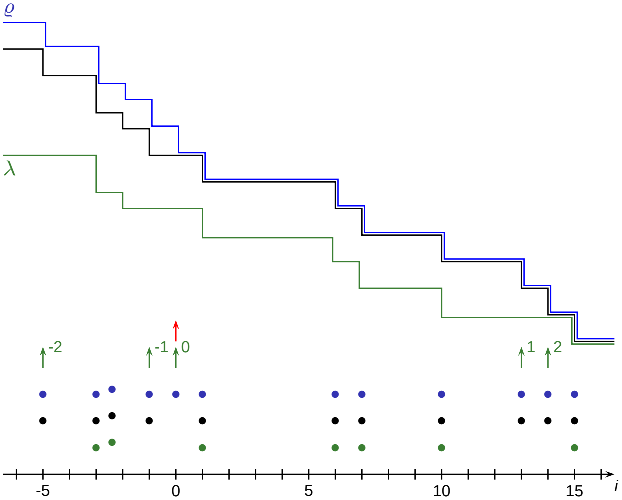

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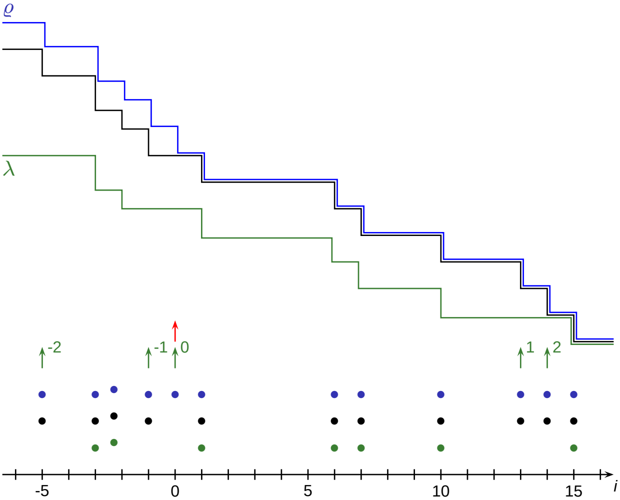

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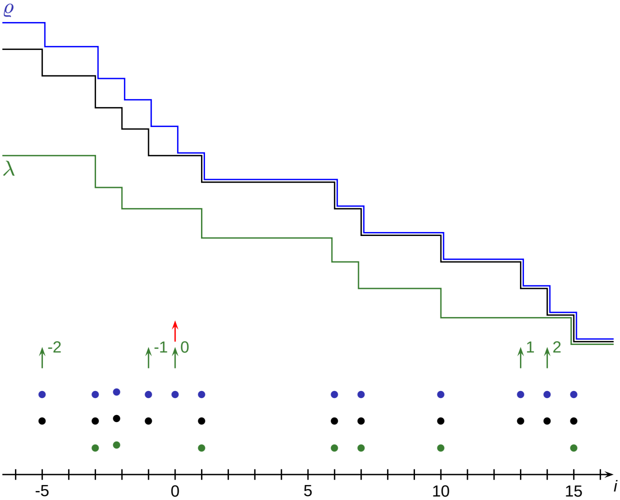

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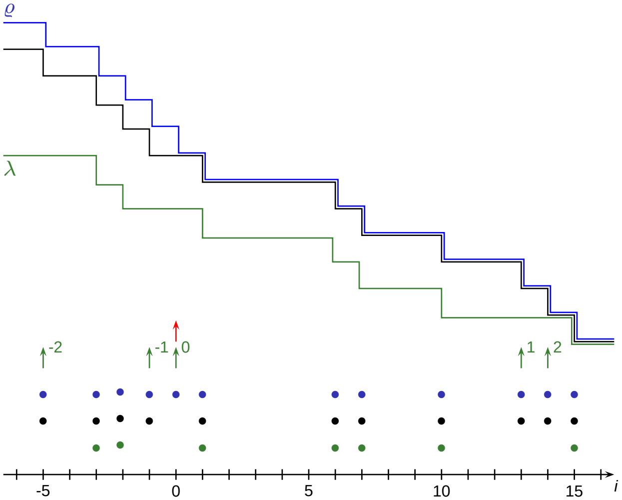

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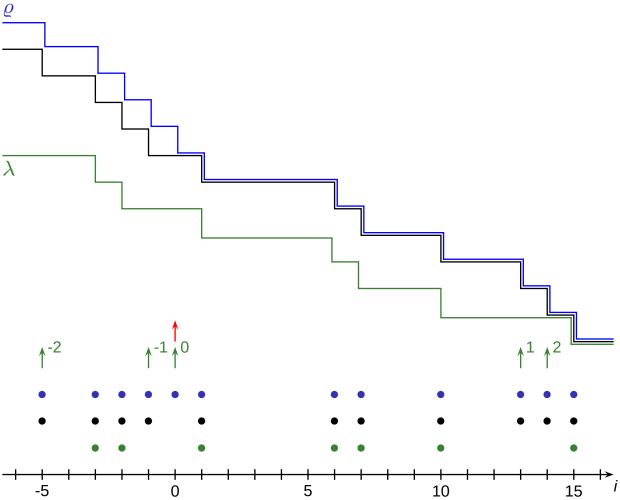

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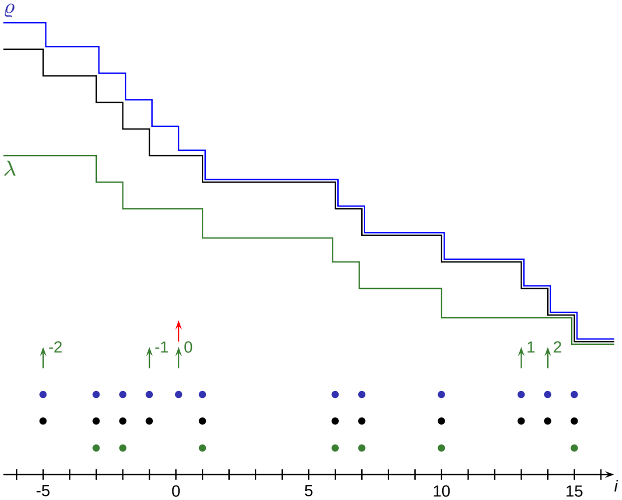

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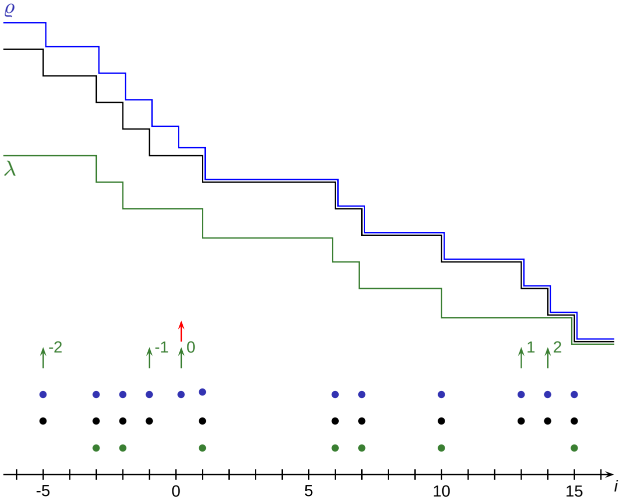

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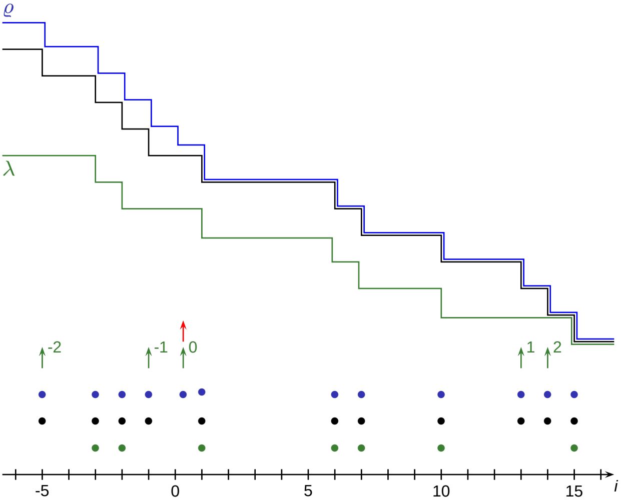

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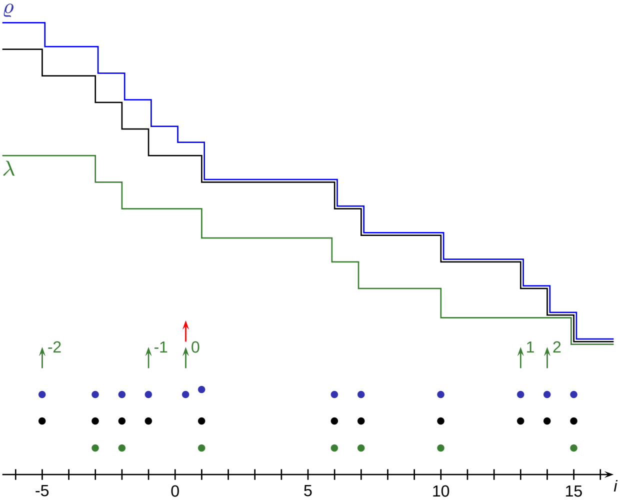

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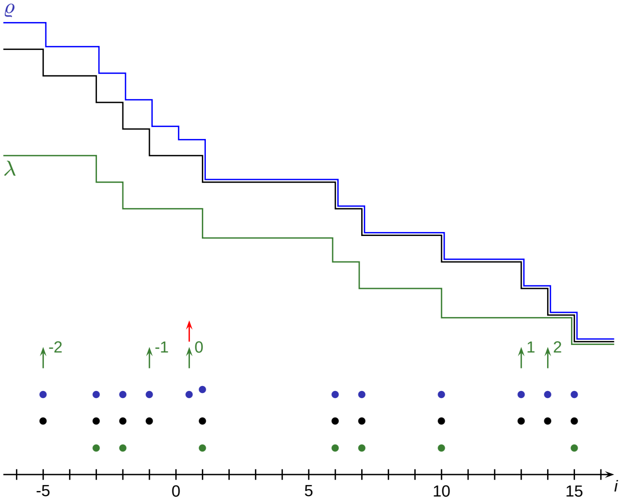

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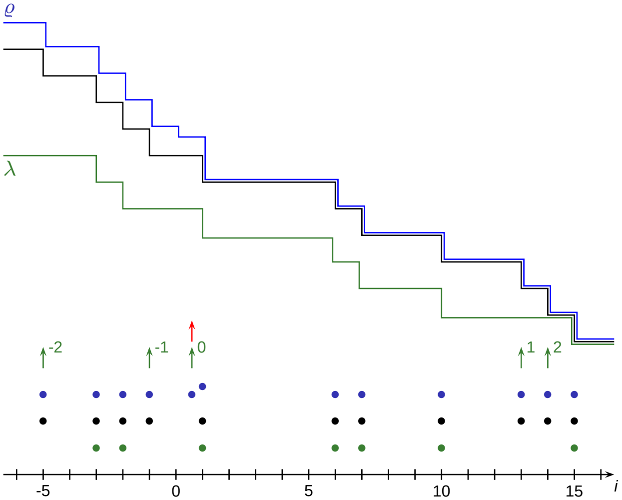

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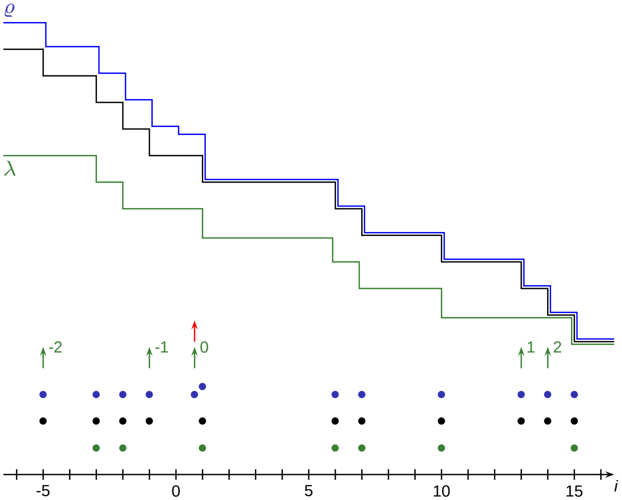

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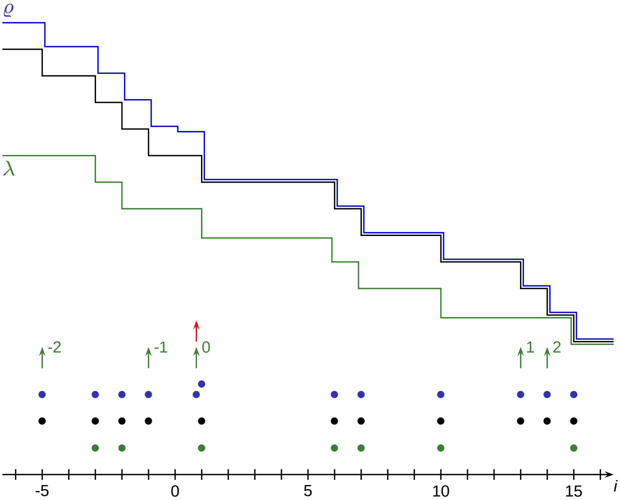

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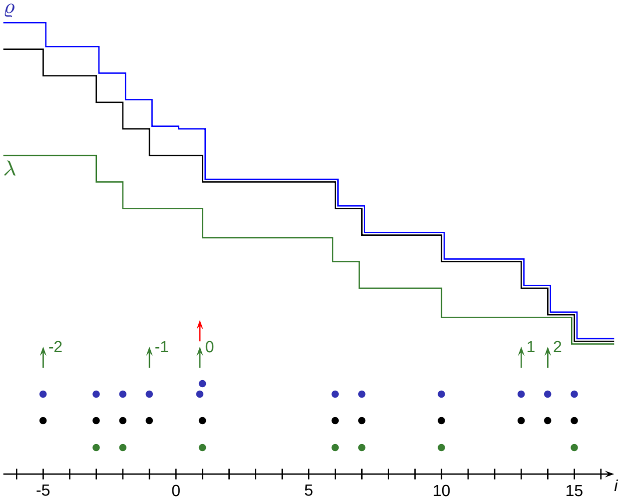

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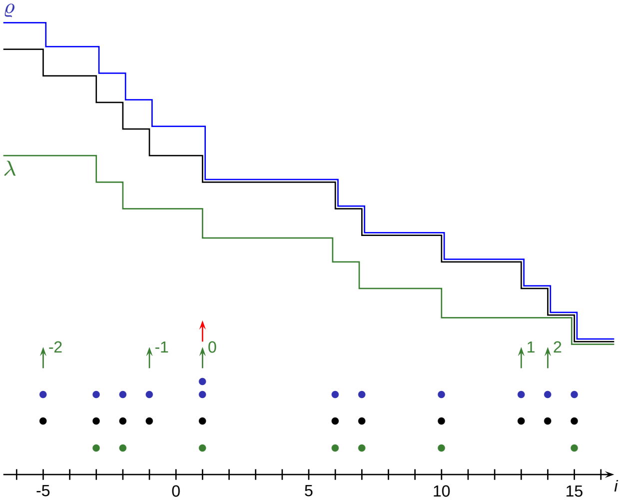

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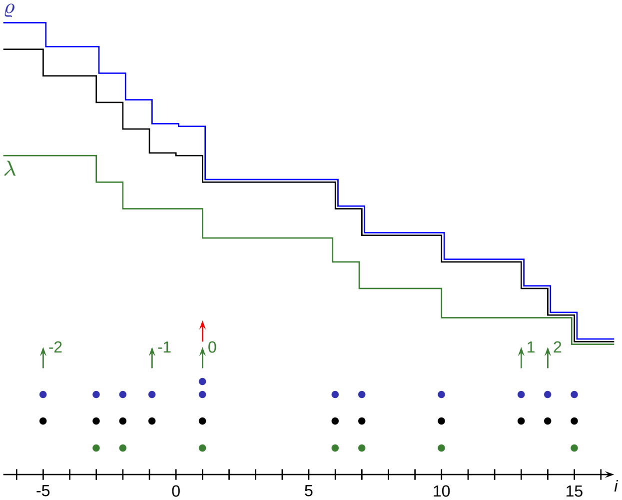

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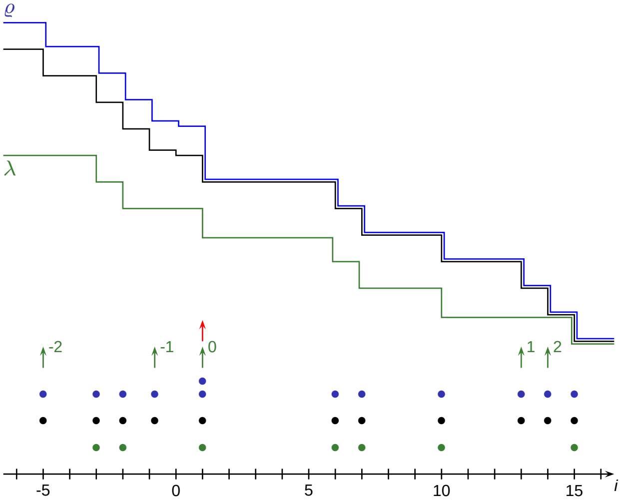

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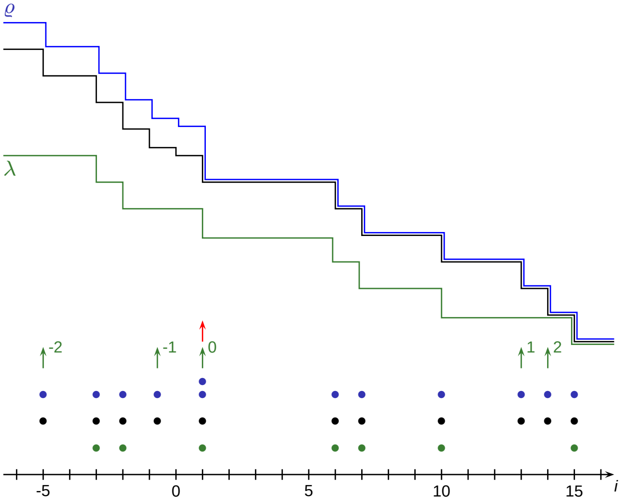

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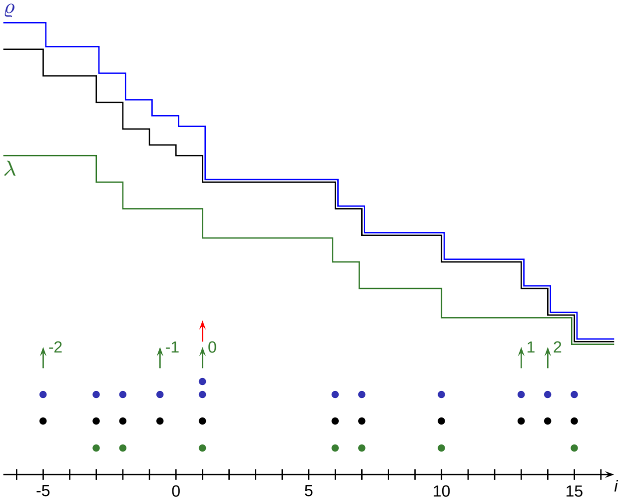

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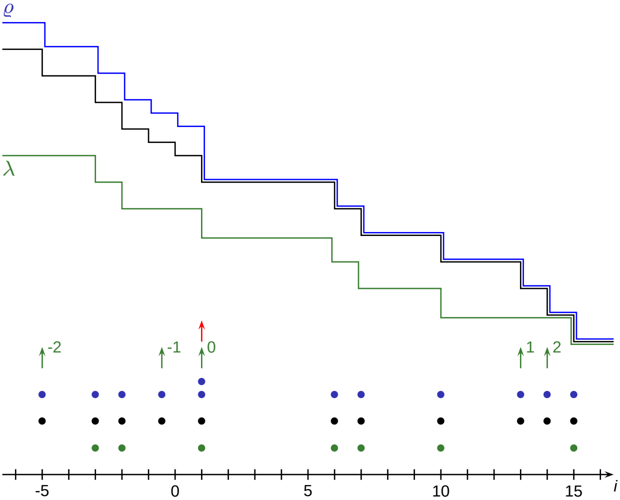

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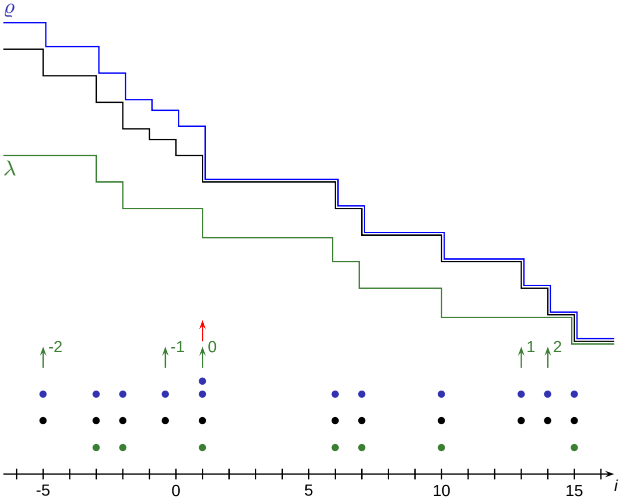

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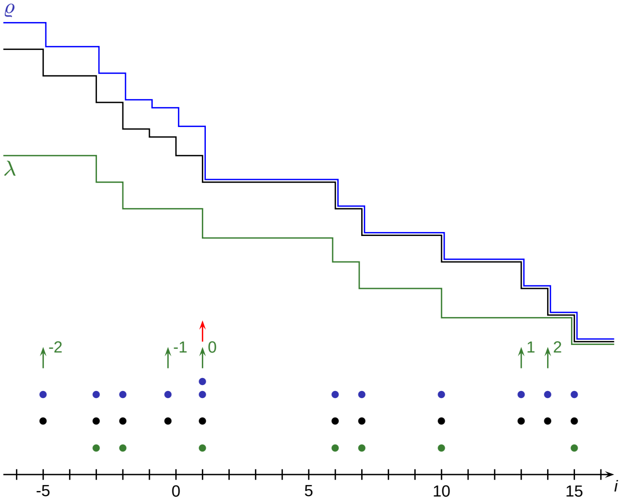

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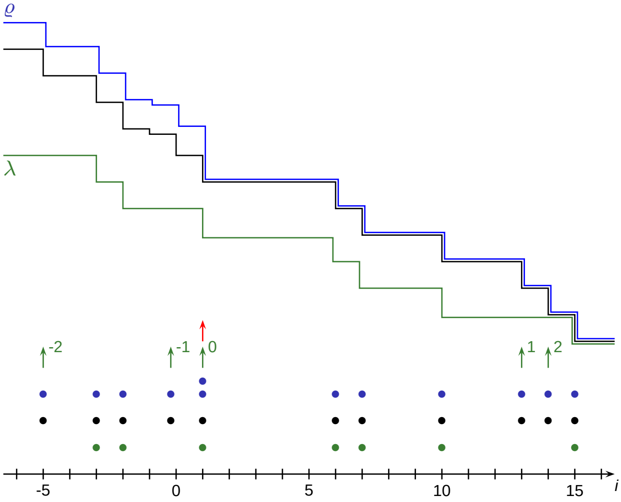

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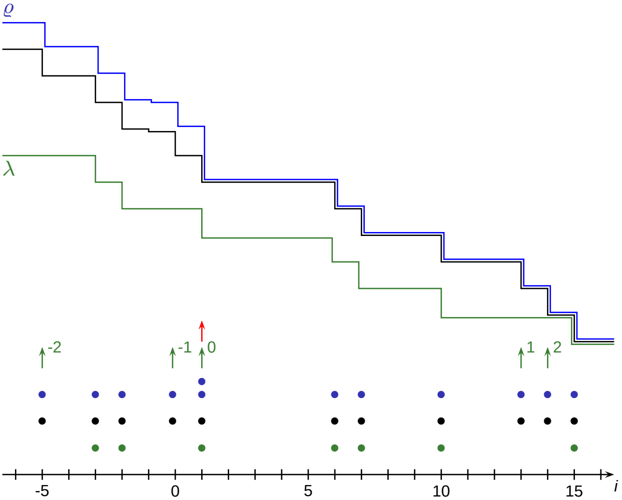

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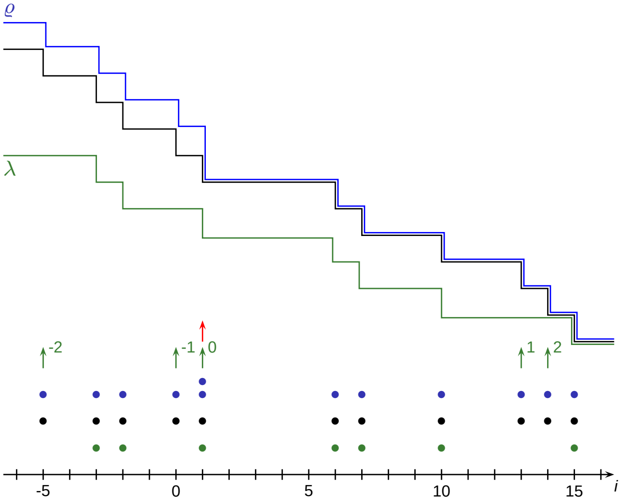

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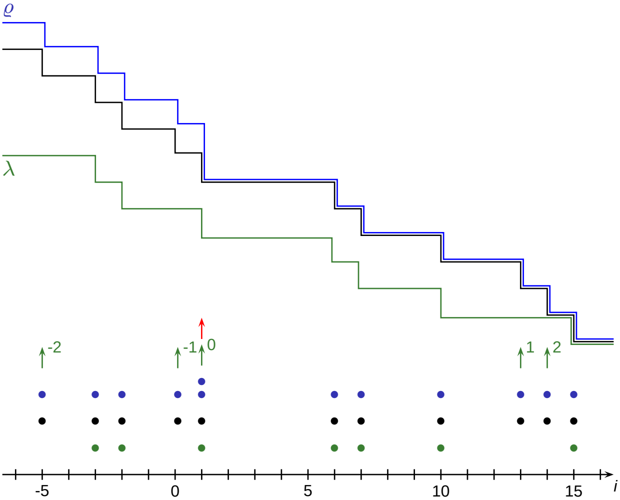

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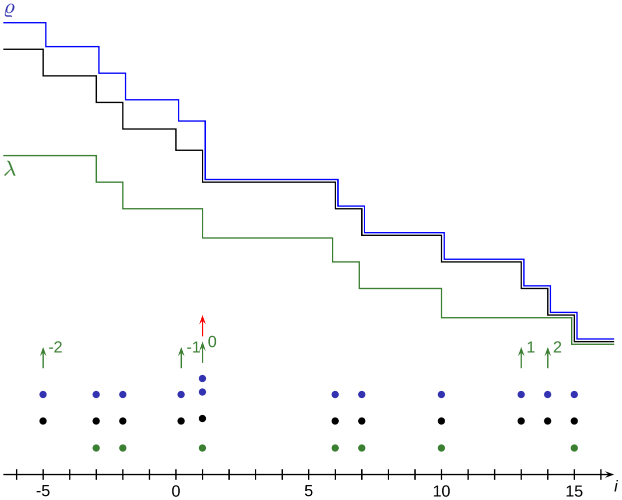

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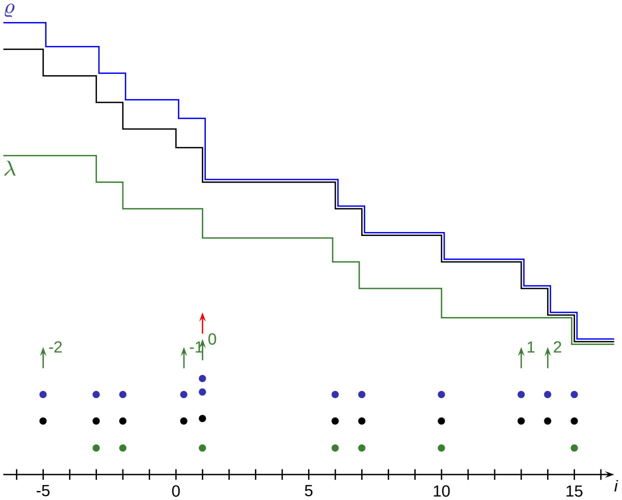

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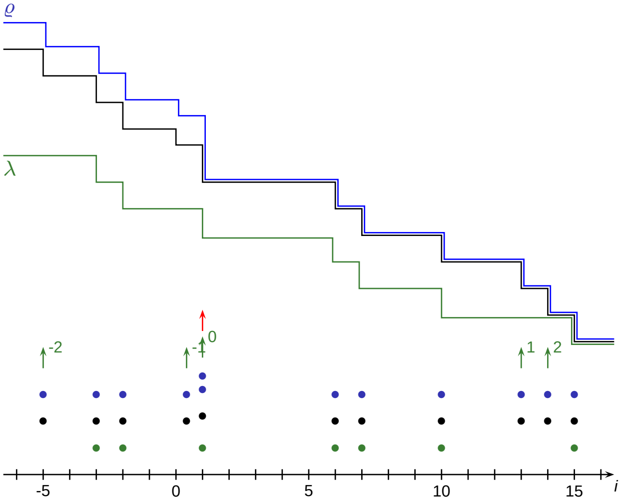

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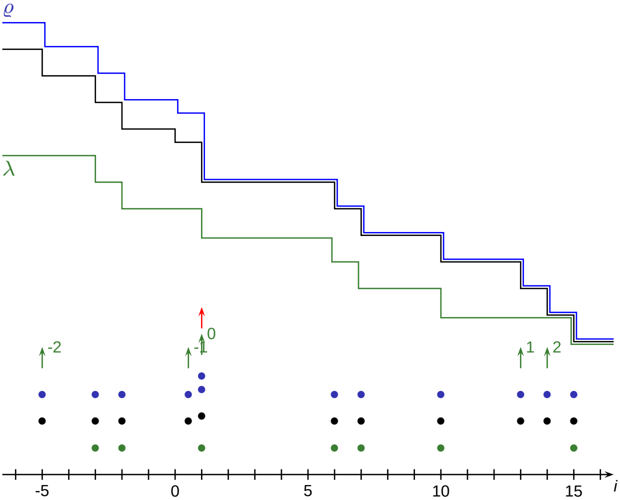

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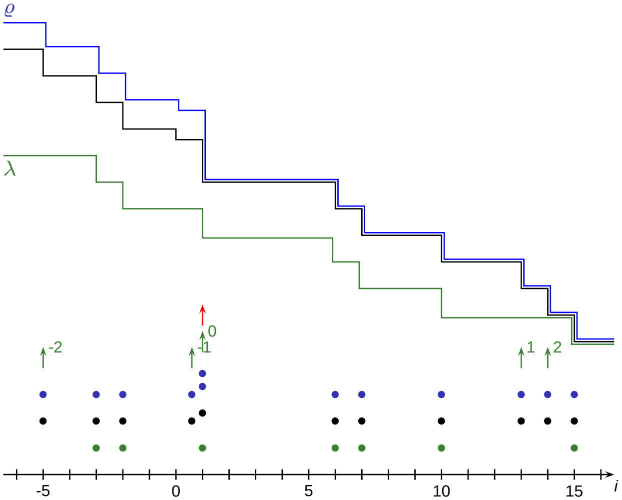

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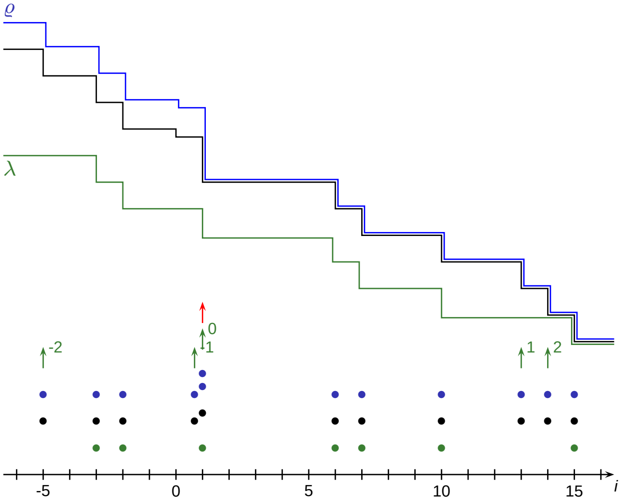

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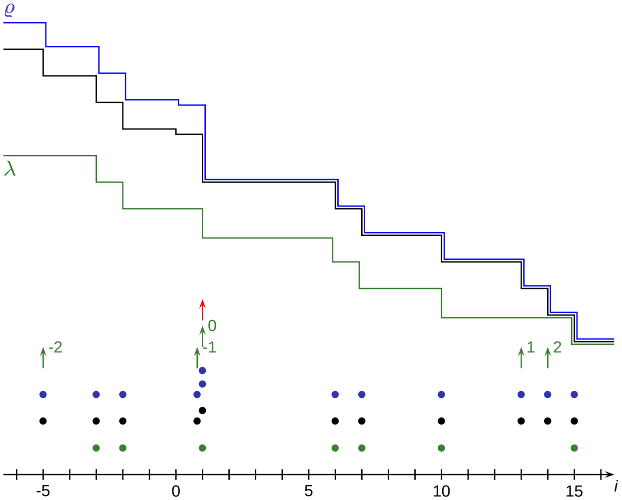

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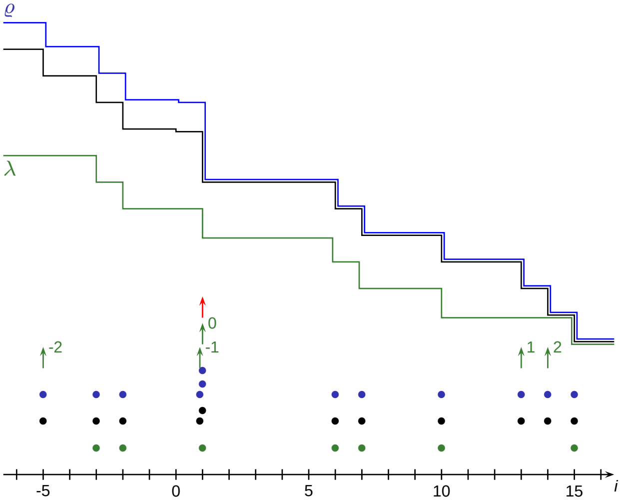

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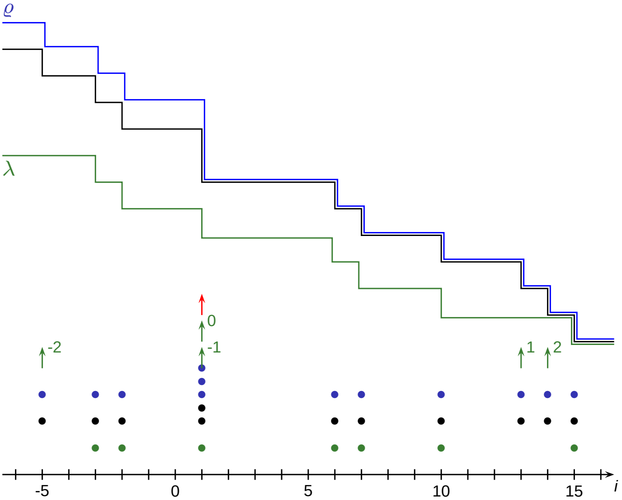

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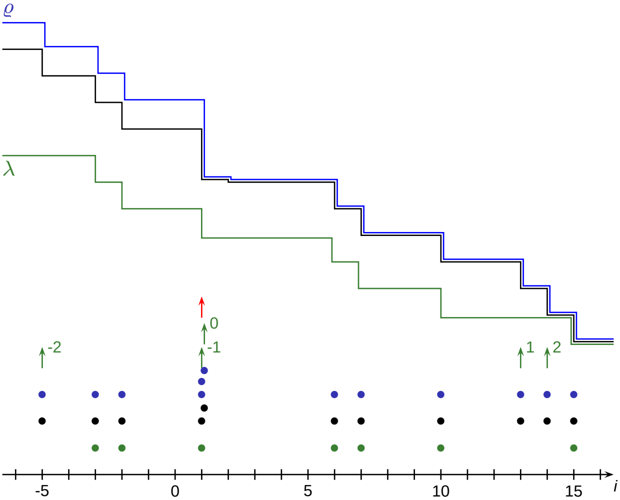

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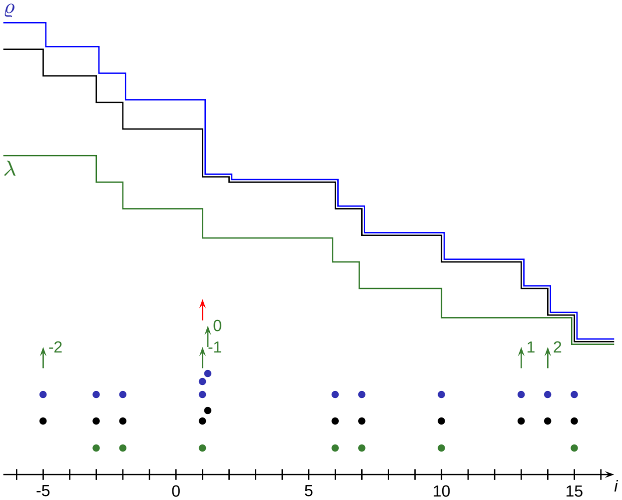

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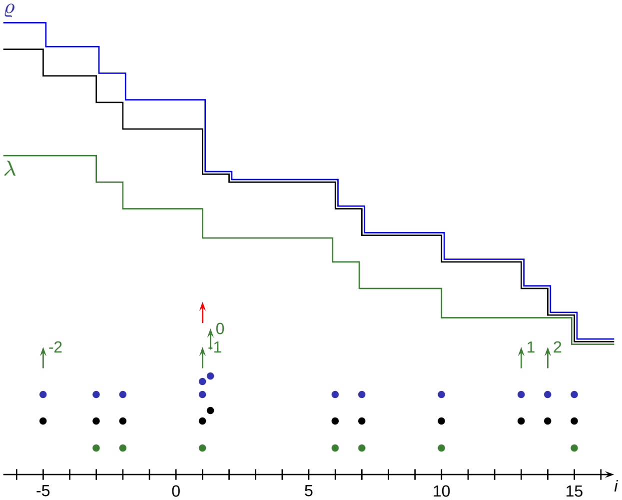

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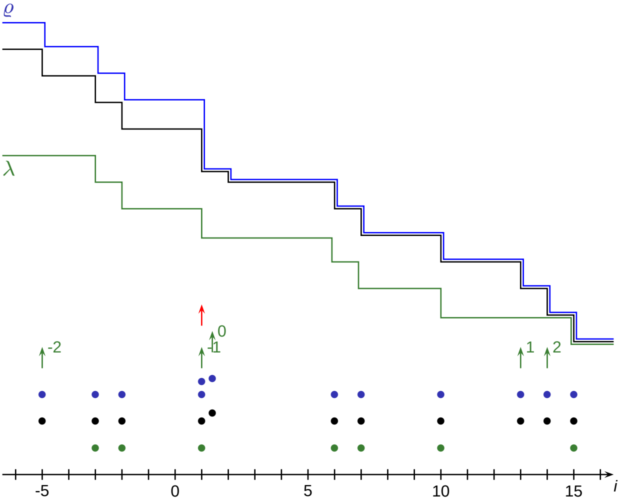

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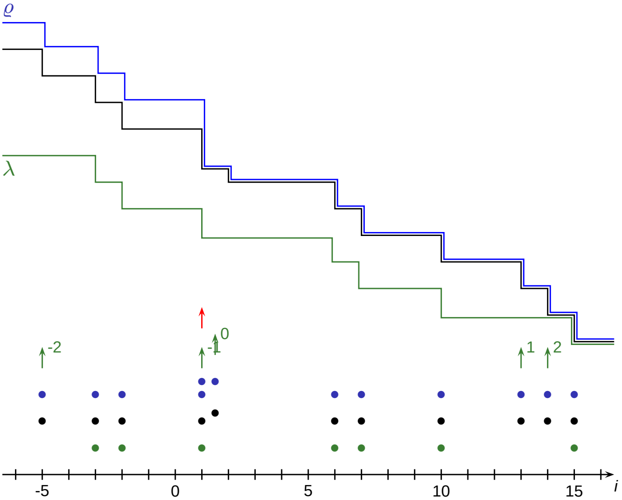

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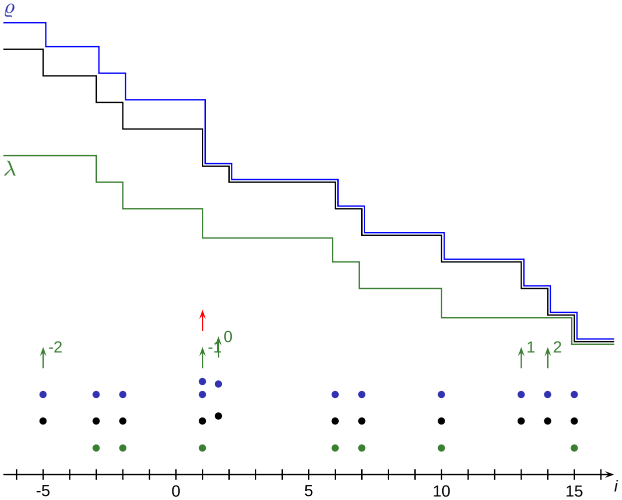

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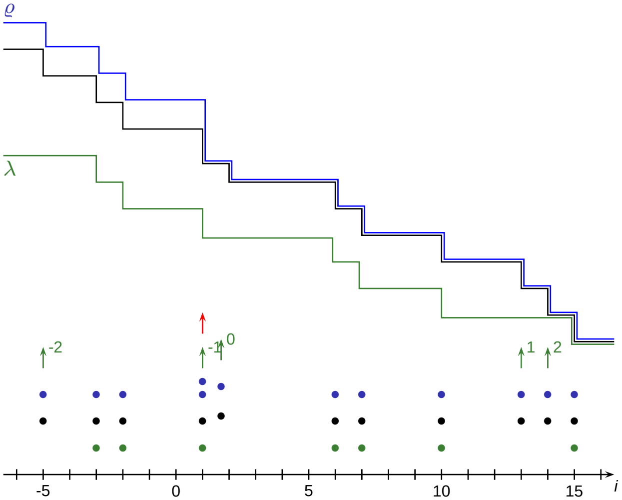

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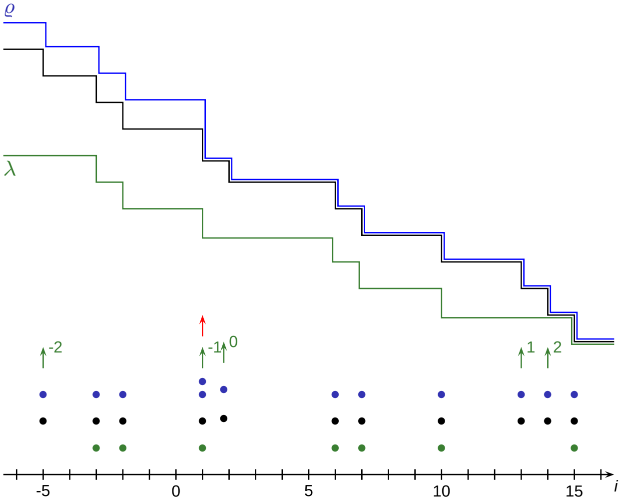

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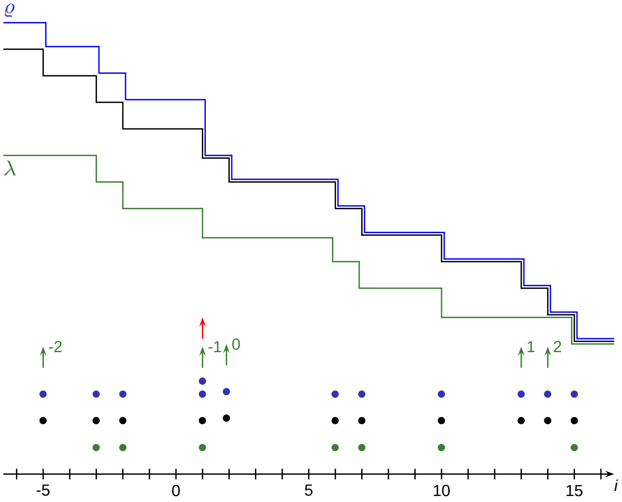

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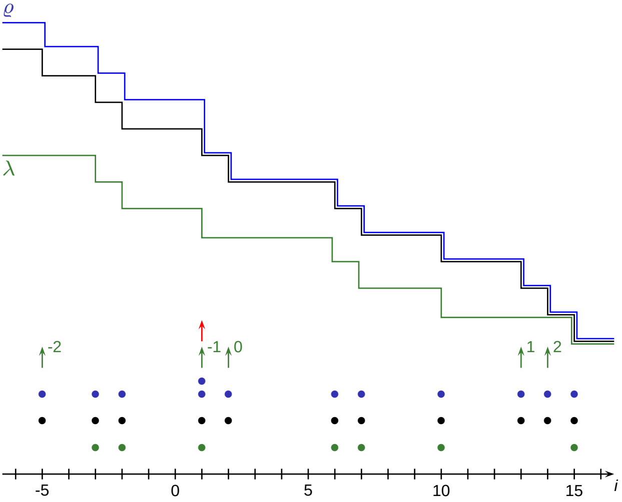

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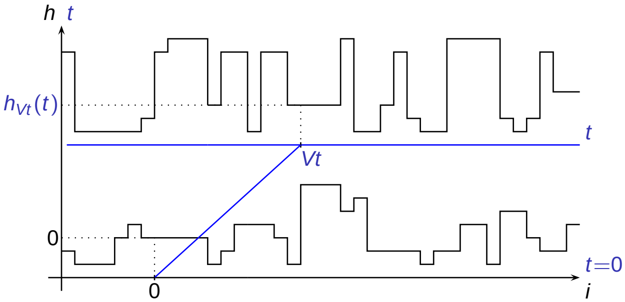

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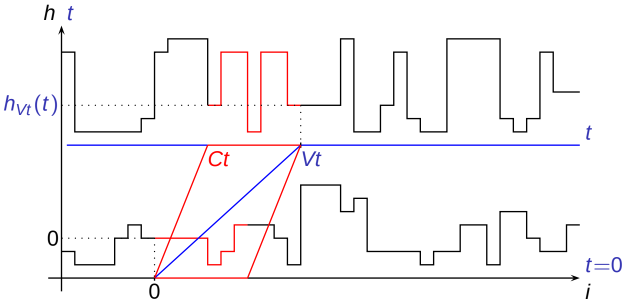

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There is a huge literature now on limit distribution results, using combinatorial and asymptotic analytic tools.

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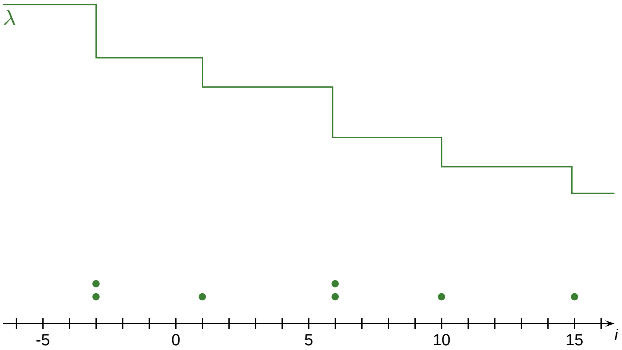

Second class particle current: difference in growth.

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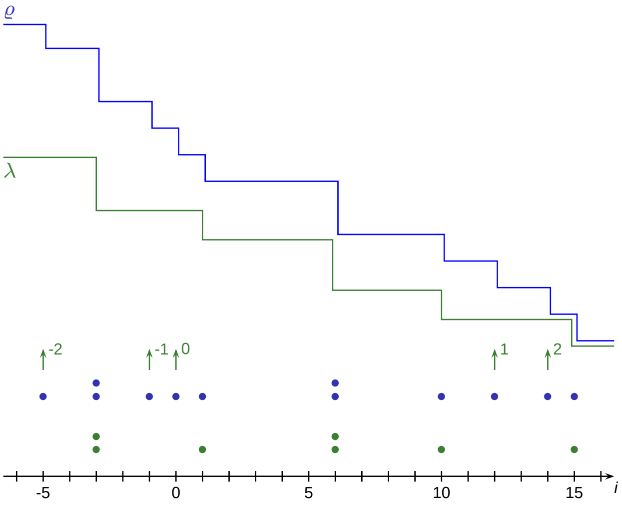

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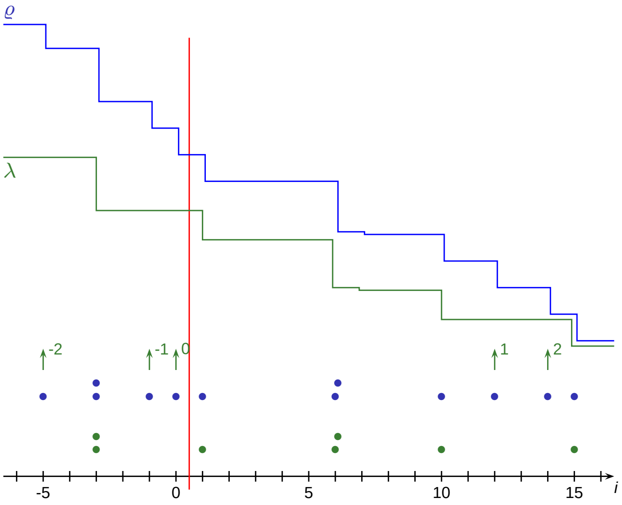

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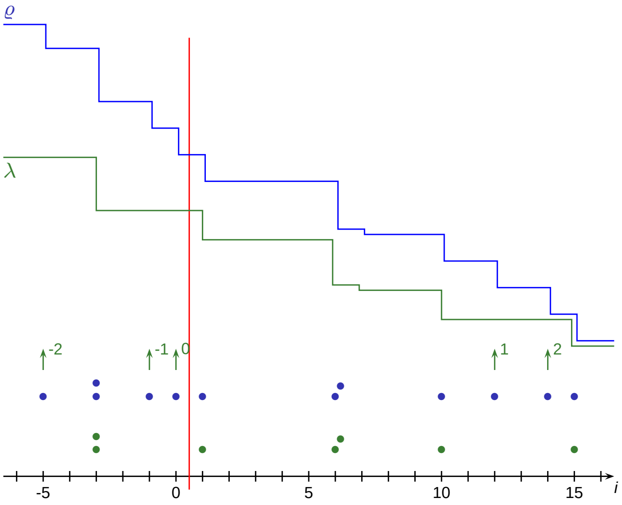

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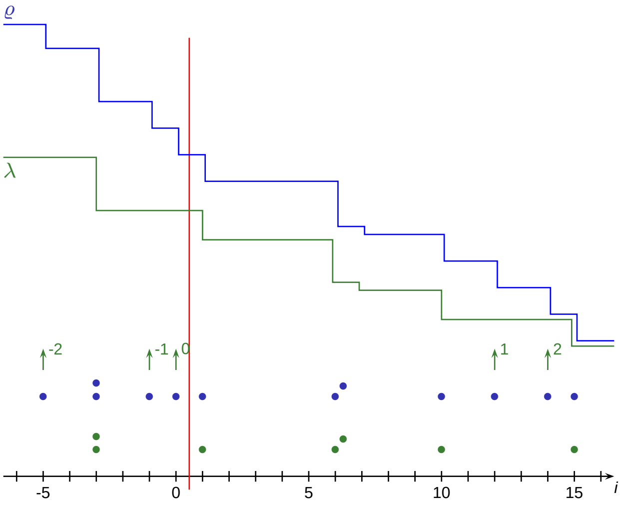

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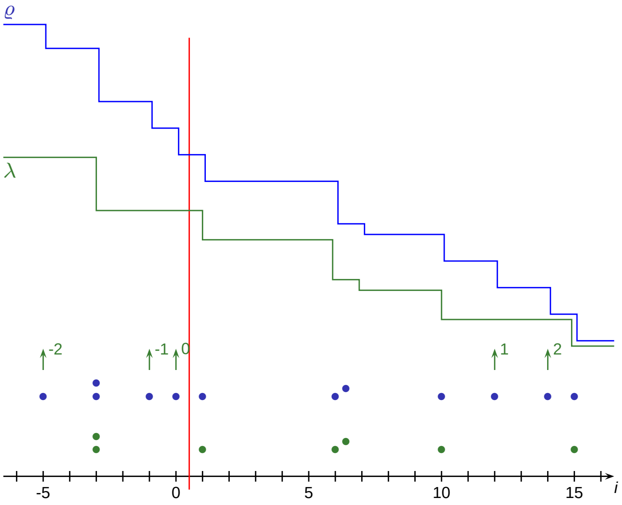

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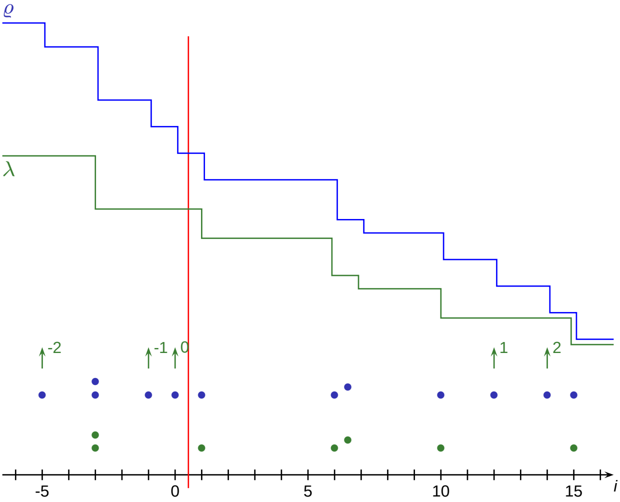

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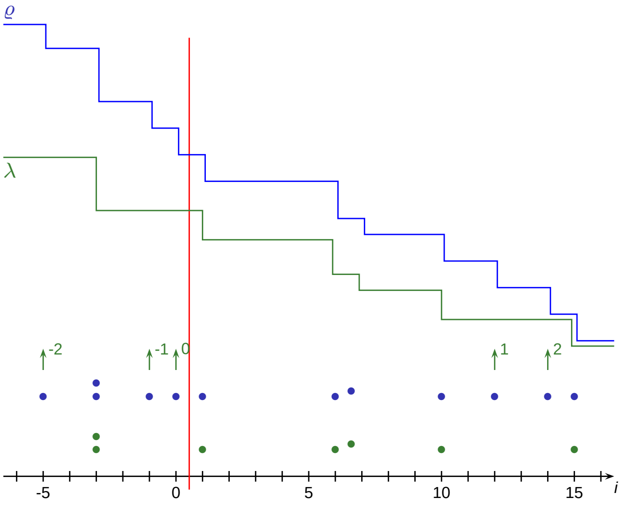

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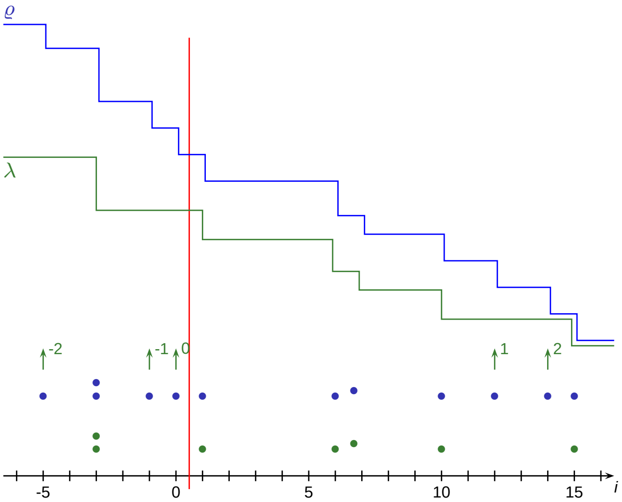

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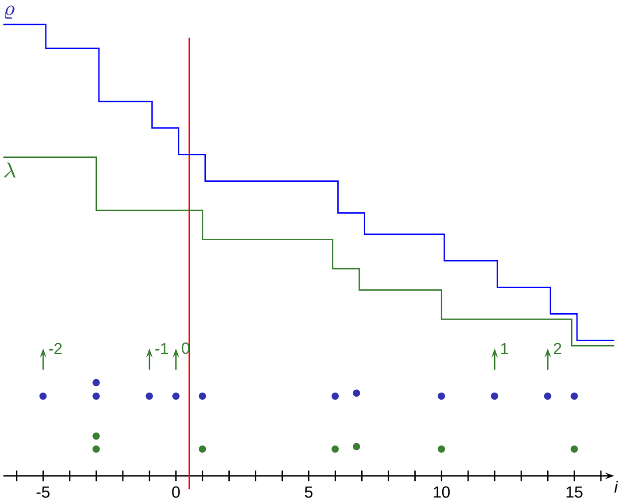

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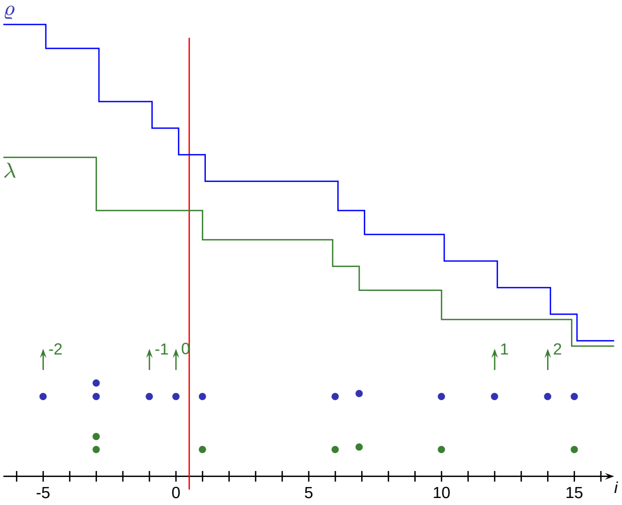

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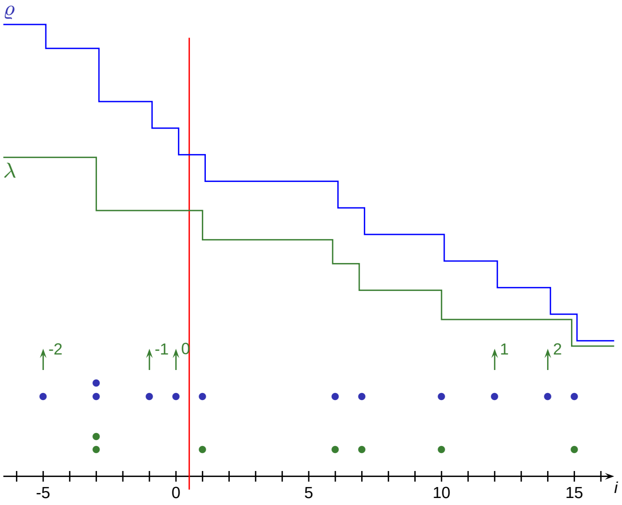

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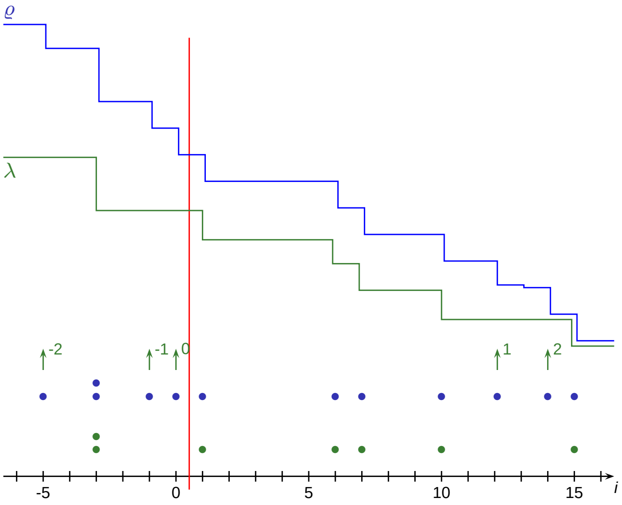

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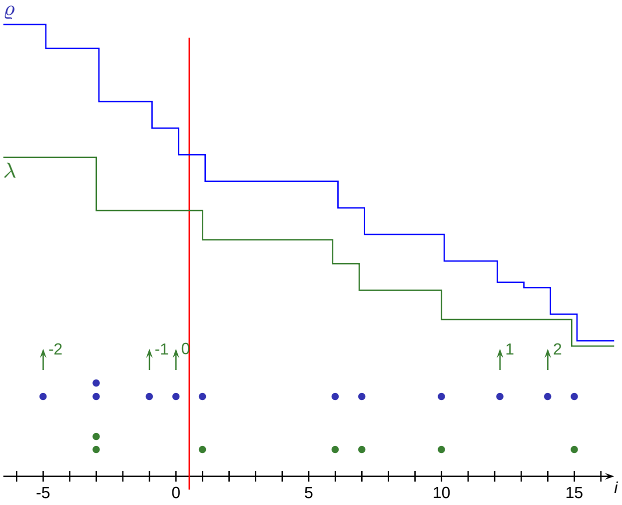

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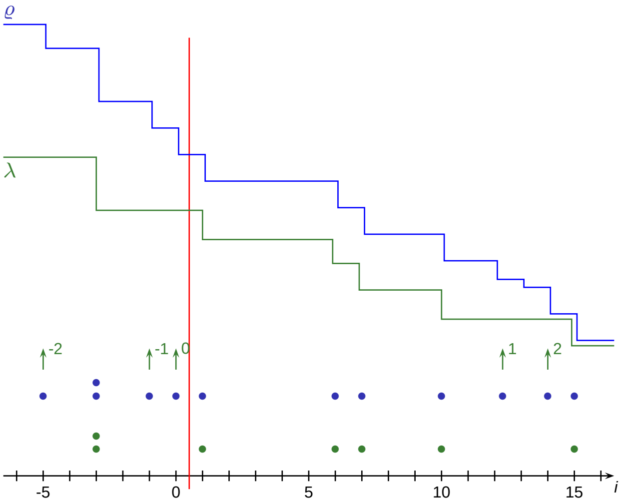

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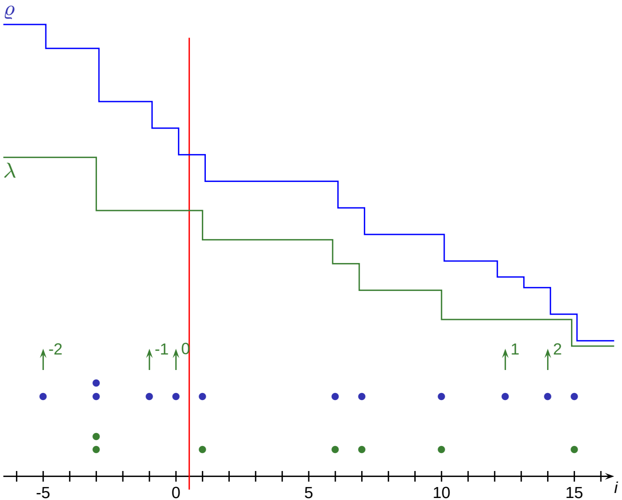

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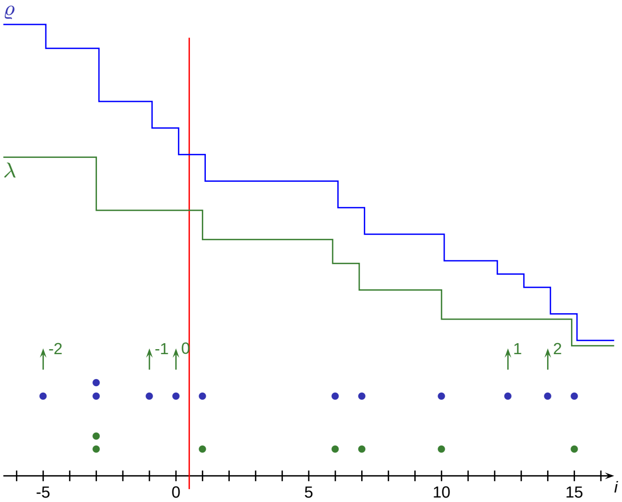

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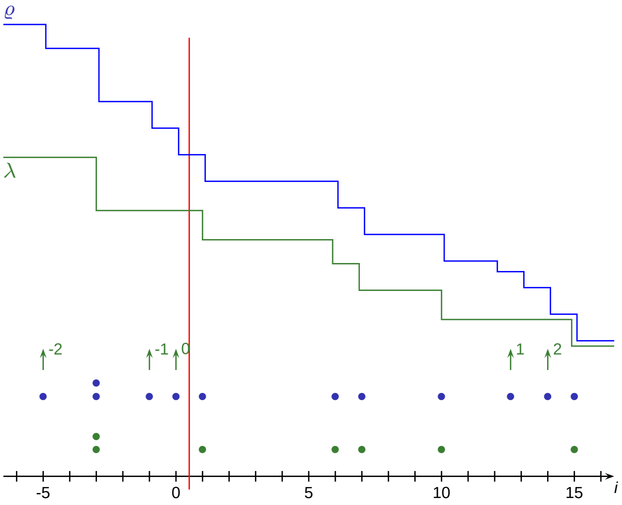

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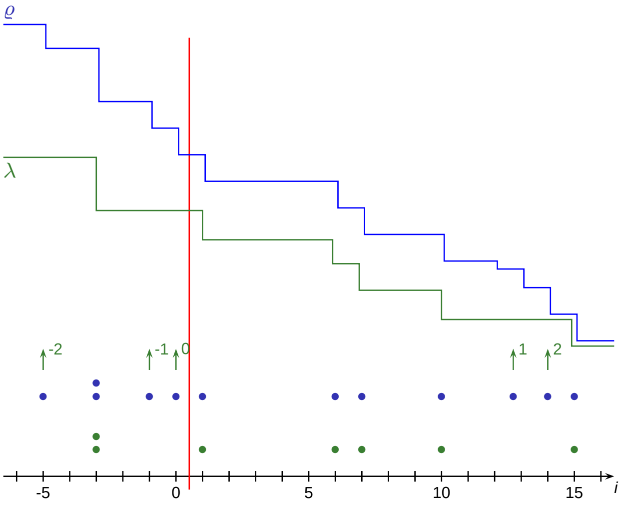

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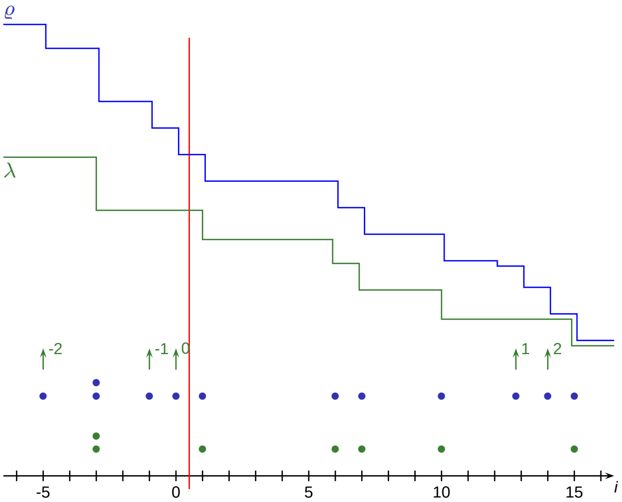

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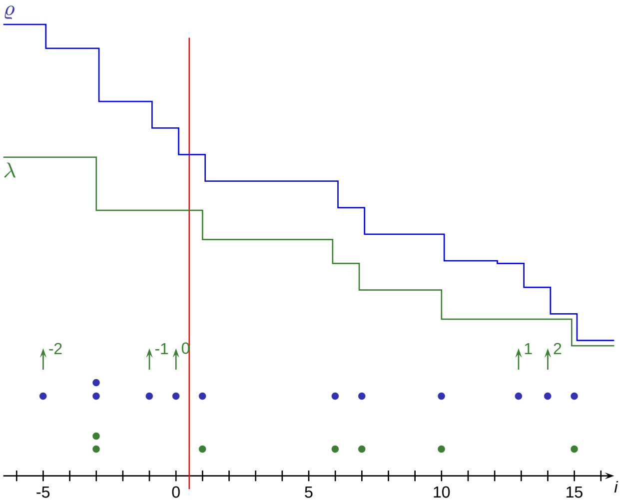

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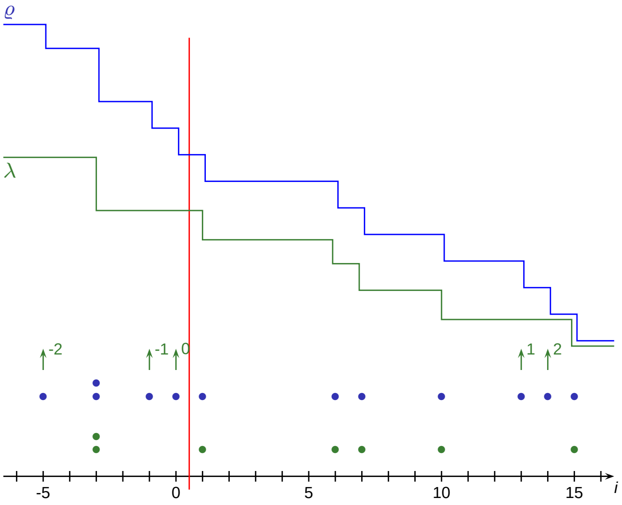

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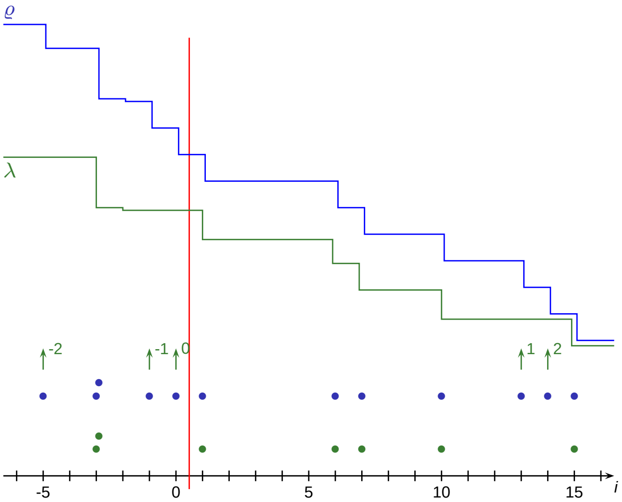

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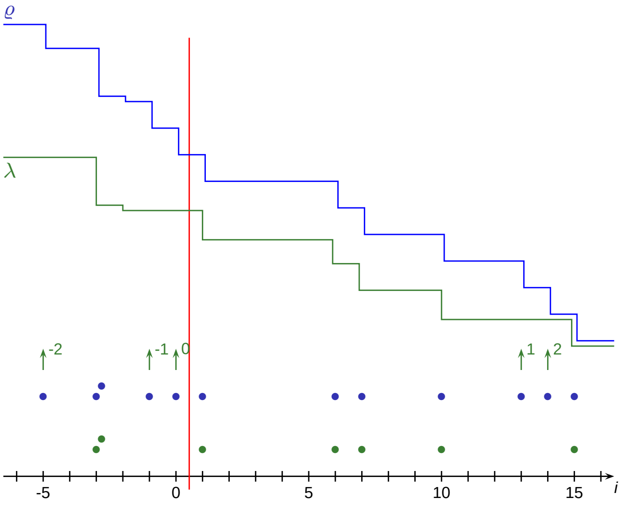

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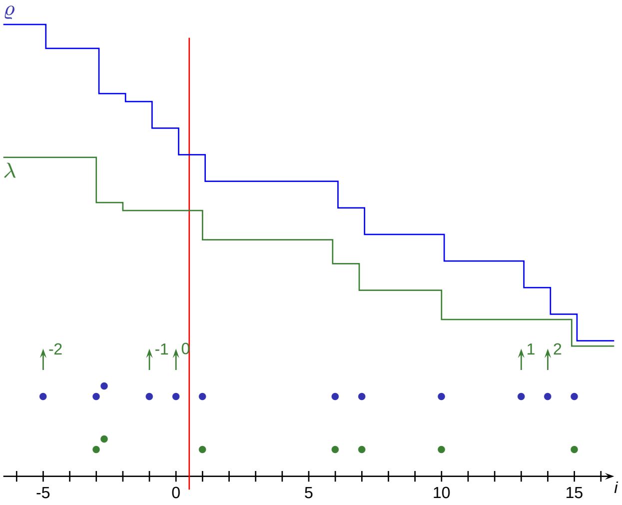

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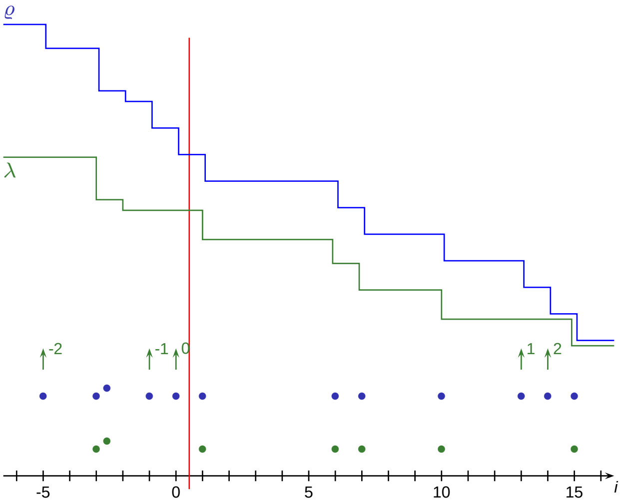

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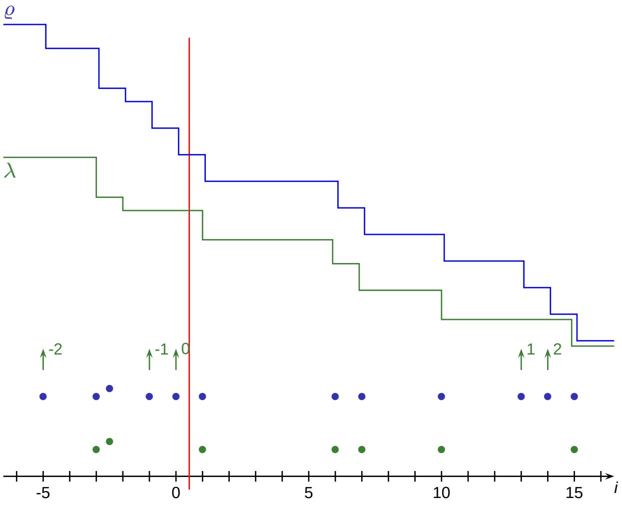

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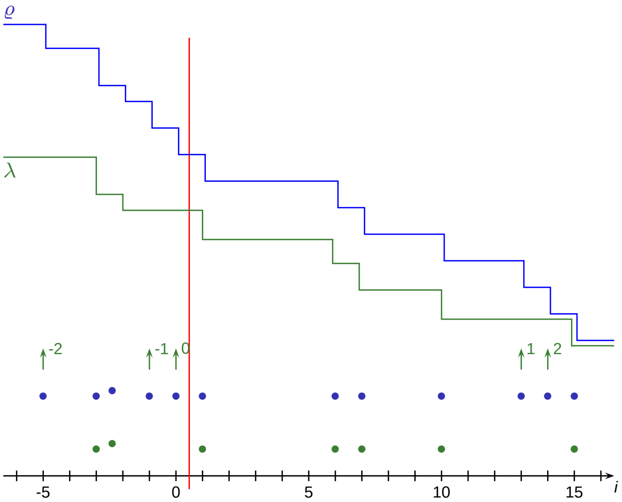

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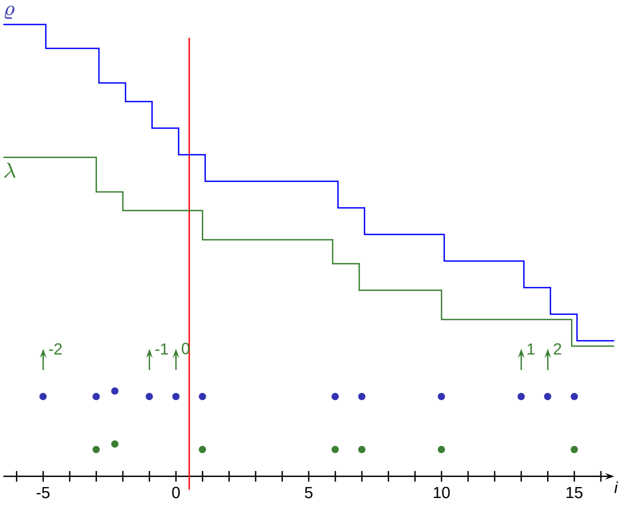

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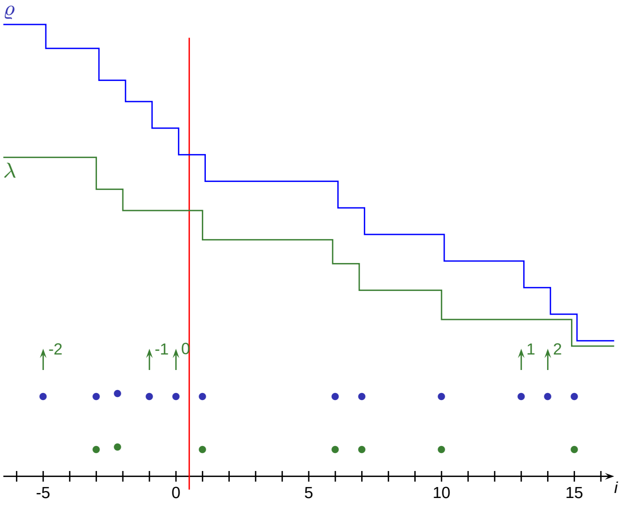

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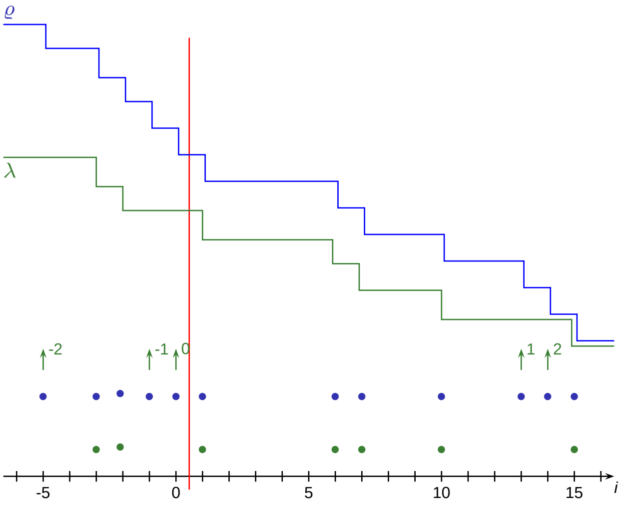

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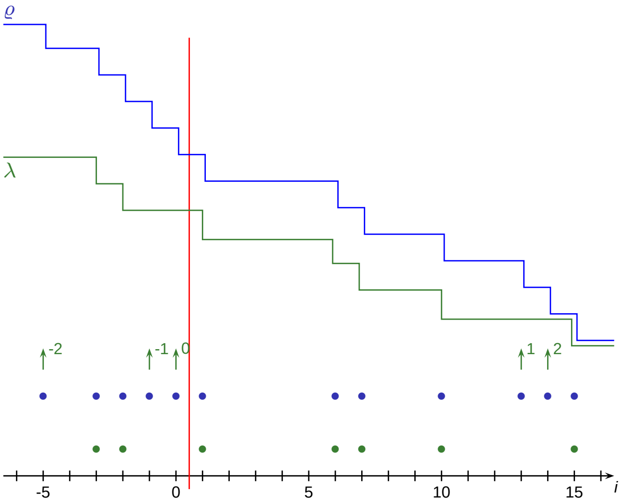

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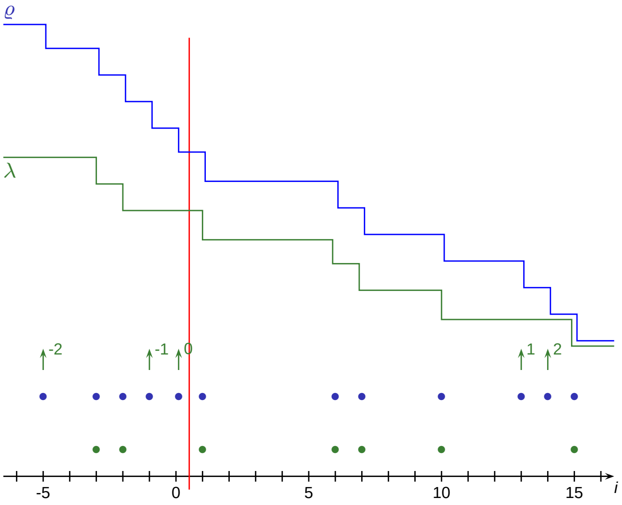

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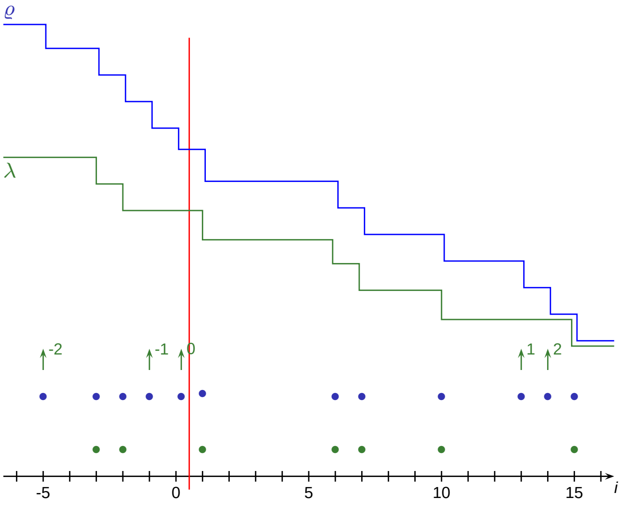

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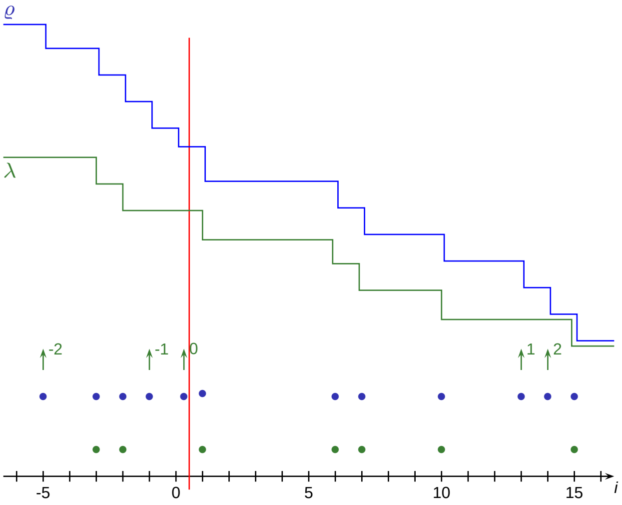

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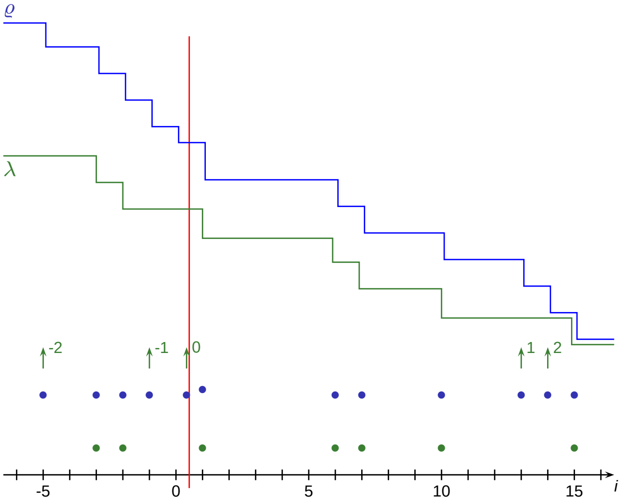

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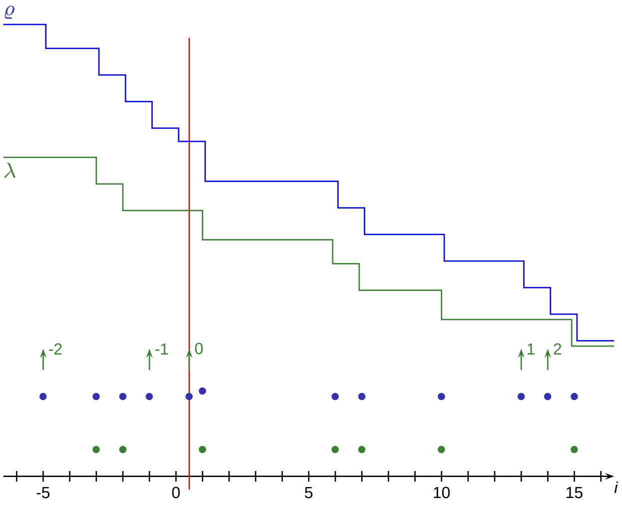

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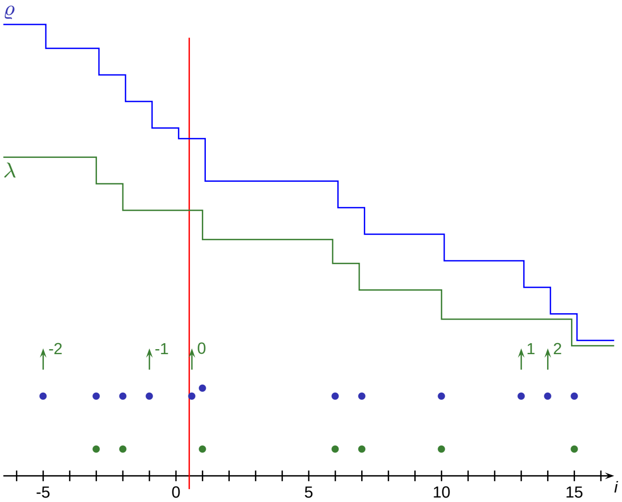

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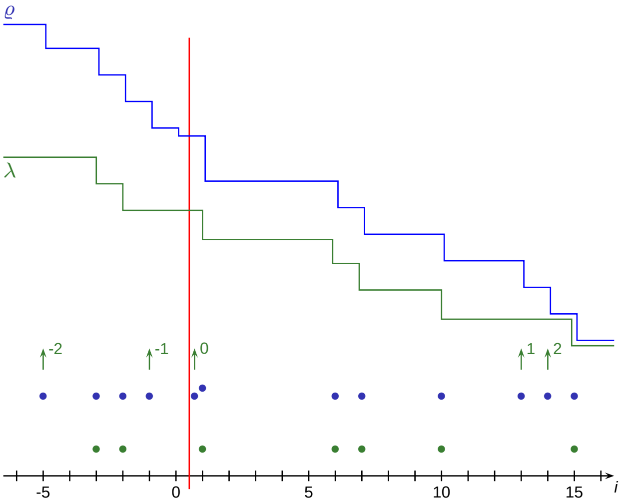

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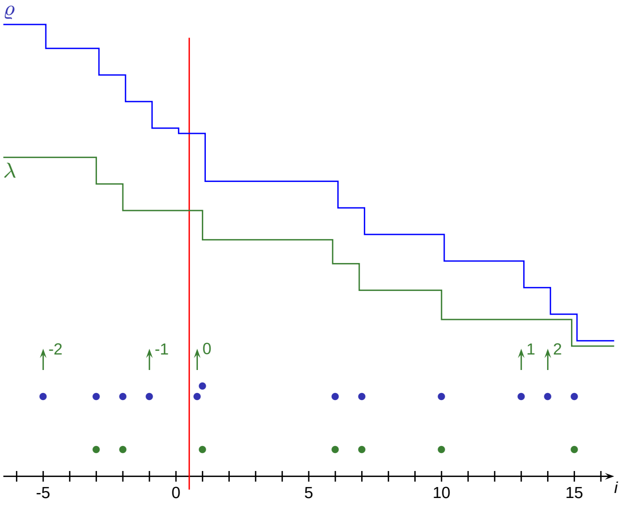

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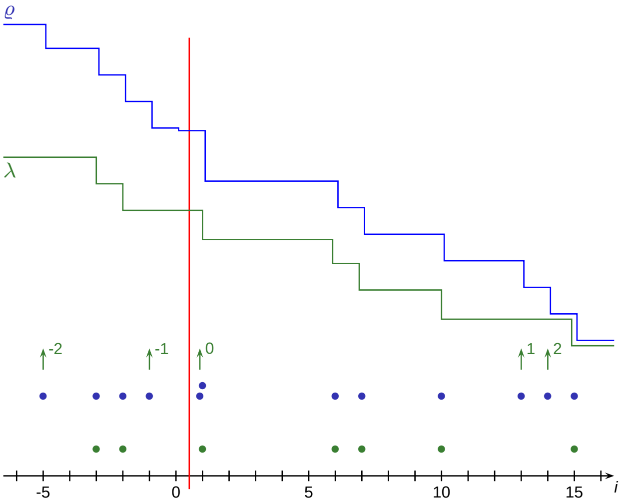

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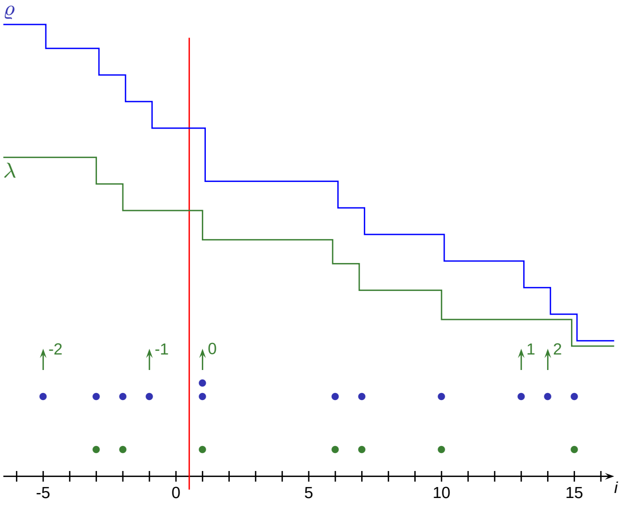

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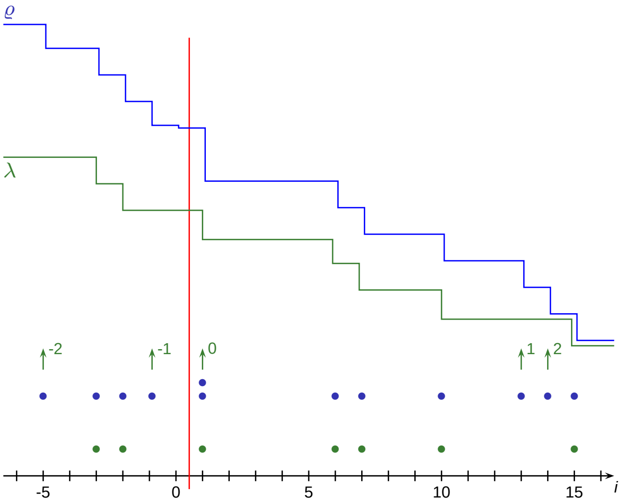

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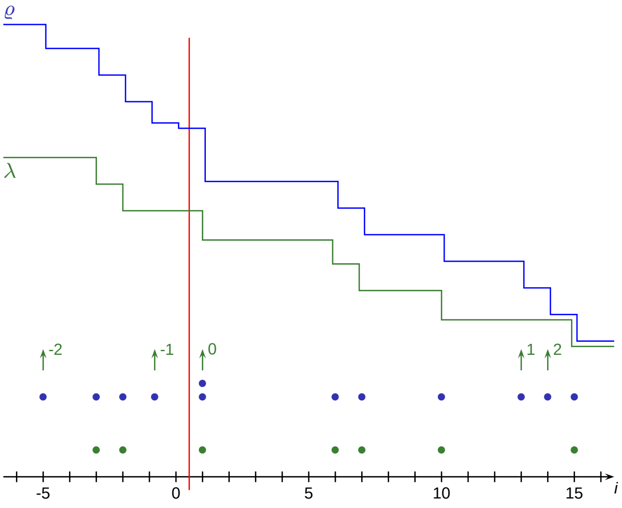

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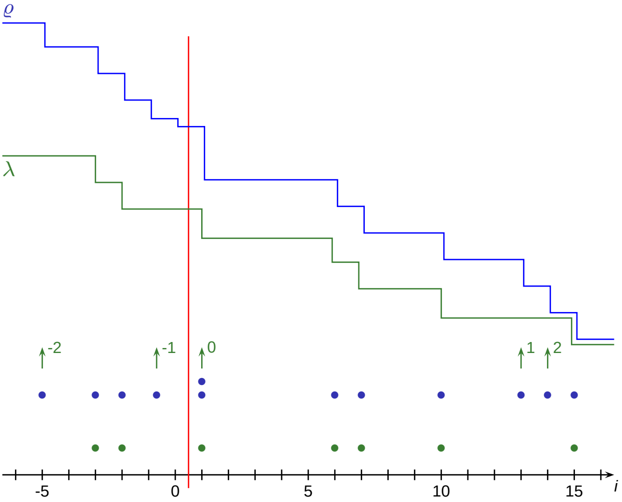

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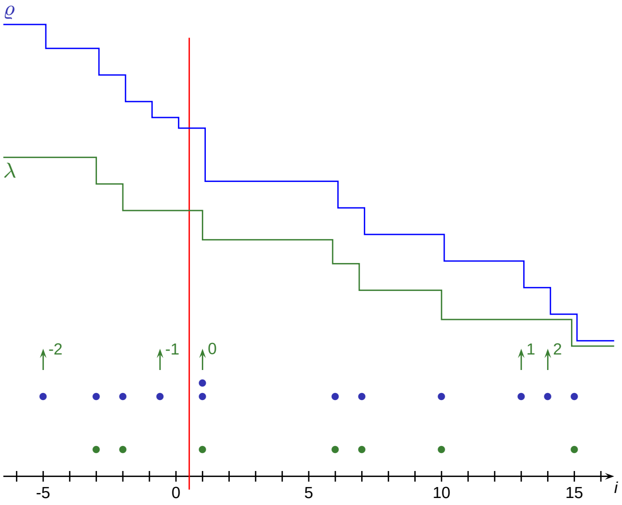

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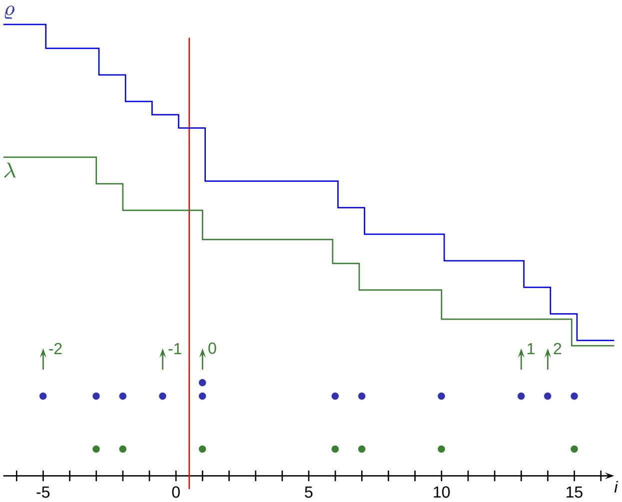

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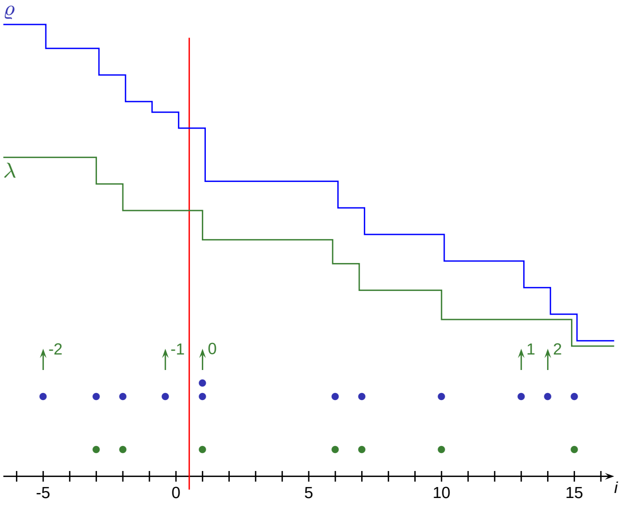

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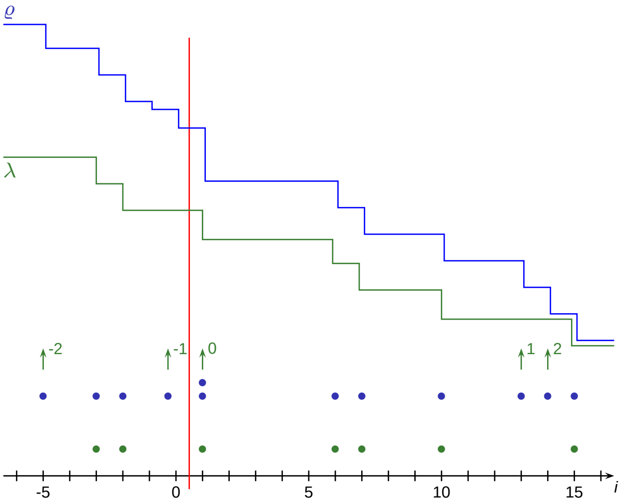

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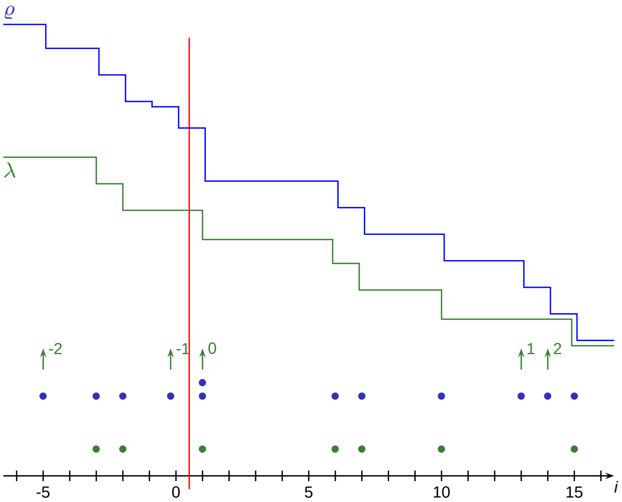

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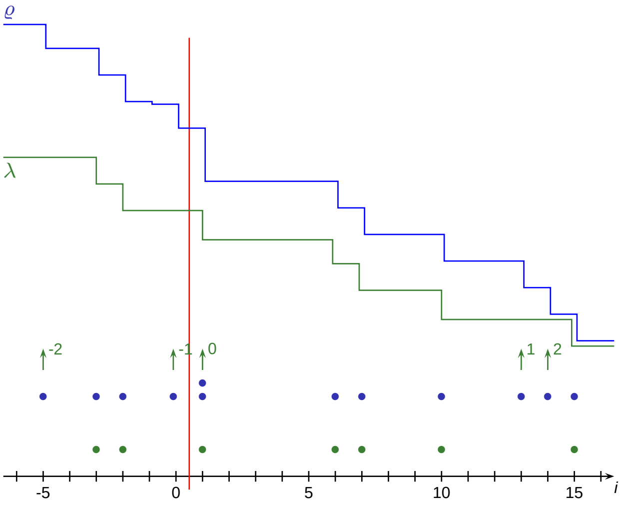

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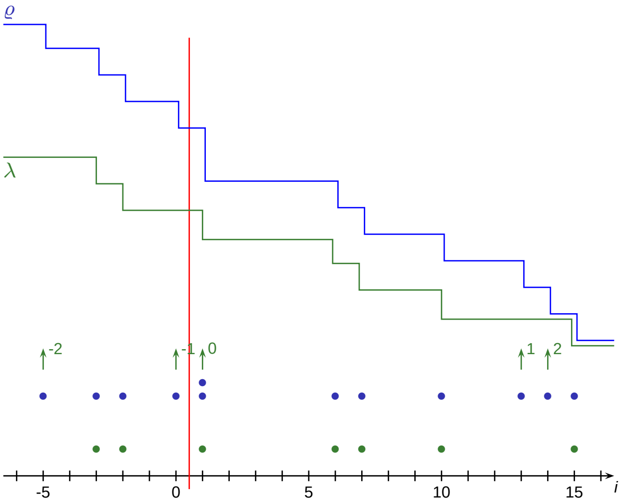

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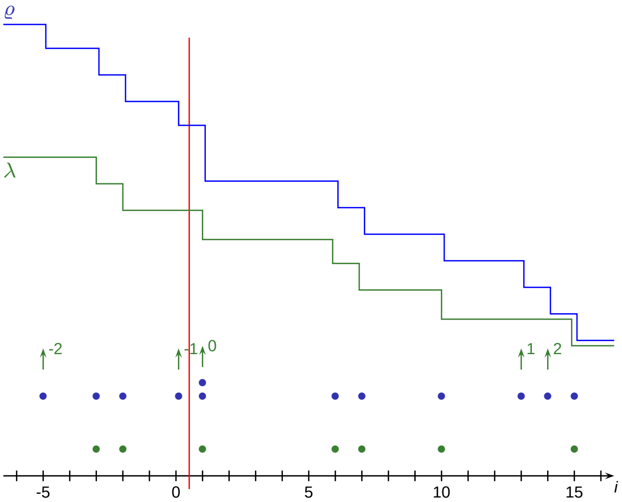

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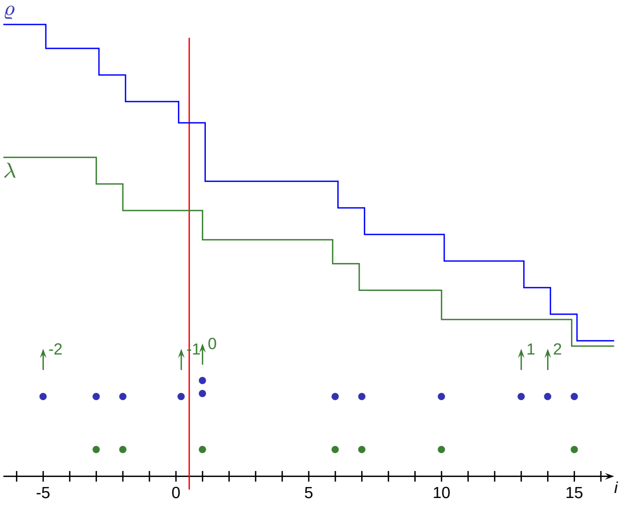

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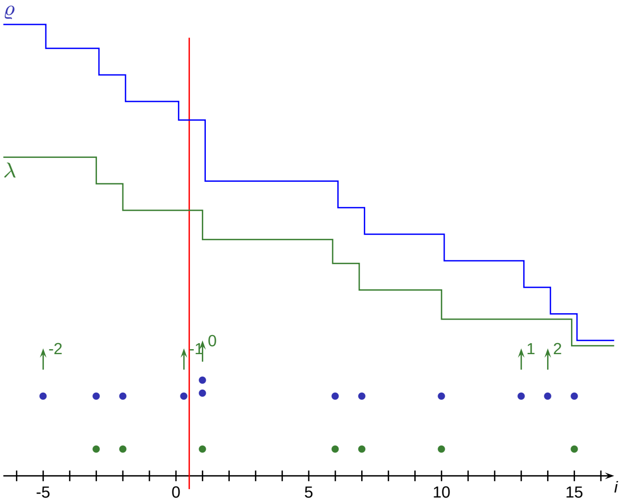

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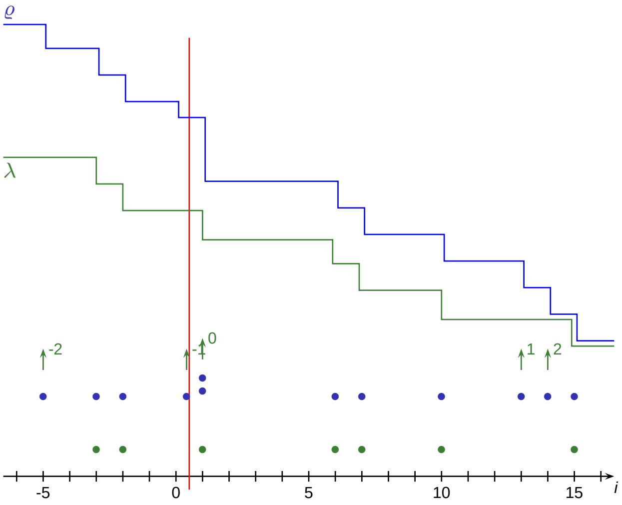

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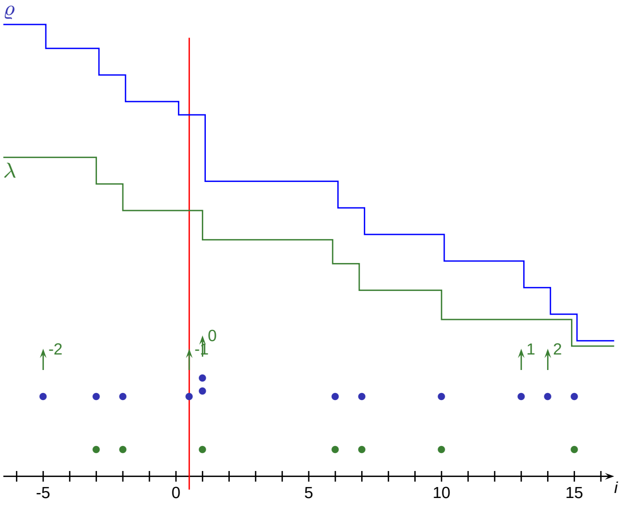

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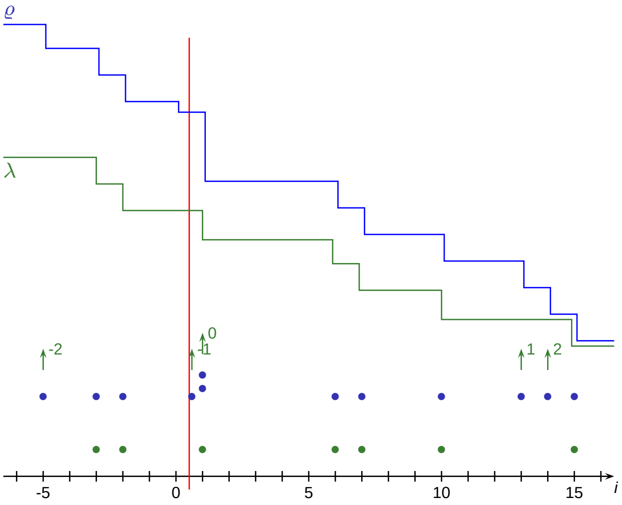

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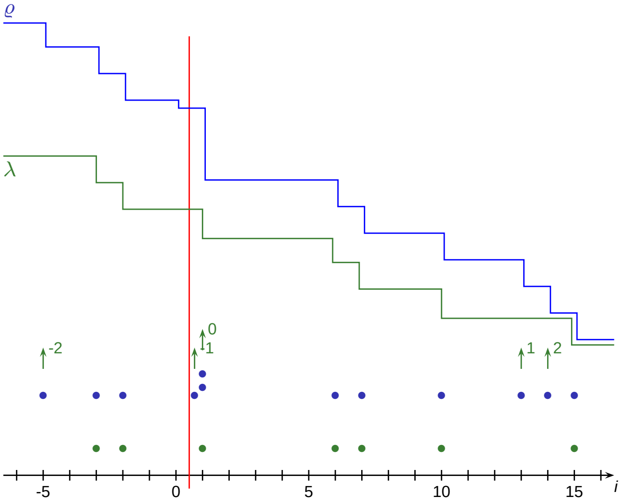

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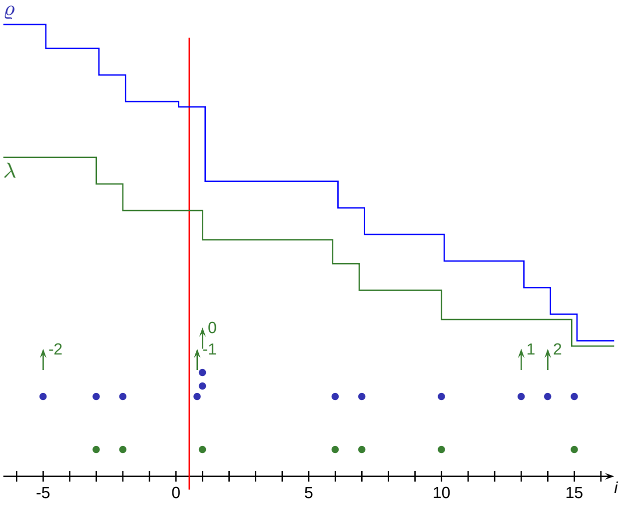

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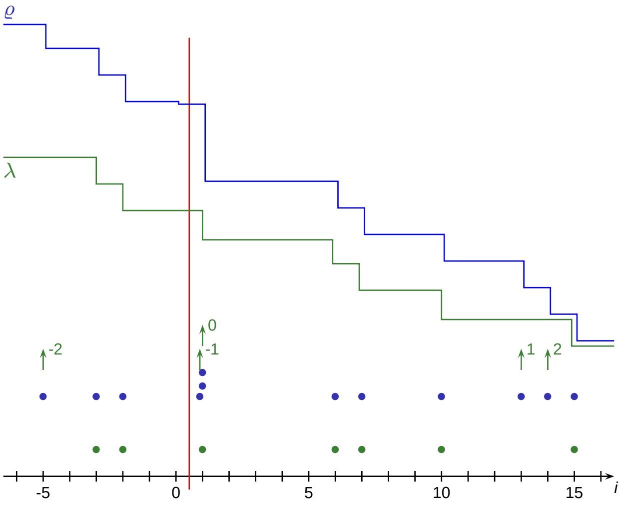

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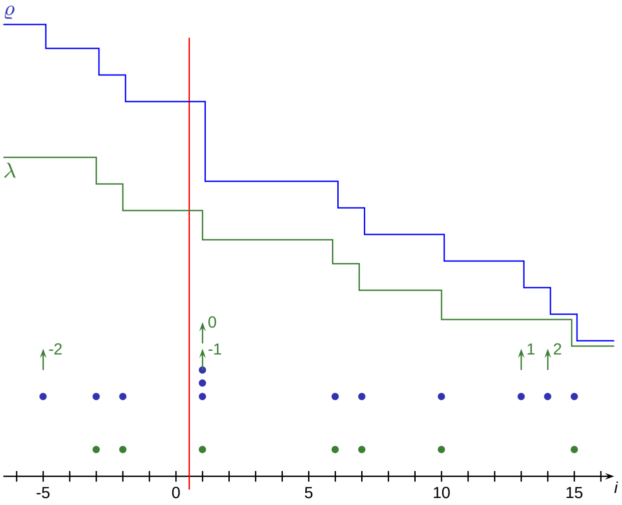

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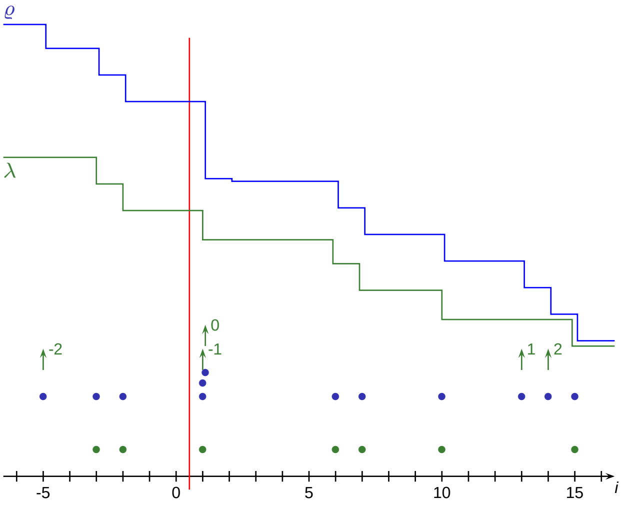

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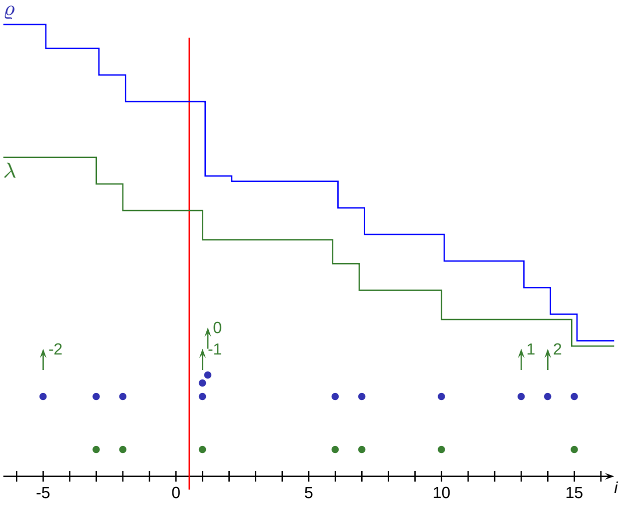

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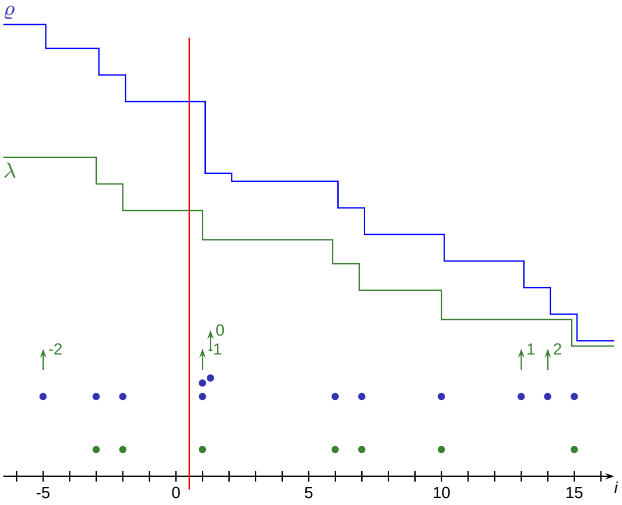

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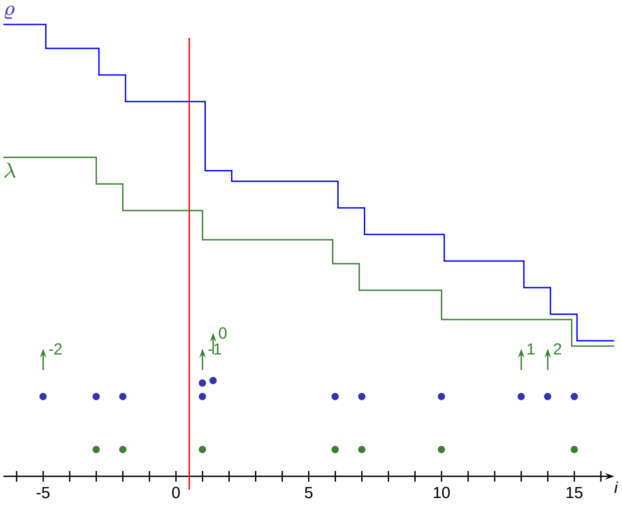

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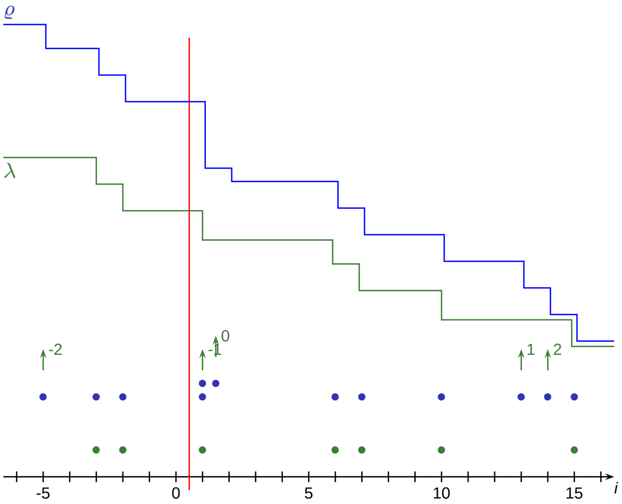

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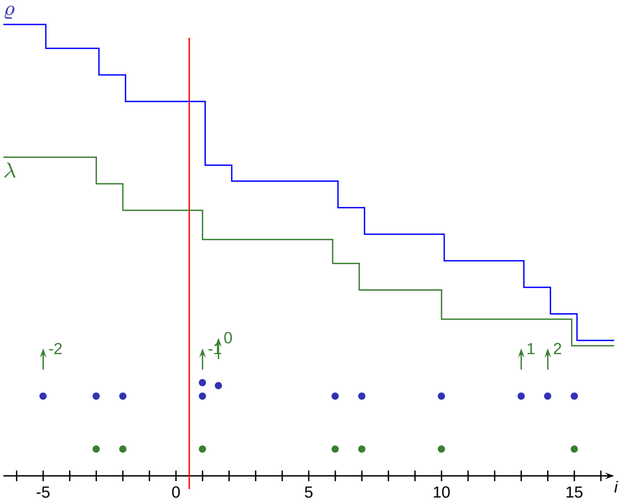

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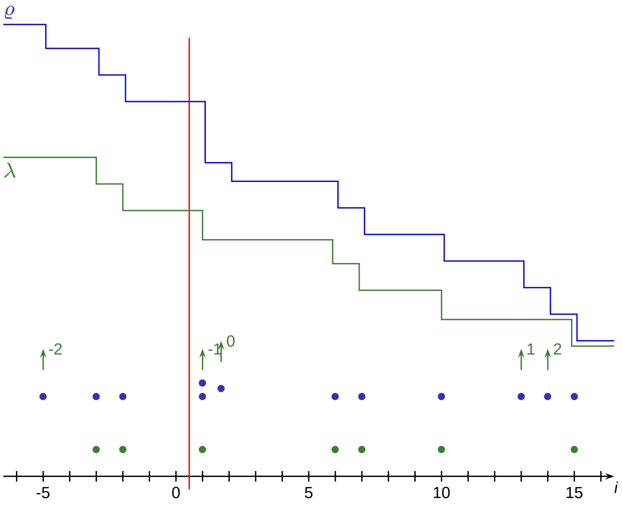

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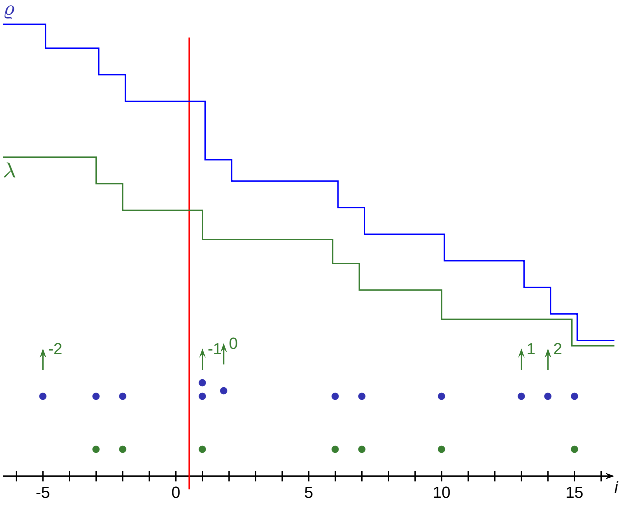

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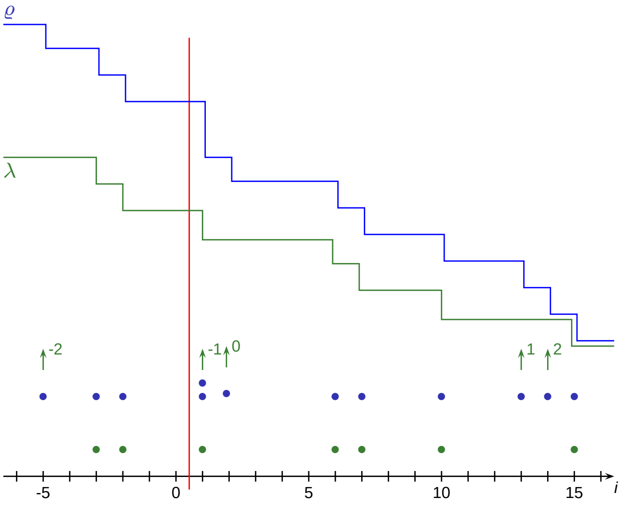

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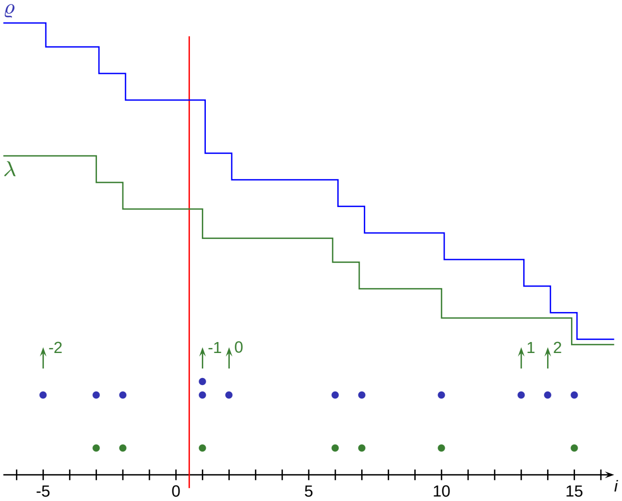

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Upper bound (concave case)

$\mathbf{P}\{Q(t) \text{ is too large}\}$

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$$\mathbf{P}\{Q(t) \text{ is too large}\} \le \mathbf{P}\{X(t) \text{ is too large}\}$$
$$\le \mathbf{P}\{\text{too many} \uparrow\text{'s have crossed } Ct\}$$

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Centering $h_{Ct}(t) - h_{Ct}(t)$ brings in a second-order Taylor-expansion of $H(\varrho)$.

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The computations result in (remember $\mathbf{E}(Q(t)) = Ct$)

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Upper bound

Theorem (B. - Seppäläinen; also ideas from B. Tóth, H. Spohn and M. Prähofer)

Started from (almost) equilibrium,

$$\mathbf{Var}(h_{Ct}(t)) = c \cdot \mathbf{E}|Q(t) - C \cdot t|$$

in the whole family of processes.

$\mathbf{P}\{Q(t) - Ct \geq u\} \leq c \cdot \frac{t^2}{u^4} \cdot \mathbf{Var}(h_{Ct}(t))$ $\mathbf{E}Q(t) = Ct$

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$$\mathbf{P}\{|\widetilde{Q}(t)| > u\} \leq c \cdot \frac{t^2}{u^4} \cdot E$$

with $\widetilde{Q}(t) := Q(t) - Ct$ and $E := \mathbf{E}|\widetilde{Q}(t)|$.

$\mathbf{P}\{Q(t) - Ct \geq u\} \leq c \cdot \frac{t^2}{u^4} \cdot \mathbf{Var}(h_{Ct}(t))$ $\mathbf{E}Q(t) = Ct$

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$$\mathbf{P}\{Q(t) - Ct \geq u\} \leq c \cdot \frac{t^2}{u^4} \cdot \mathbf{E}|Q(t) - C \cdot t|$$

$$\mathbf{P}\{|\widetilde{Q}(t)| > u\} \leq c \cdot \frac{t^2}{u^4} \cdot E$$

with $\widetilde{Q}(t) := Q(t) - Ct$ and $E := \mathbf{E}|\widetilde{Q}(t)|$.

$\mathbf{P}\{|\widetilde{Q}(t)| > u\} \leq c \cdot \frac{t^2}{u^4} \cdot E$

Upper bound

We had $\mathbf{P}\{|\widetilde{Q}(t)| > u\} \leq c \cdot \frac{t^2}{u^4} \cdot E$.

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&\le c \cdot \frac{t^2}{E^2} + \frac{1}{2}E,
\end{aligned}
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that is, $E^3 \le c \cdot t^2$.

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$$\begin{aligned} \mathbf{Var}(h_{Ct}(t)) &\overset{\text{Thm}}{=} \text{const.} \cdot \mathbf{E}|Q(t) - Ct| \\ &= \text{const.} \cdot E \le c \cdot t^{2/3}. \qquad \blacksquare \end{aligned}$$

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Lower bound

In the upper bound, the relevant orders were

$$u \text{ (deviation of } Q(t)) \sim t^{2/3}, \qquad \varrho - \lambda \sim t^{-1/3}.$$

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The critical feature in both the upper bound and lower bound was microscopic convexity/concavity: $Q(t) \geq X(t)$ (convex) or $Q(t) \leq X(t)$ (concave).

Microscopic convexity/concavity

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Microscopic convexity/concavity

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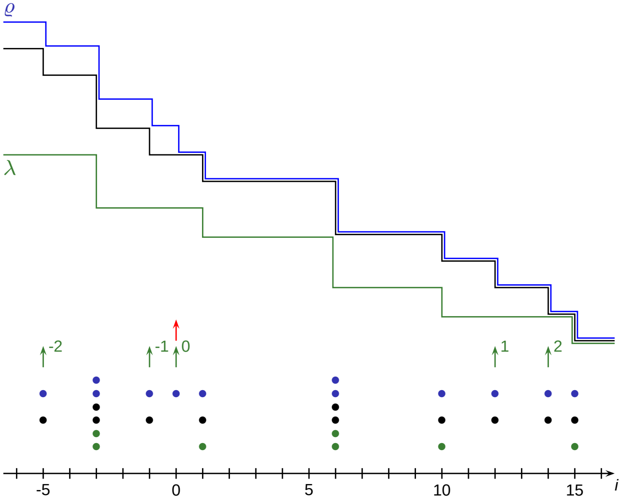
$Q(t) \leq X(t)$ + tight error



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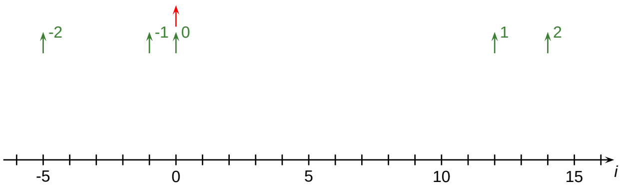

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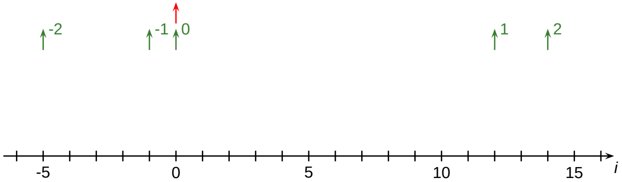

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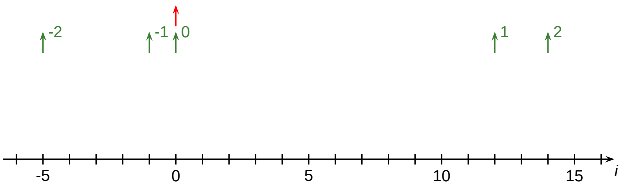

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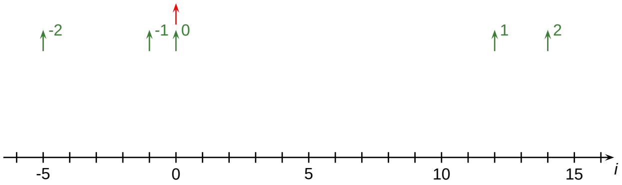

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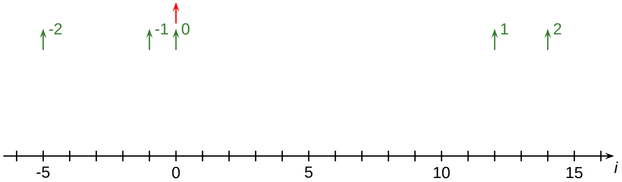

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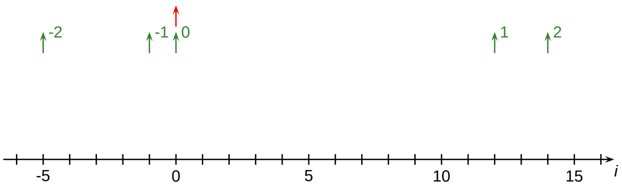

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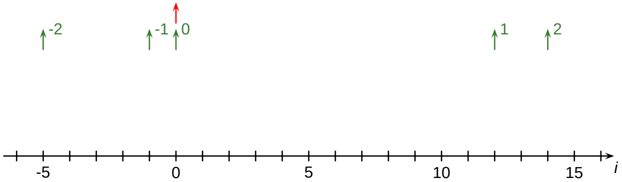

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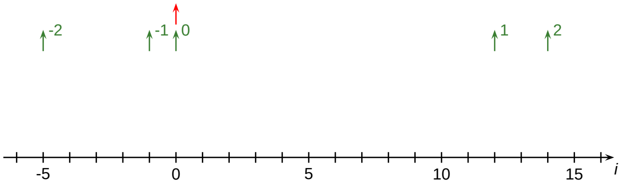

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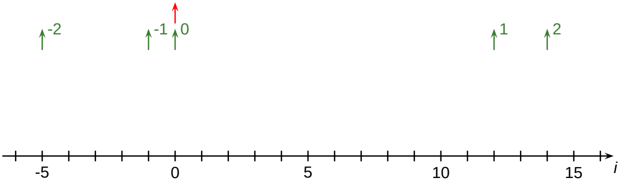

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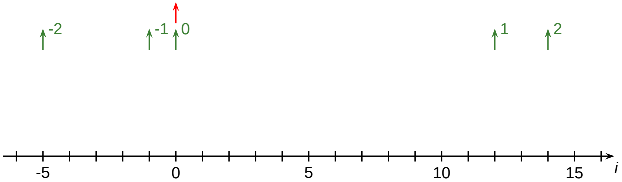

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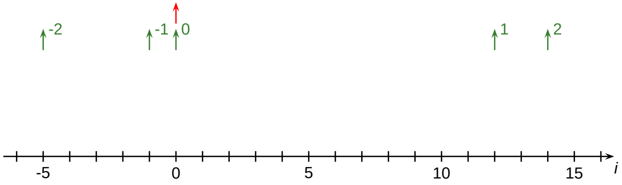

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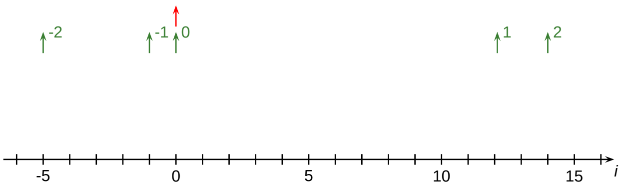

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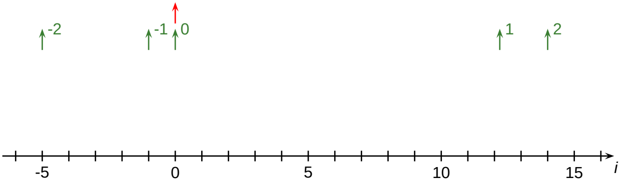

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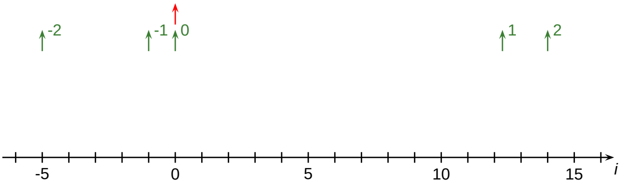

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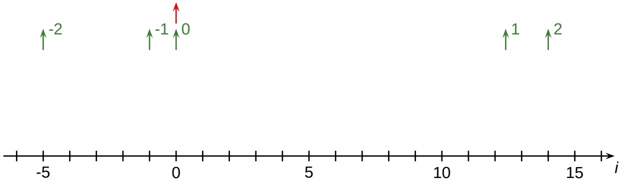

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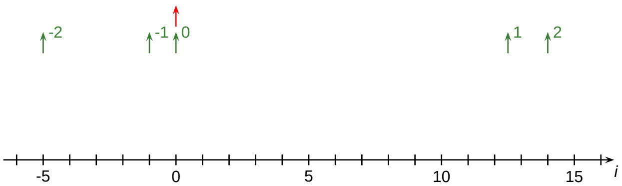

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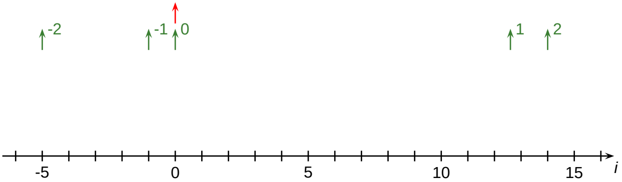

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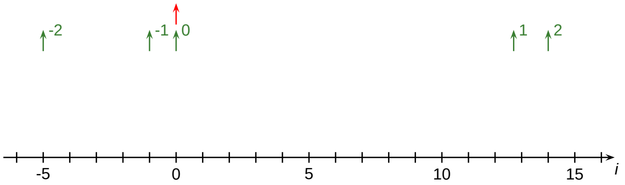

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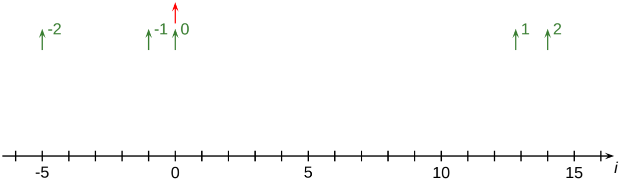

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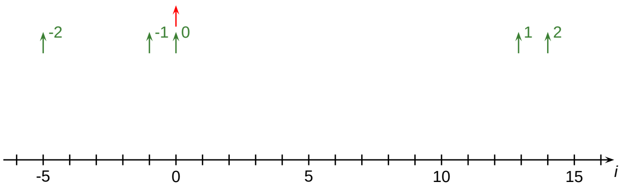

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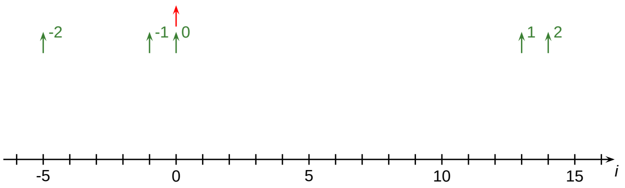

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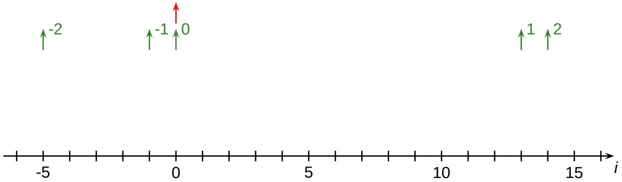

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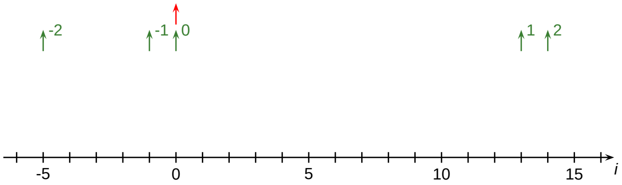

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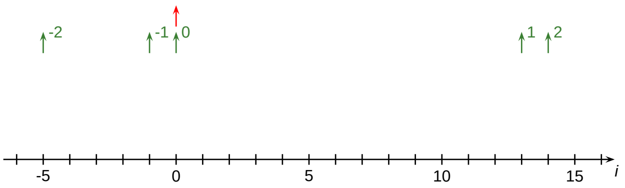

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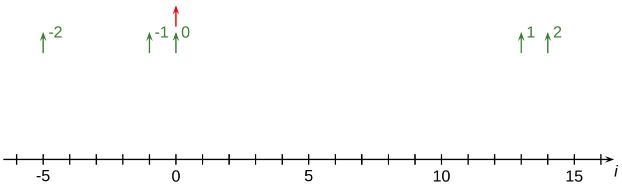

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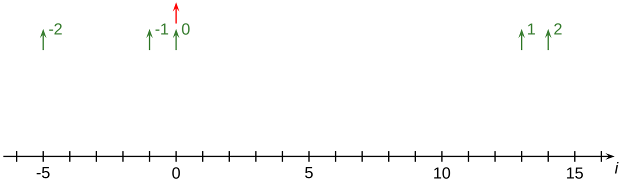

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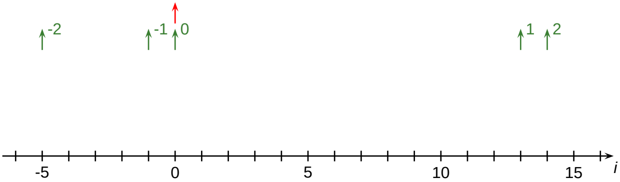

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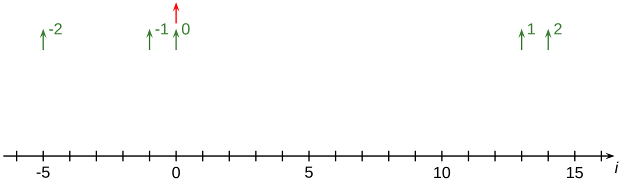

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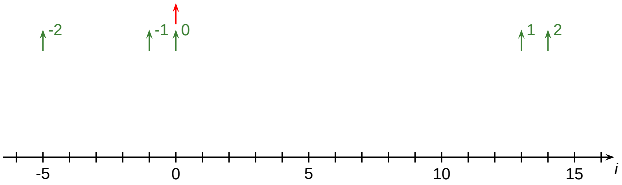

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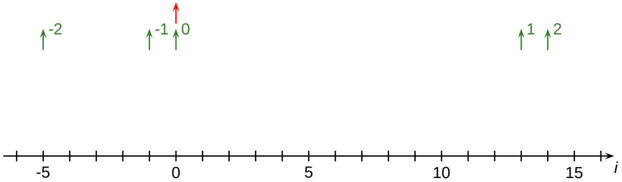

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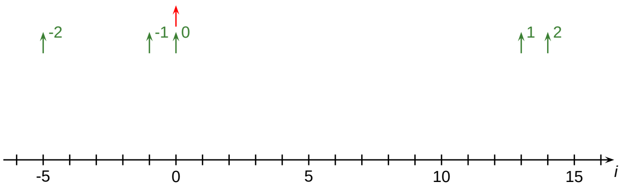

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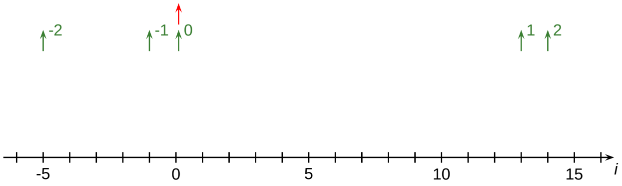

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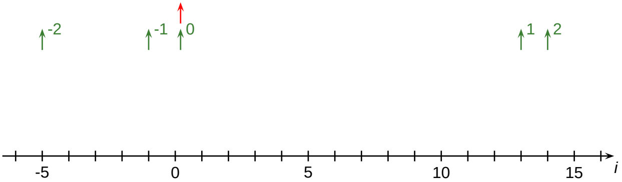

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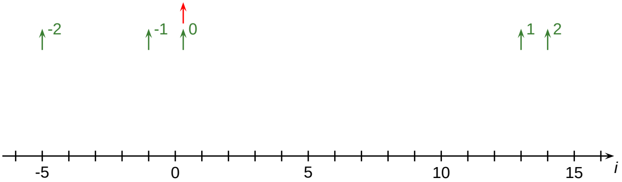

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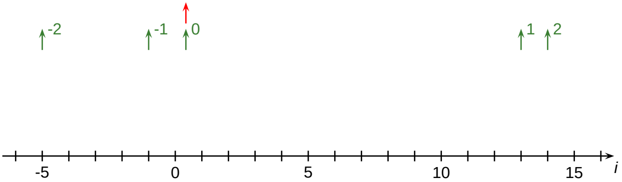

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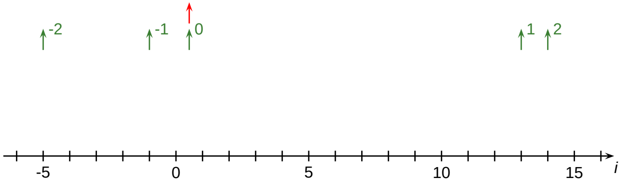

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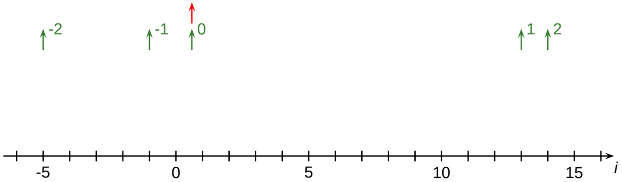

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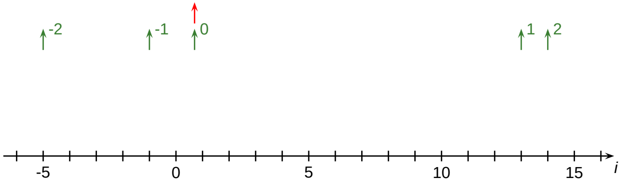

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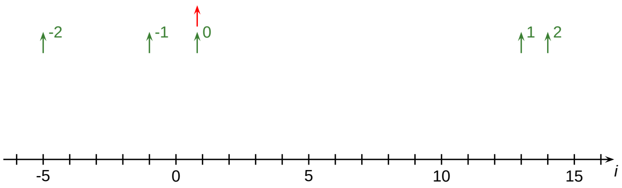

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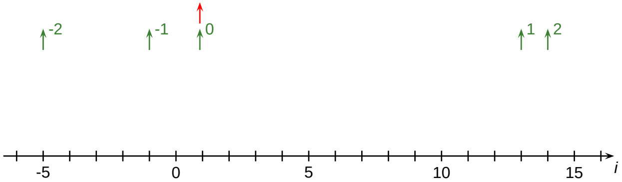

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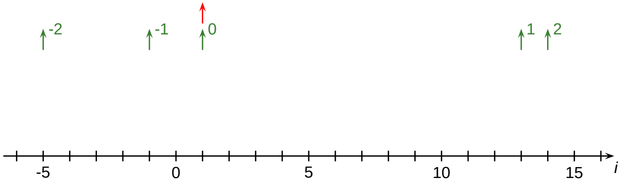

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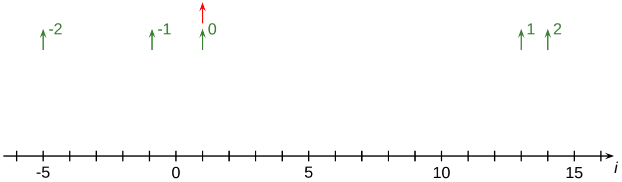

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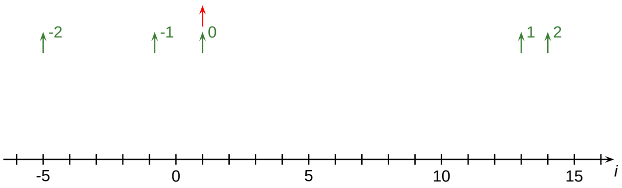

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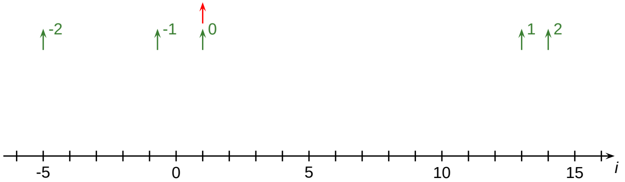

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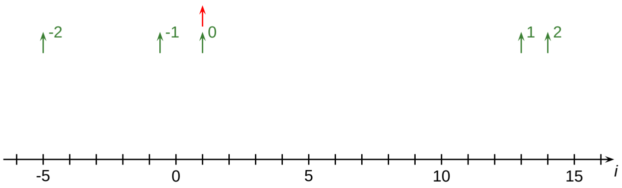

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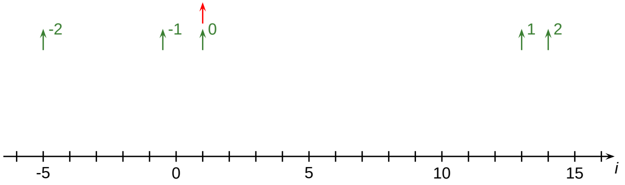

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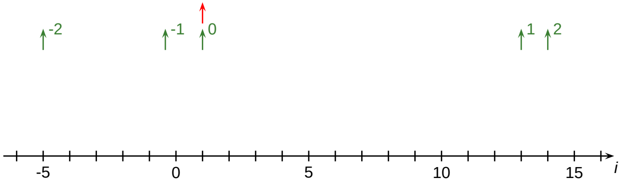

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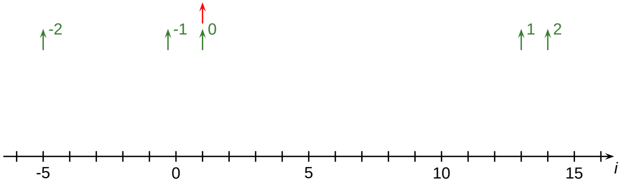

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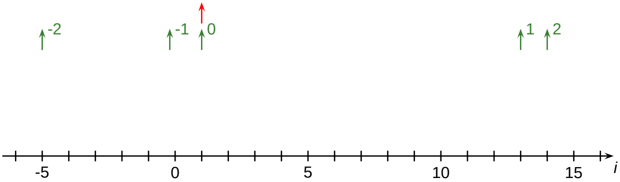

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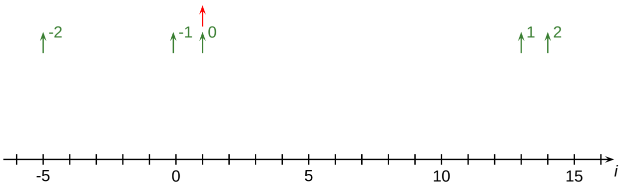

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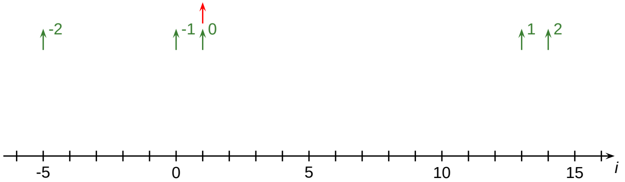

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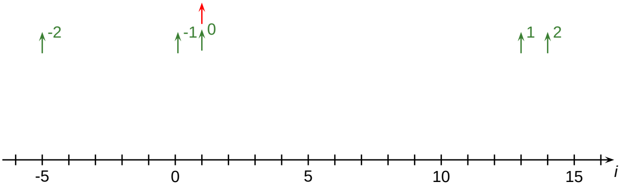

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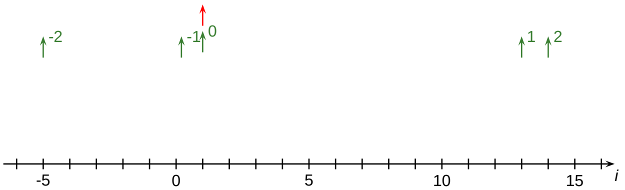

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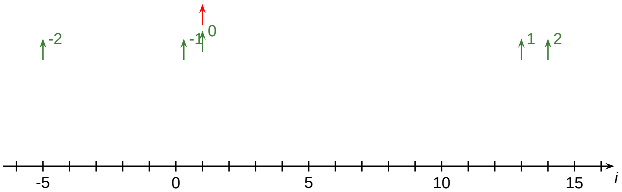

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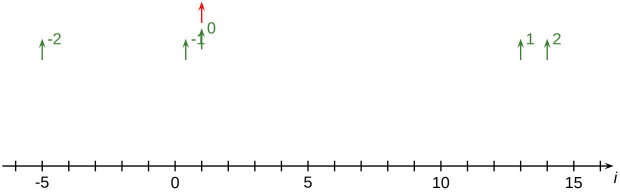

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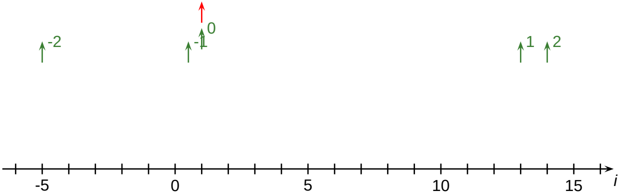

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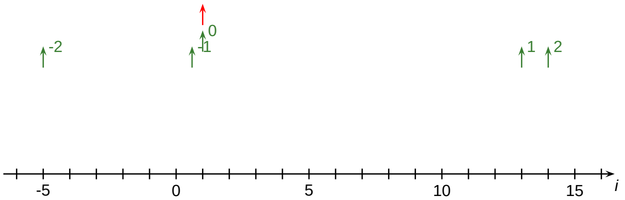

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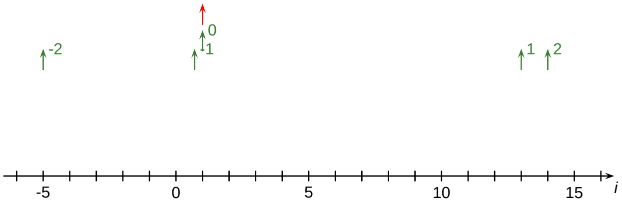

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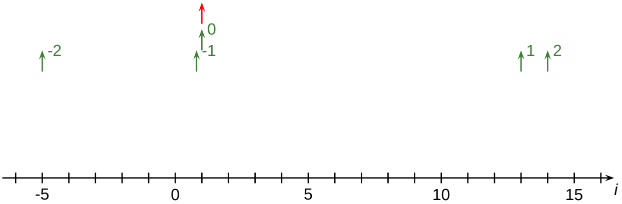

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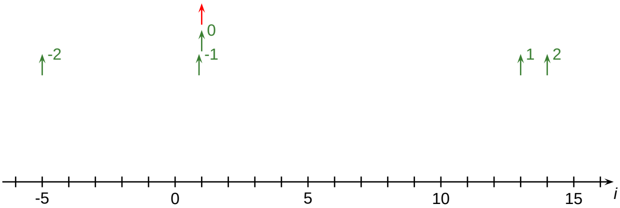

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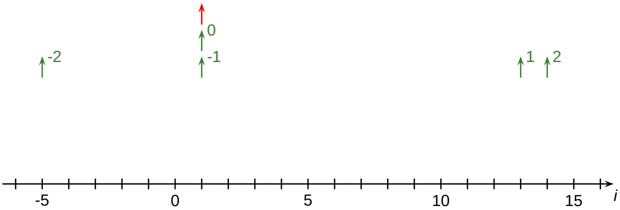

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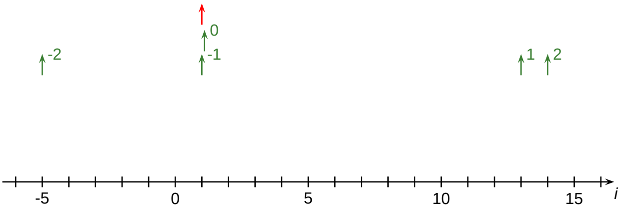

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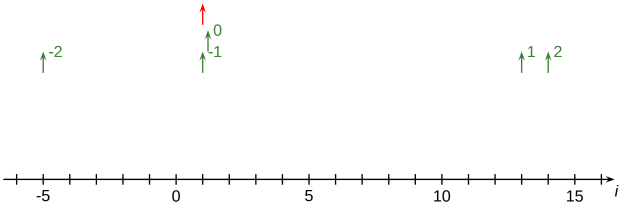

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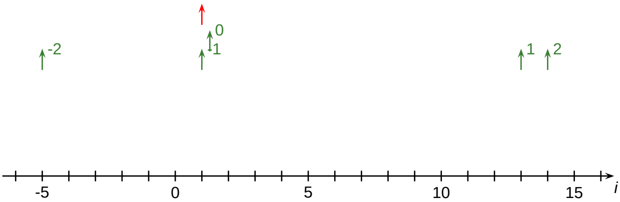

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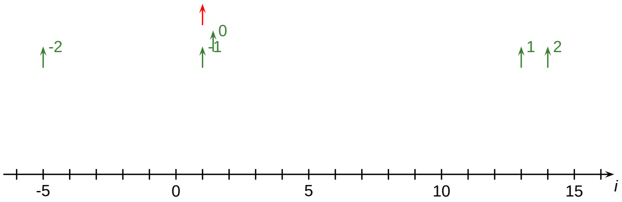

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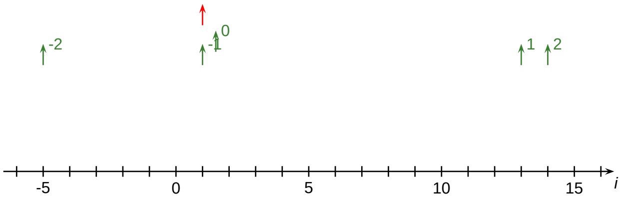

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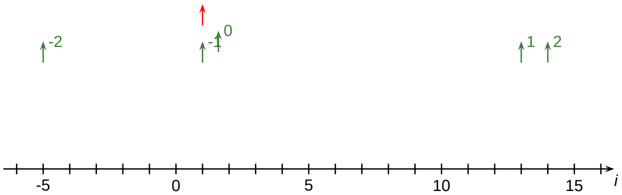

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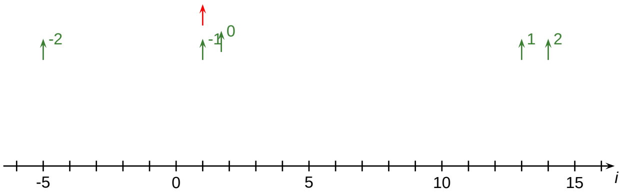

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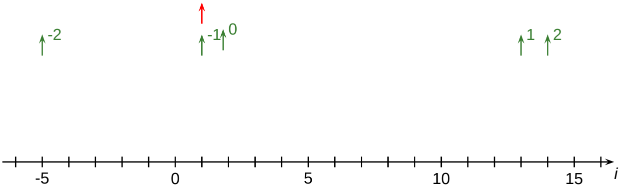

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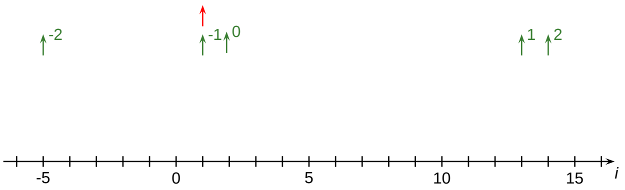

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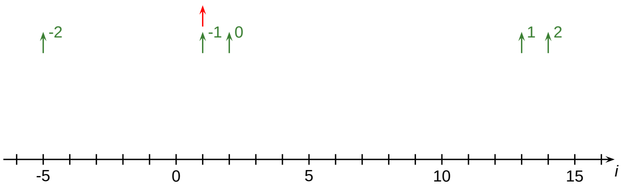

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This is the form of microscopic concavity we currently use: $m_Q(t)$ is dominated by a time-independent distribution with finite variance.

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Once this is done, we could proceed with less and less convex/concave models to see how $t^{1/3}$ scaling turns to $t^{1/4}$ for linear models (random average process, linear rate AZRP)...

Thank you.