Markov chains from a distance: shocking particles

Márton Balázs

School of Mathematics

Matrix, University of Bristol, 2 December, 2020

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$\rightarrow$ More  's, even smaller probability.

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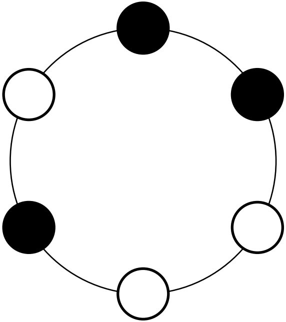
$$\begin{aligned}\mathbf{P}\{\text{none of them ring}\} &= \mathbf{P}\{\tau > t\}^k \\ &= \mathrm{e}^{-kt} \\ &\simeq (1 - kt) + \text{error}.\end{aligned}$$

The Totally Asymmetric Simple Exclusion Process

TASEP



m balls in N possible slots.

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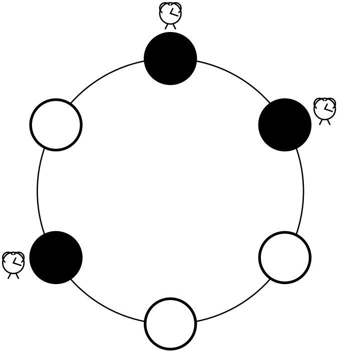

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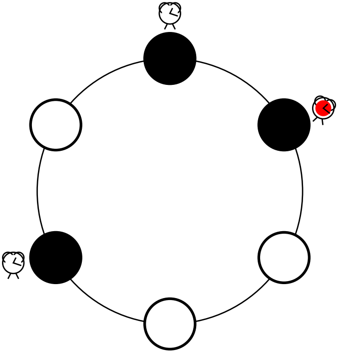

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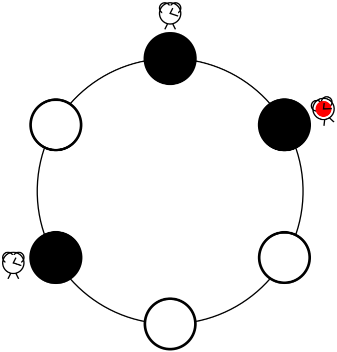

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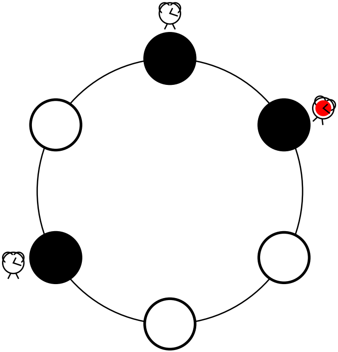

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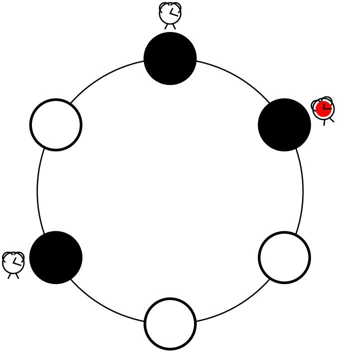

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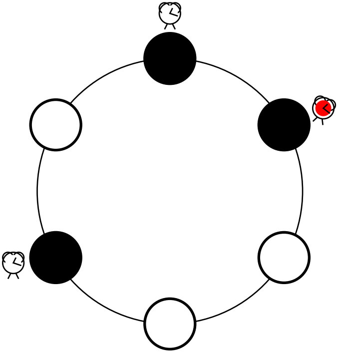

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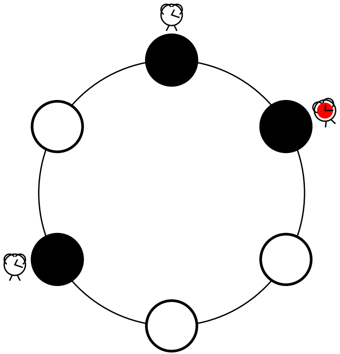

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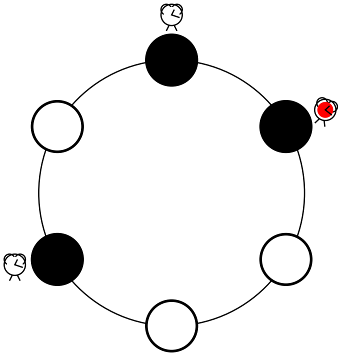

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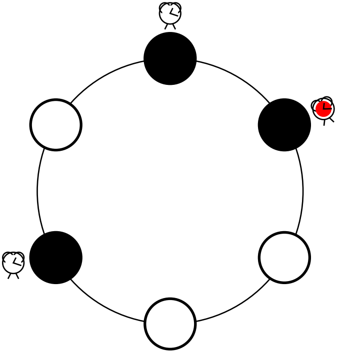

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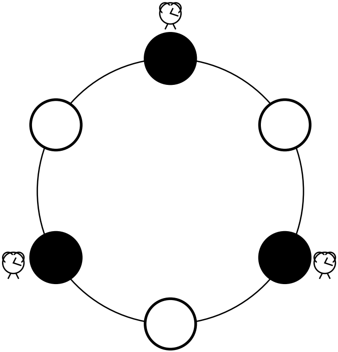

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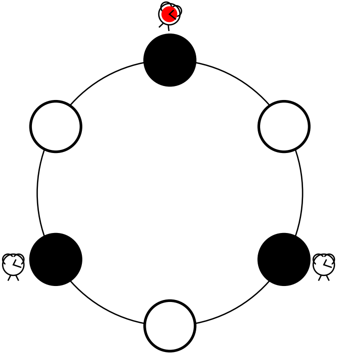

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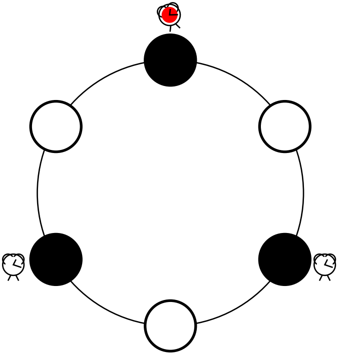

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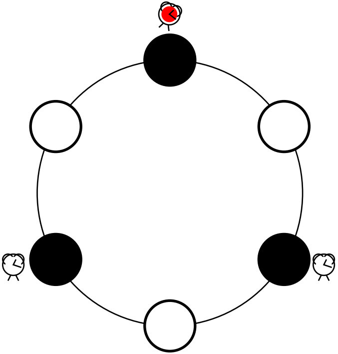

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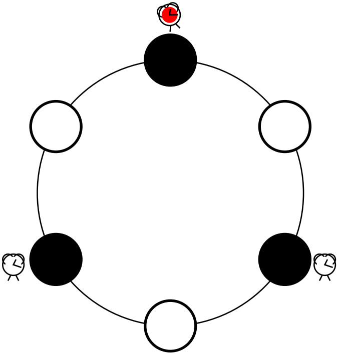

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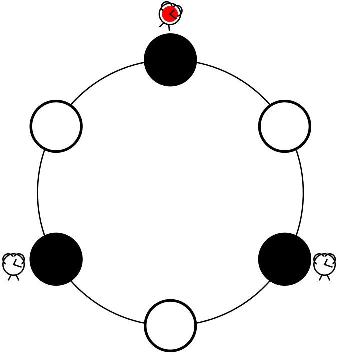

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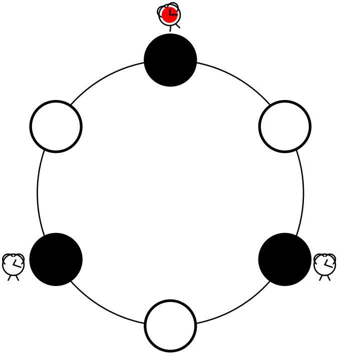

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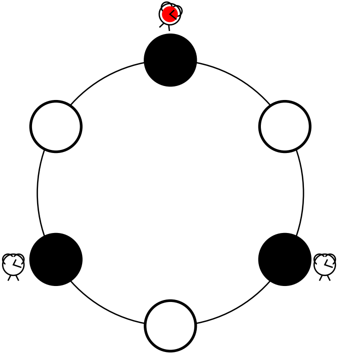

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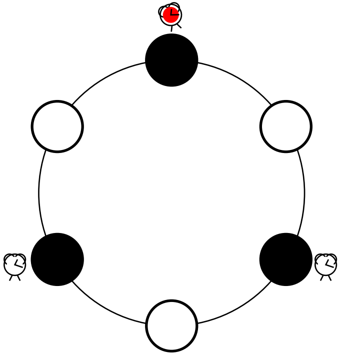

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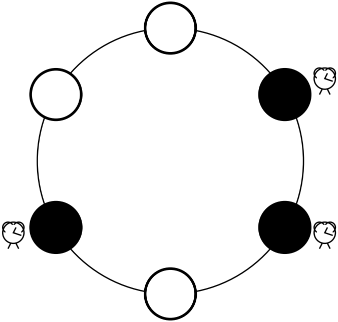

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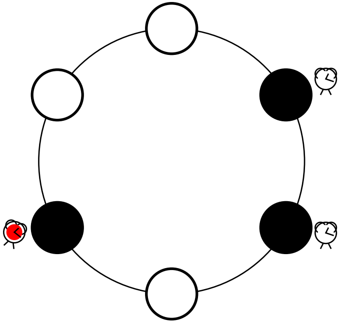

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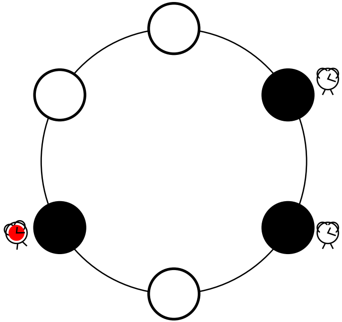

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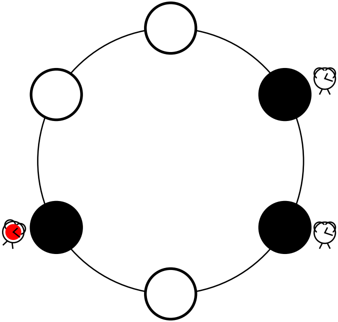

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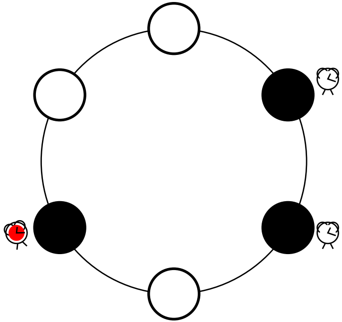

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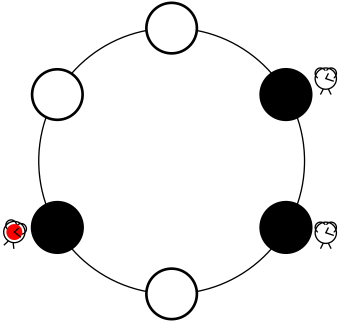

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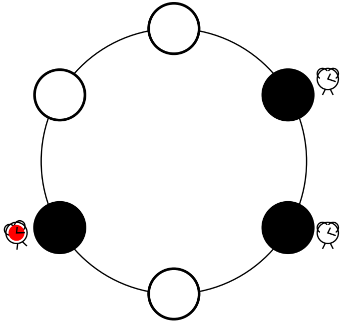

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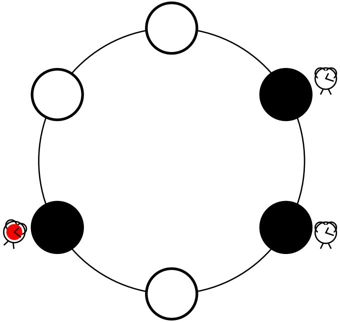

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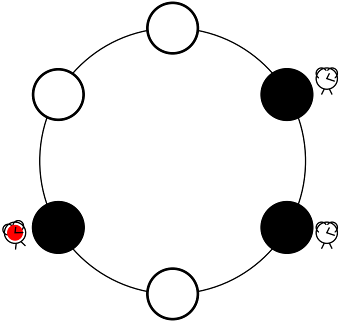

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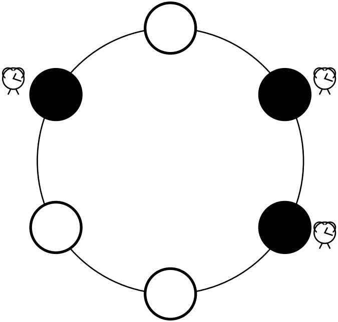

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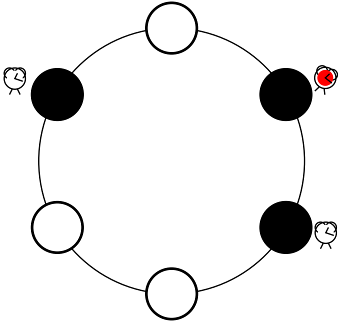

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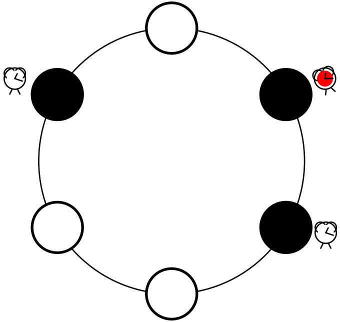

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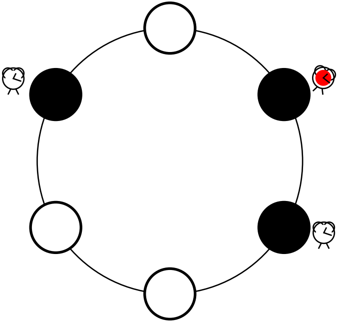

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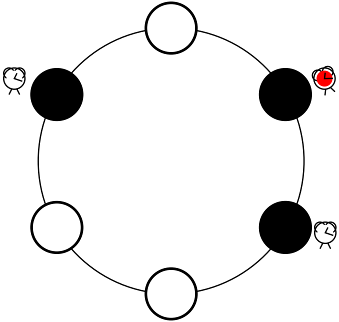

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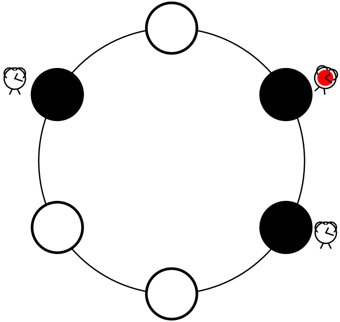

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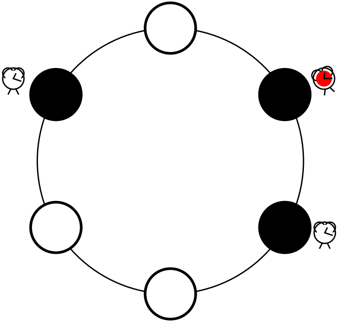

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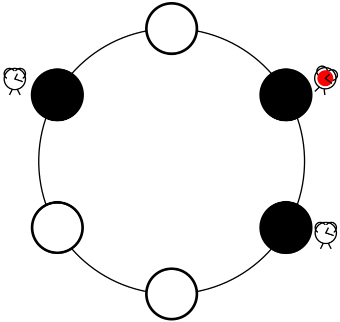

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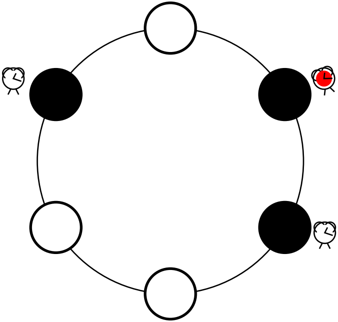

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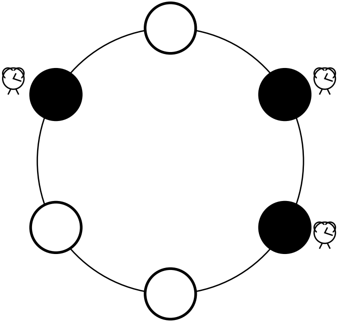

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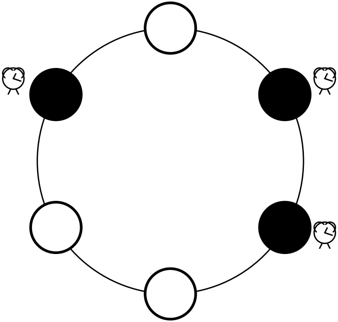

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Memoryless, independent 's ⇒ if we know the present, no need to know the past. *Markov property*, makes things handy.

Stationary distribution

Random process $\rightsquigarrow$ need to talk about *distributions*.

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Theorem

With N and m fixed, the distribution that gives equal chance to each (m-ball) configuration, is stationary.

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1st remark.

In this case every configuration occurs with probability $1/\binom{N}{m}$.

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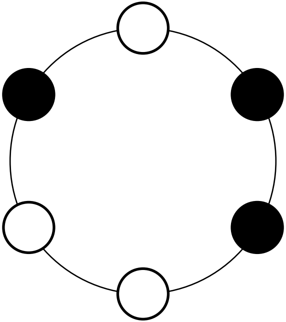
2nd remark.

With fixed N, m, there is no other stationary distribution.

Stationary distribution

Almost proof

A state ω



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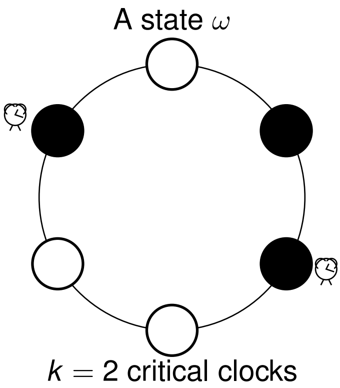

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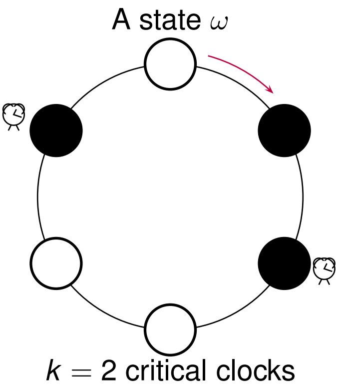

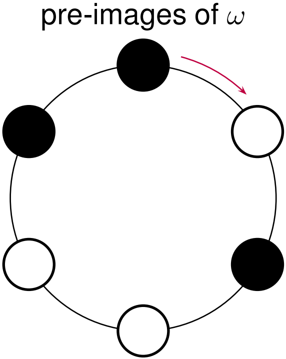

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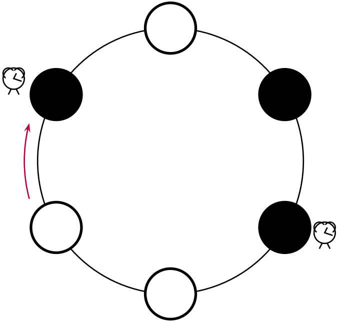
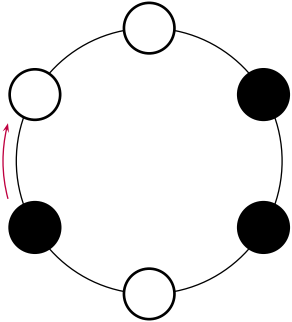


$k = 2$ critical clocks

pre-images of ω



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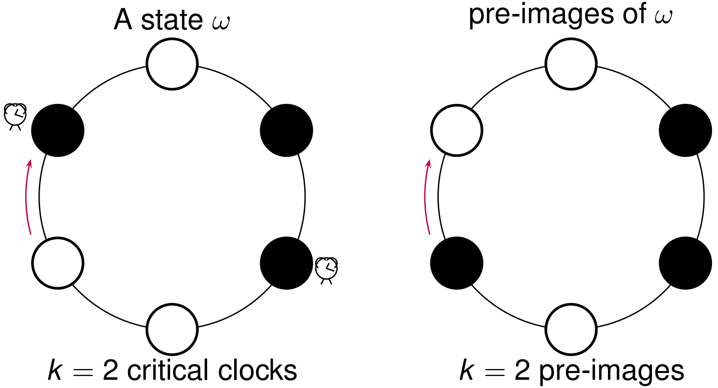

The number of critical clocks for ω = the number of pre-images of $\omega = k$

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Suppose that each configuration has the same probability p at time s. What is the probability of the state ω after a small time t?

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$$\mathbf{P}\{\omega \text{ at time } s + t\}$$

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$$\begin{aligned}
&\mathbf{P}\{\omega \text{ at time } s+t\} \\
&= \mathbf{P}\{\omega \text{ at time } s \text{ and no jumps within time } t\} \\
&\quad + \mathbf{P}\{\text{was a pre-image of } \omega \text{ at time } s\text{, and jumps to } \omega\} \\
&\quad + \text{error (at least two jumps occur within the small time } t\text{)}
\end{aligned}$$

Stationary distribution

Almost proof

Suppose that each configuration has the same probability p at time s. What is the probability of the state ω after a small time t?

$$\mathbf{P}\{\omega \text{ at time } s+t\}$$
$$= \mathbf{P}\{\omega \text{ at time } s \text{ and no jumps within time } t\}$$
$$+ \mathbf{P}\{\text{was a pre-image of } \omega \text{ at time } s\text{, and jumps to } \omega\}$$
$$+ \text{error (at least two jumps occur within the small time } t)$$

$= \mathbf{P}\{\omega$ at time s and none of the k critical  's ring$\}$

$+ \sum_{\eta \text{ is a pre-image of } \omega} \mathbf{P}\{\eta$ at time s and the right critical  rings$\}$

$+$ error

Stationary distribution

Almost proof

$\mathbf{P}\{\omega$ at time $s+t\}$

$= \mathbf{P}\{\omega$ at time s and none of the k critical  's ring$\}$

$+ \sum_{\eta \text{ is a pre-image of } \omega} \mathbf{P}\{\eta$ at time s and the right critical  rings$\}$

$+$ error

Stationary distribution

Almost proof

$\mathbf{P}\{\omega \text{ at time } s+t\}$

$= \mathbf{P}\{\omega$ at time s and none of the k critical  's ring$\}$

$+ \sum_{\eta \text{ is a pre-image of } \omega} \mathbf{P}\{\eta$ at time s and the right critical  rings$\}$

$+ \text{error}$

$$= p \cdot (1-kt) + \sum_{\eta \text{ is a pre-image of } \omega} p \cdot t + \text{error}$$

Stationary distribution

Almost proof

$\mathbf{P}\{\omega \text{ at time } s+t\}$

$= \mathbf{P}\{\omega \text{ at time } s \text{ and none of the } k \text{ critical}$  $\text{'s ring}\}$

$+ \sum_{\eta \text{ is a pre-image of } \omega} \mathbf{P}\{\eta \text{ at time } s \text{ and the right critical}$  $\text{rings}\}$

$+ \text{error}$

$$= p \cdot (1 - kt) + \sum_{\eta \text{ is a pre-image of } \omega} p \cdot t + \text{error}$$

$$= p \cdot (1 - kt) + k \cdot p \cdot t + \text{error}$$

Stationary distribution

Almost proof

$$\begin{aligned}
&\mathbf{P}\{\omega \text{ at time } s+t\}\\
&= \mathbf{P}\{\omega \text{ at time } s \text{ and none of the } k \text{ critical ⏰'s ring}\}\\
&\quad + \sum_{\eta \text{ is a pre-image of } \omega} \mathbf{P}\{\eta \text{ at time } s \text{ and the right critical ⏰ rings}\}\\
&\quad + \text{error}\\
&= p \cdot (1-kt) + \sum_{\eta \text{ is a pre-image of } \omega} p \cdot t + \text{error}\\
&= p \cdot (1-kt) + k \cdot p \cdot t + \text{error} = p + \text{error}.
\end{aligned}$$

Stationary distribution

Almost proof

$$\mathbf{P}\{\omega \text{ at time } s+t\}$$

$= \mathbf{P}\{\omega$ at time s and none of the k critical  's ring$\}$

$+ \sum_{\eta \text{ is a pre-image of } \omega} \mathbf{P}\{\eta$ at time s and the right critical  rings$\}$

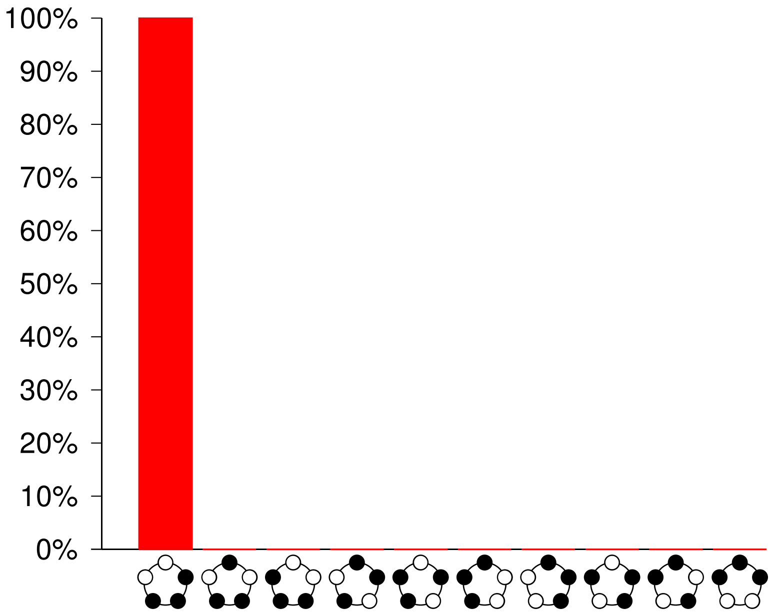
$+$ error

$$= p\cdot(1-kt) + \sum_{\eta \text{ is a pre-image of } \omega} p\cdot t + \text{error}$$

$$= p\cdot(1-kt) + k\cdot p\cdot t + \text{error} = p + \text{error}.$$

In fact error $\simeq t^2$, stays small if summed up for more and more smaller and smaller intervals of length t. □

Stationary distribution



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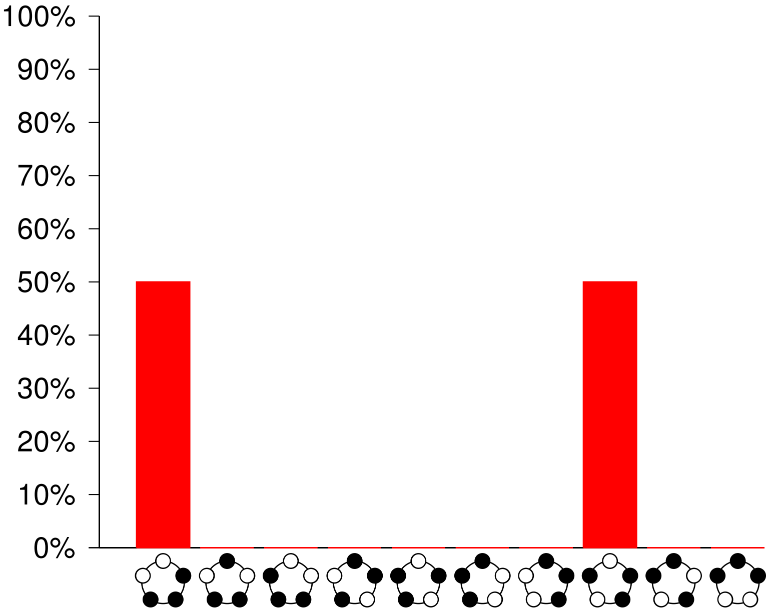

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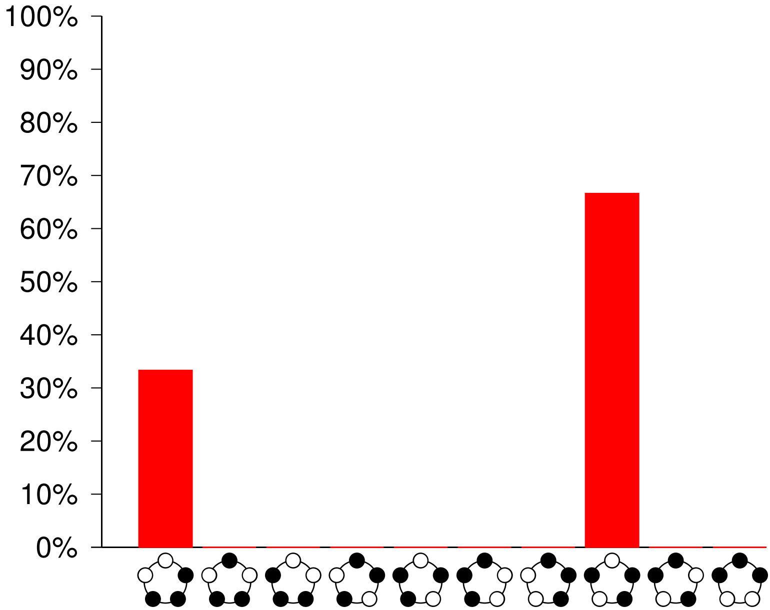

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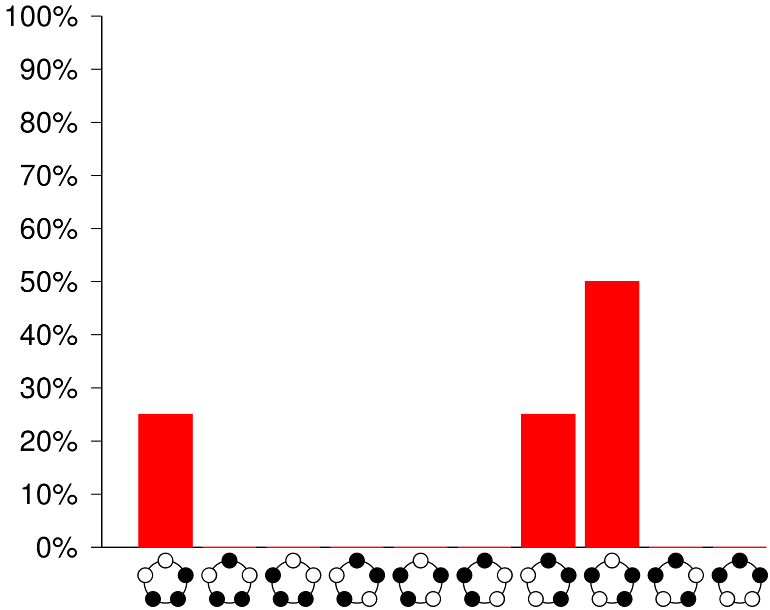

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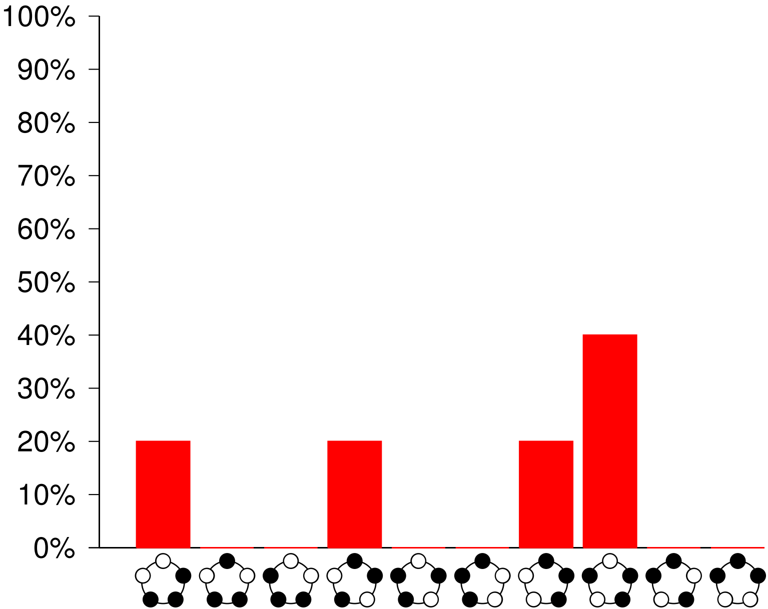

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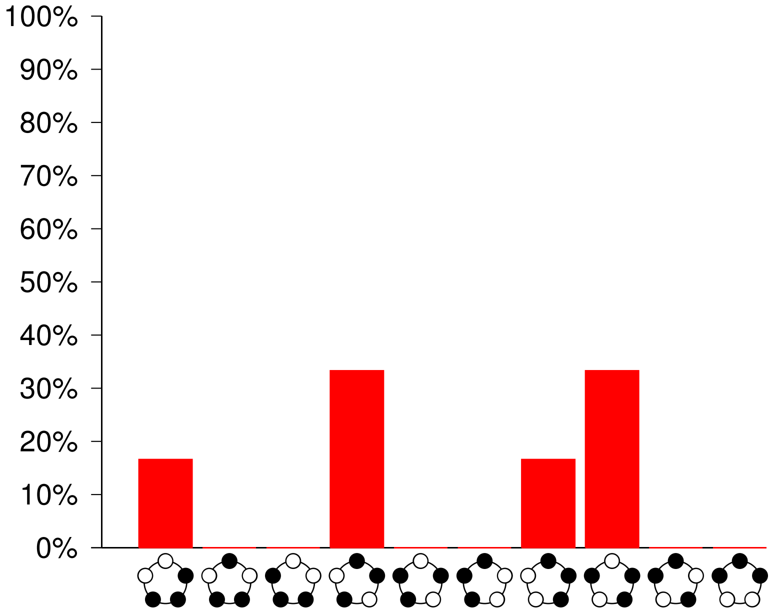

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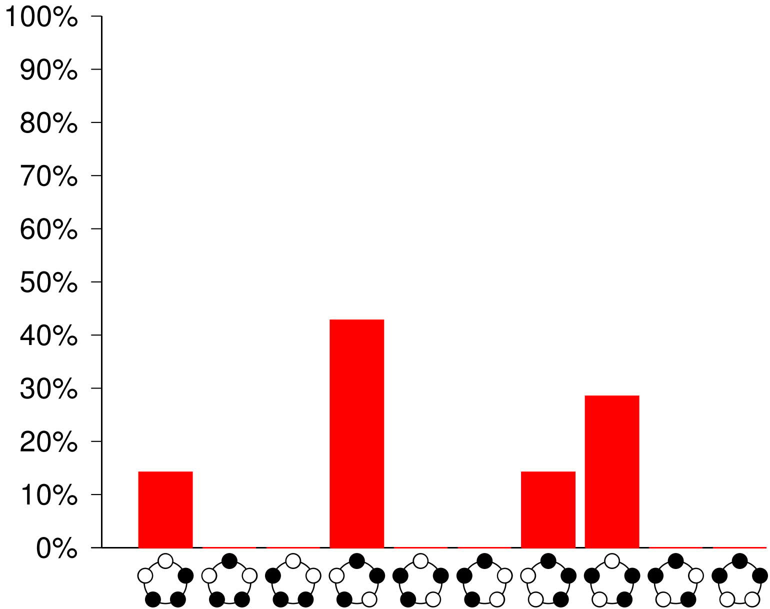

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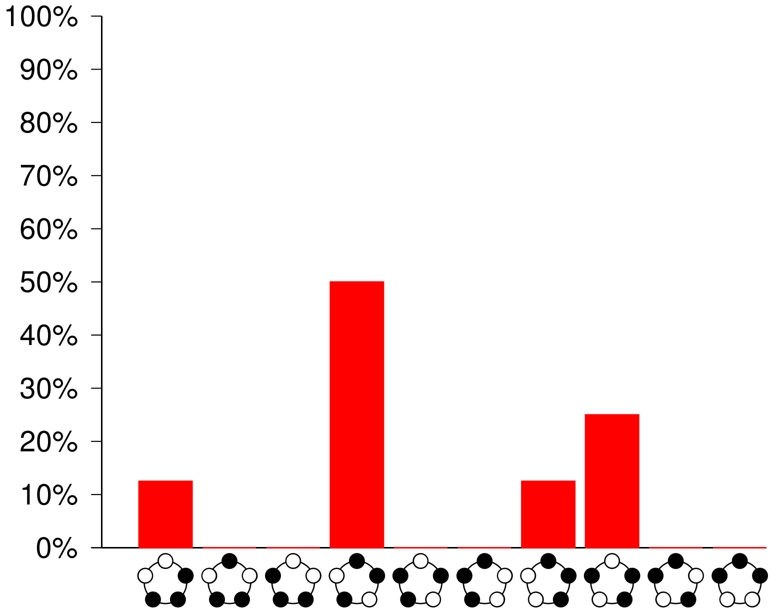

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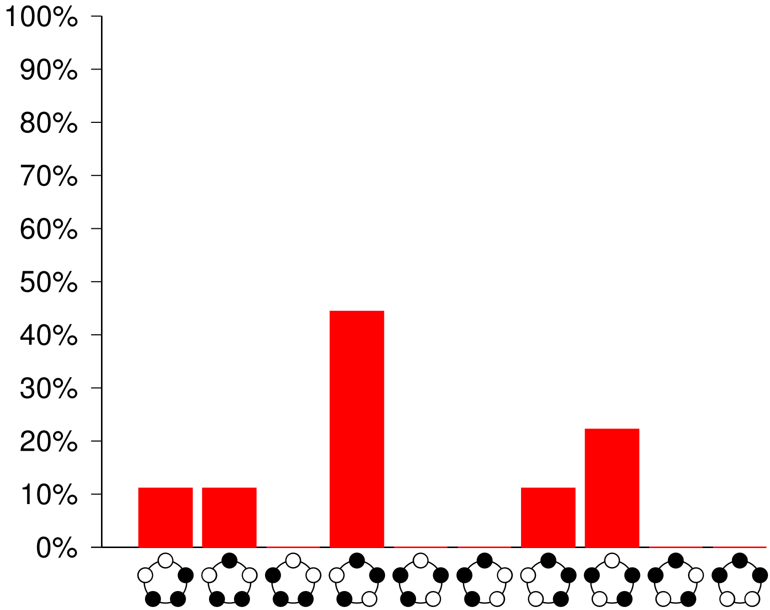

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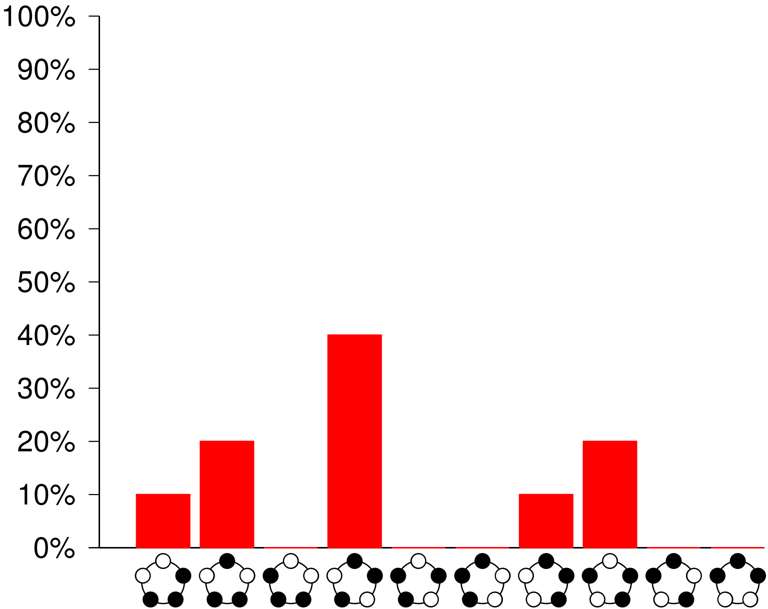

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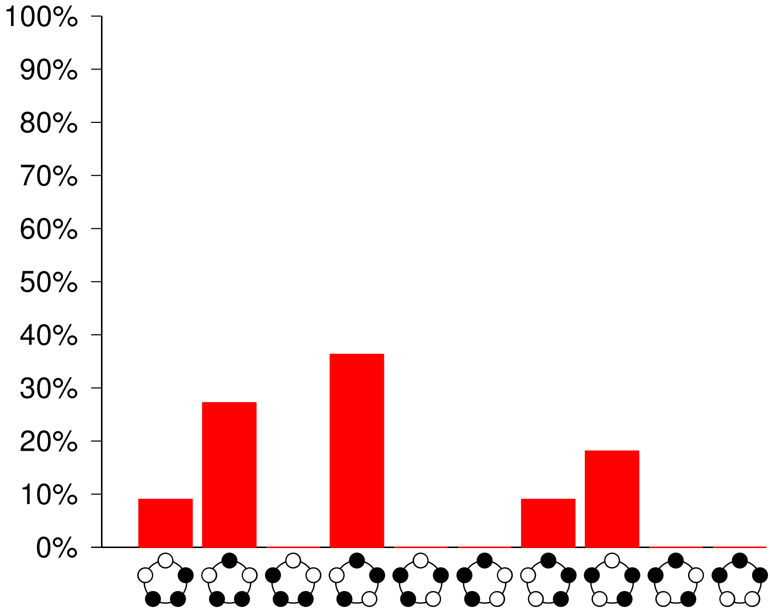

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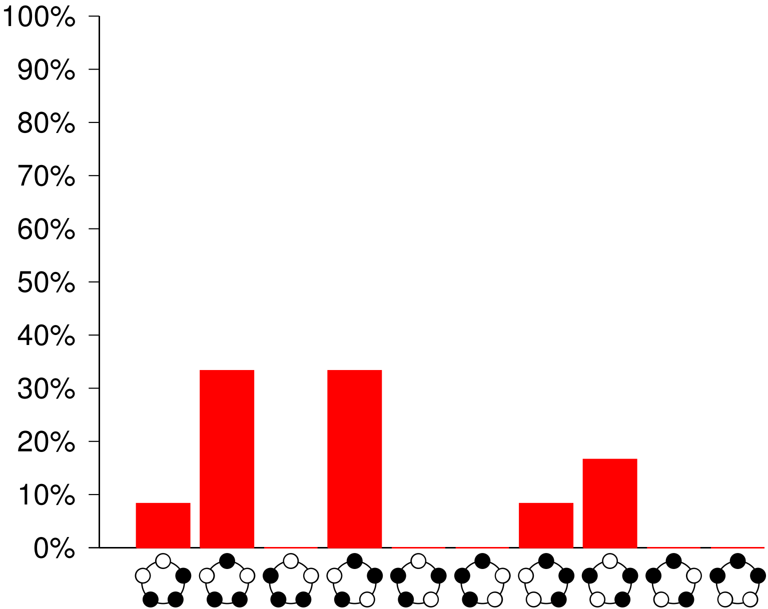

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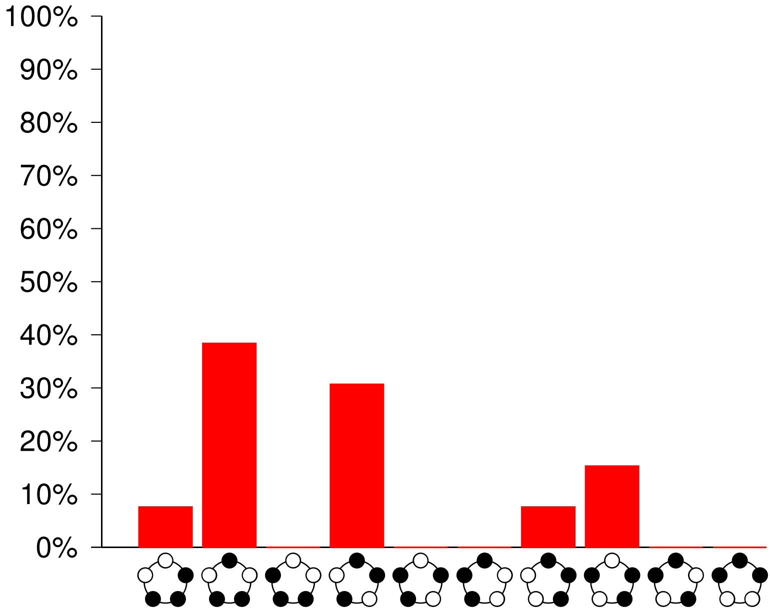

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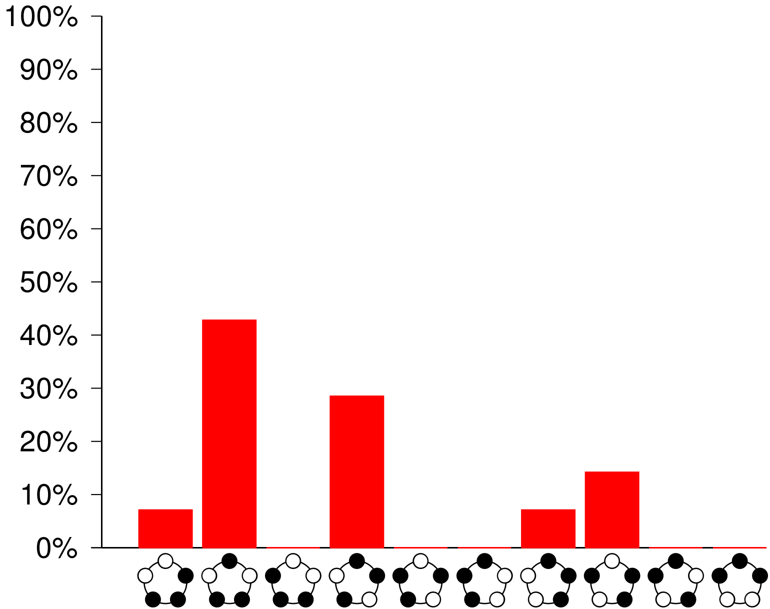

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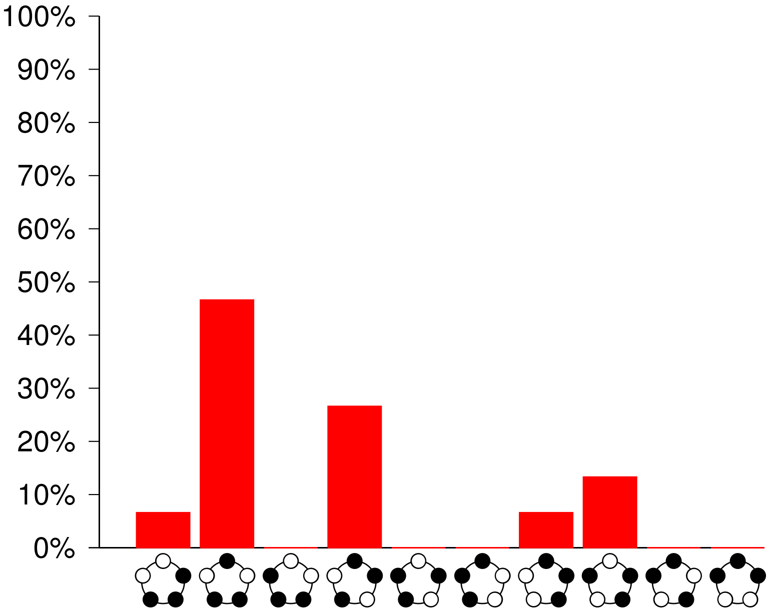

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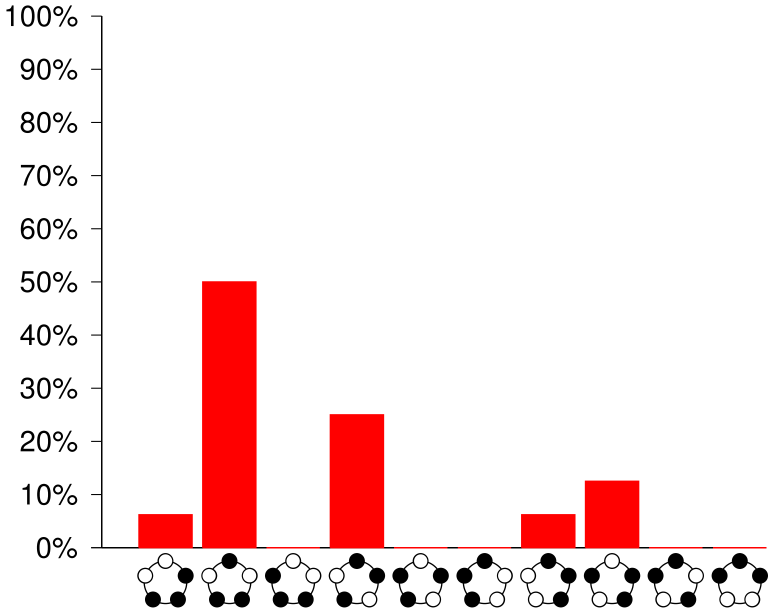

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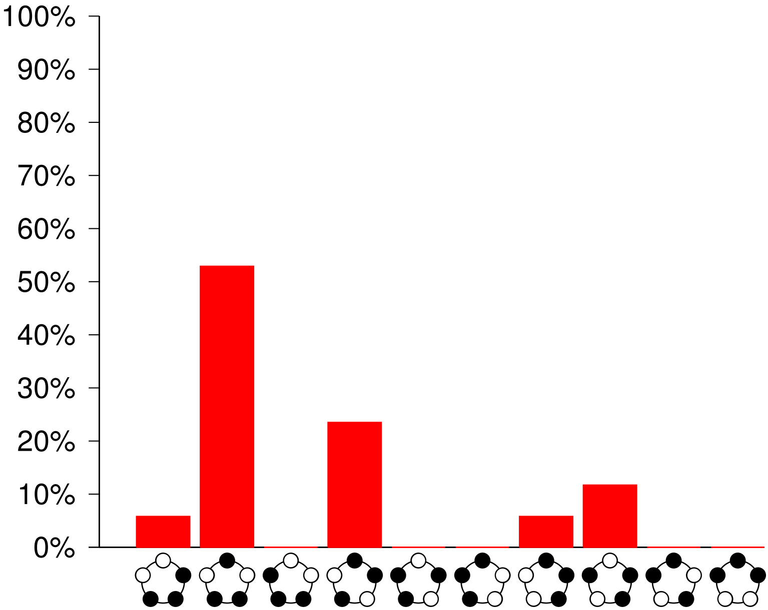

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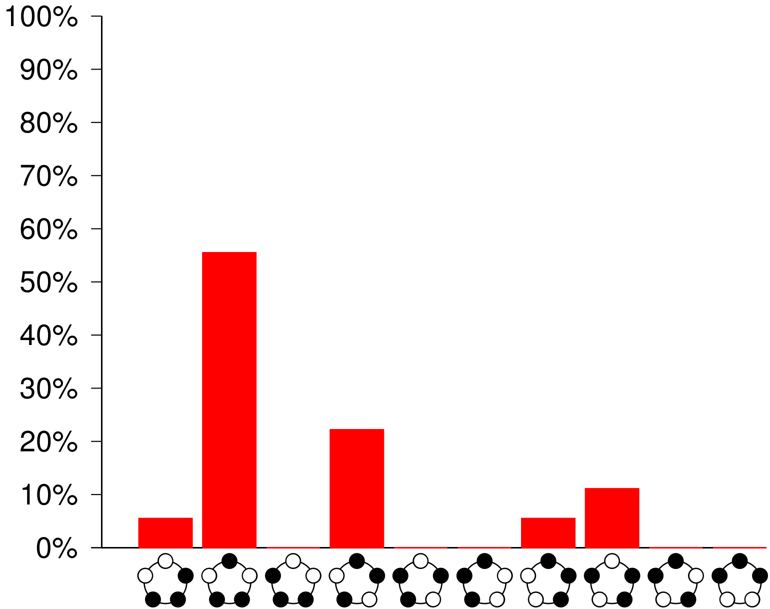

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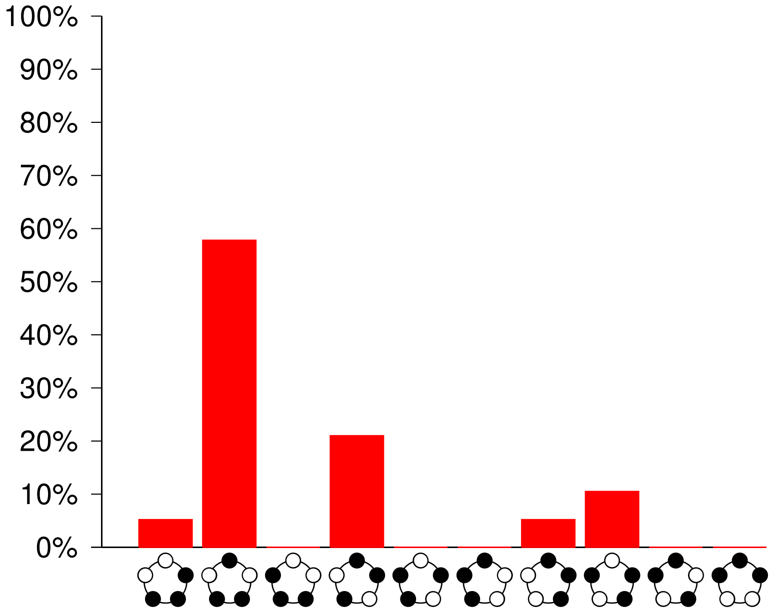

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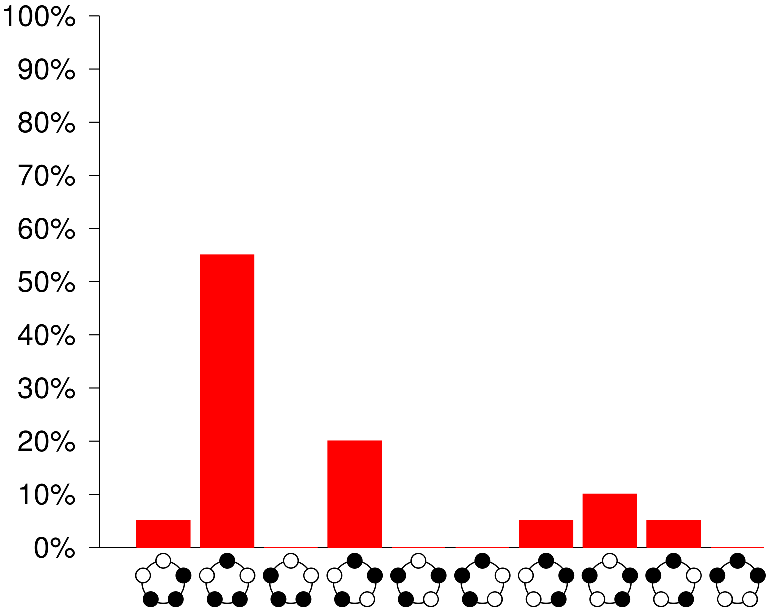

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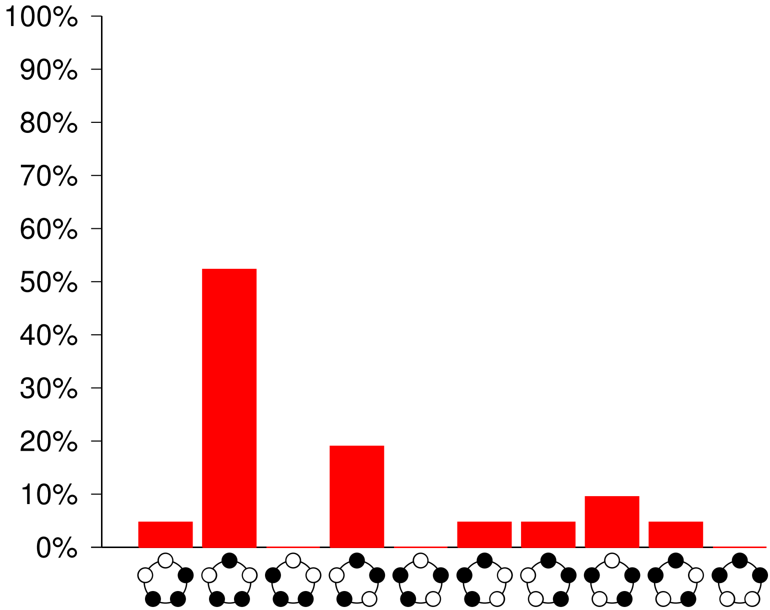

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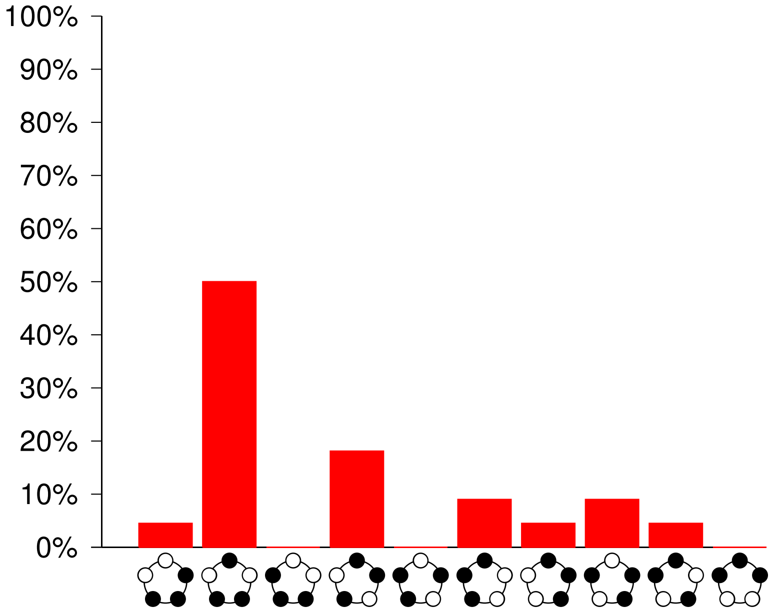

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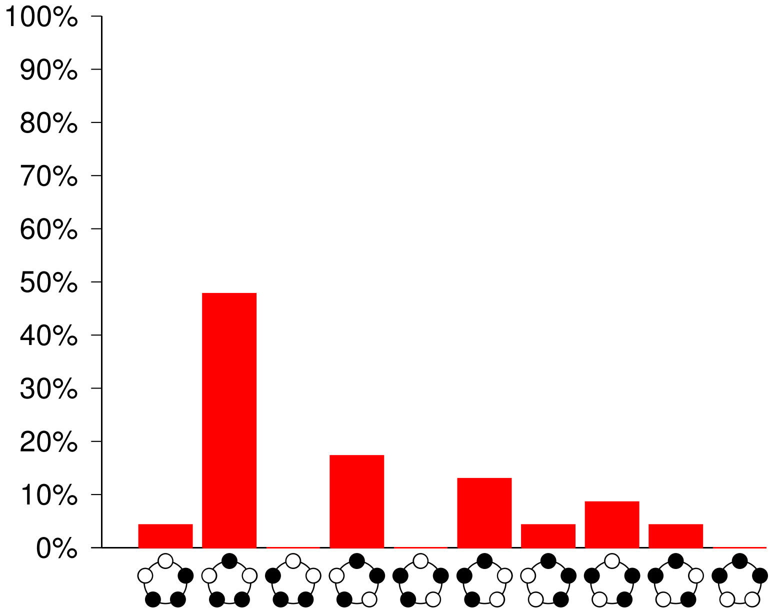

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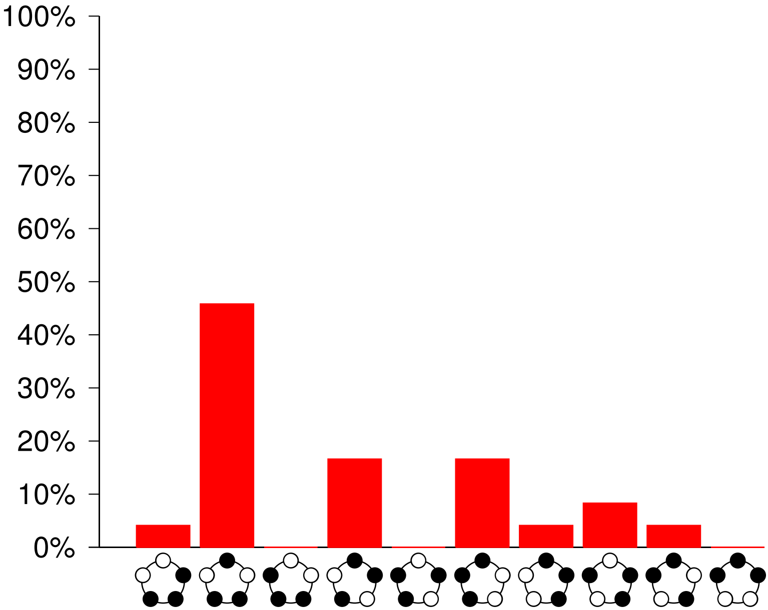

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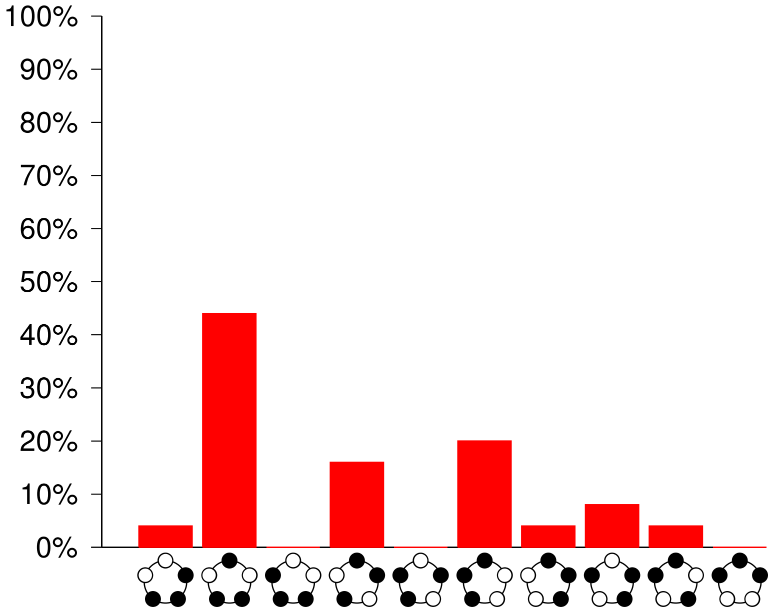

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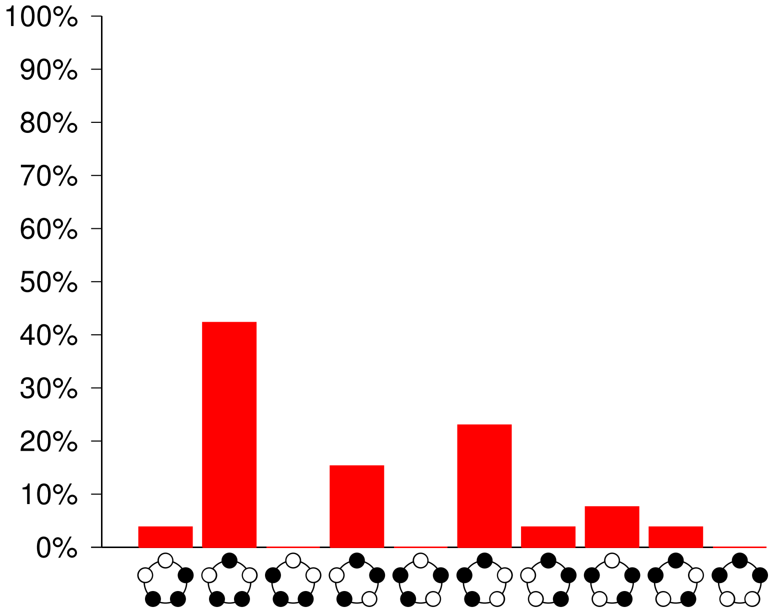

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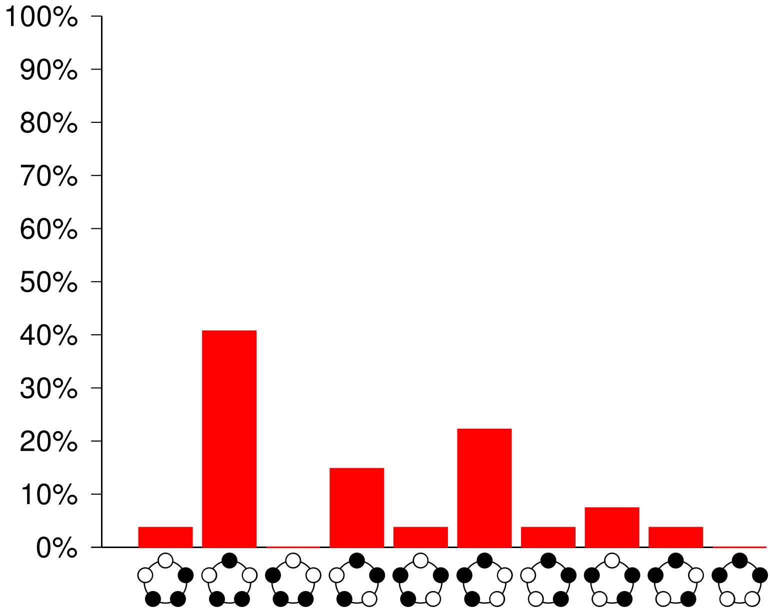

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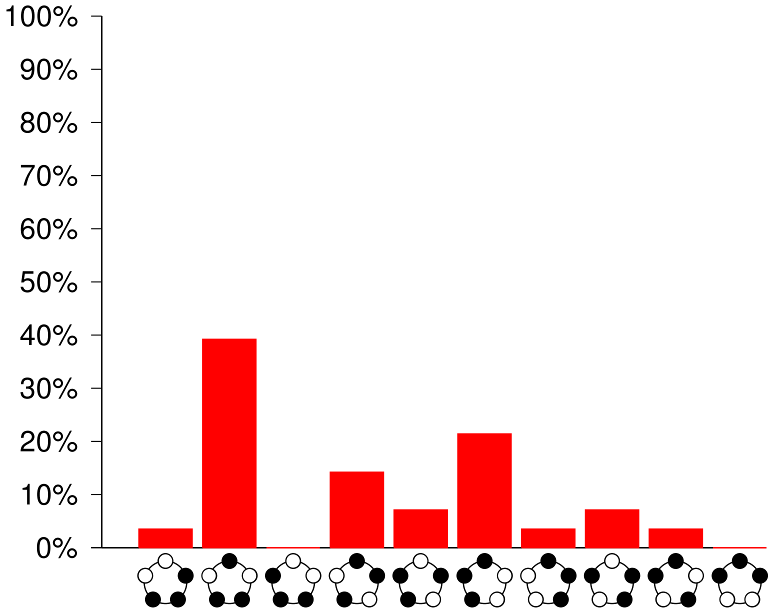

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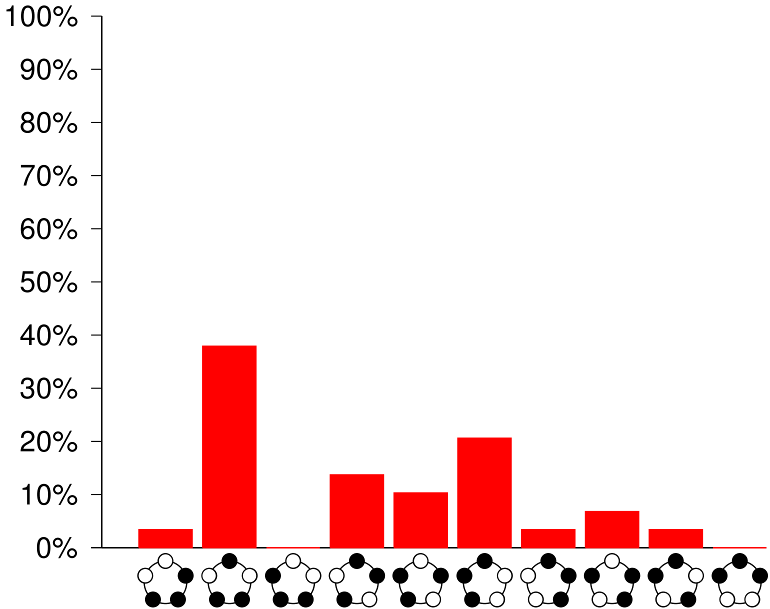

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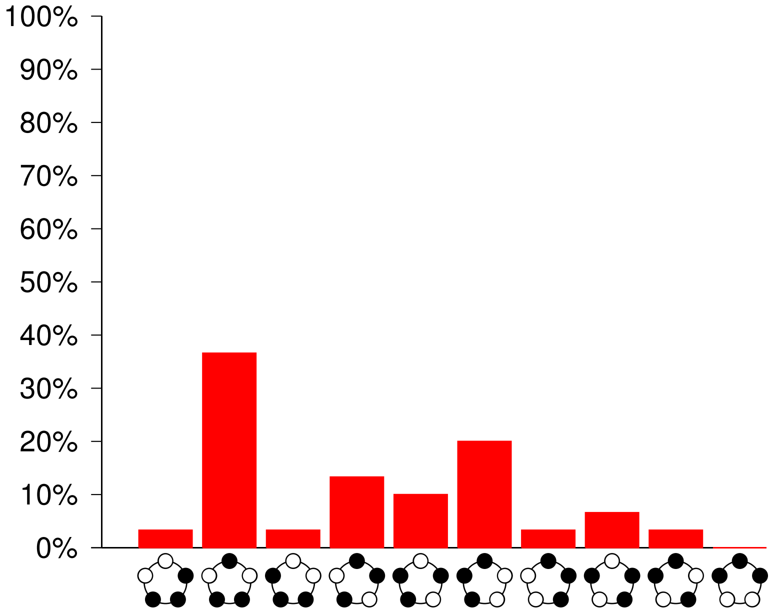

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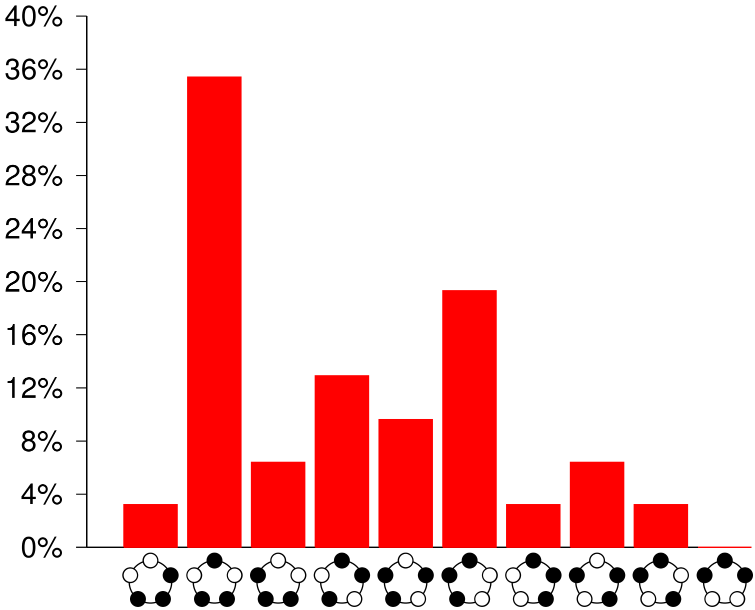

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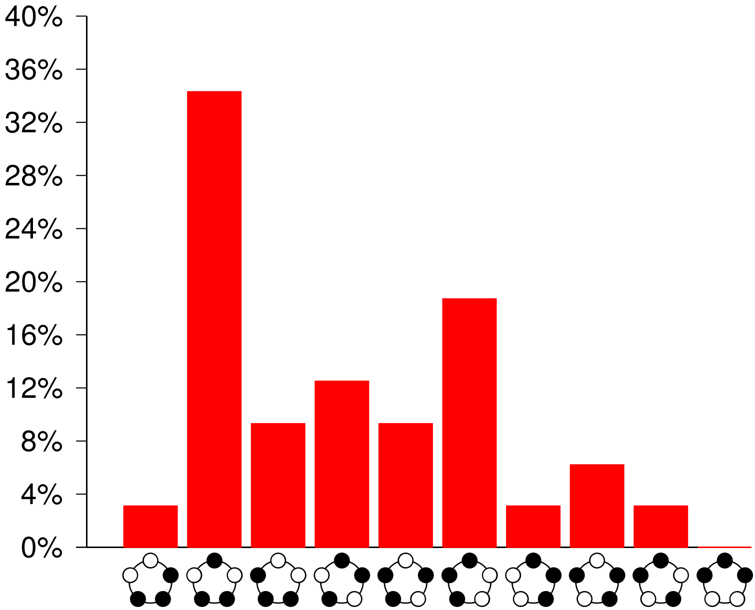

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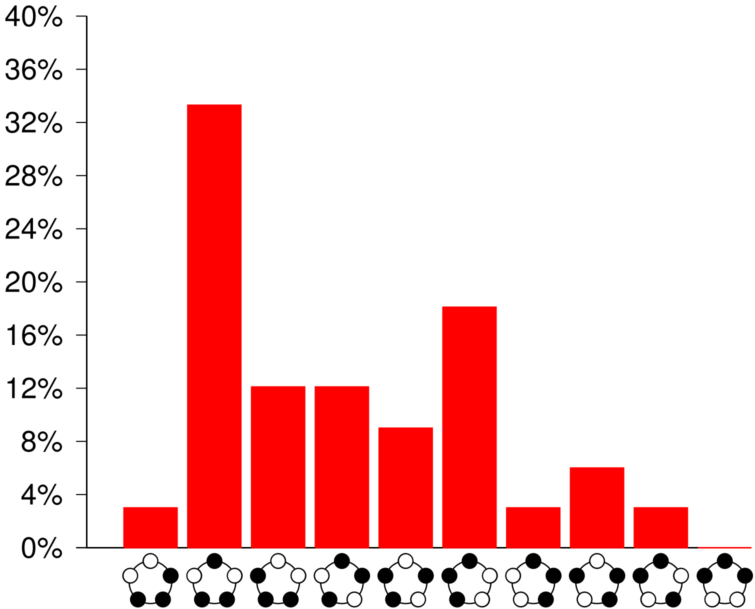

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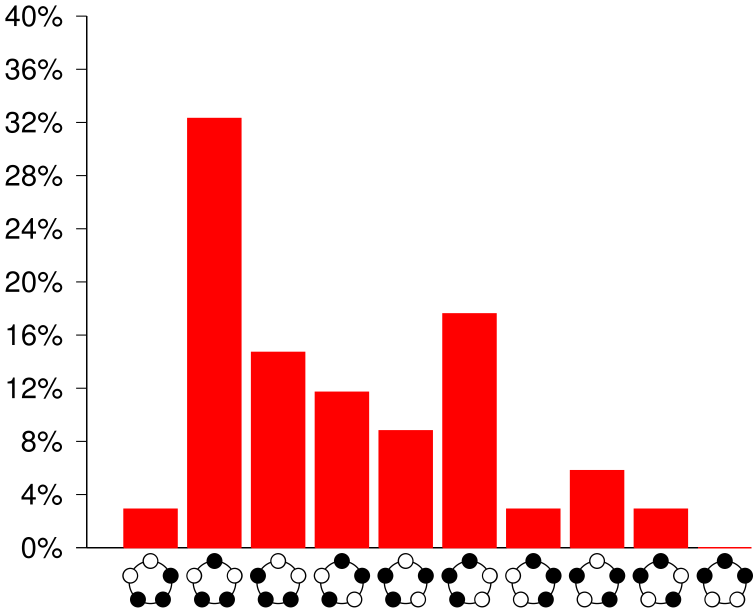

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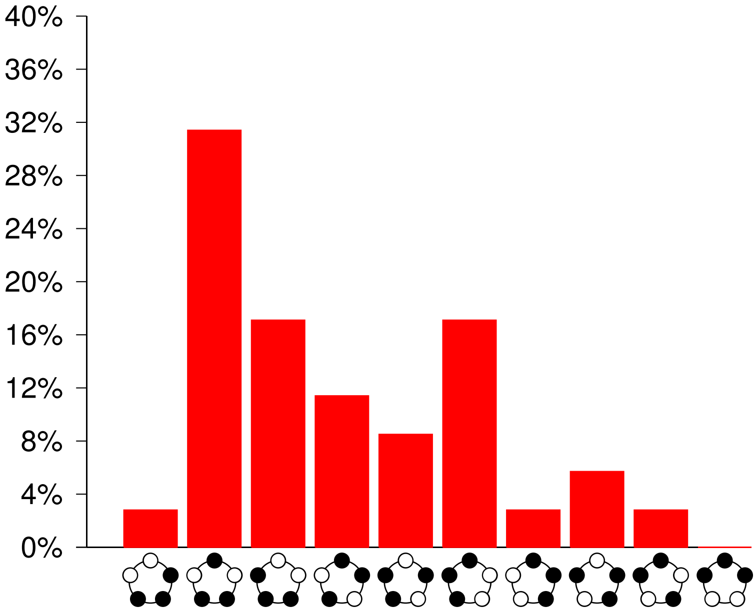

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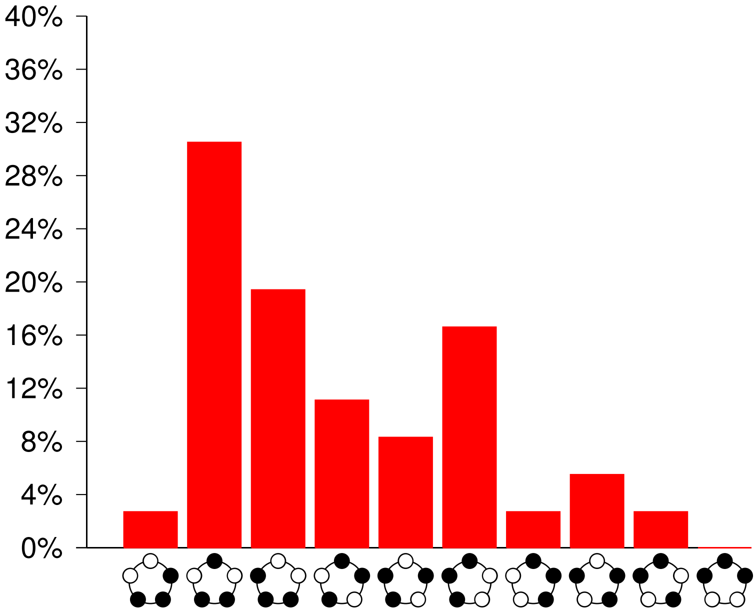

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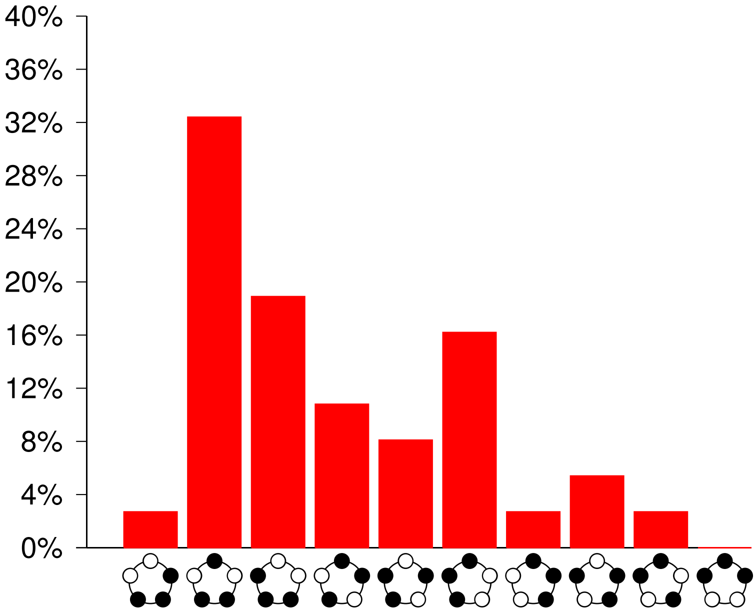

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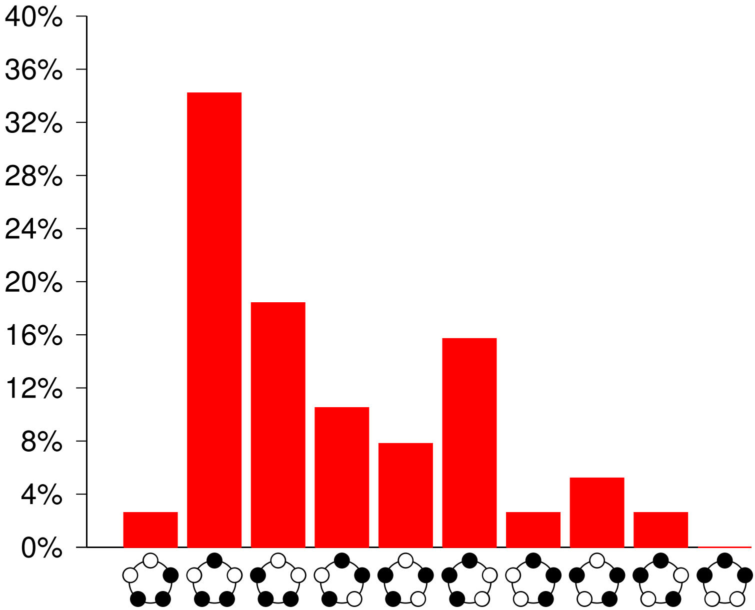

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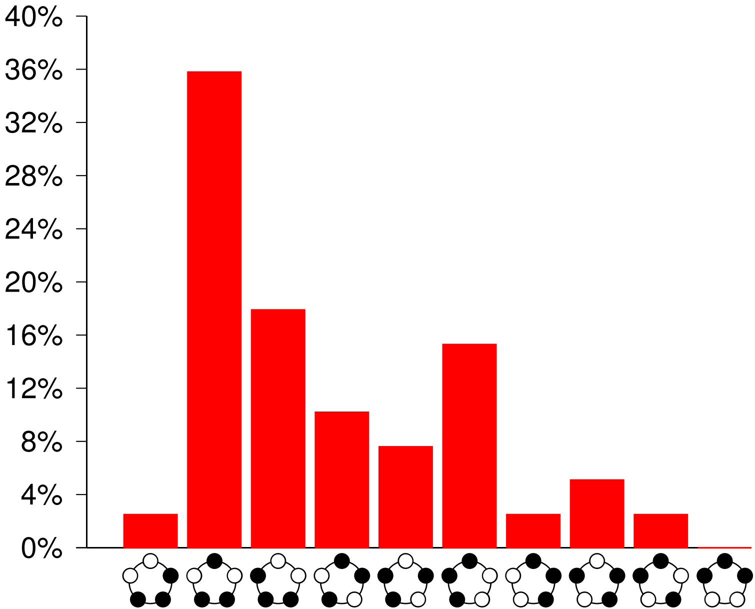

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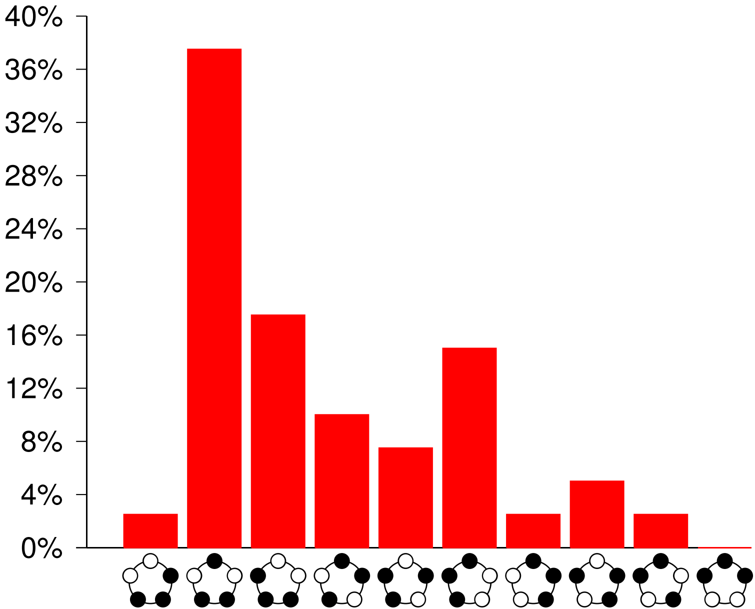

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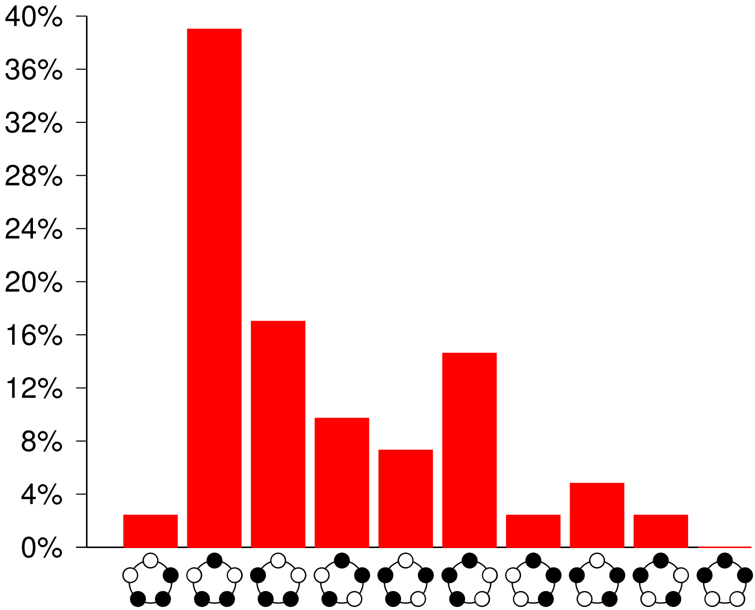

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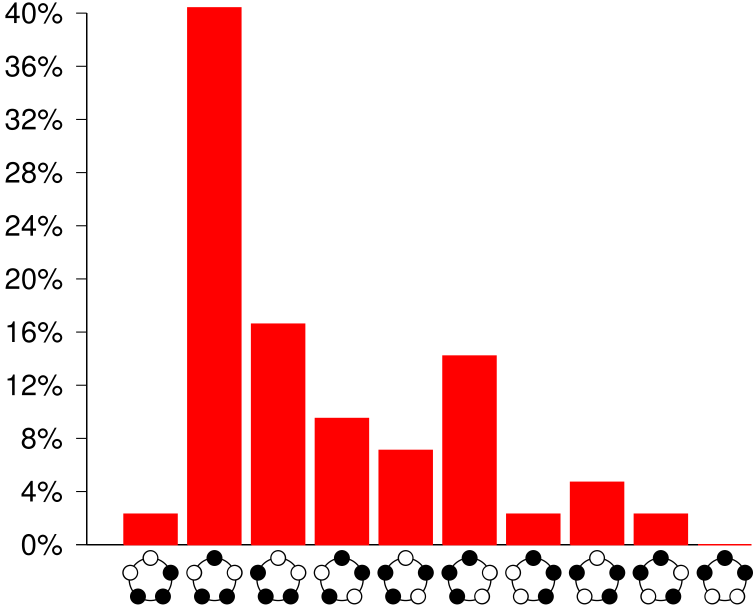

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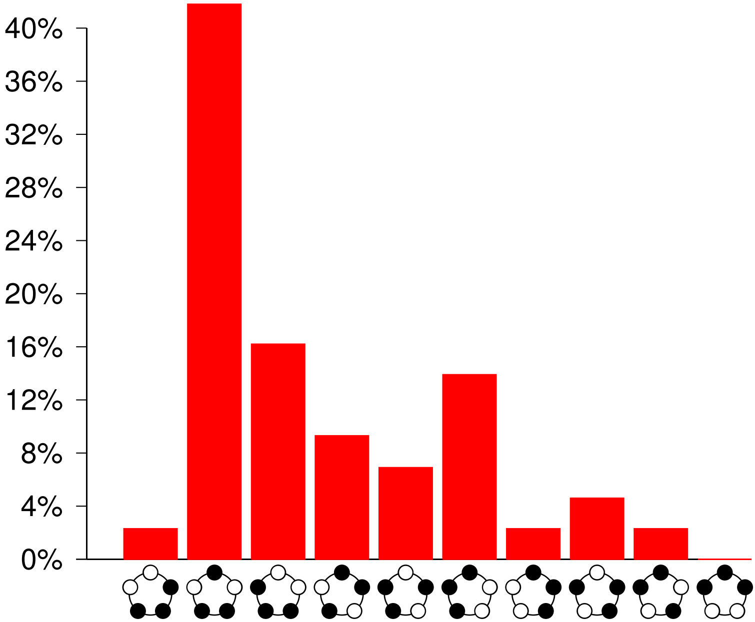

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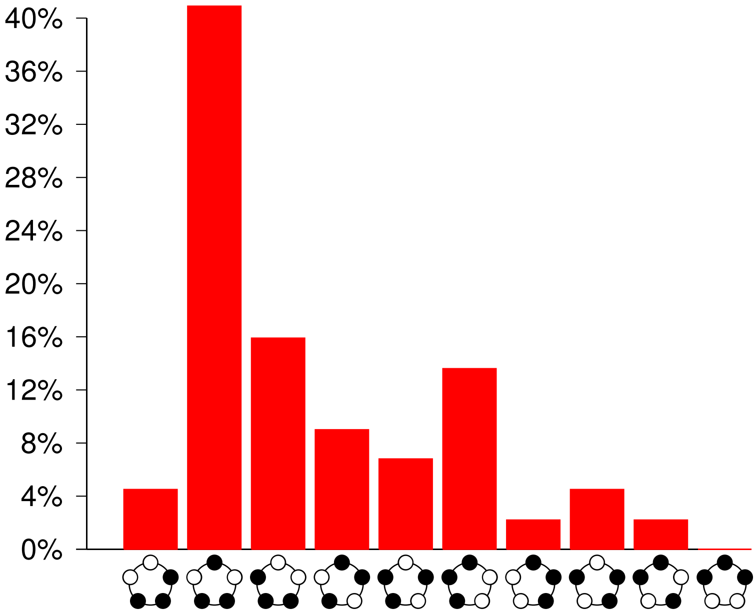

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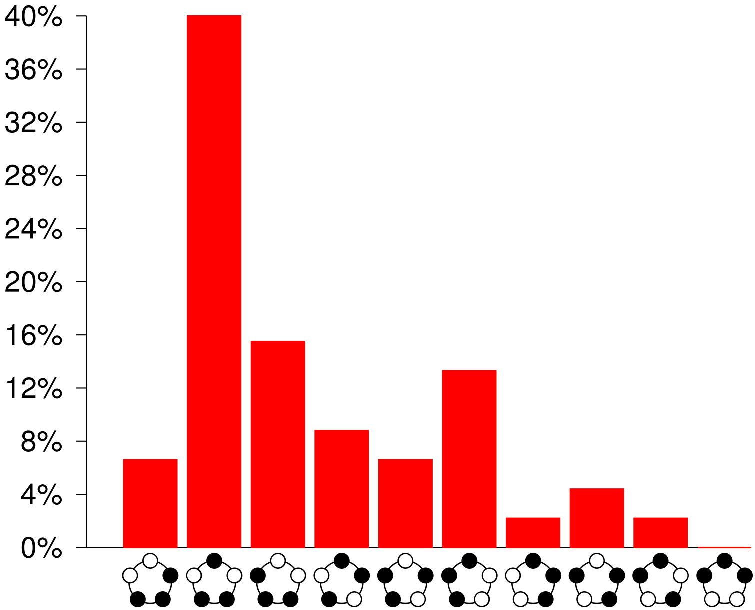

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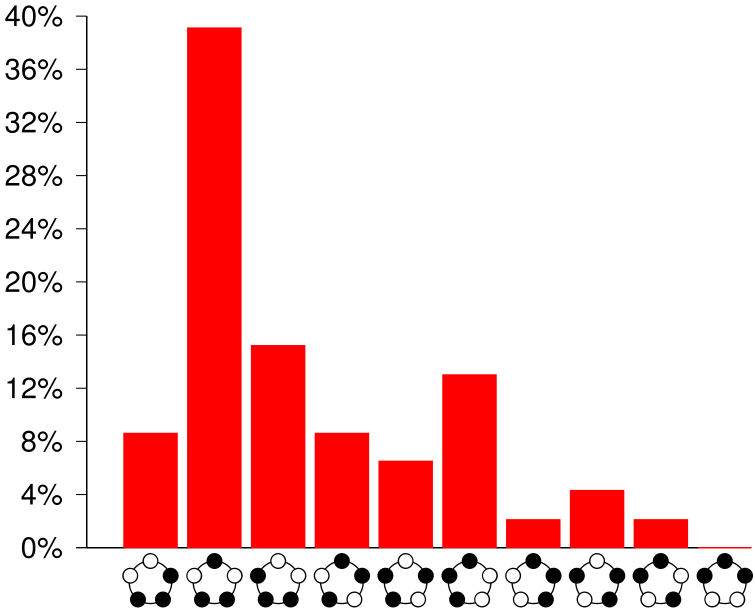

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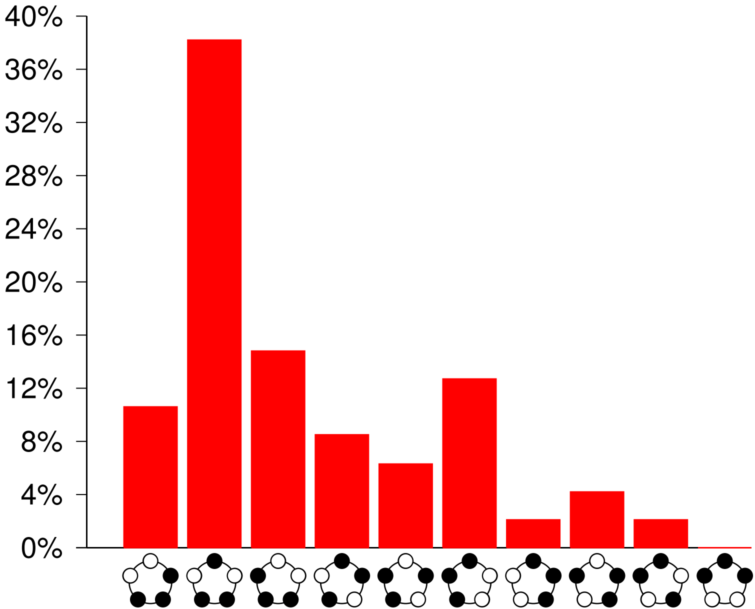

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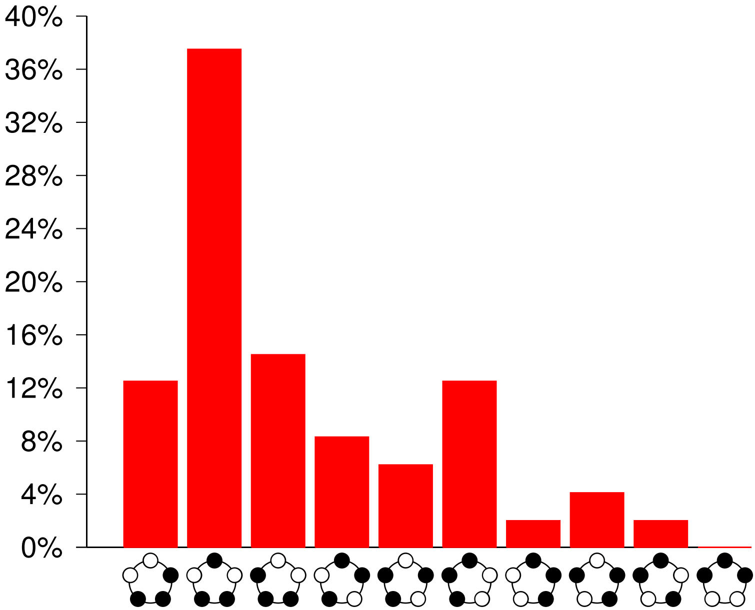

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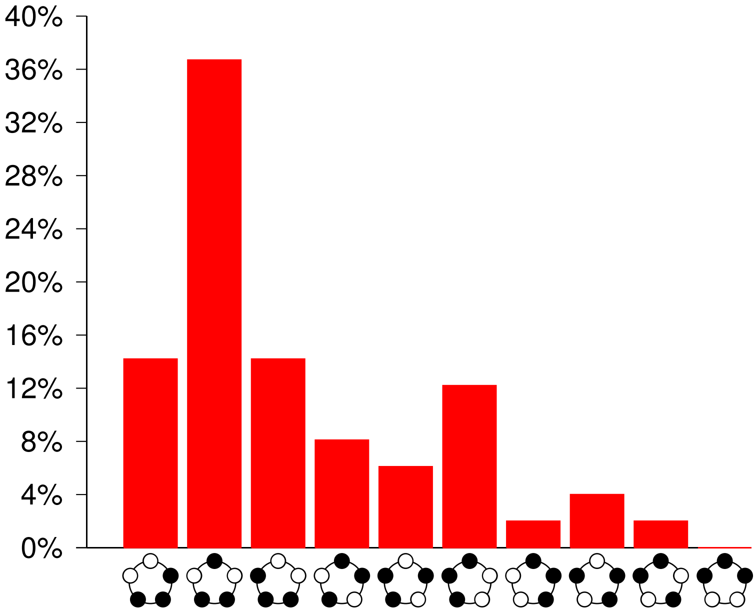

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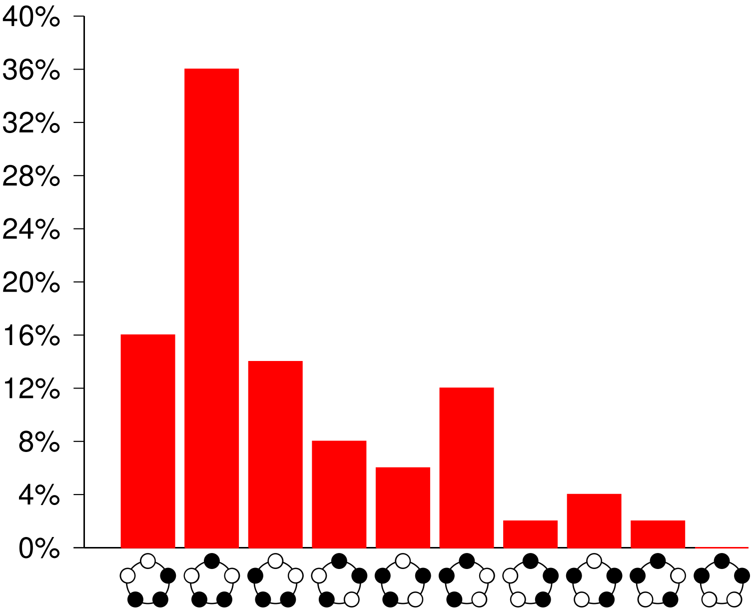

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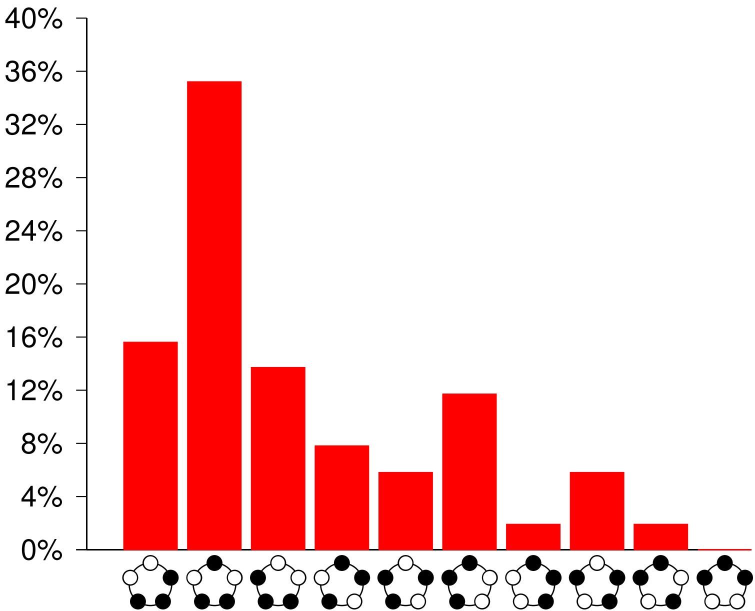

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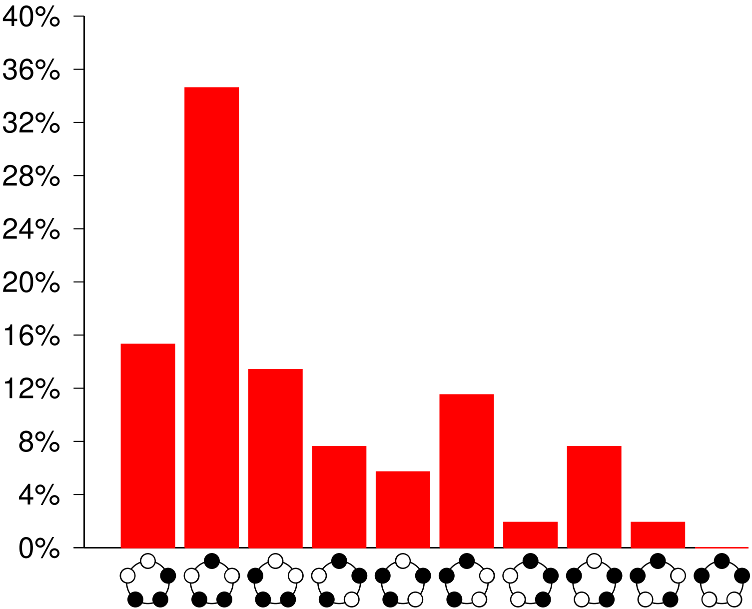

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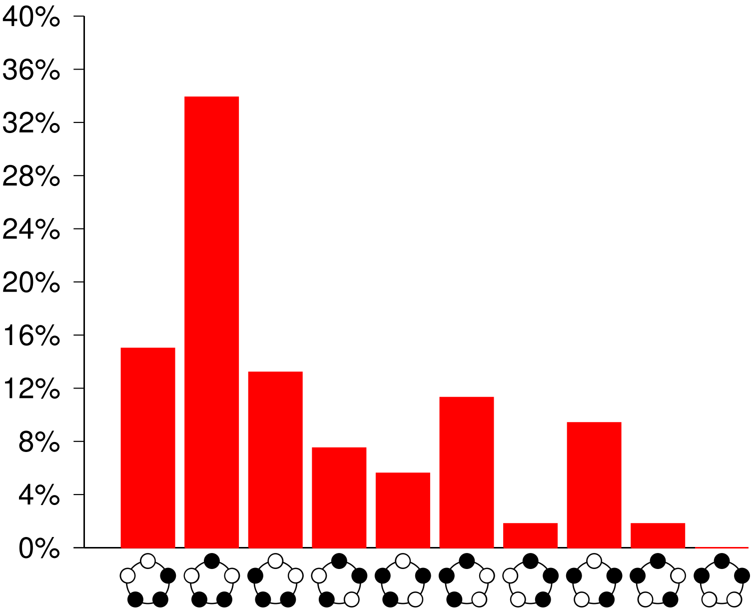

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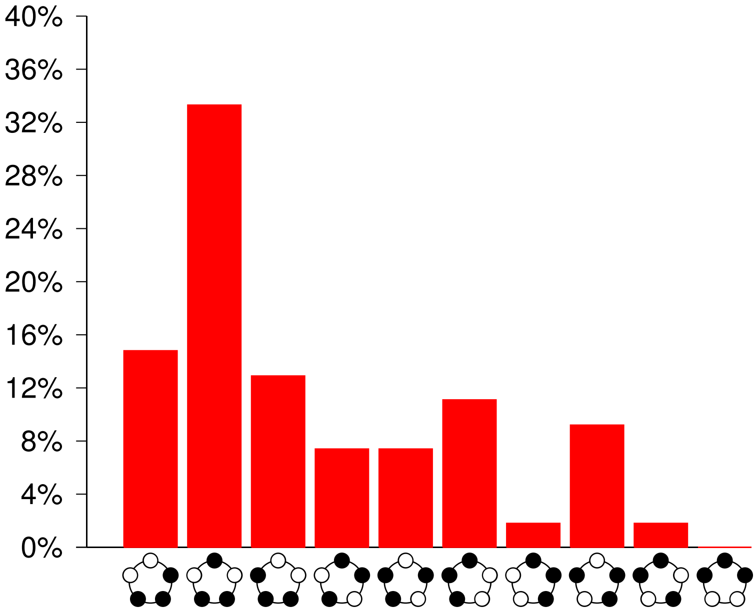

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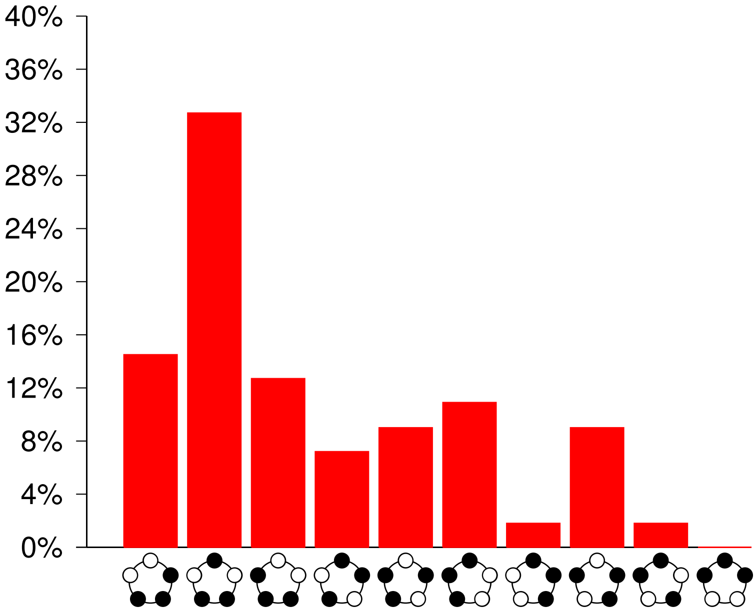

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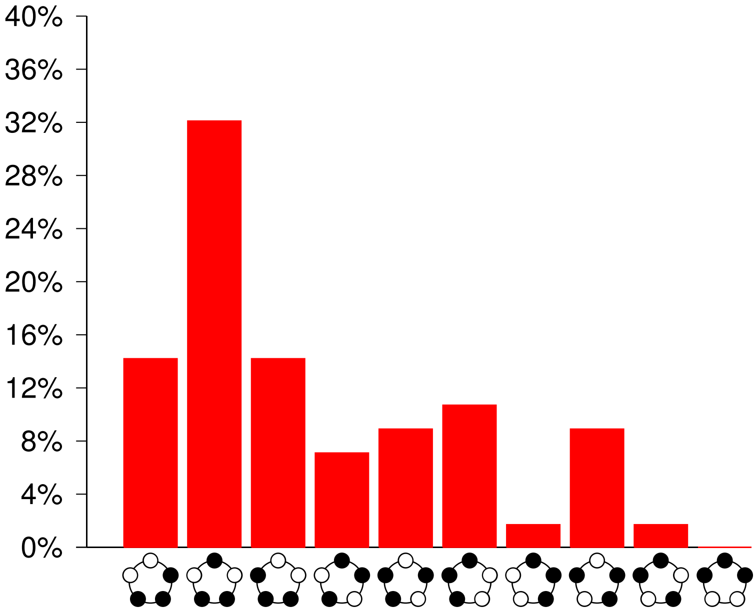

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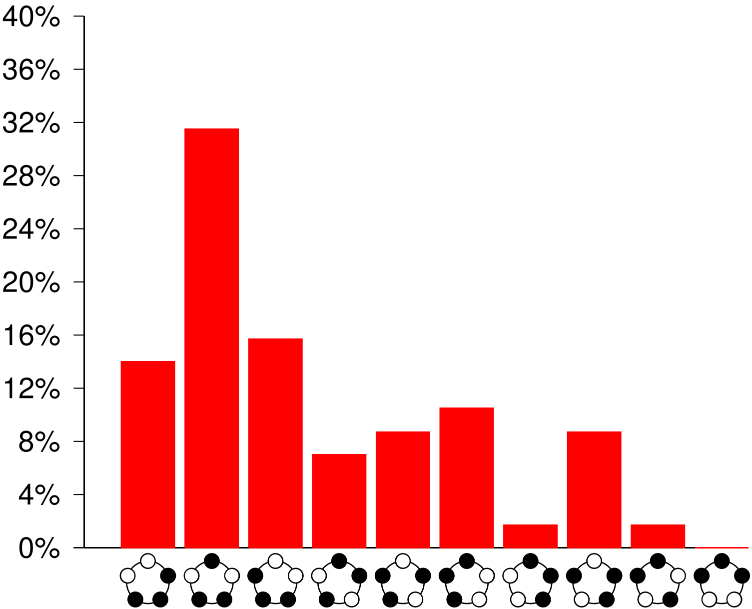

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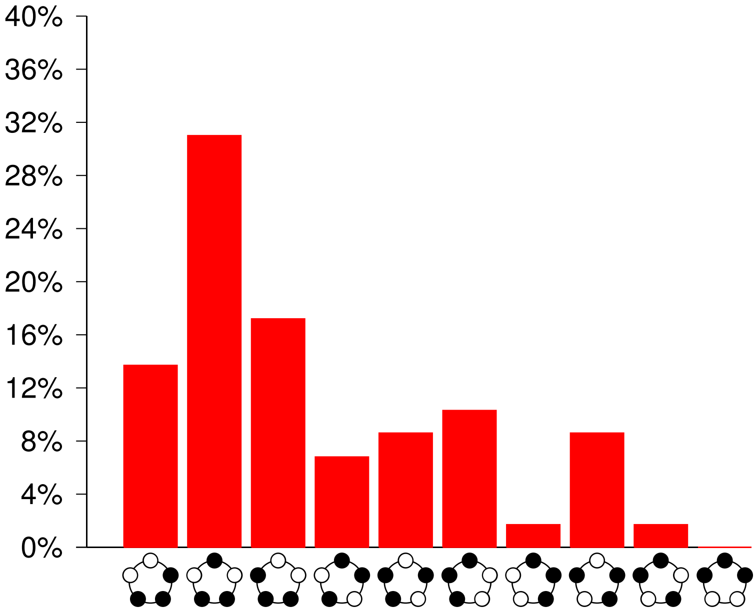

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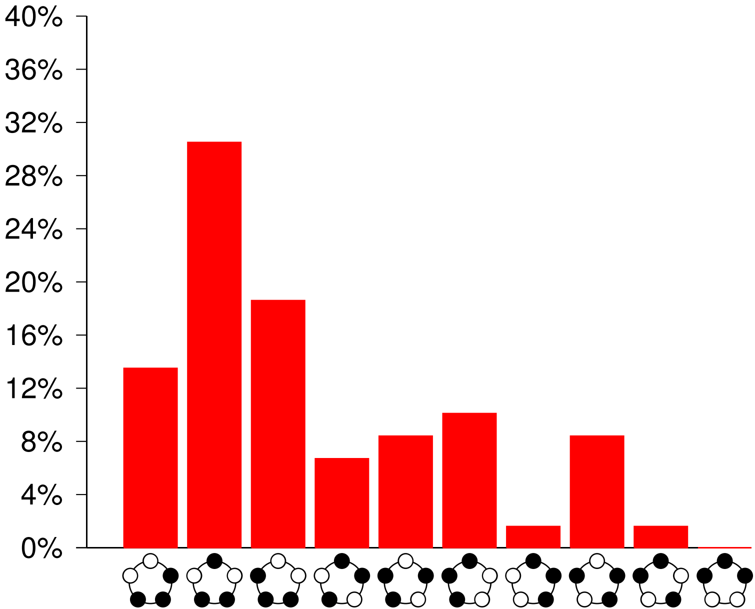

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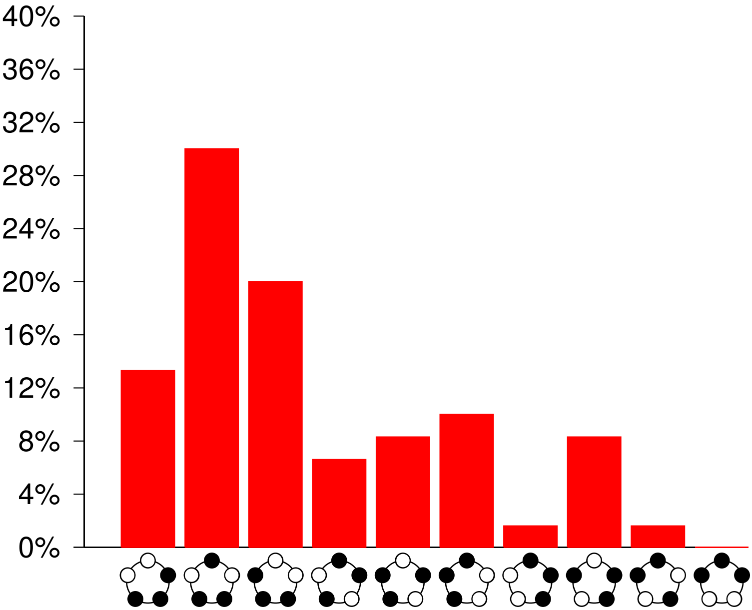

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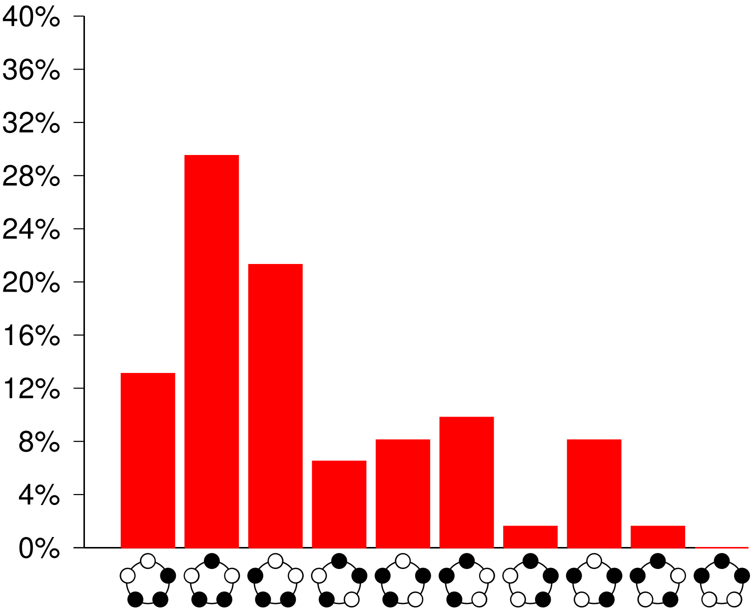

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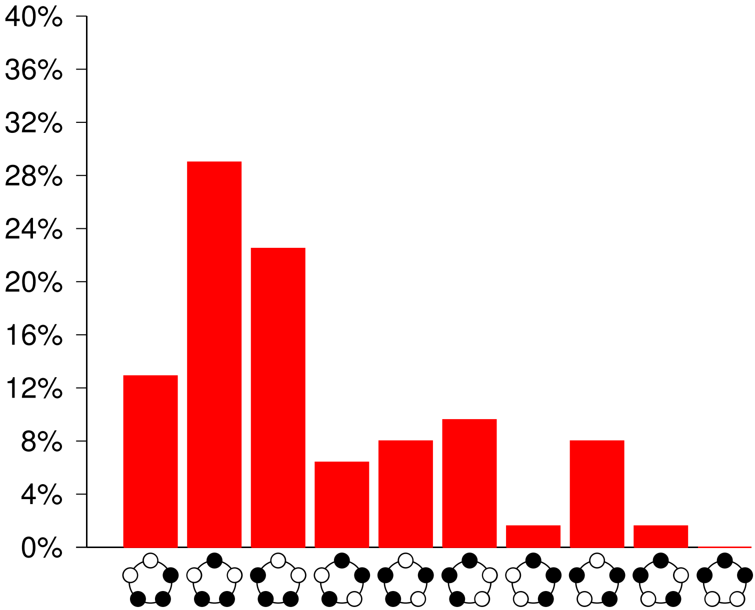

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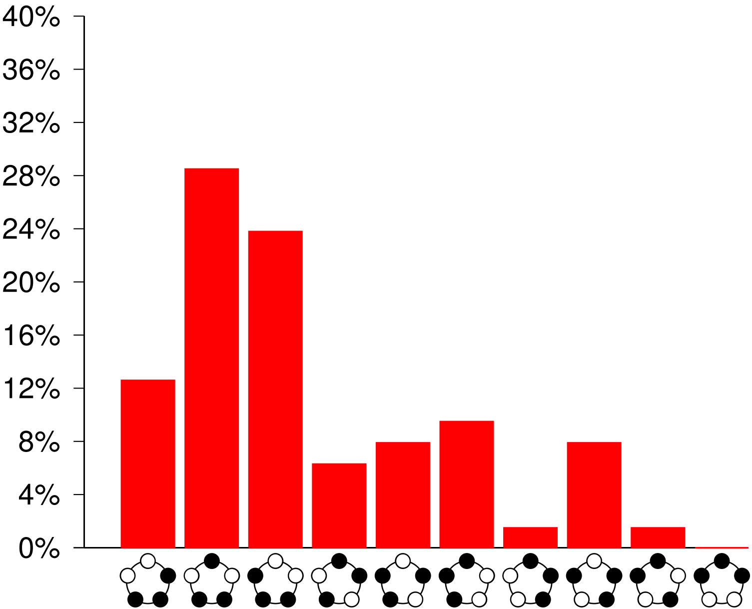

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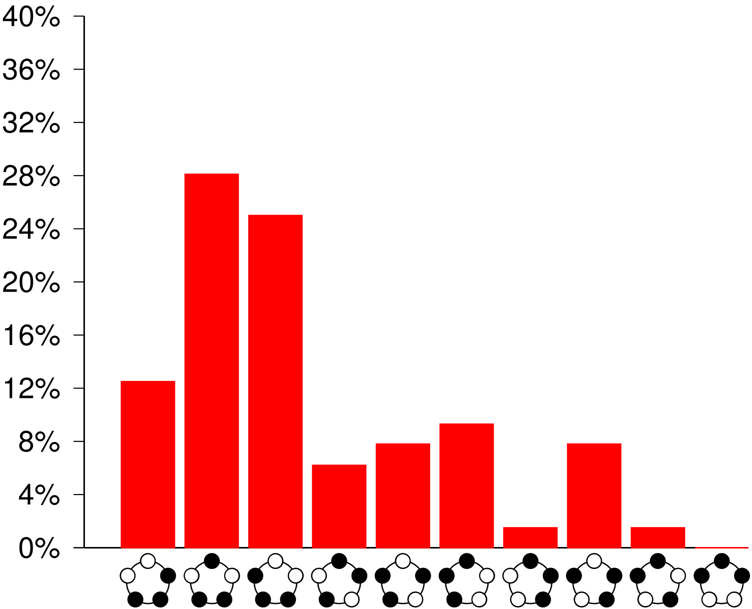
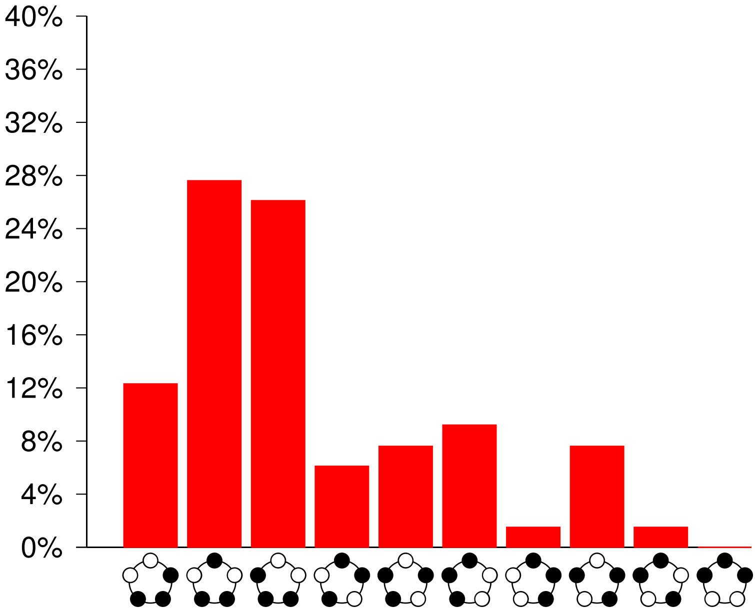


Stationary distribution



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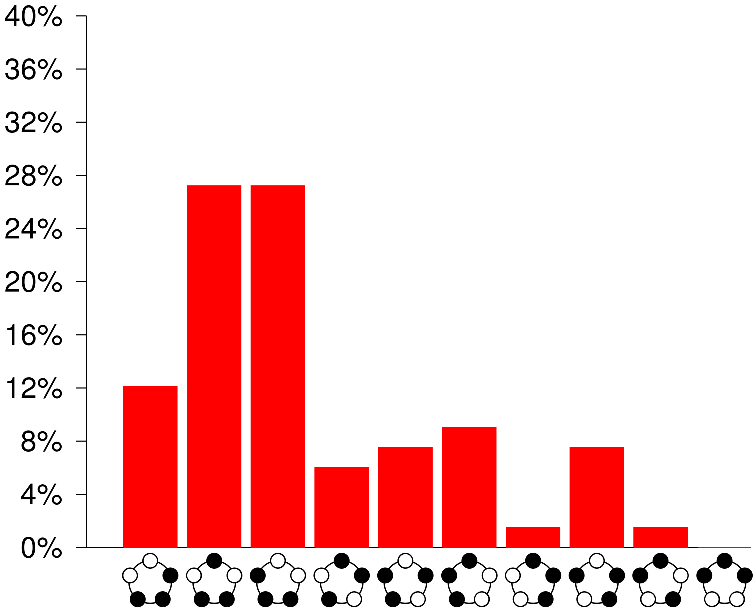

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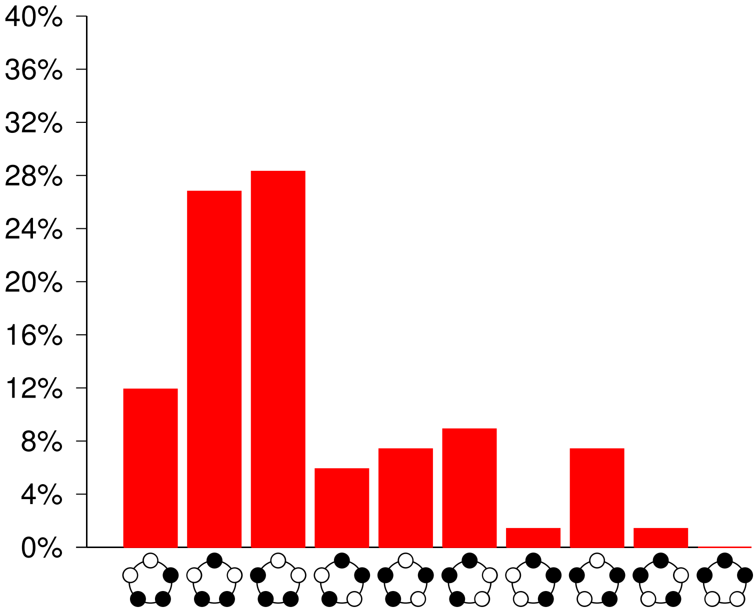

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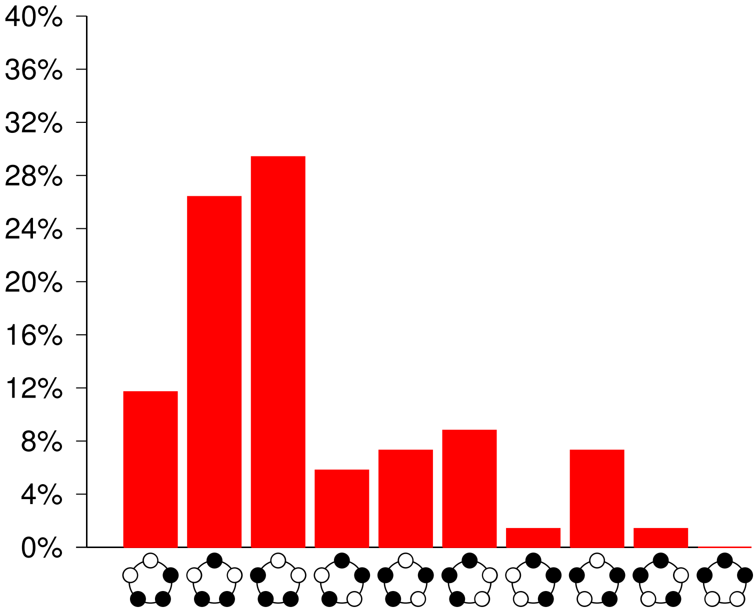

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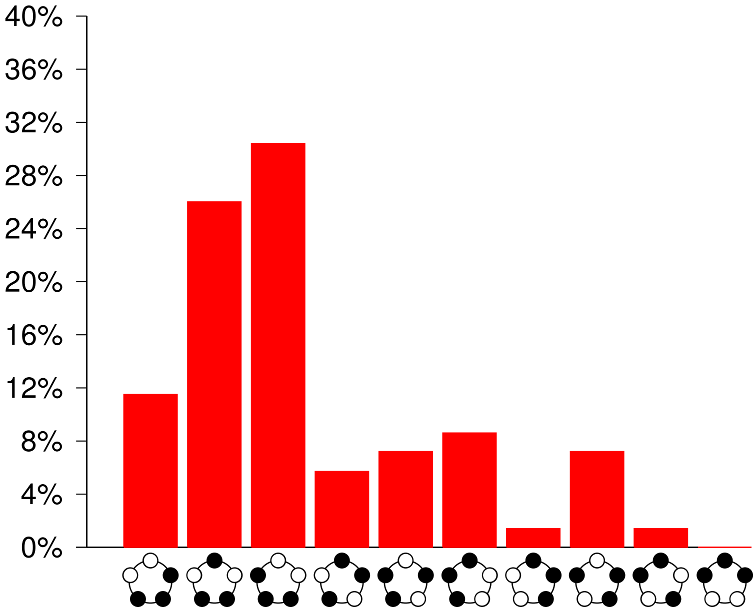

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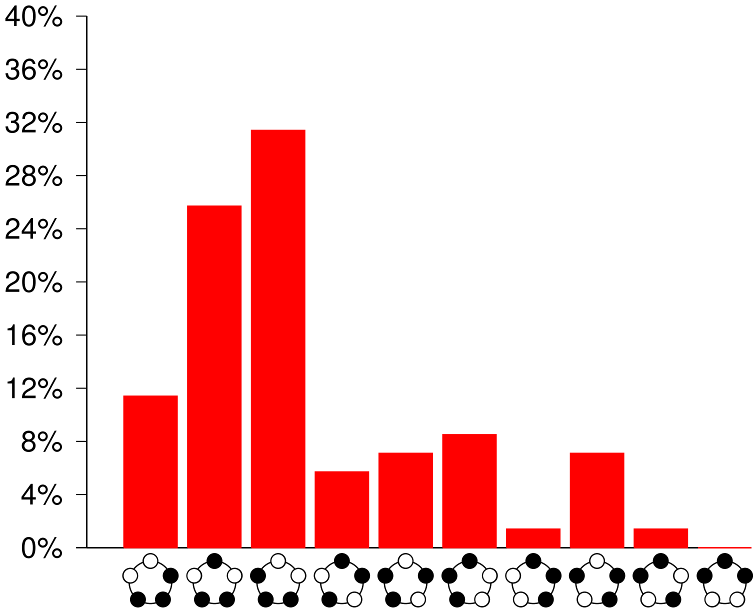

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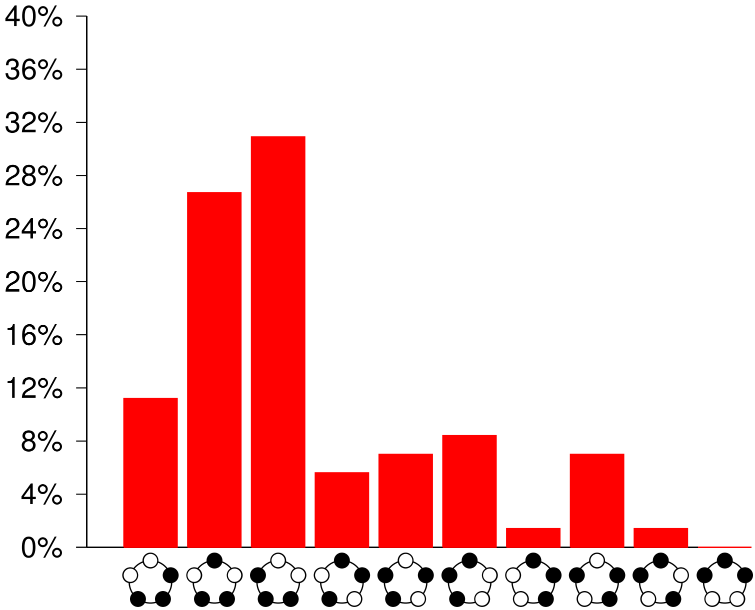

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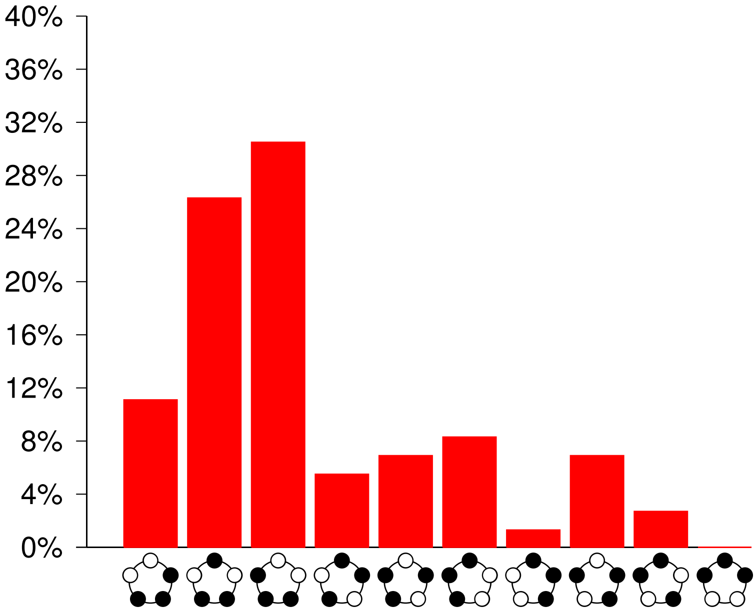

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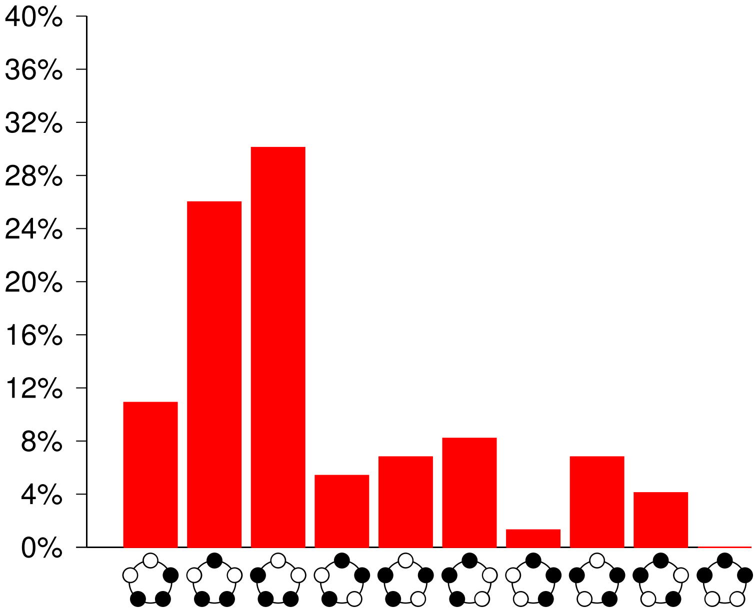

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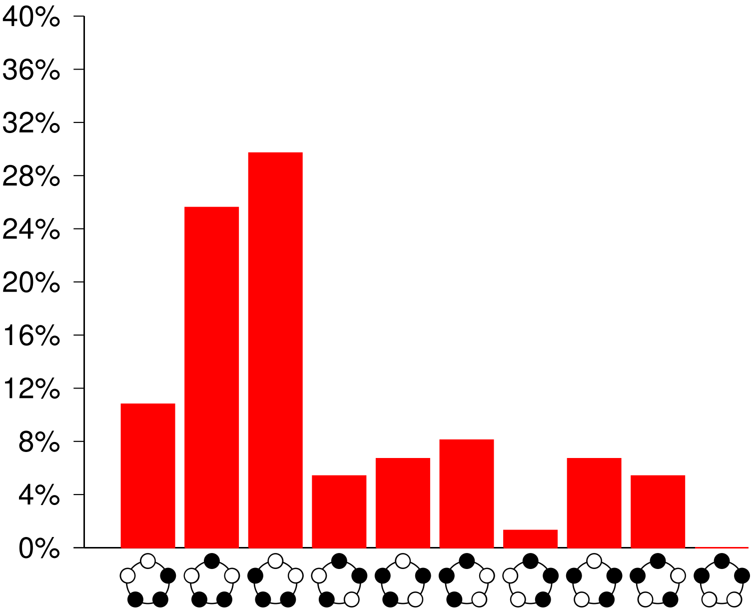

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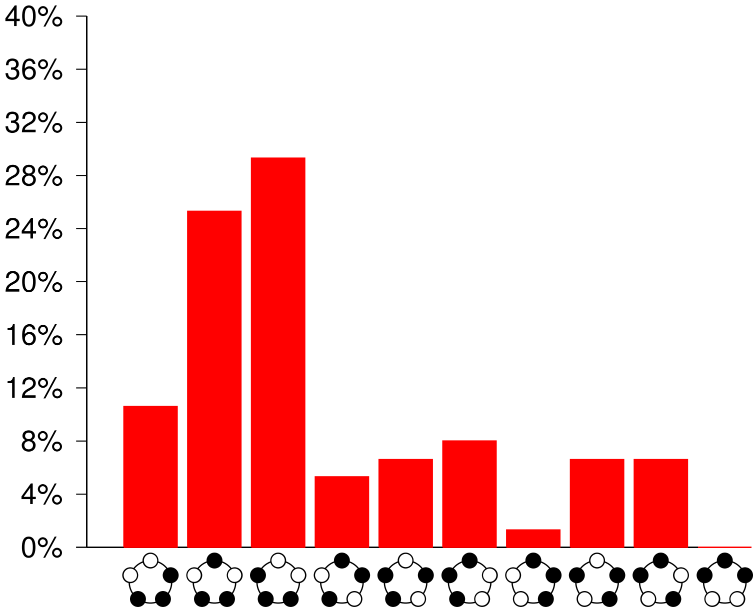

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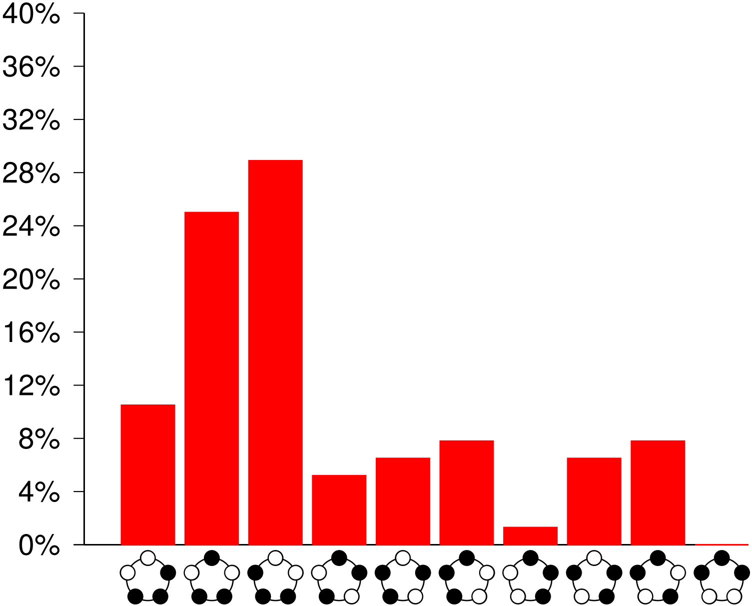

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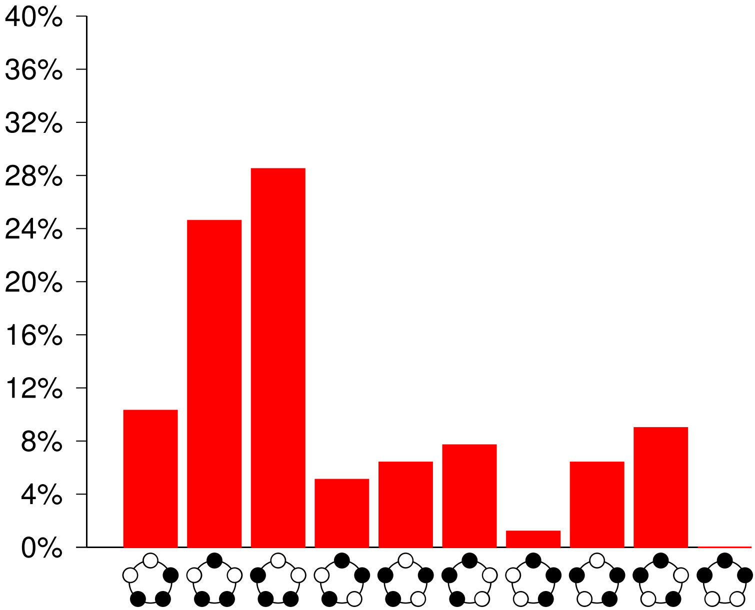

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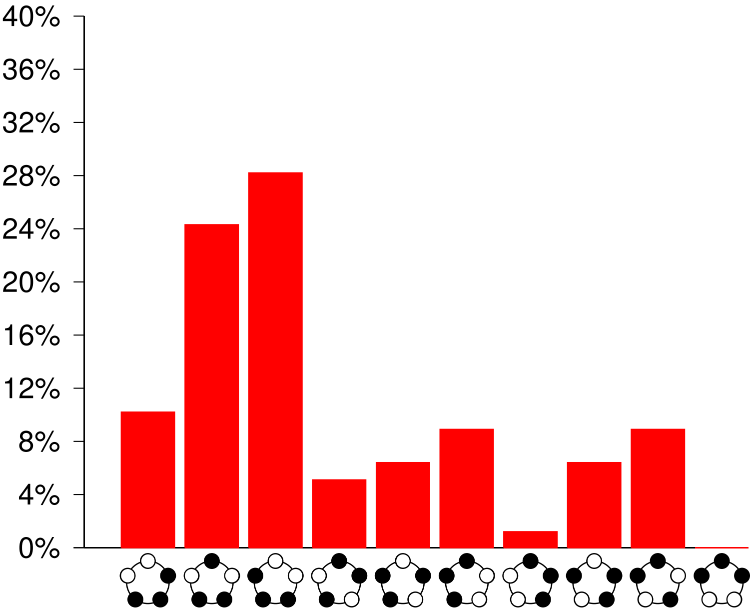

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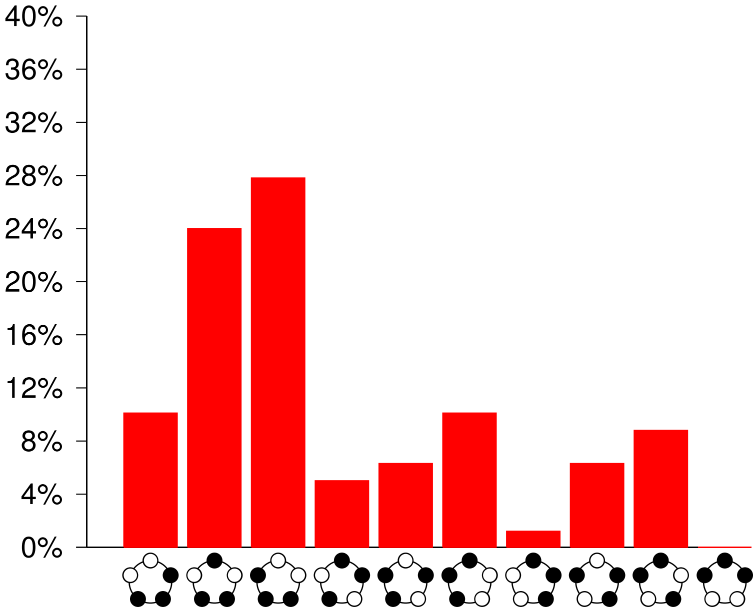

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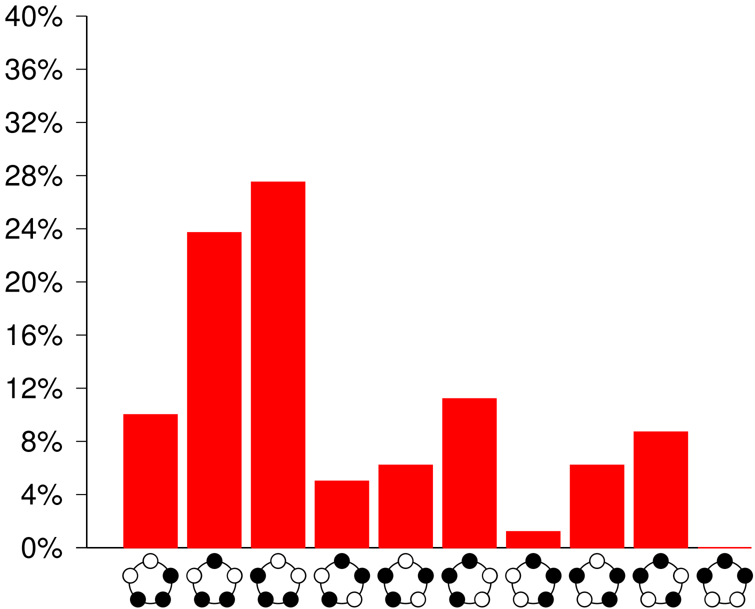

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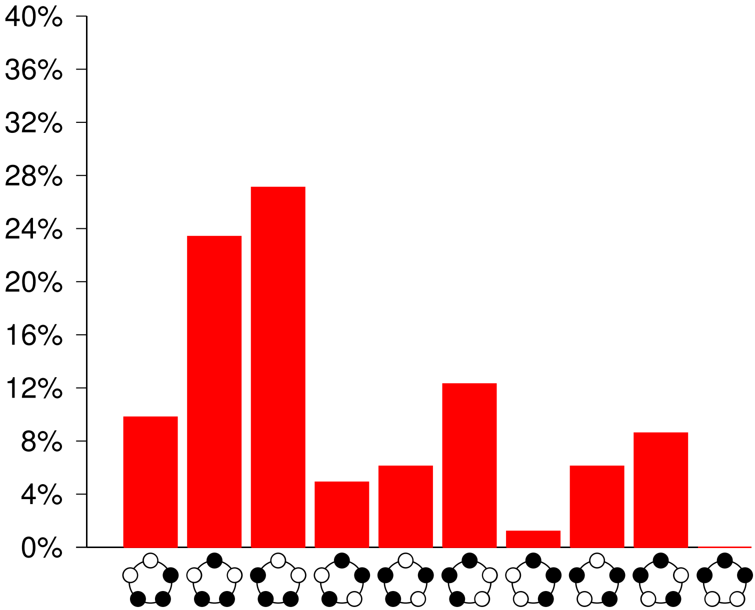

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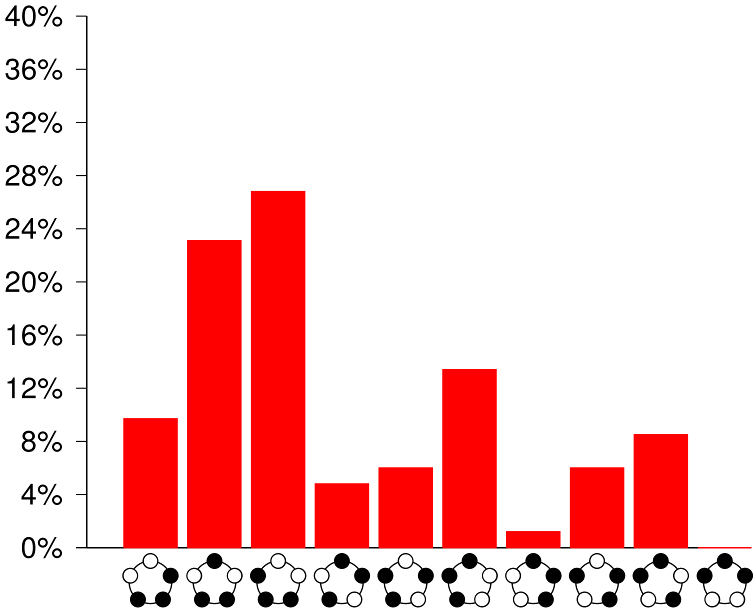

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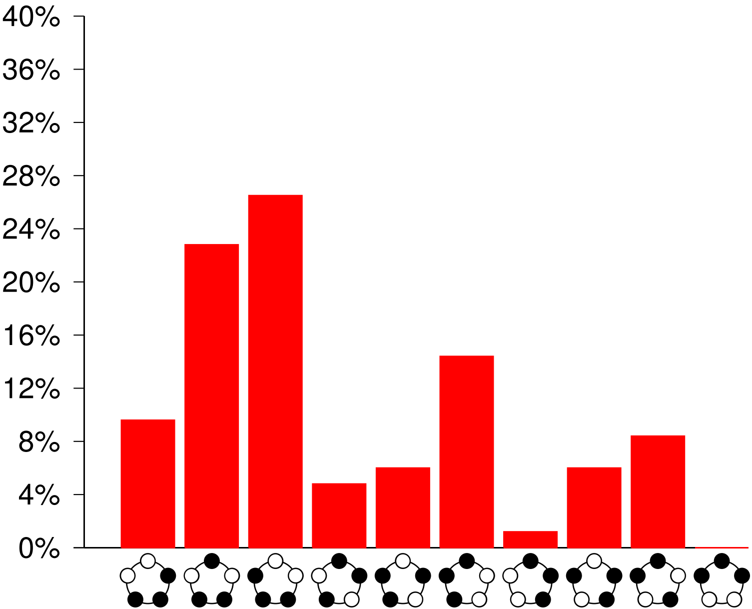

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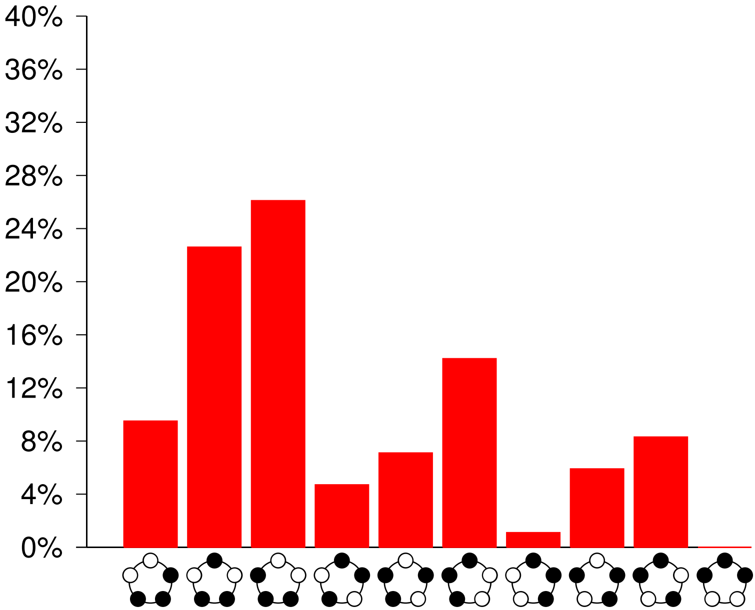

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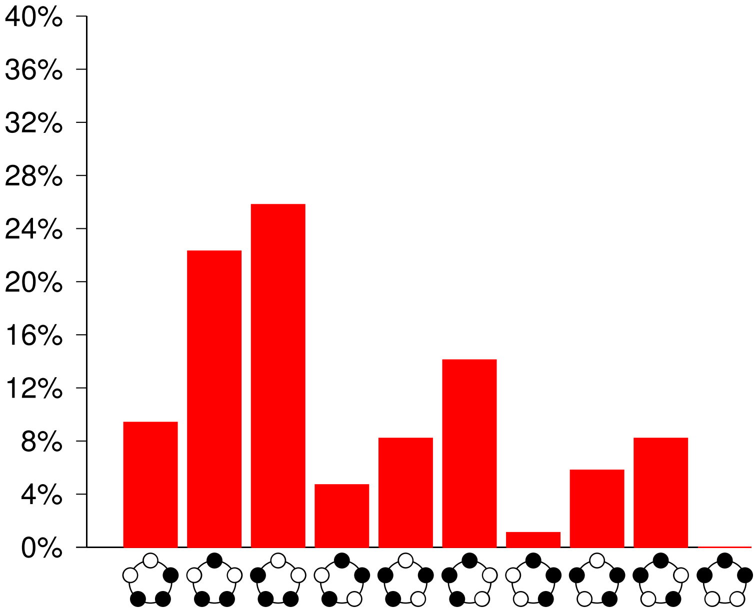

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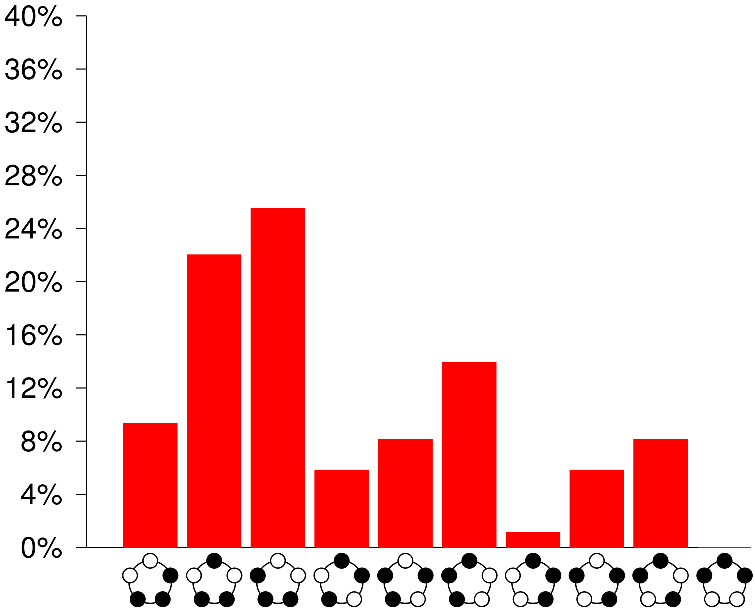

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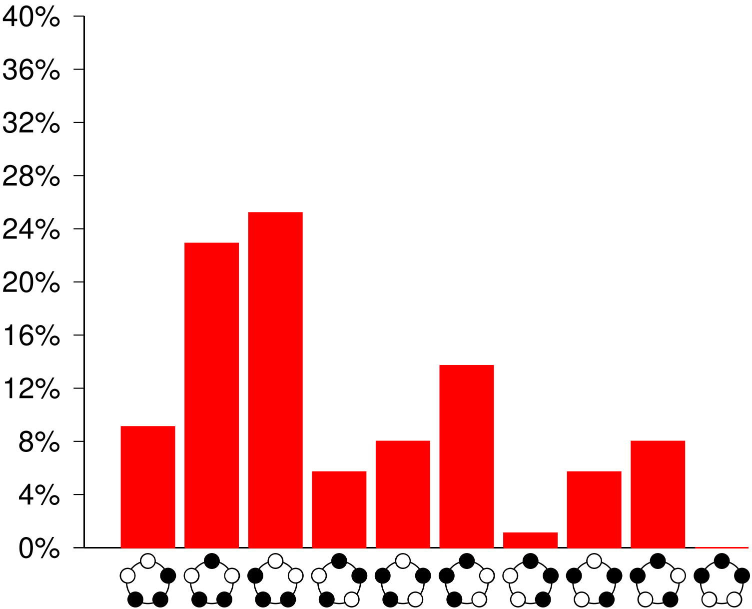

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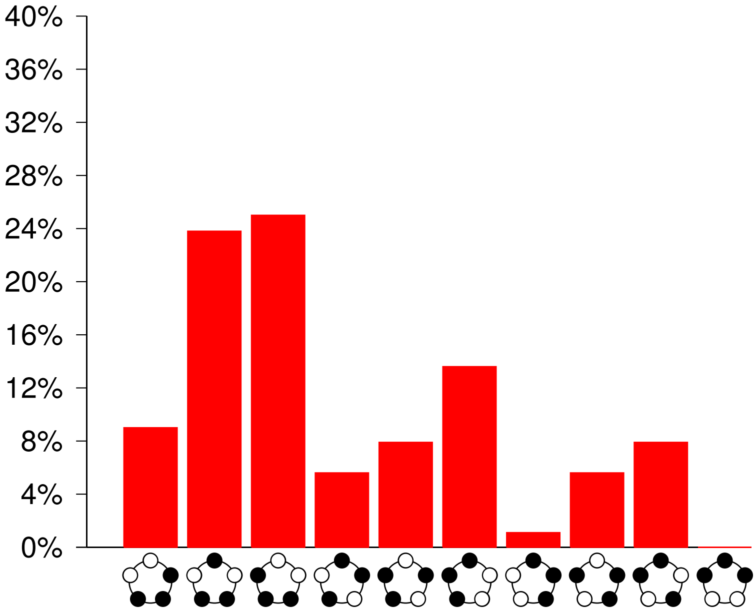

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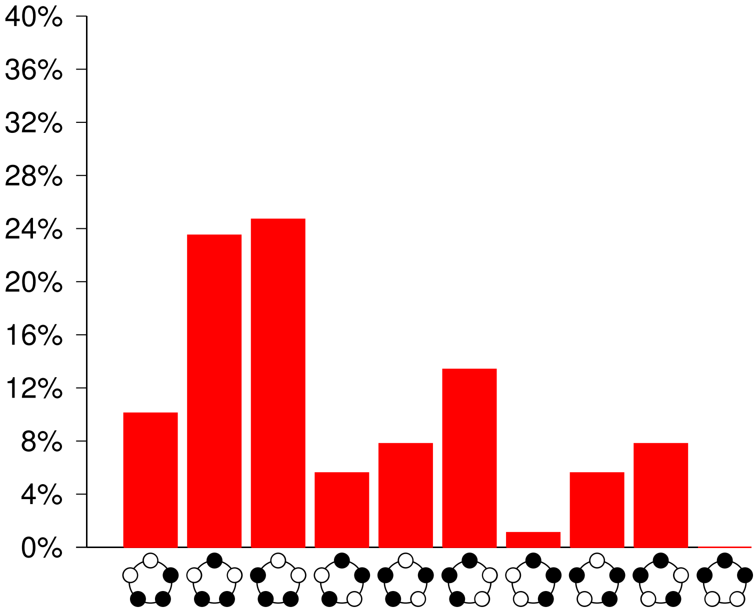

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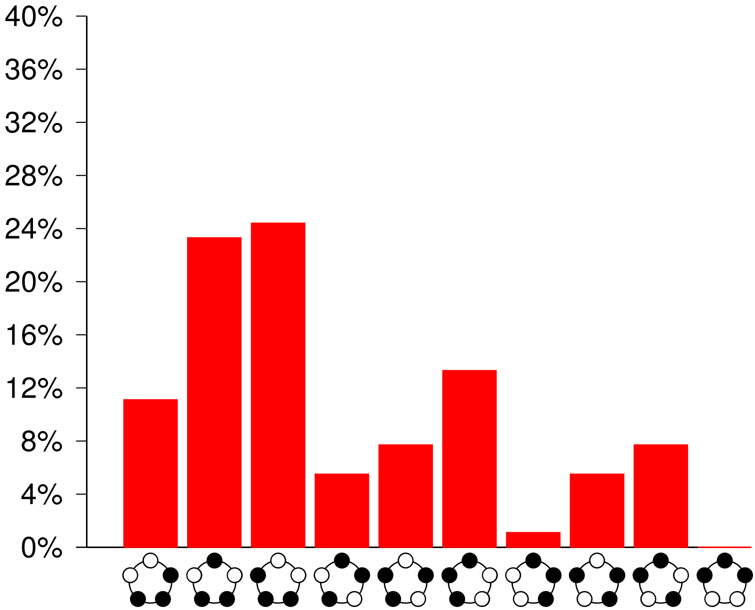

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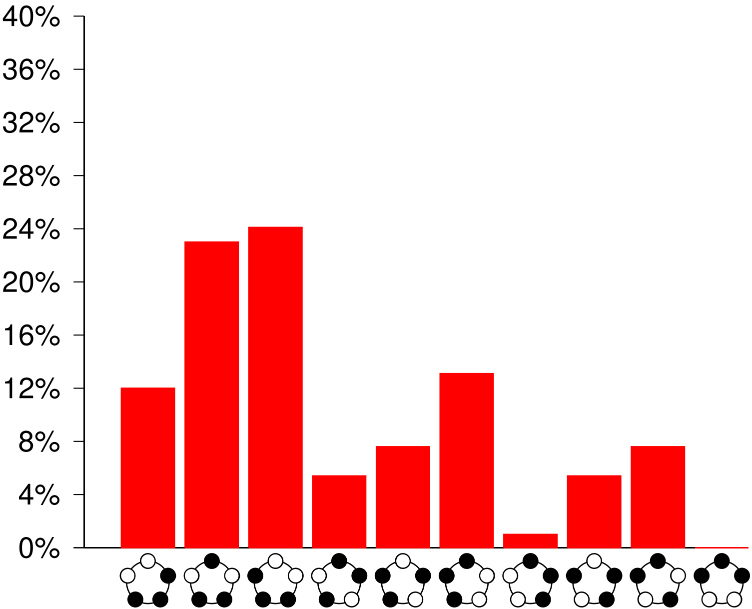

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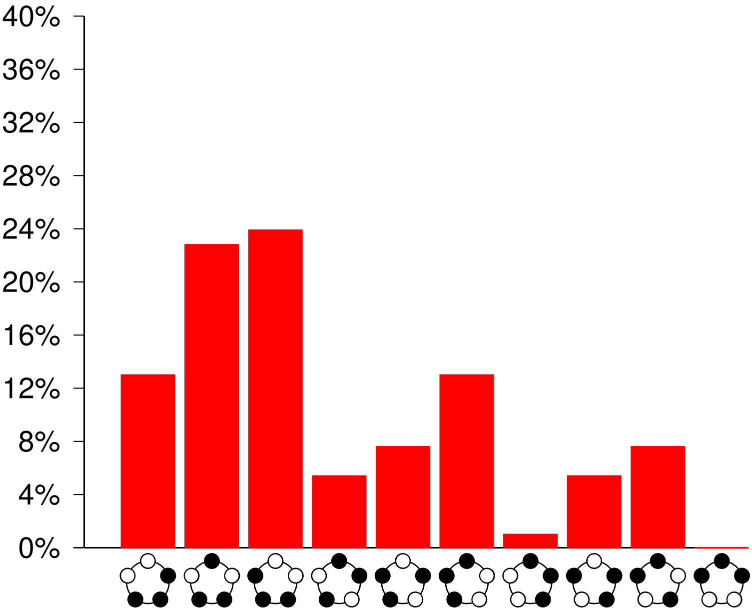

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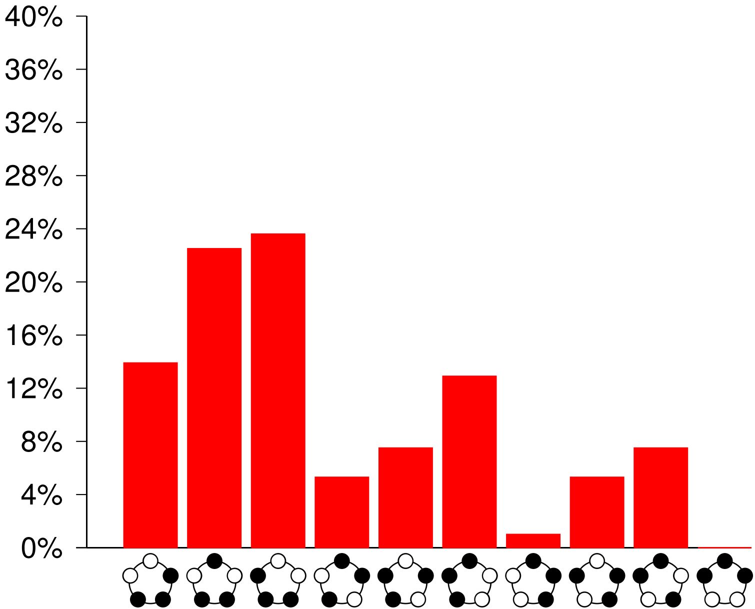

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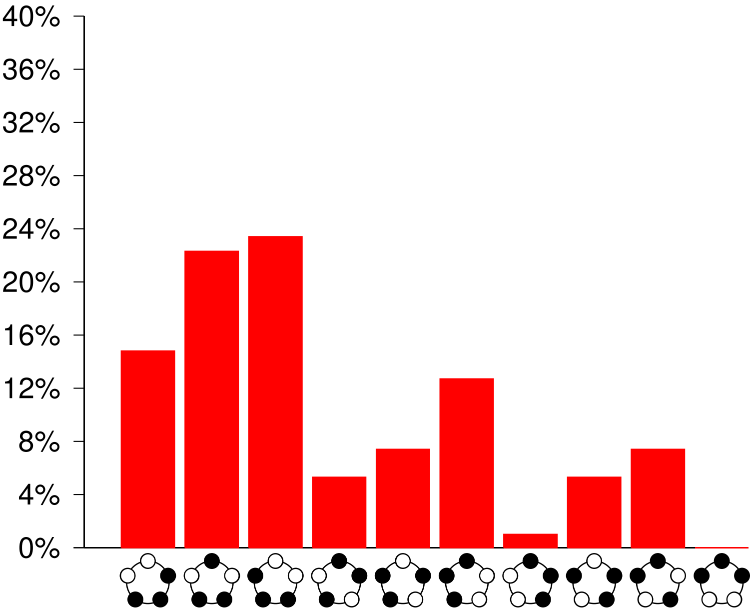

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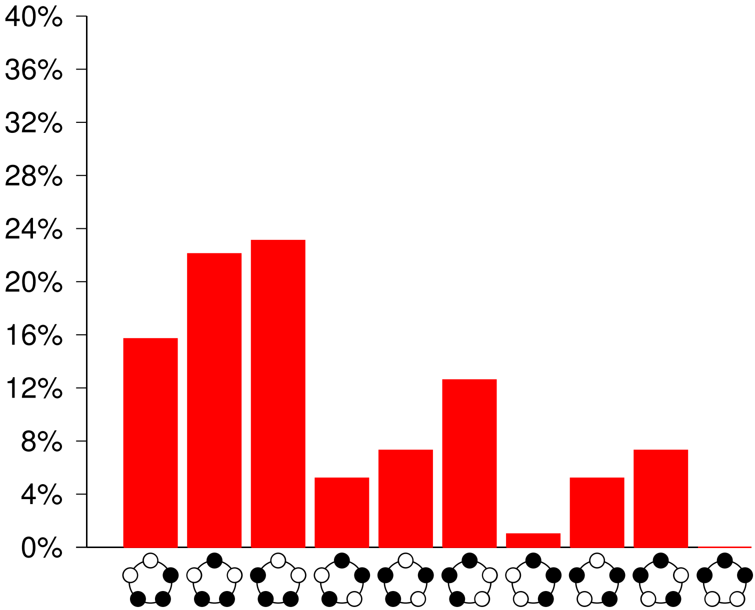

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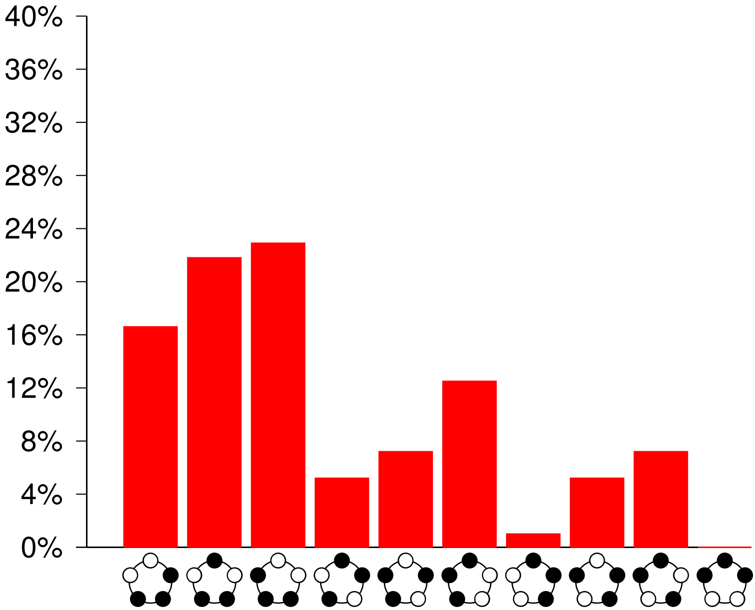

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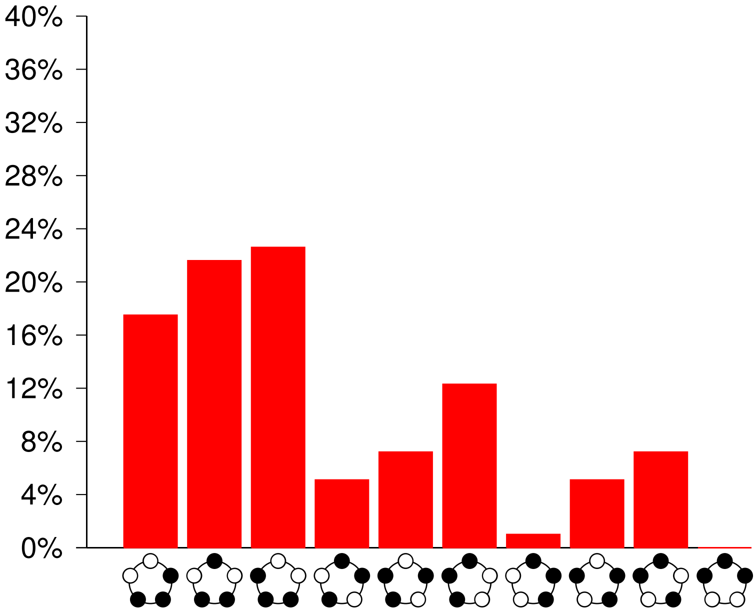

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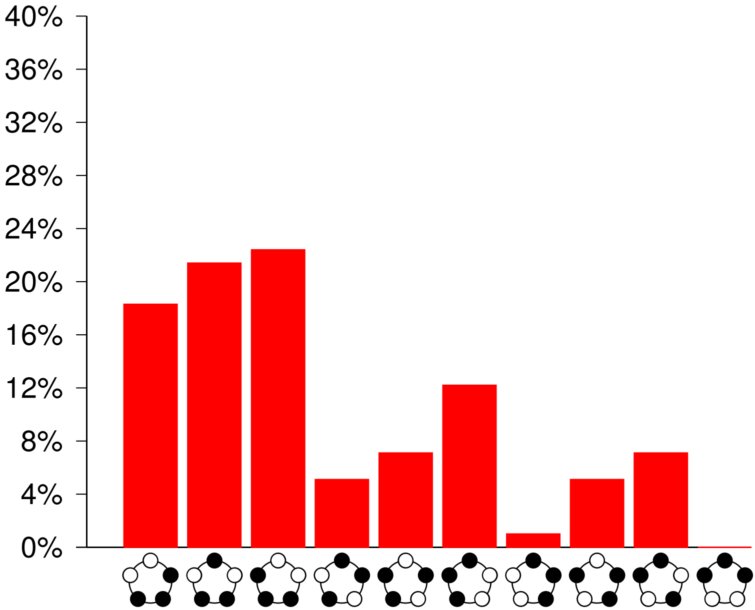

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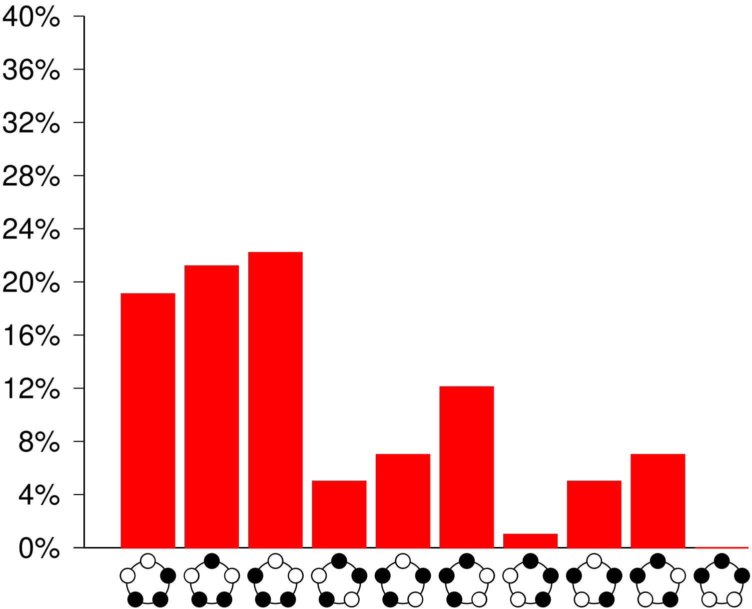

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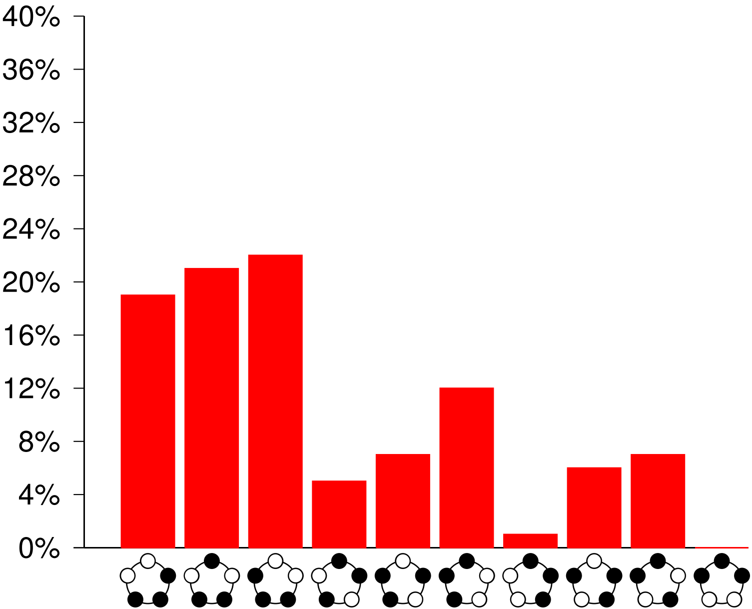

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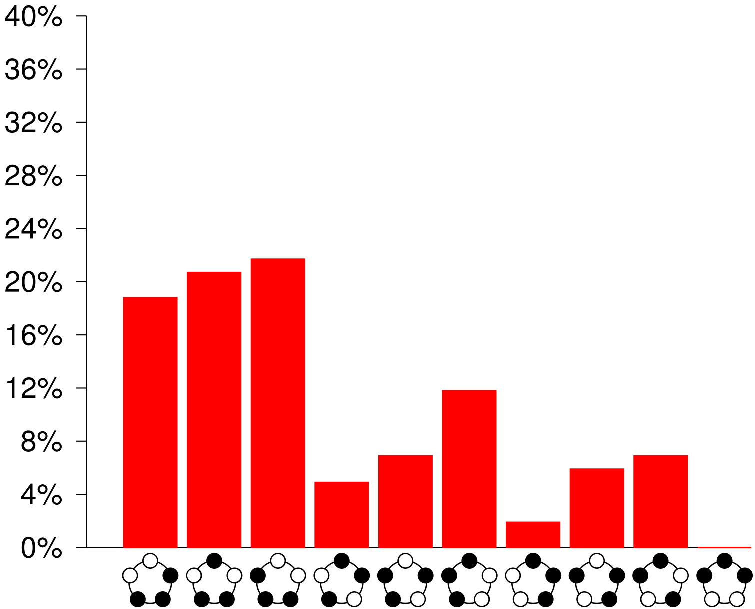

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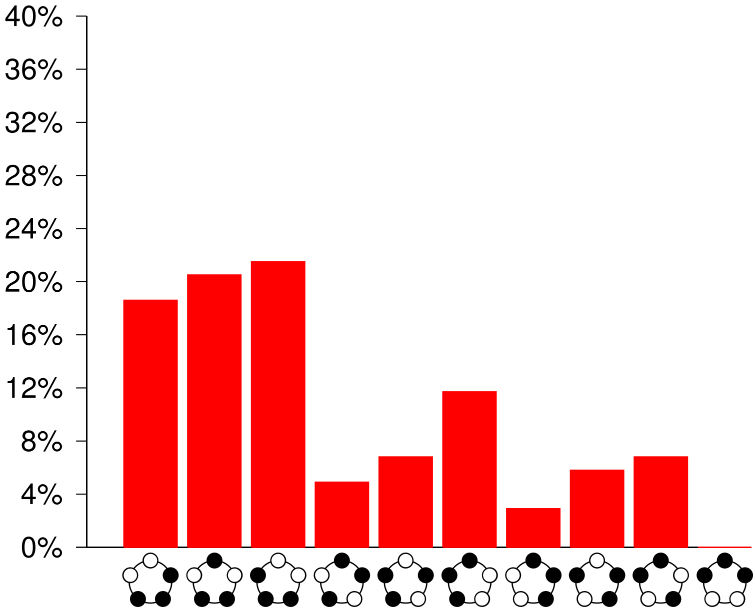

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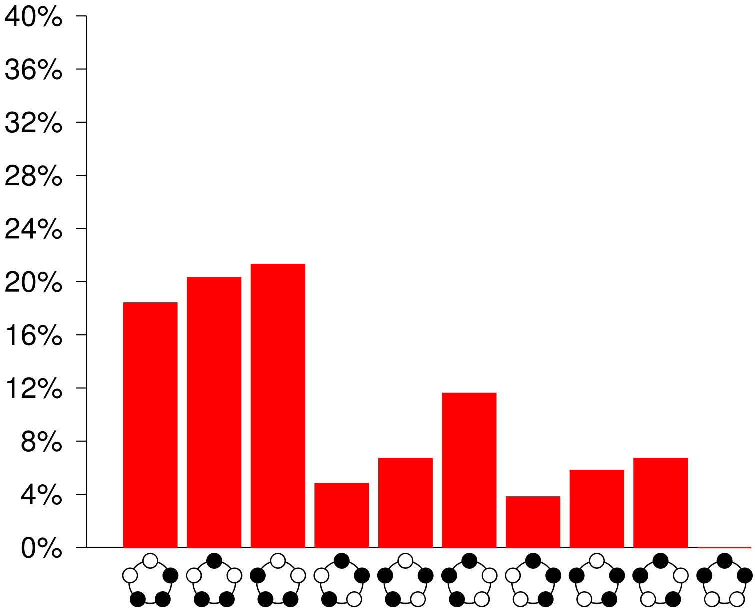

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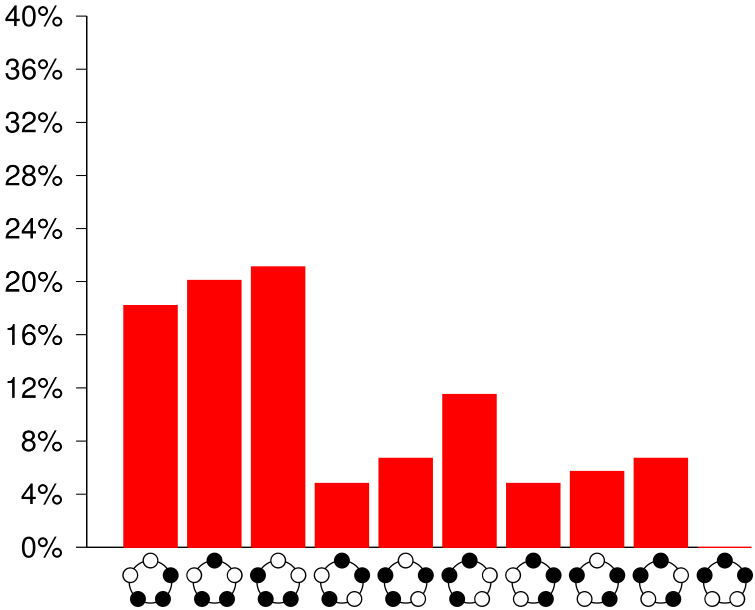

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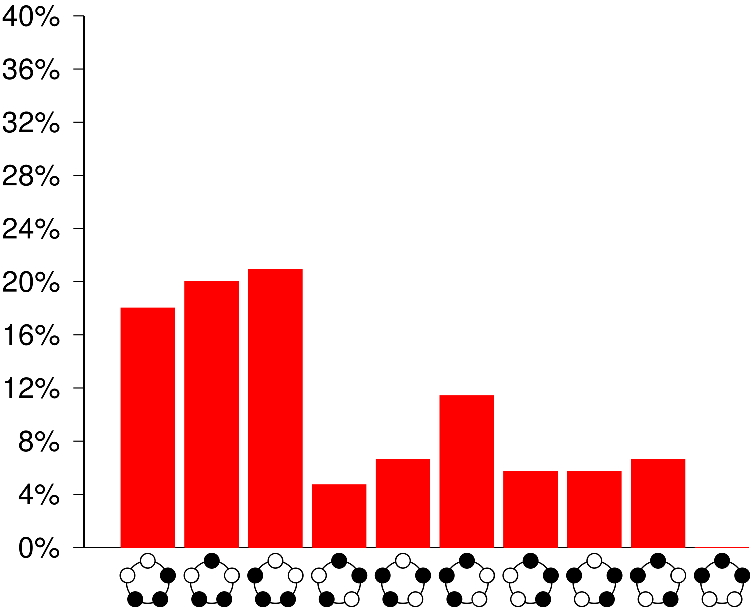

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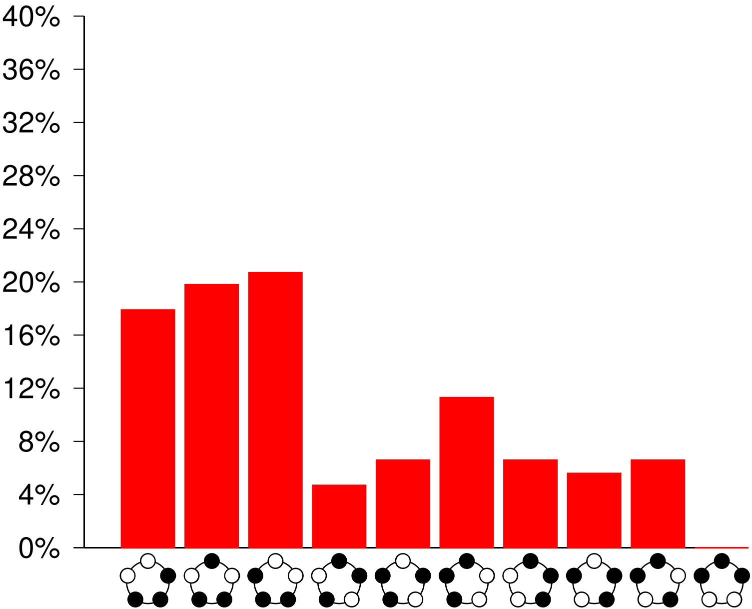

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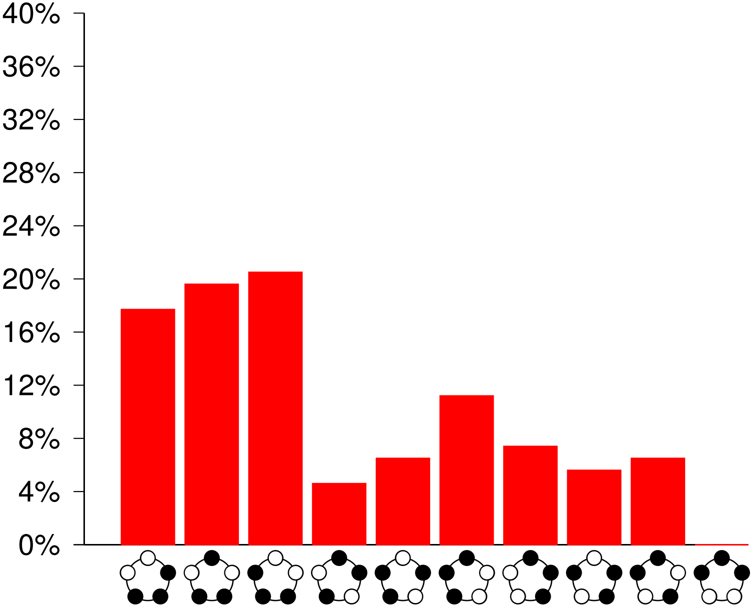

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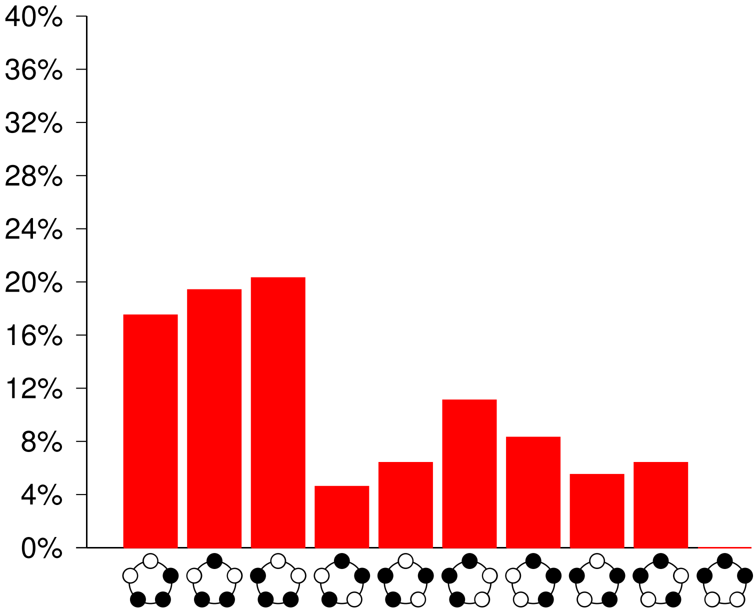

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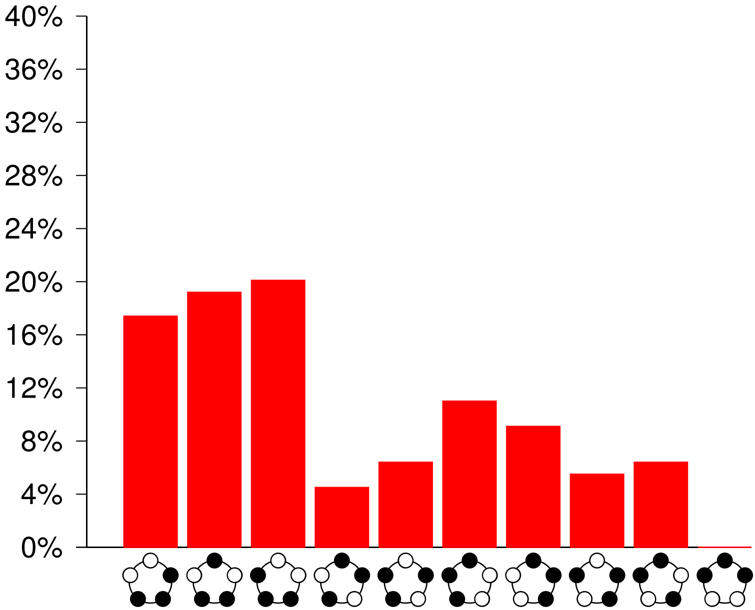

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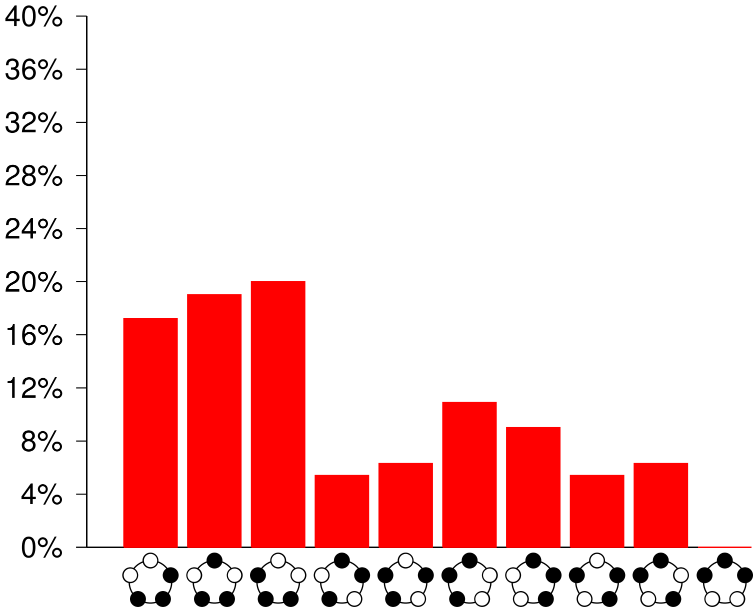

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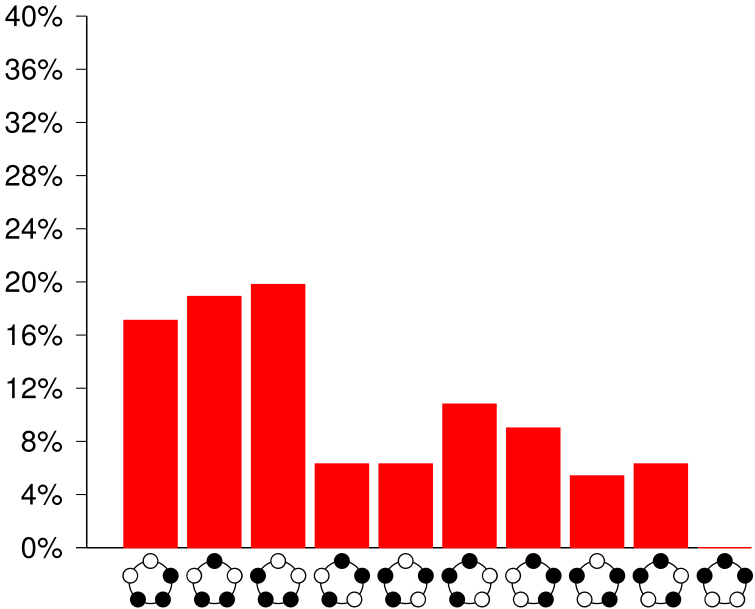

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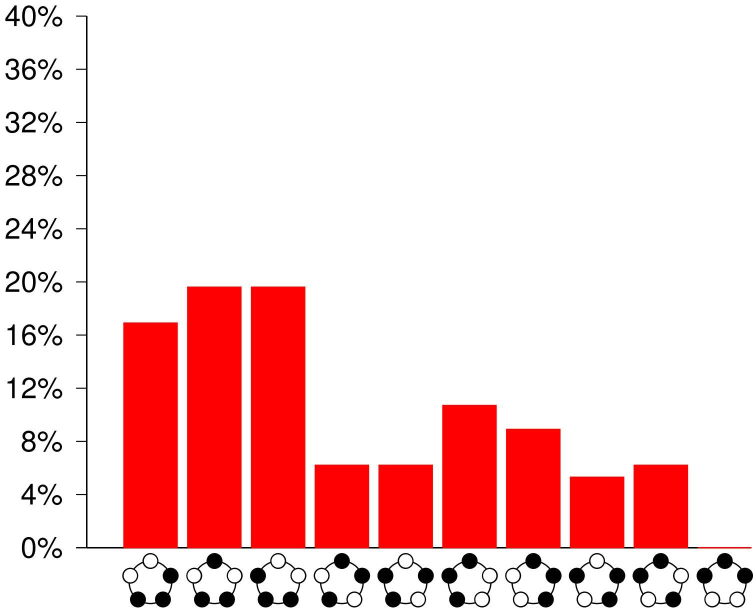

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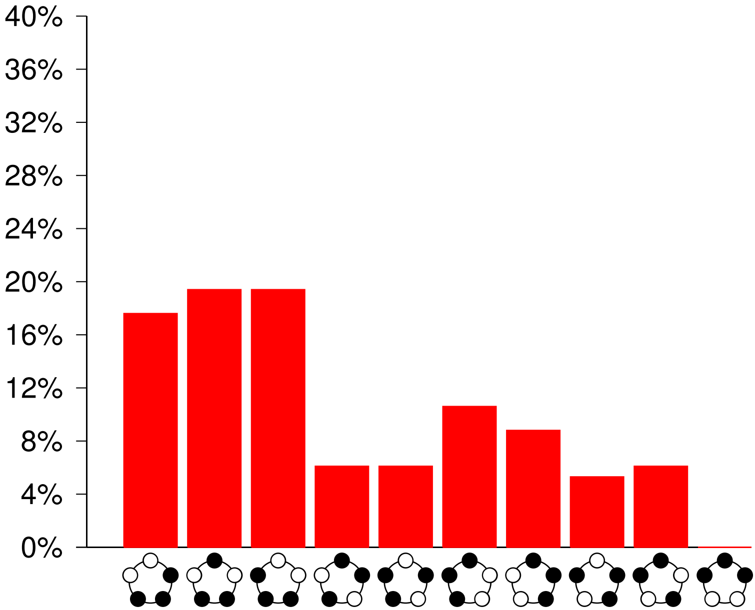

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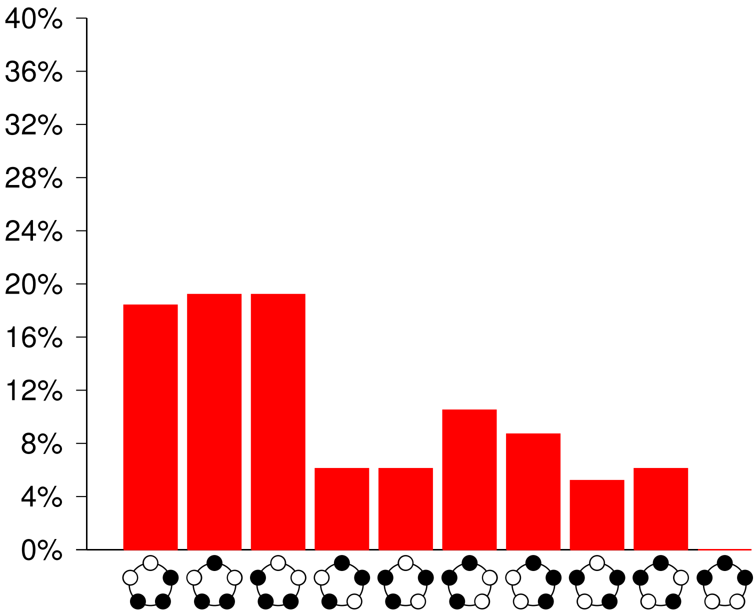

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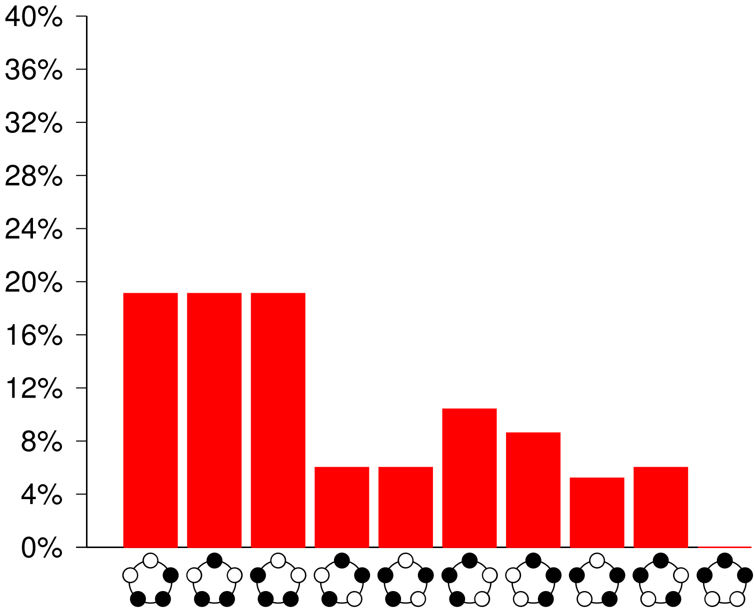

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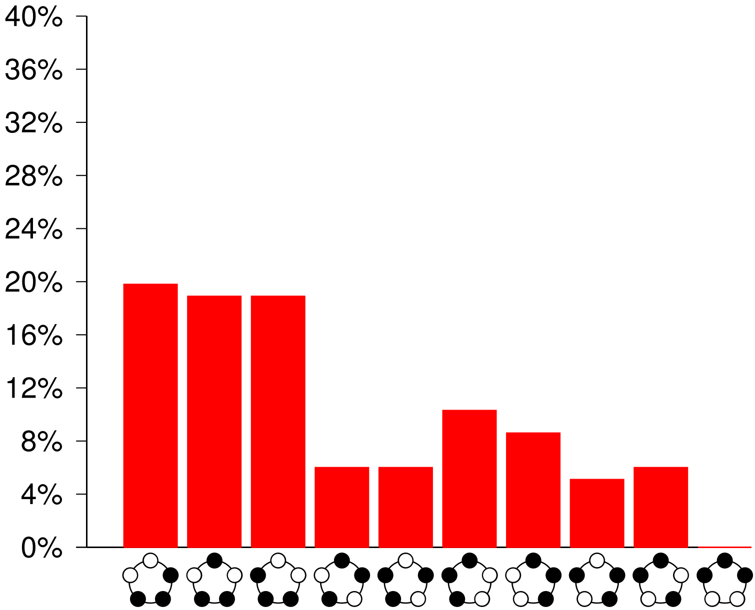

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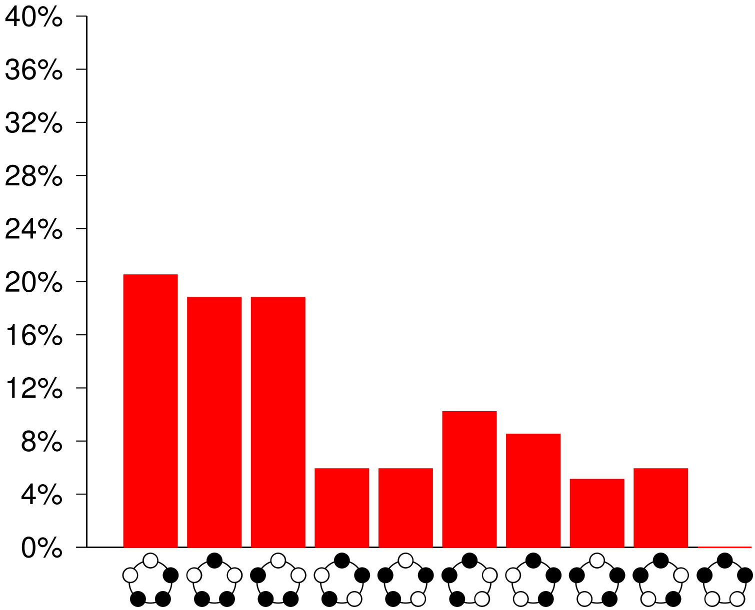

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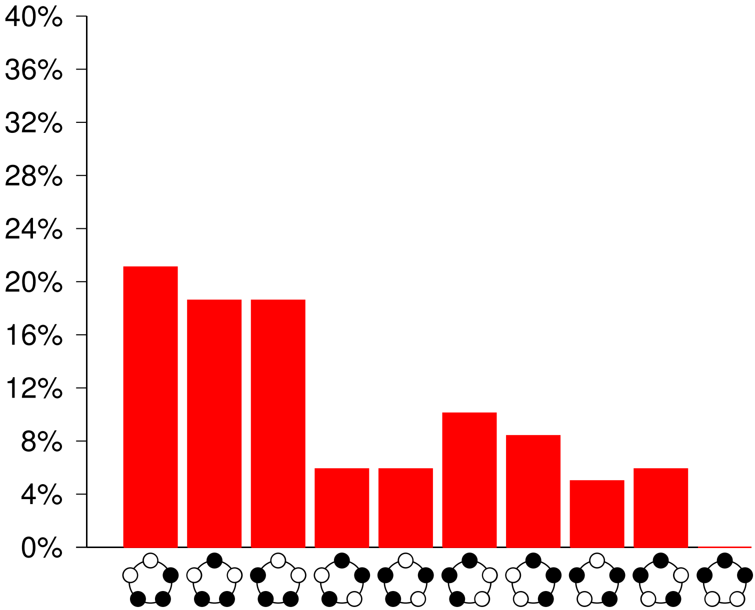

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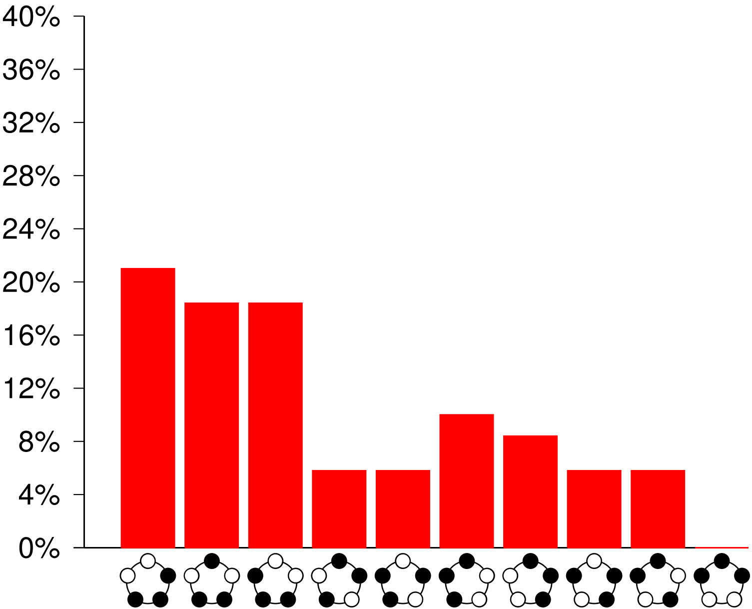

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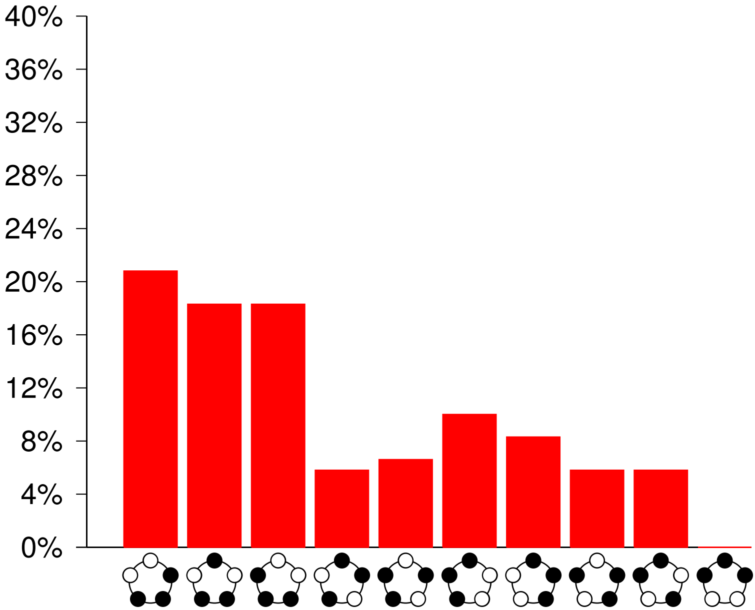

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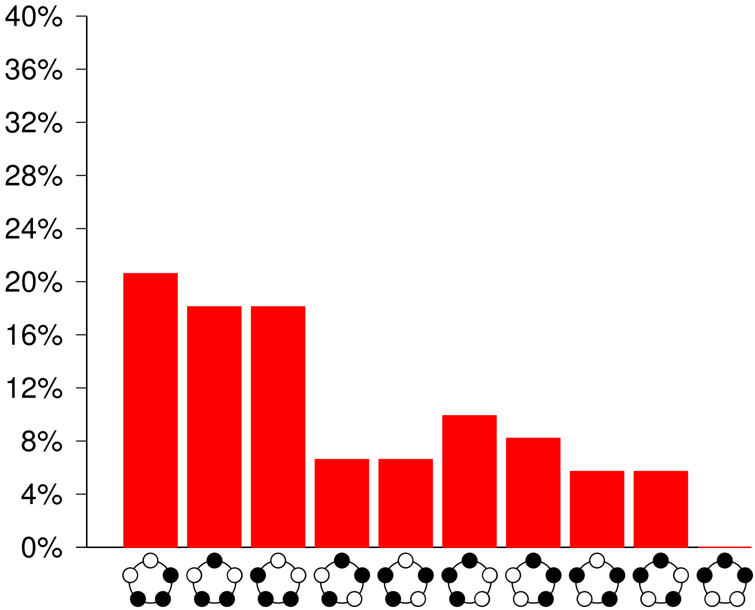

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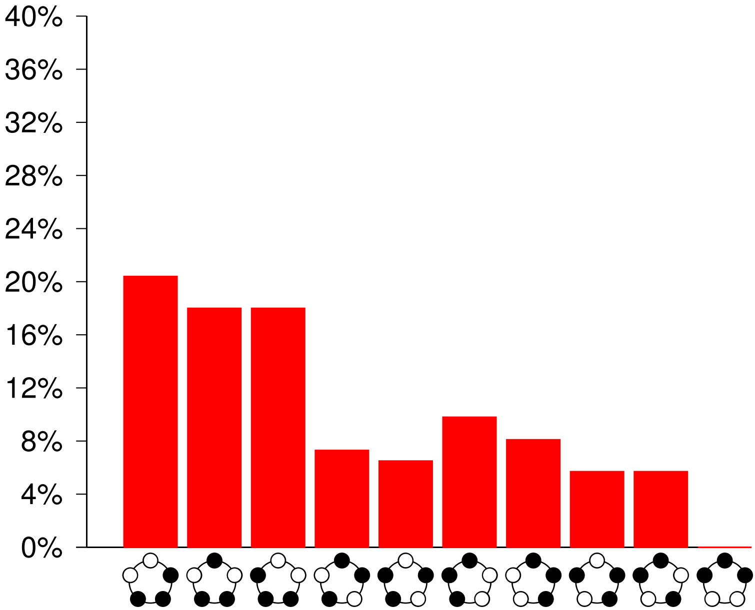

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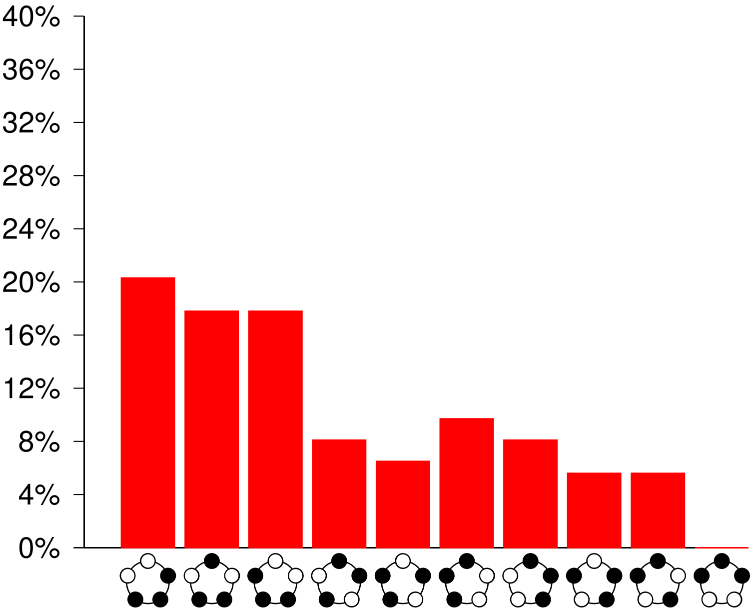

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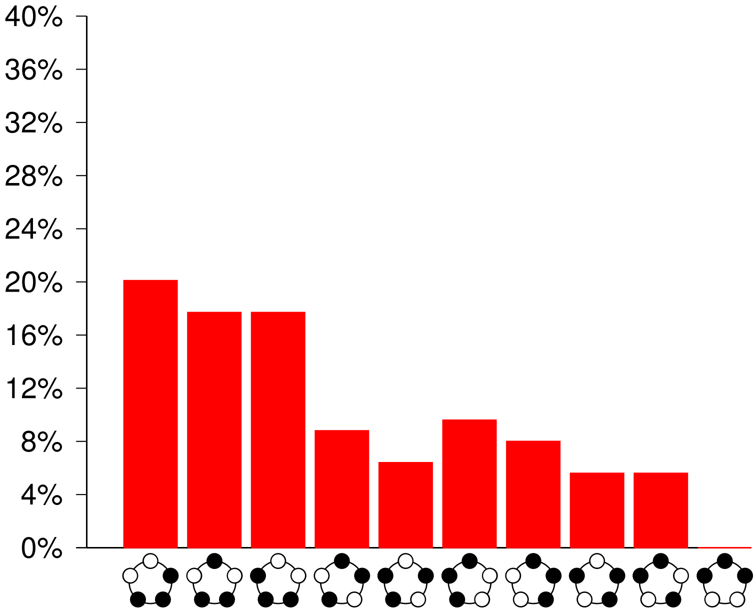

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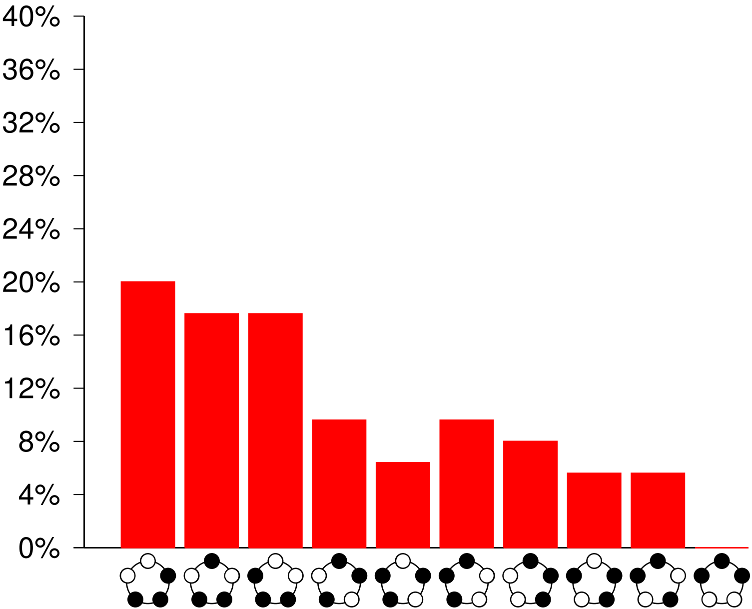

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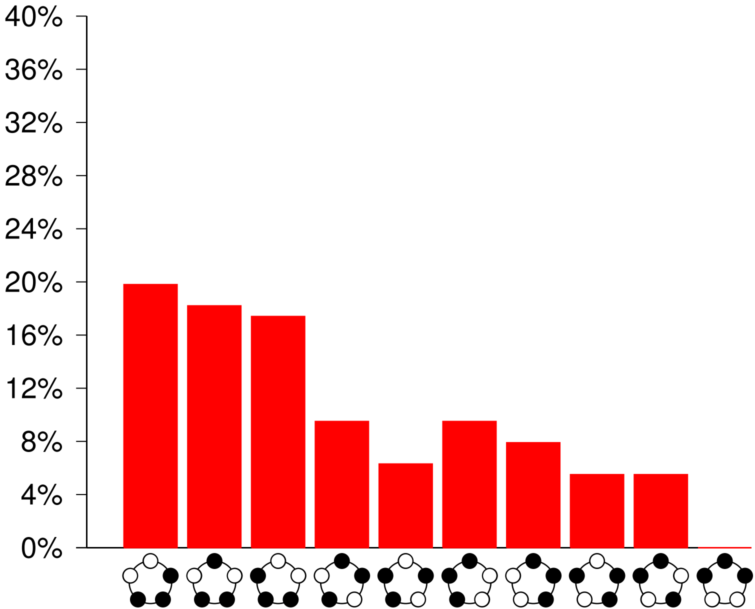

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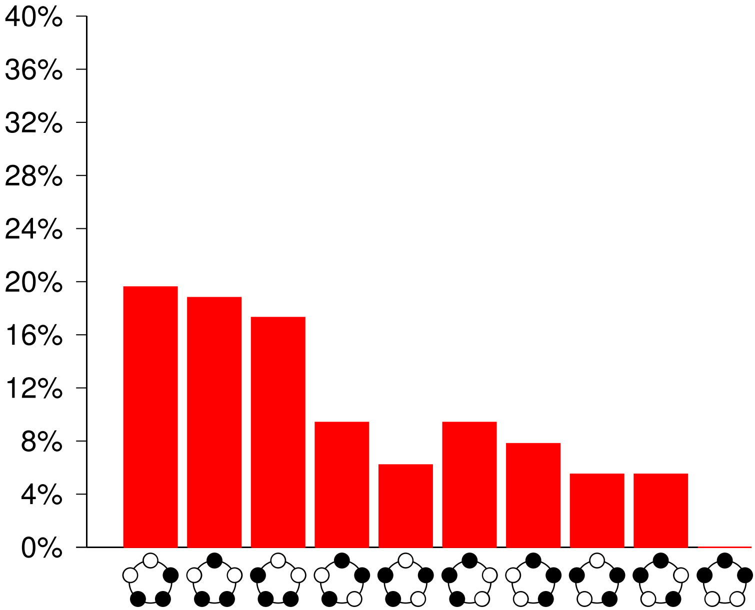

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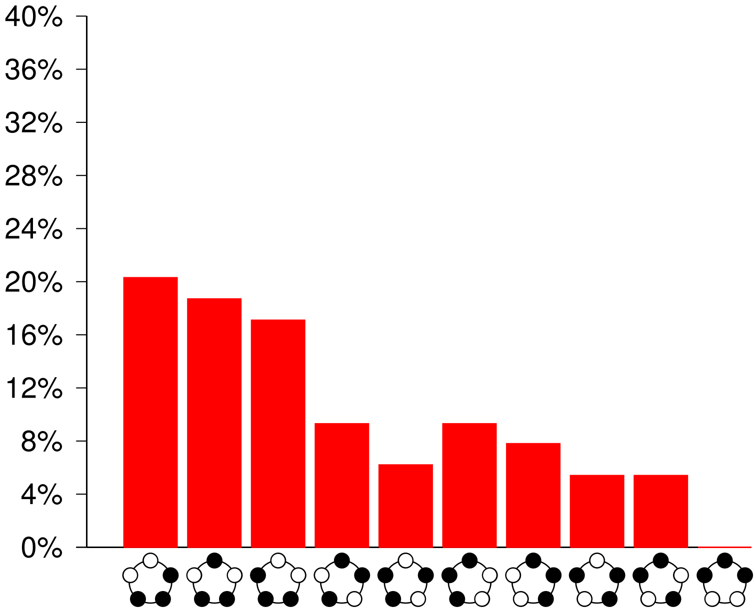

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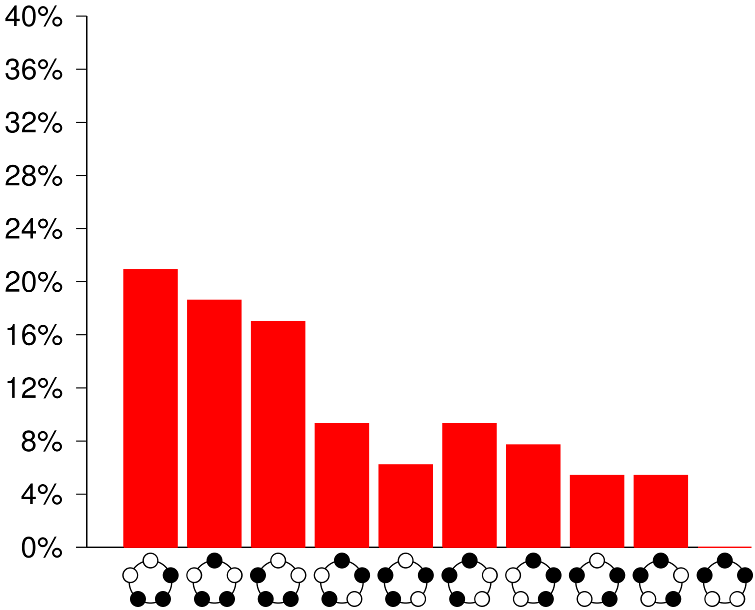

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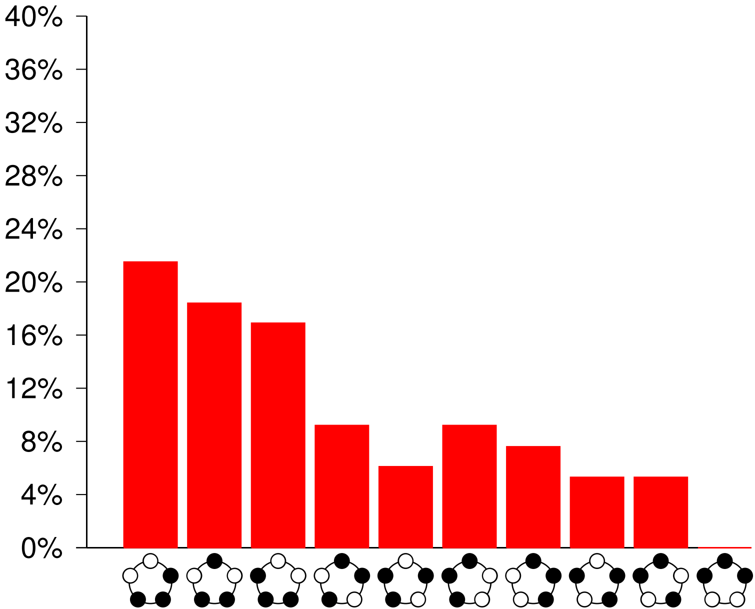

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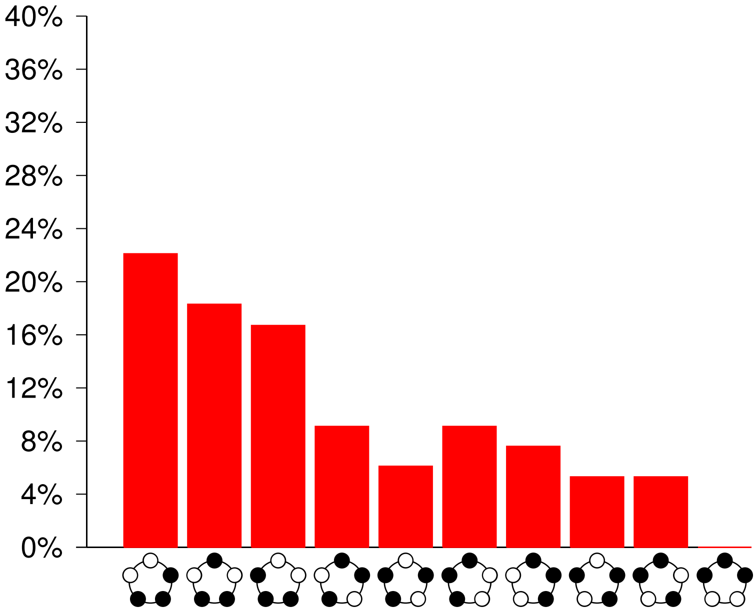

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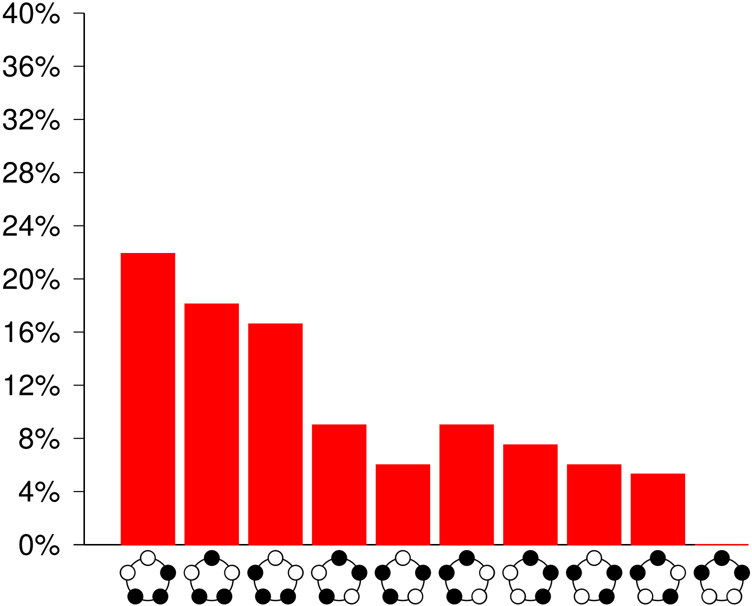

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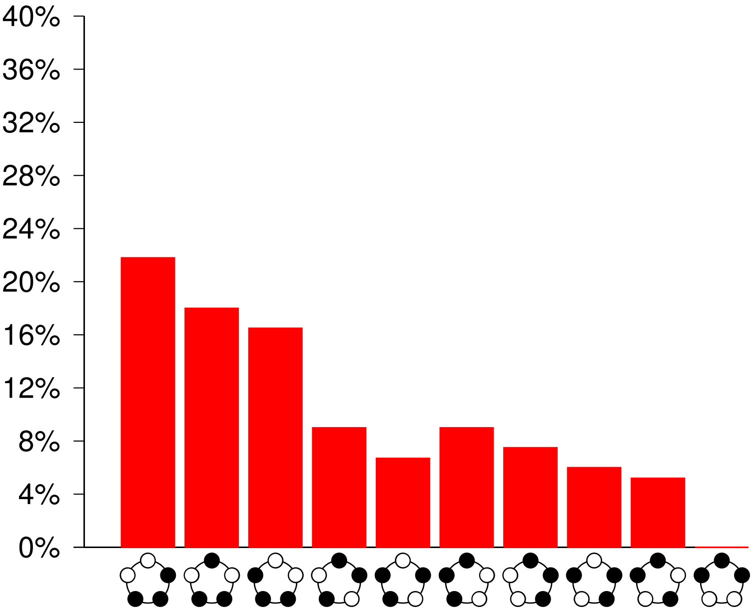

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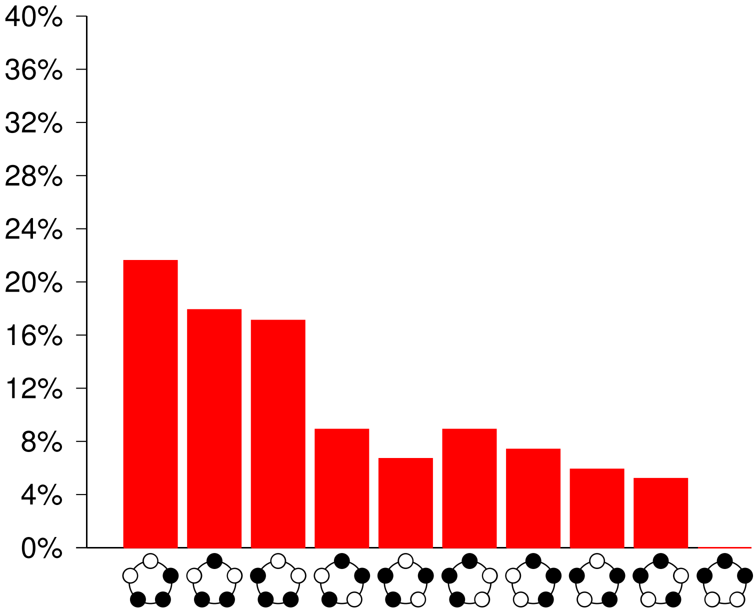

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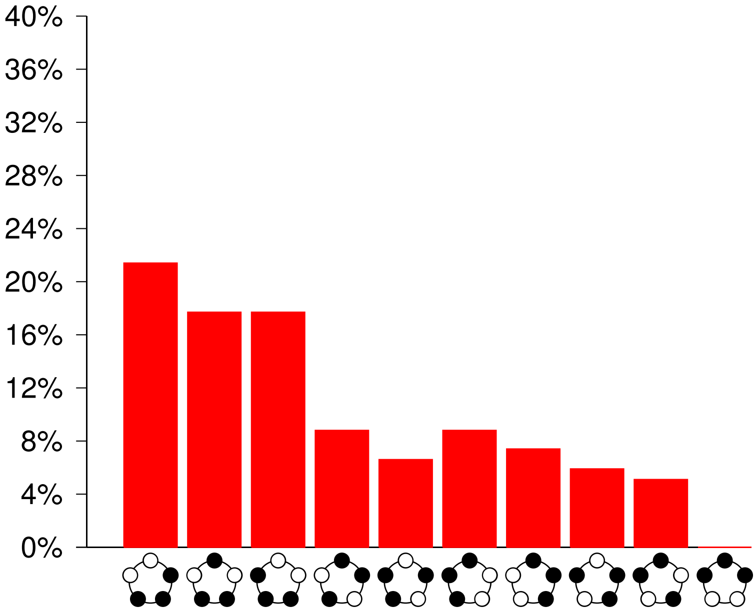

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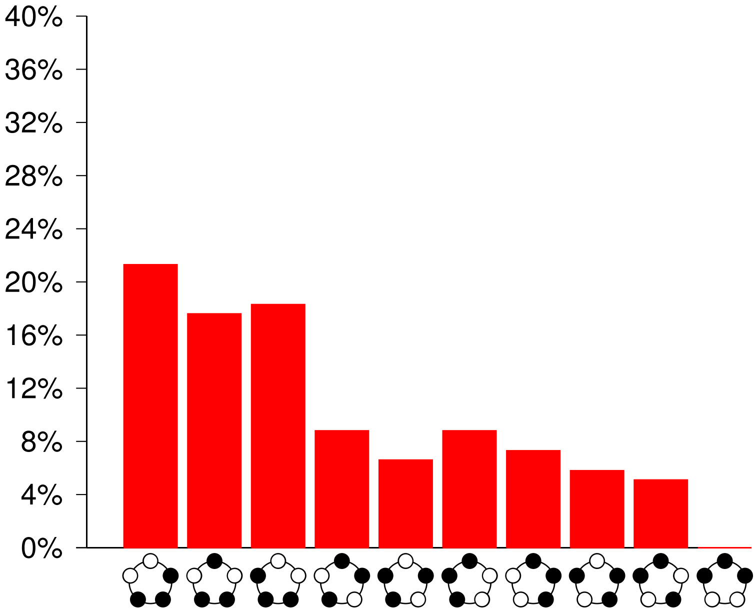

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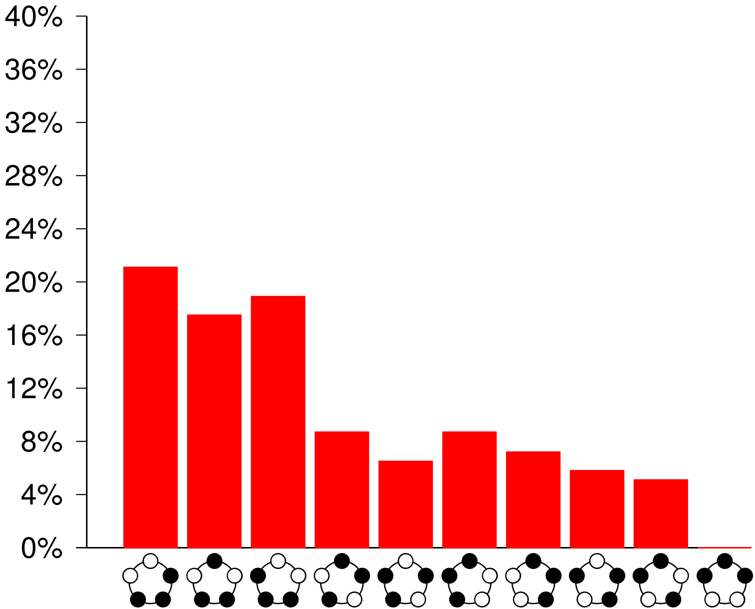

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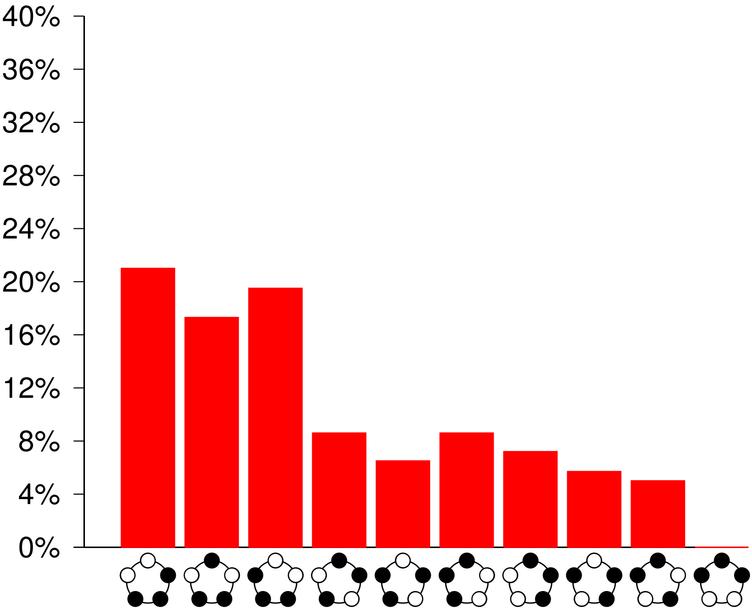

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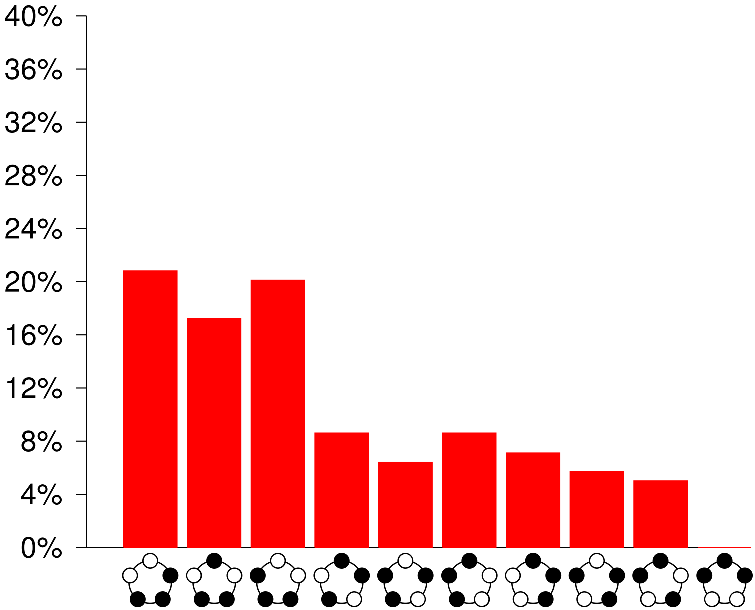

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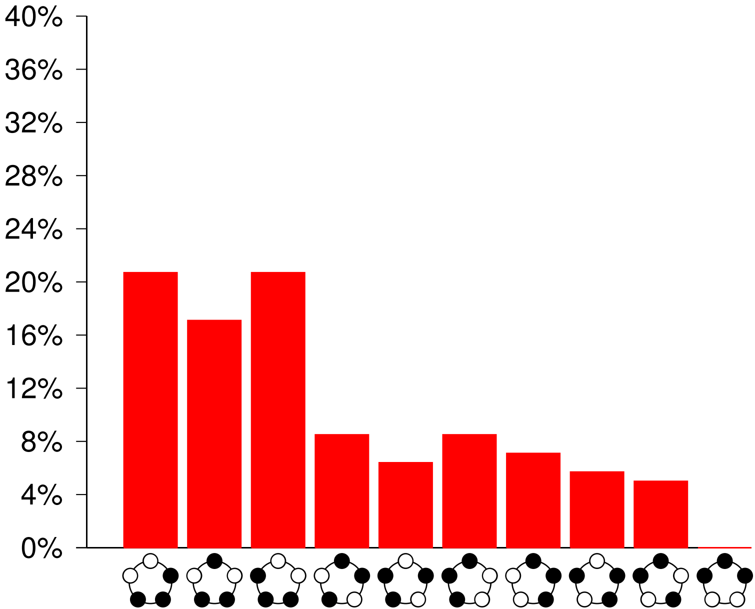

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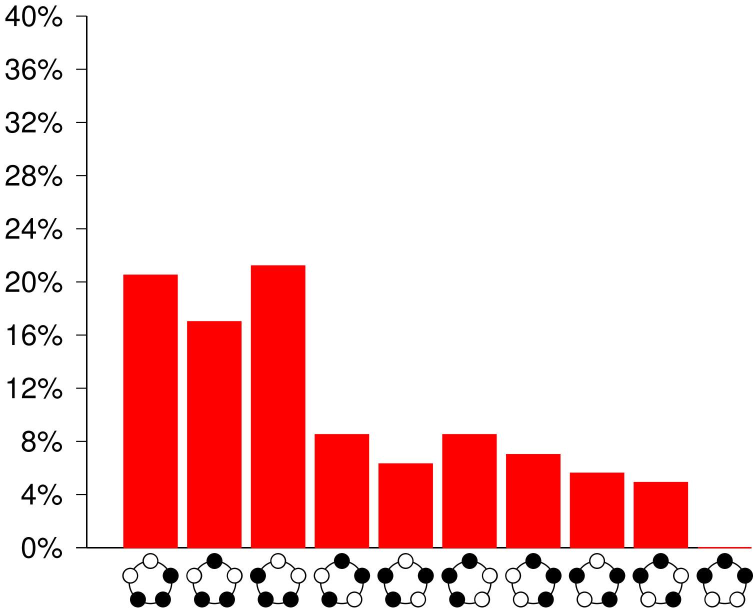

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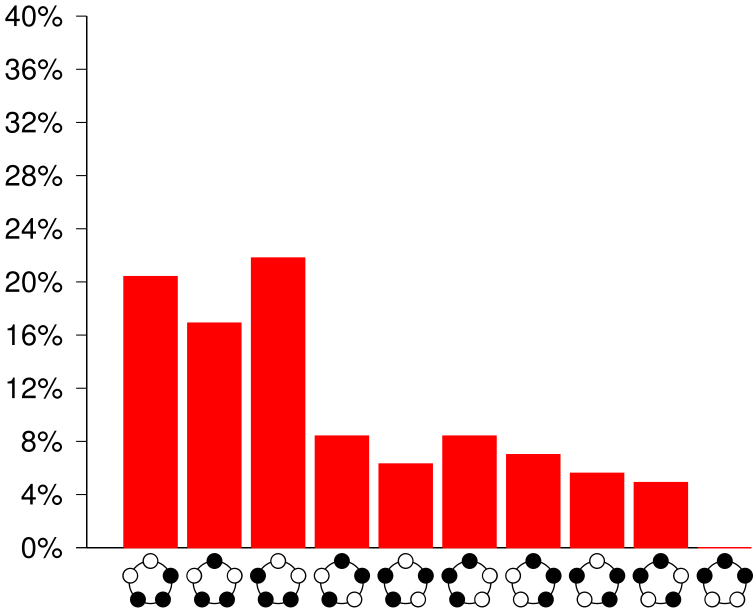

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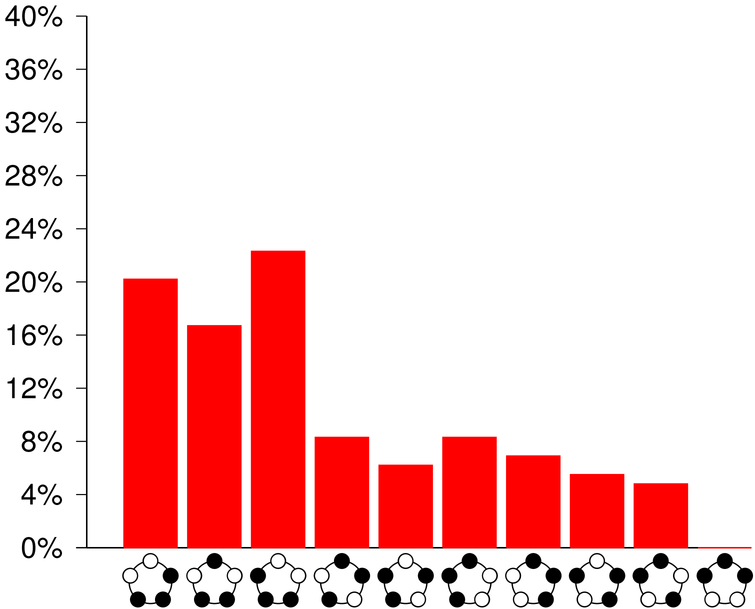

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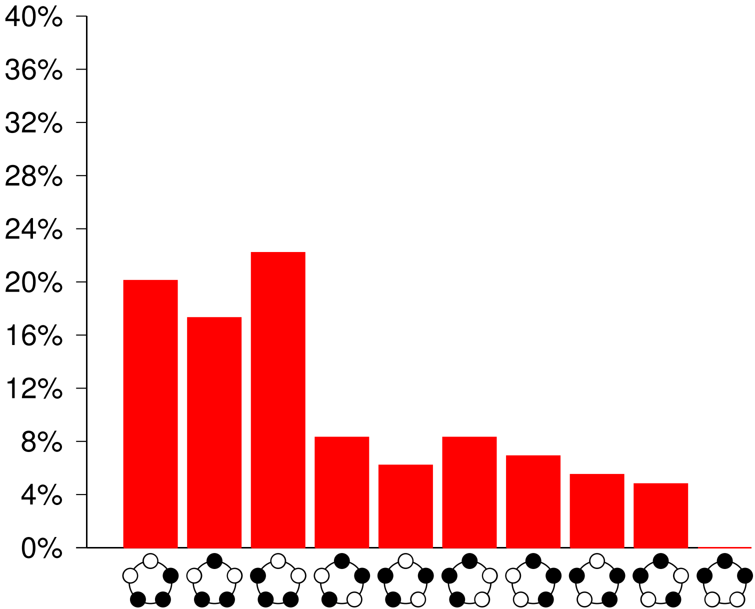

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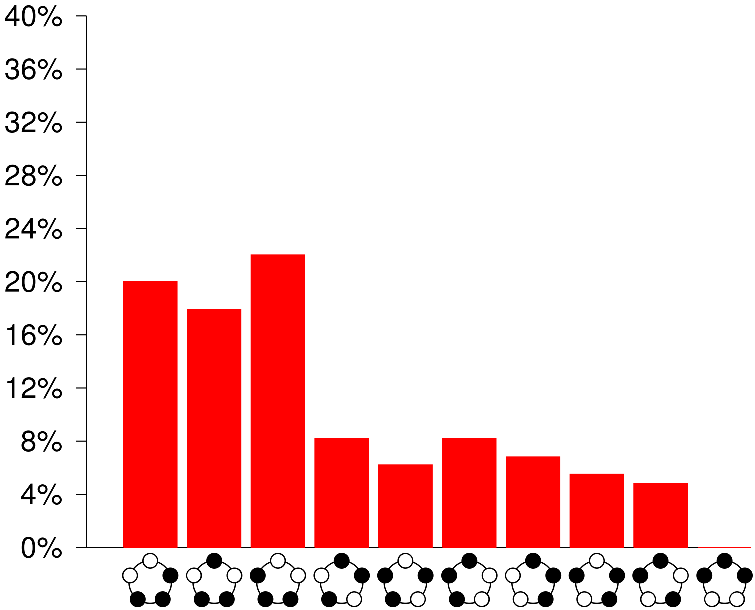

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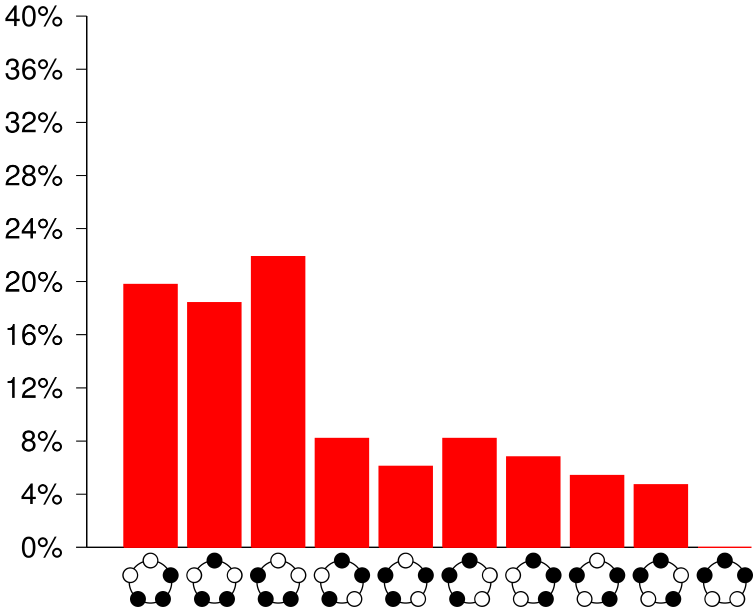

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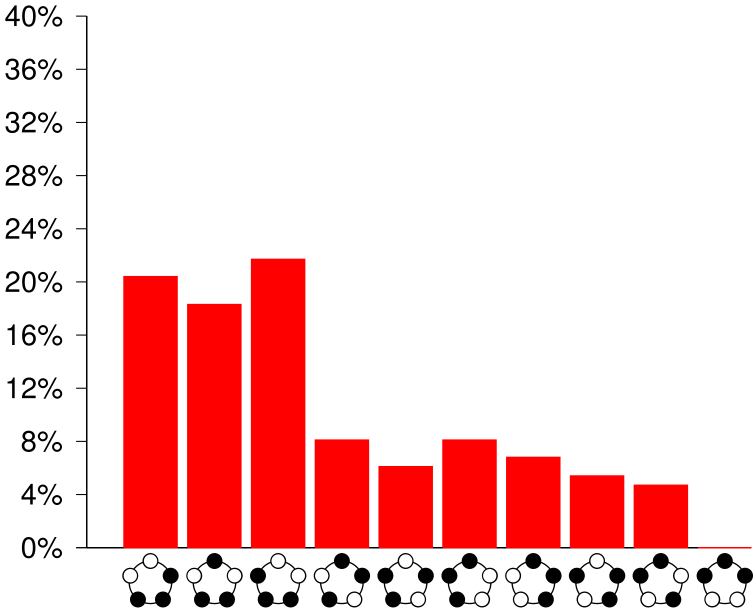

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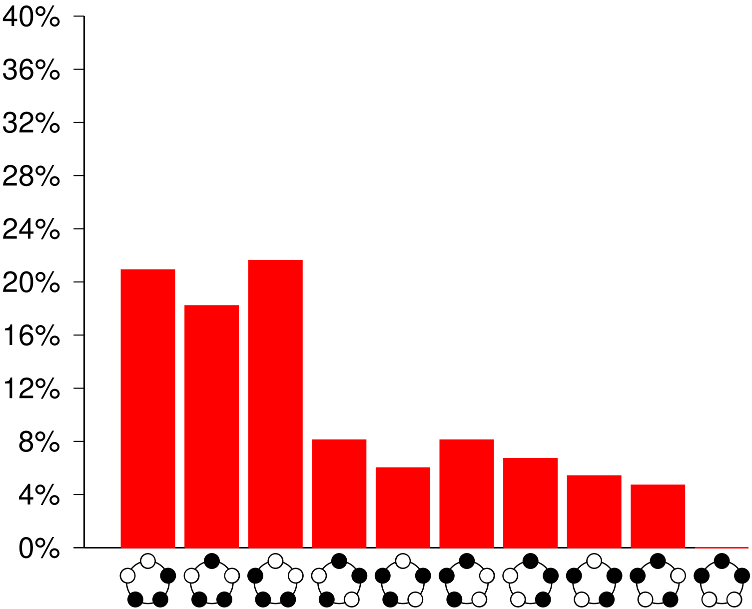

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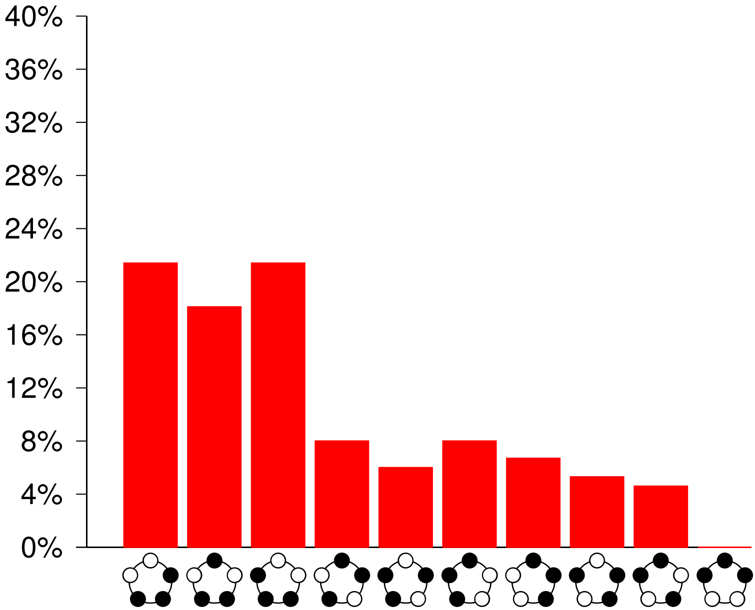

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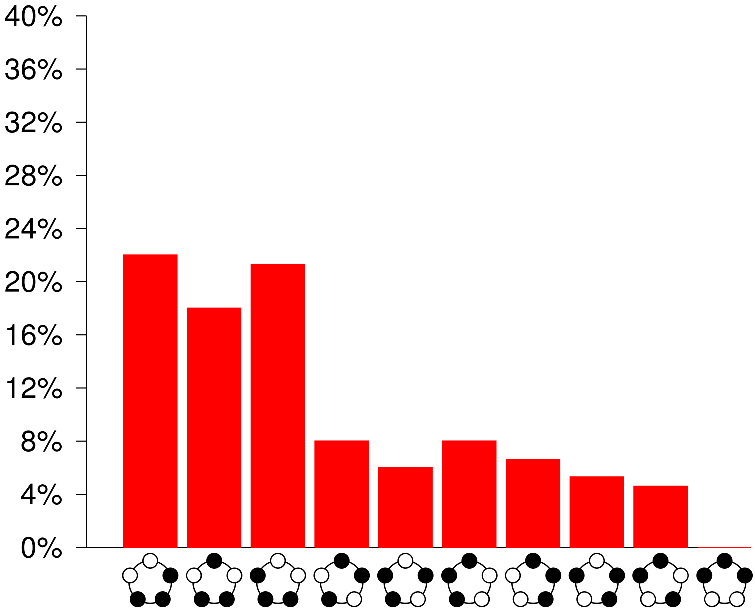

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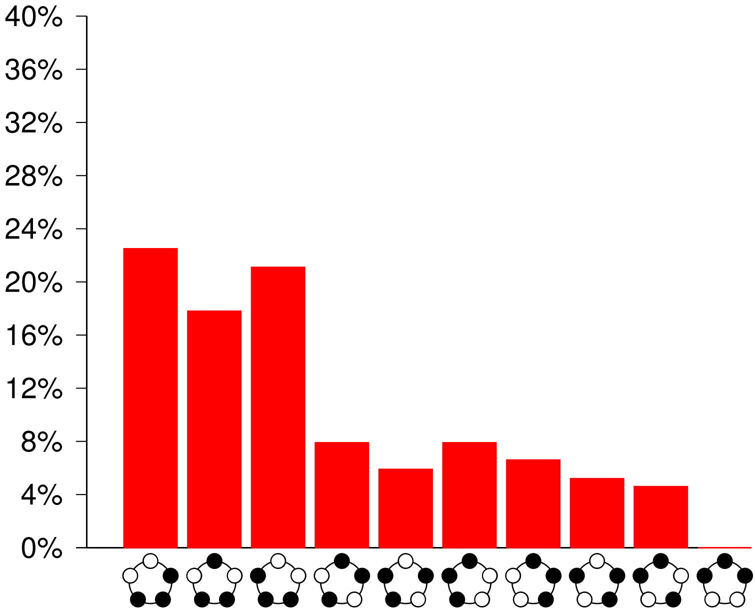

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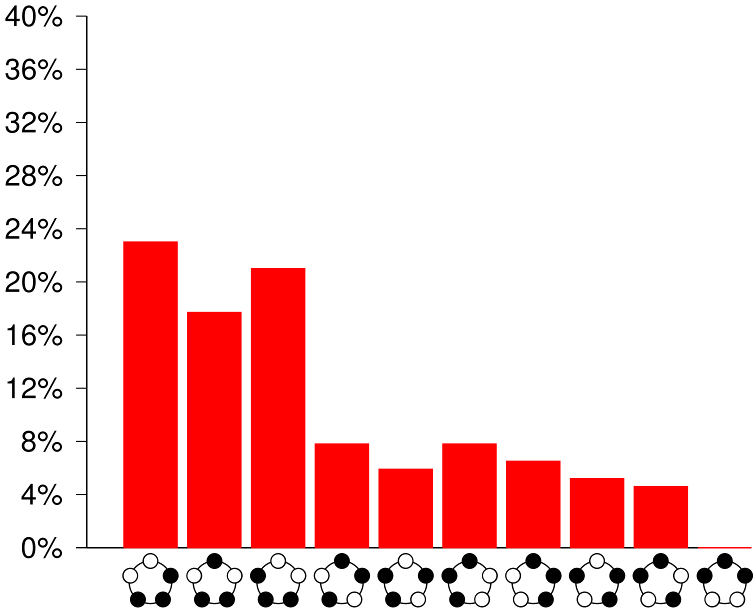

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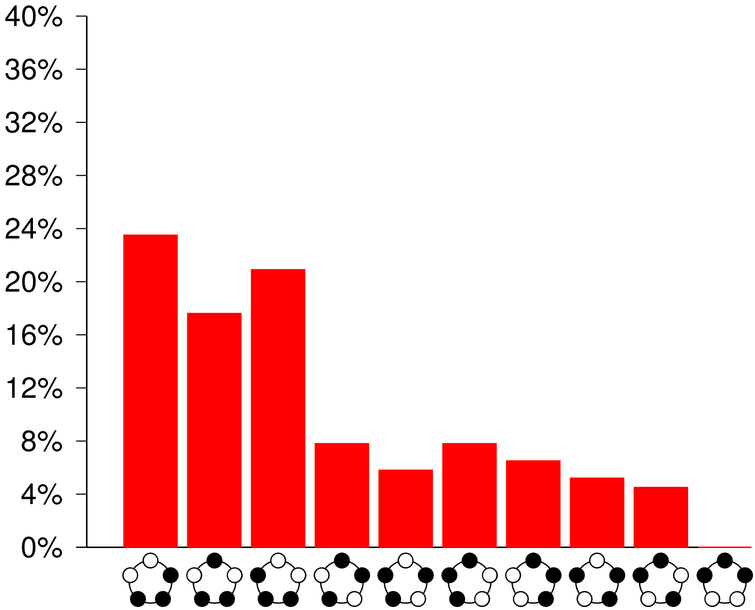

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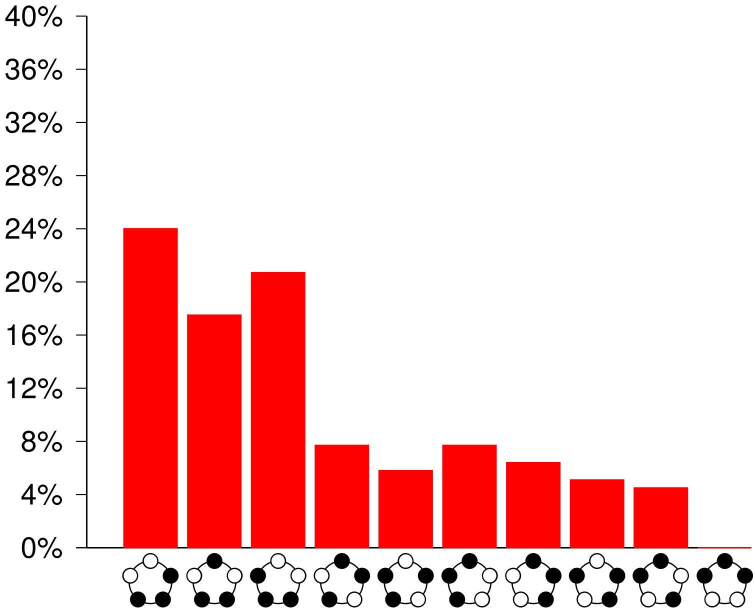

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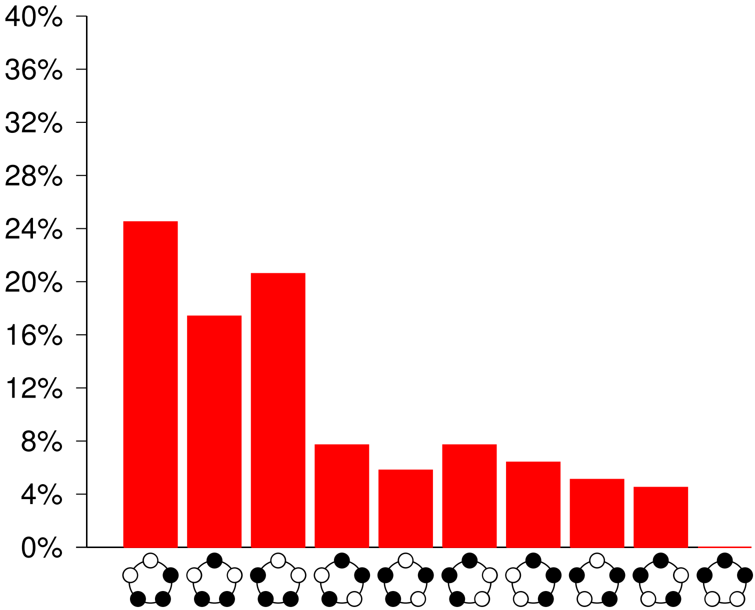

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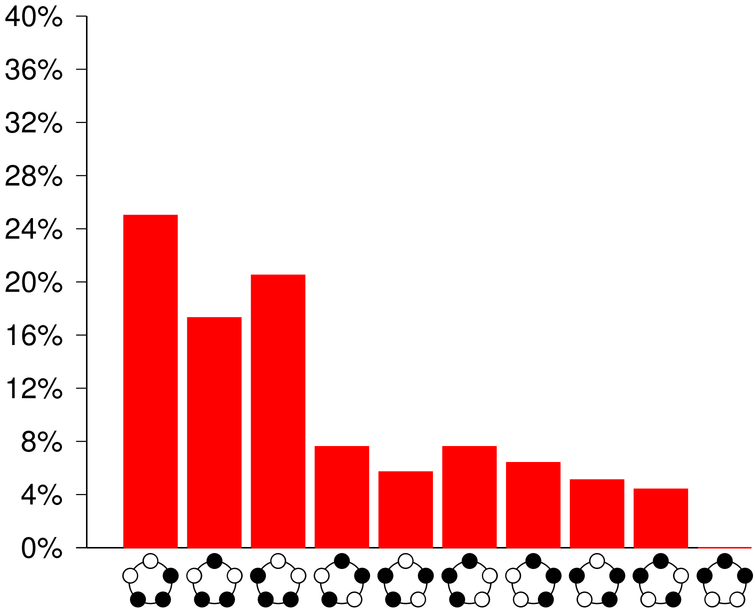

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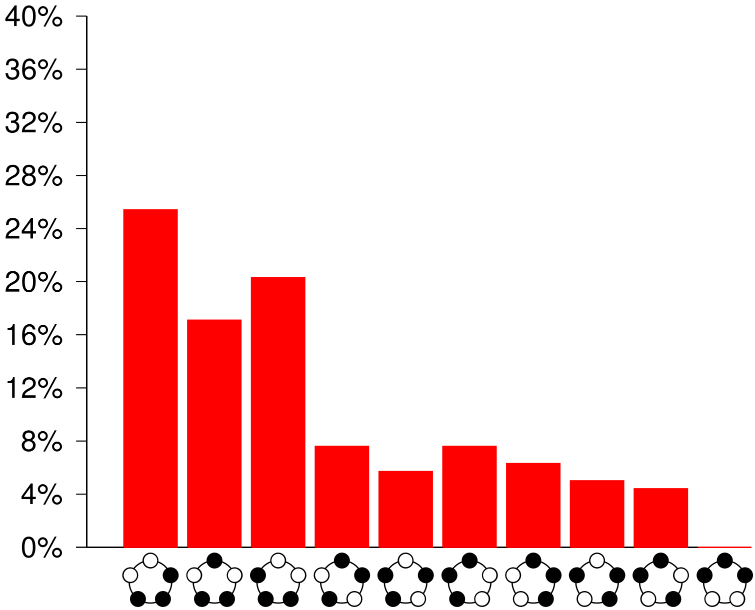

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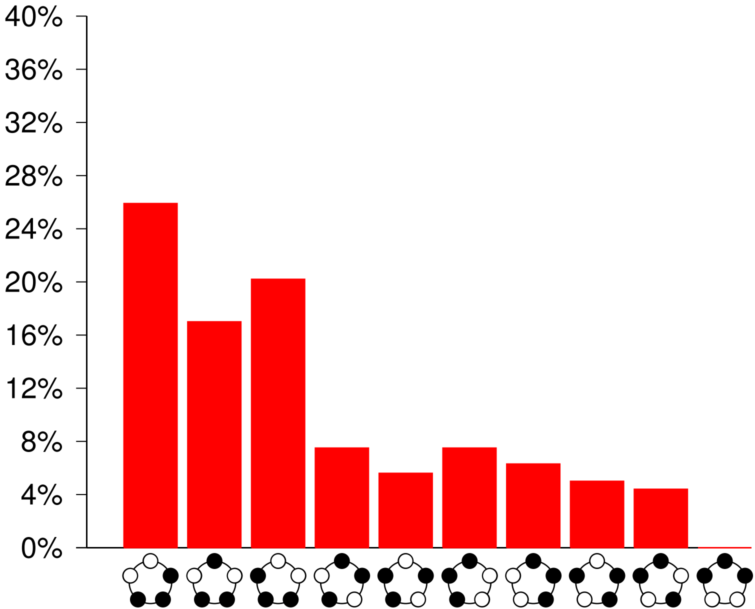

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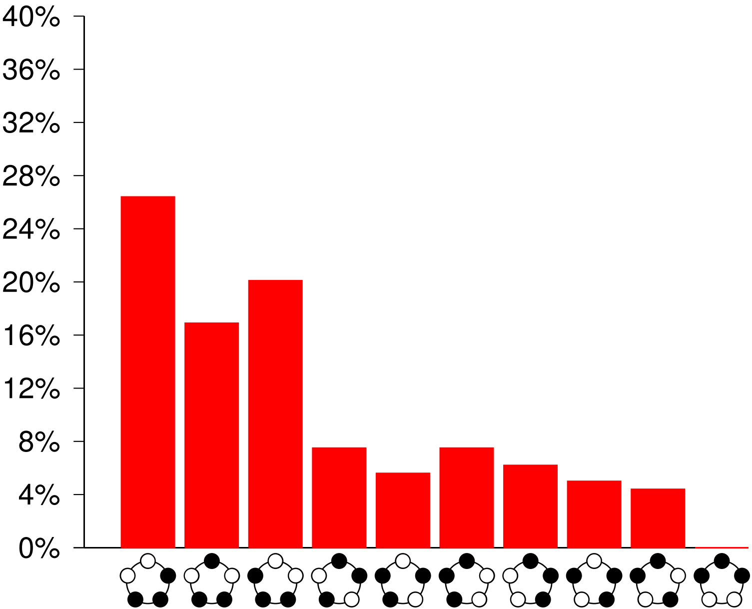

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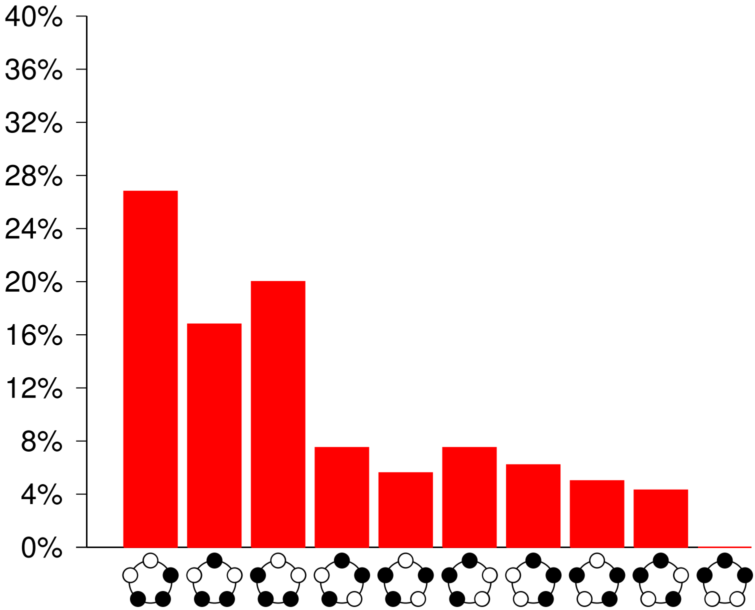

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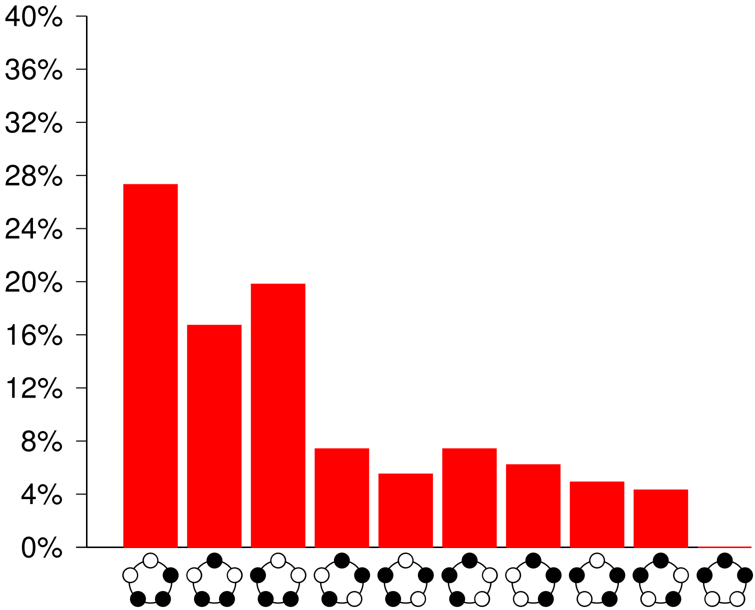

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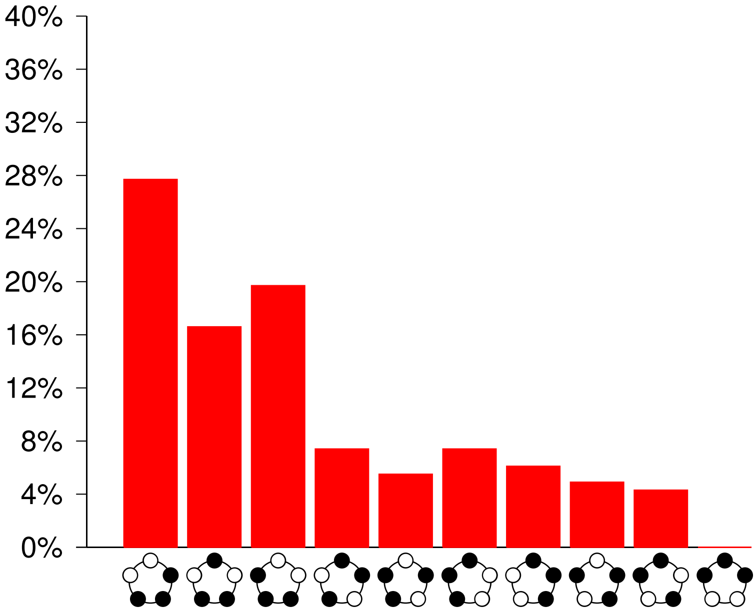

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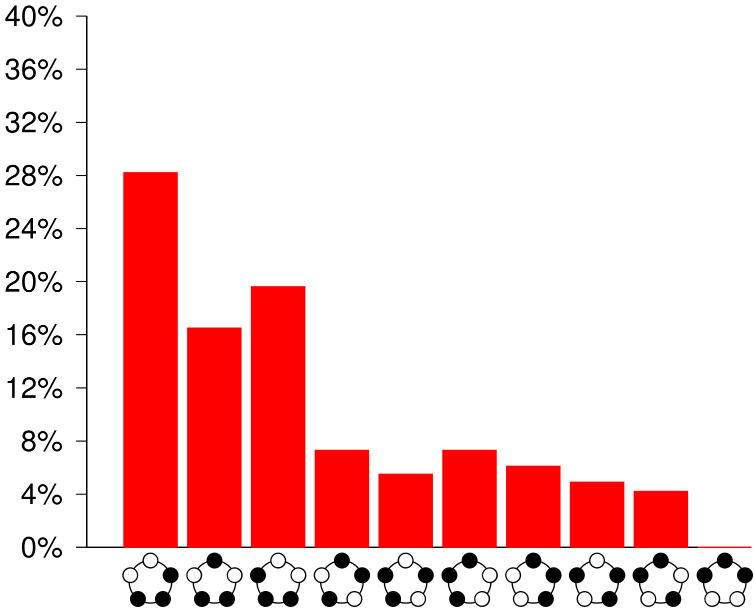

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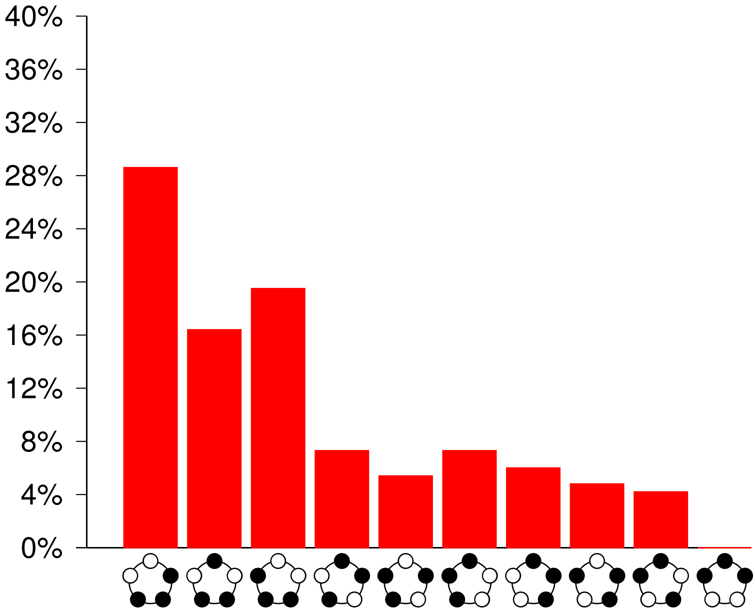

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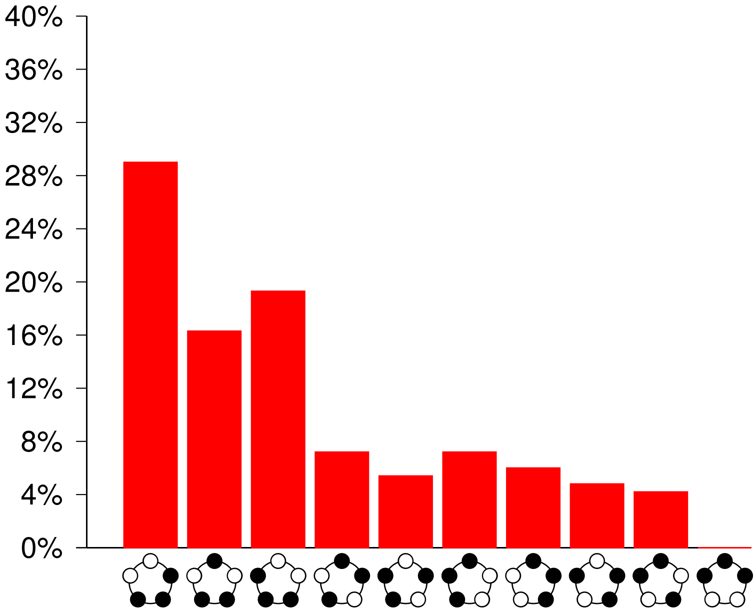

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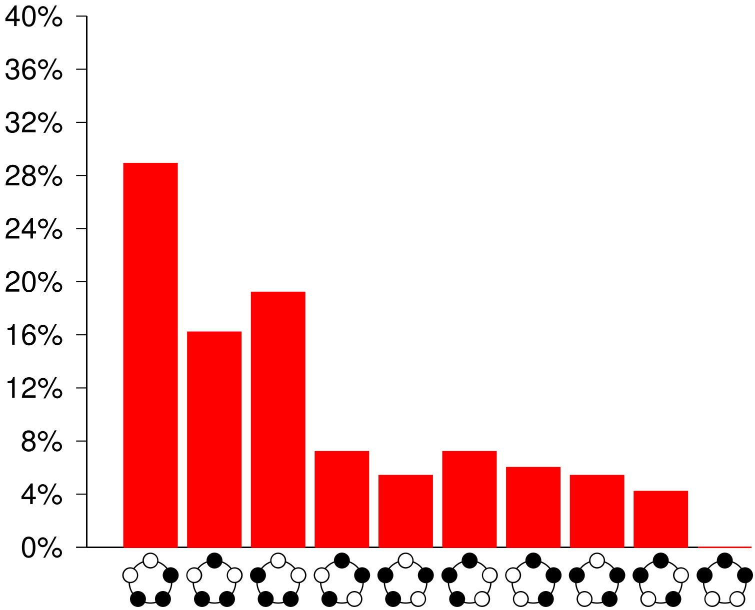

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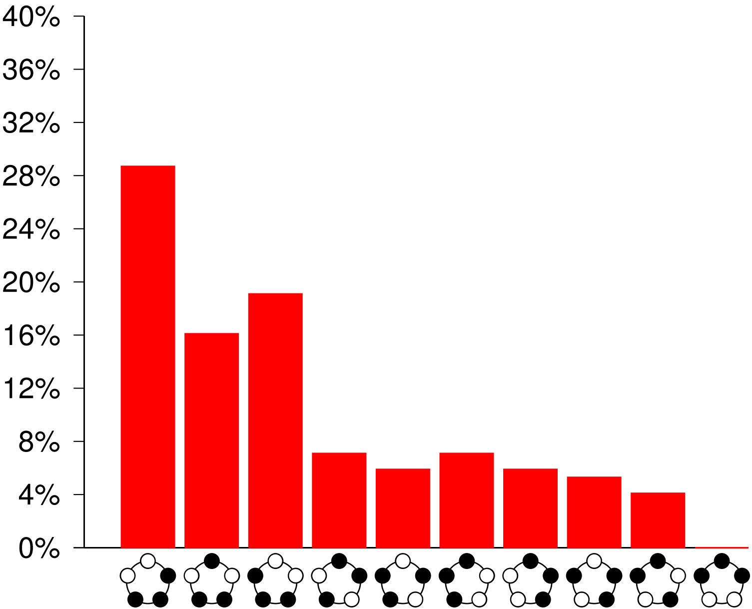

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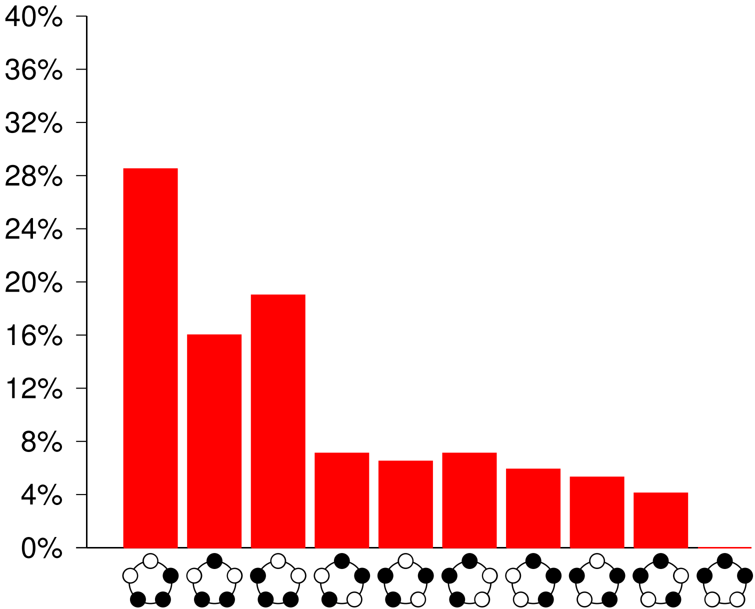

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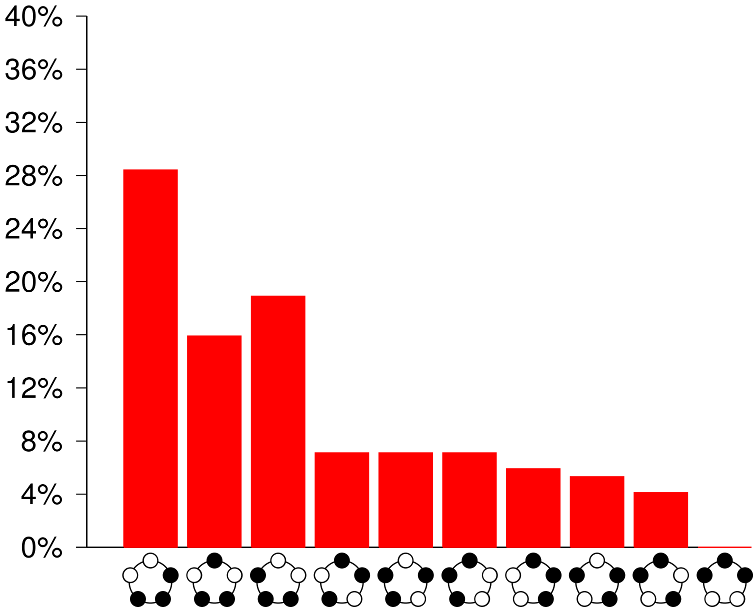

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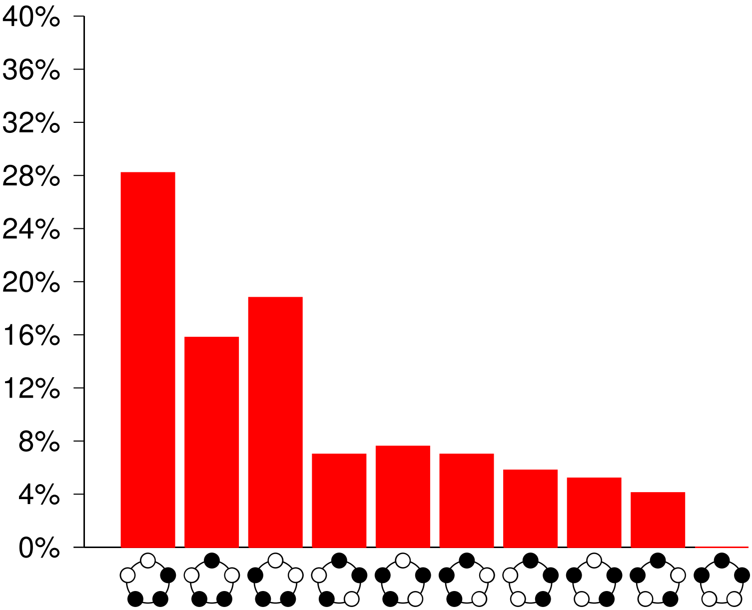

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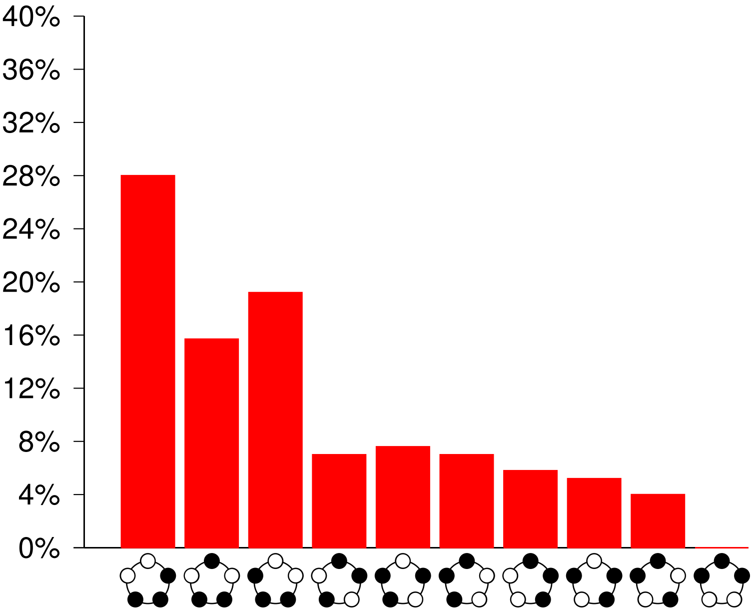

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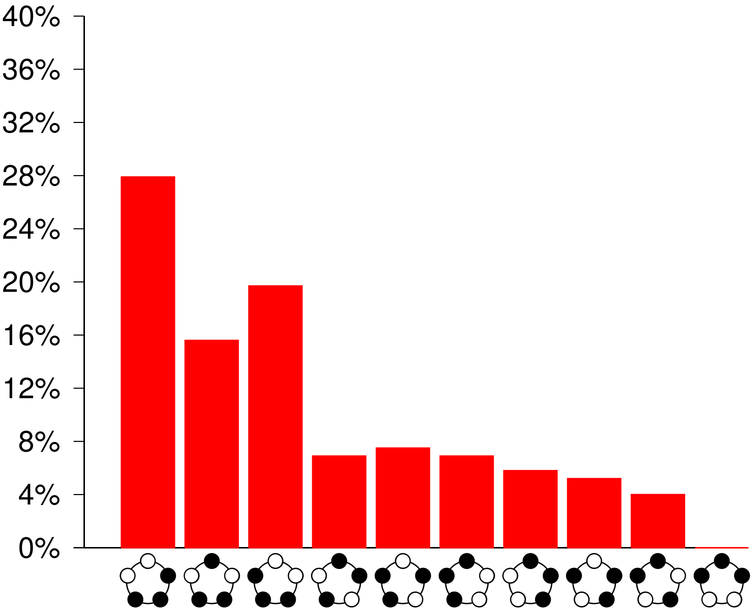

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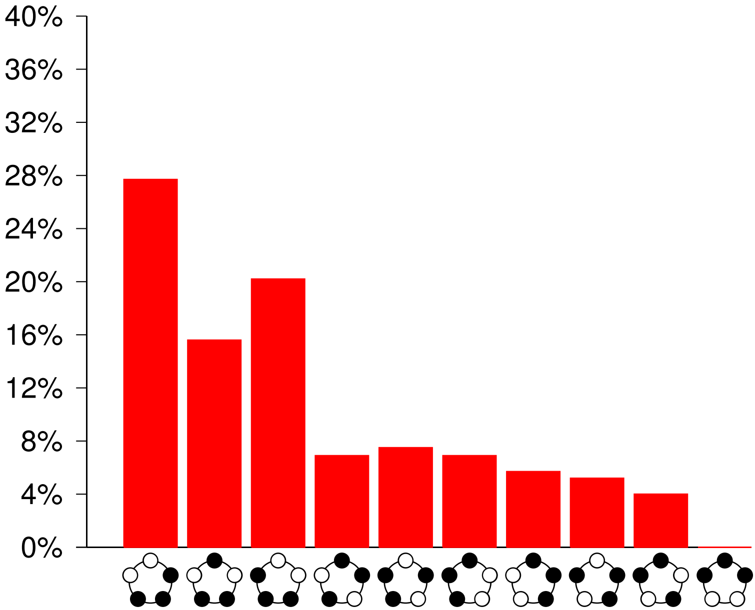

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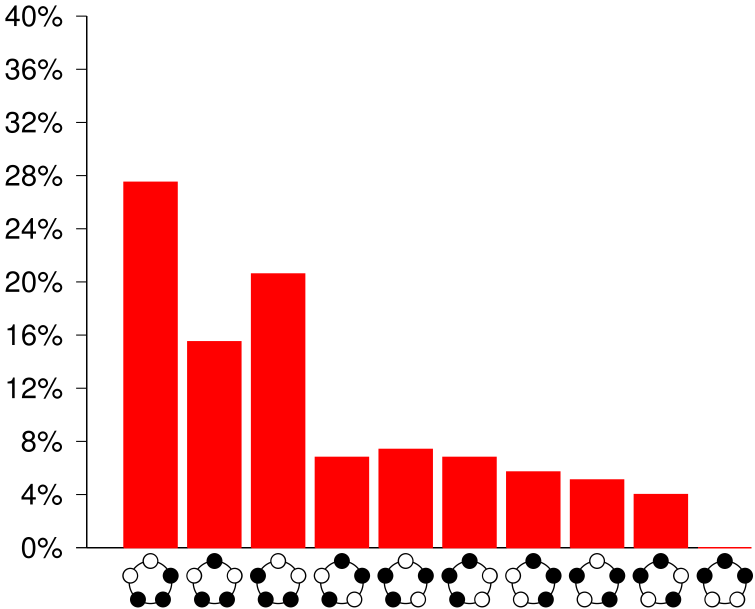

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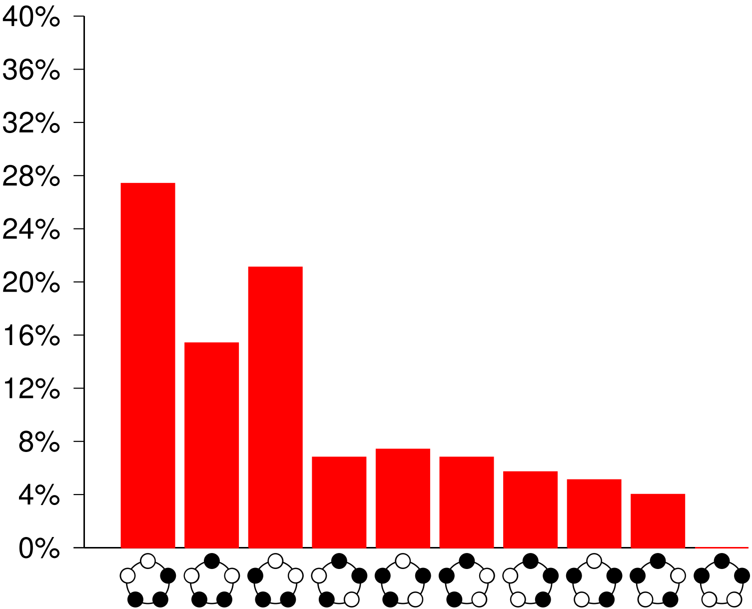

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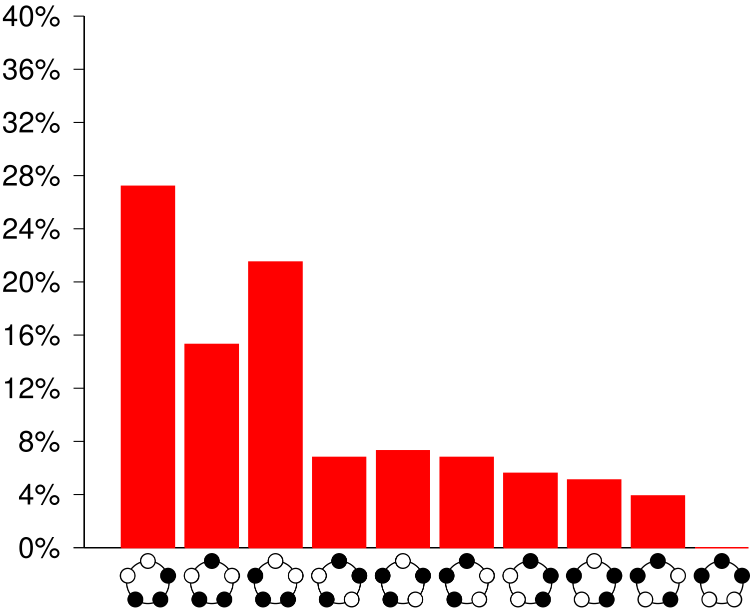

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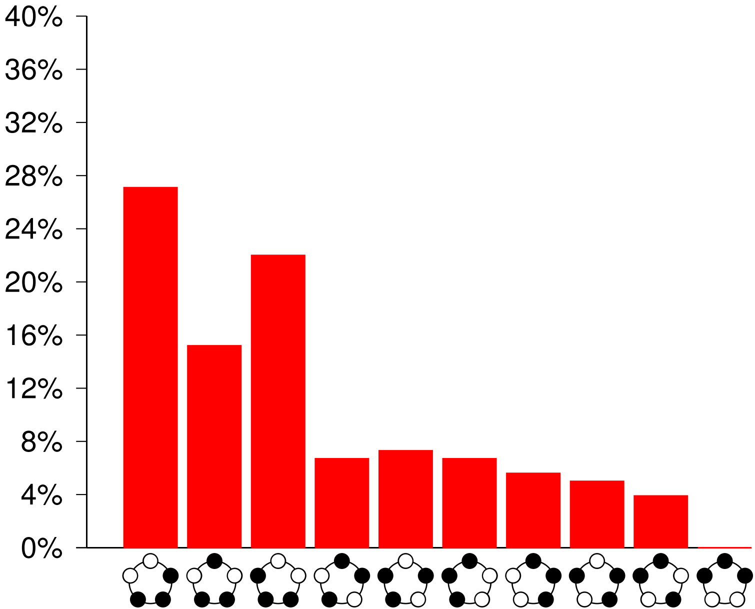

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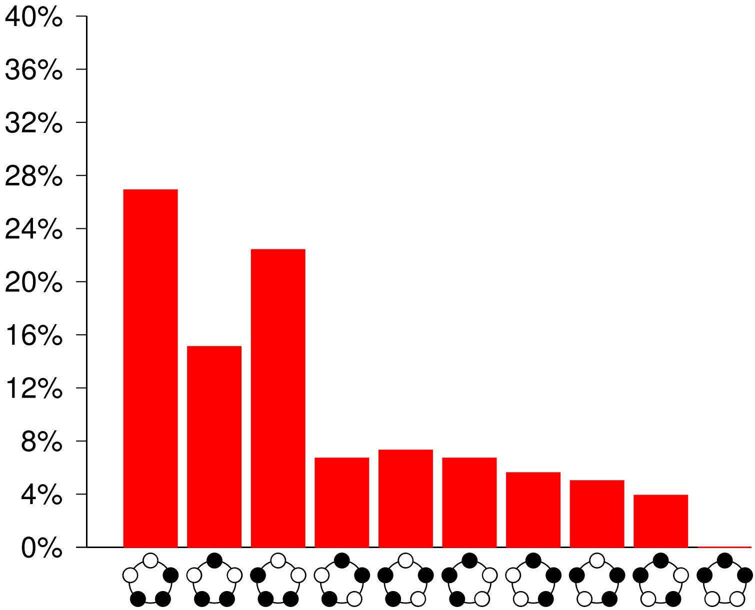

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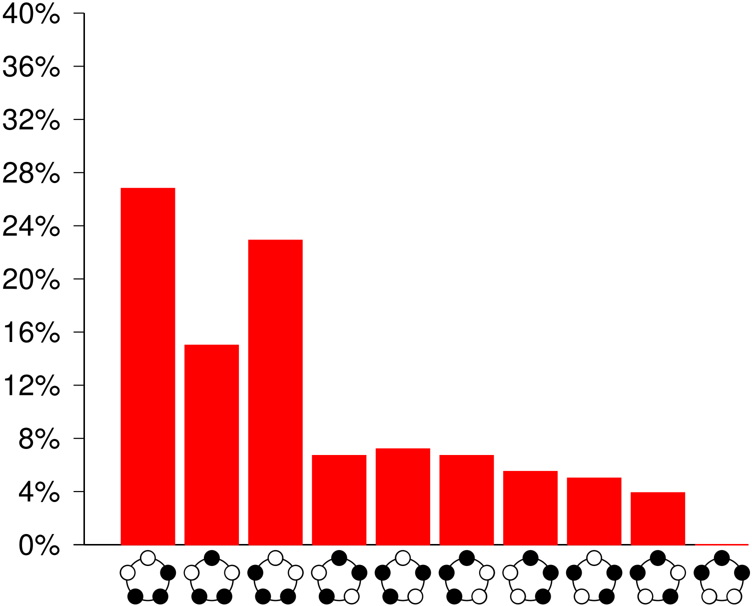

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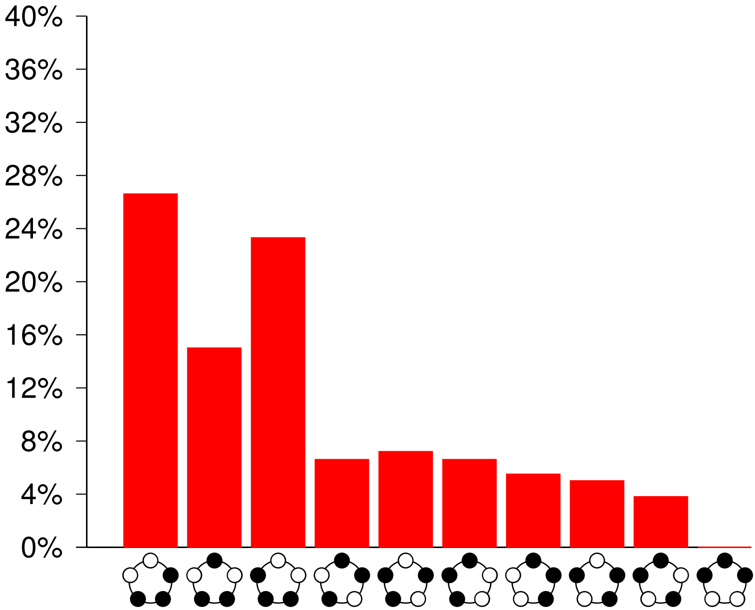

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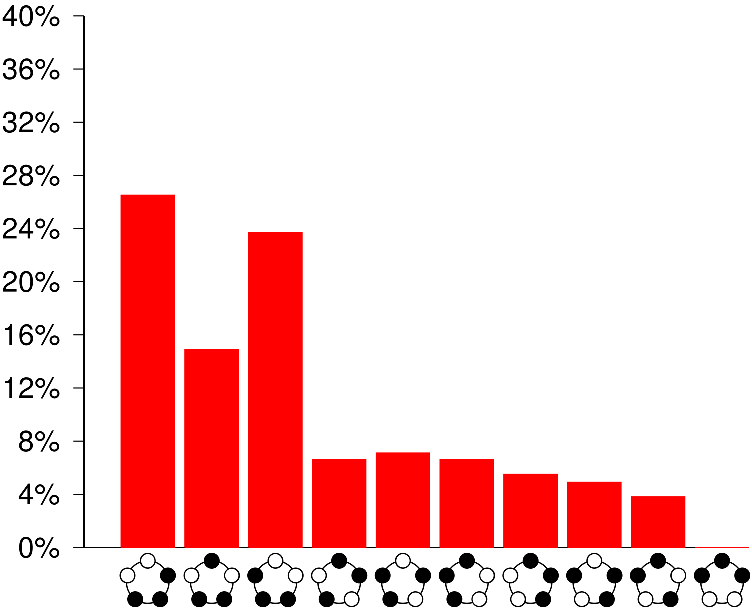

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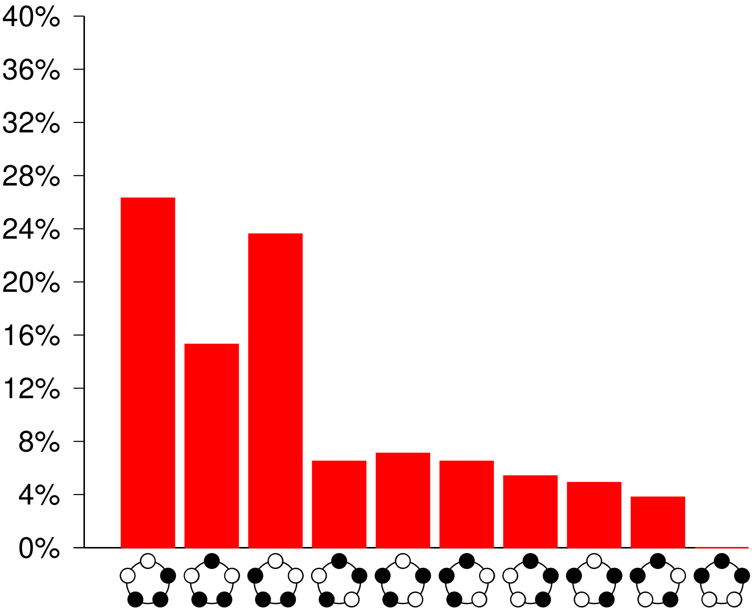

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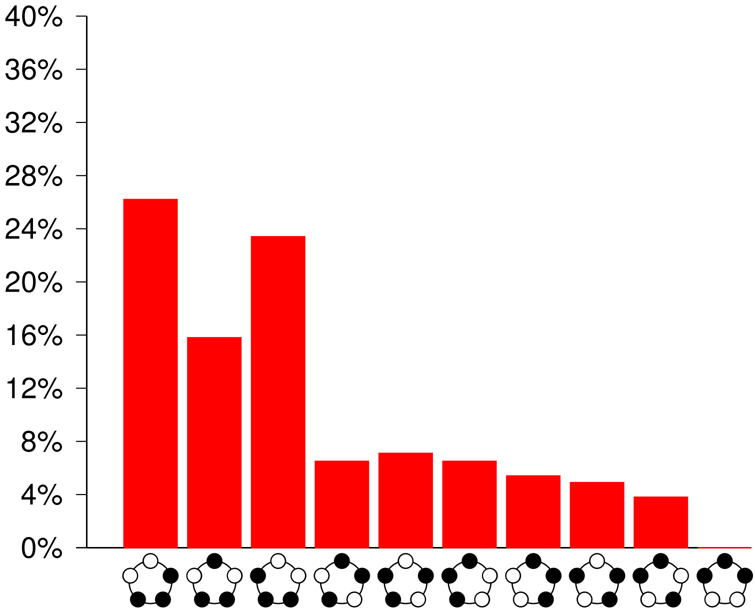

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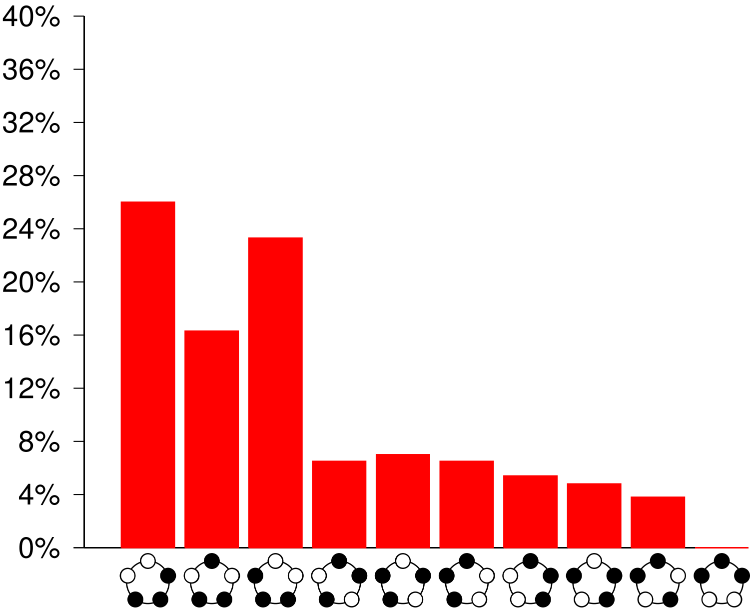

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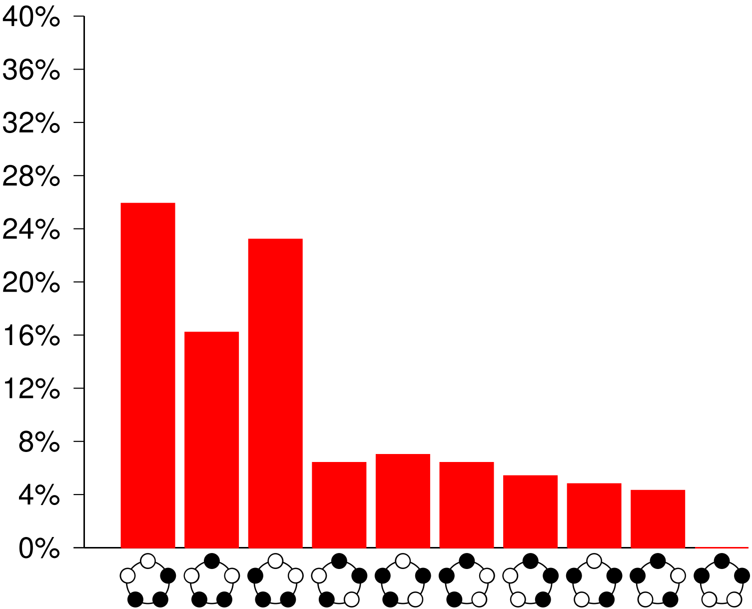

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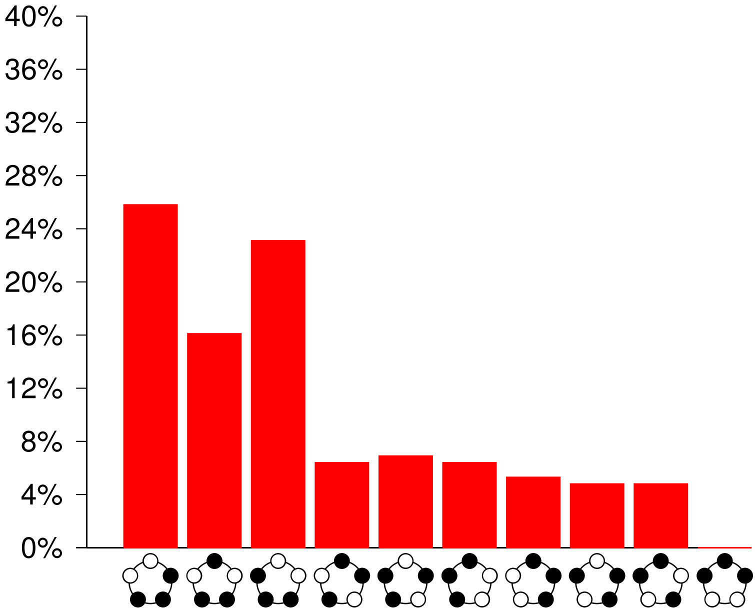

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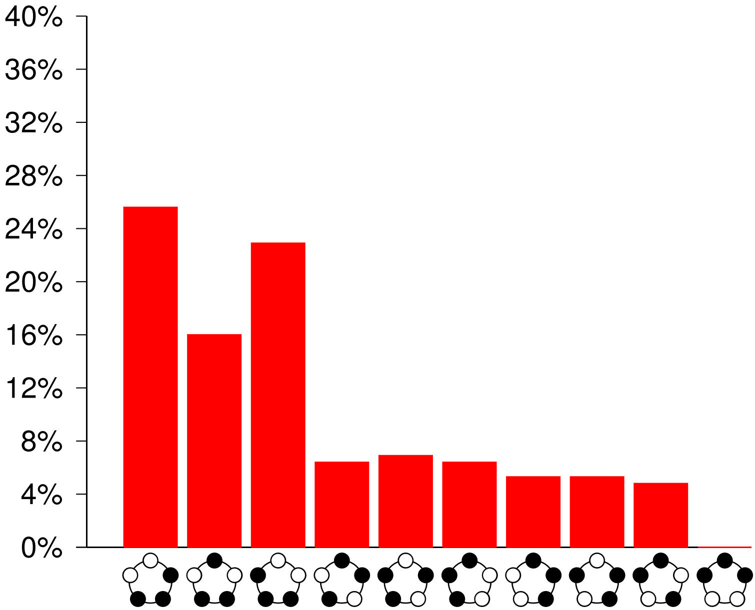

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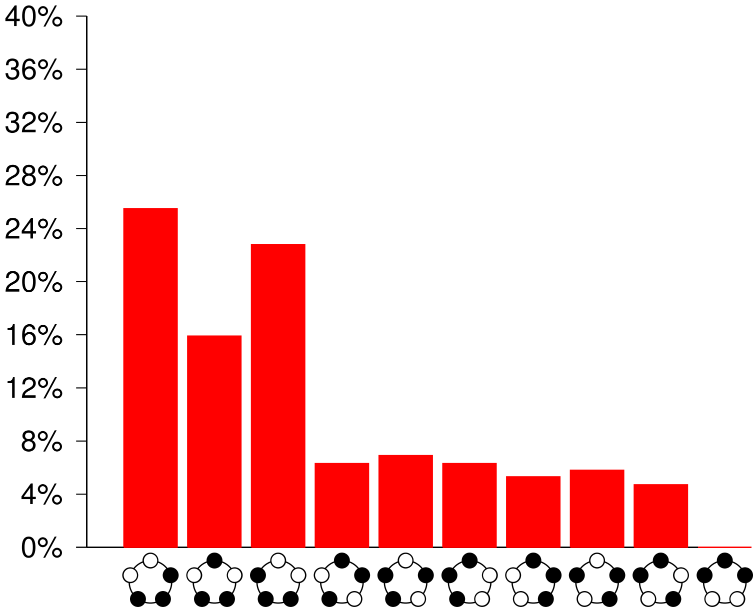

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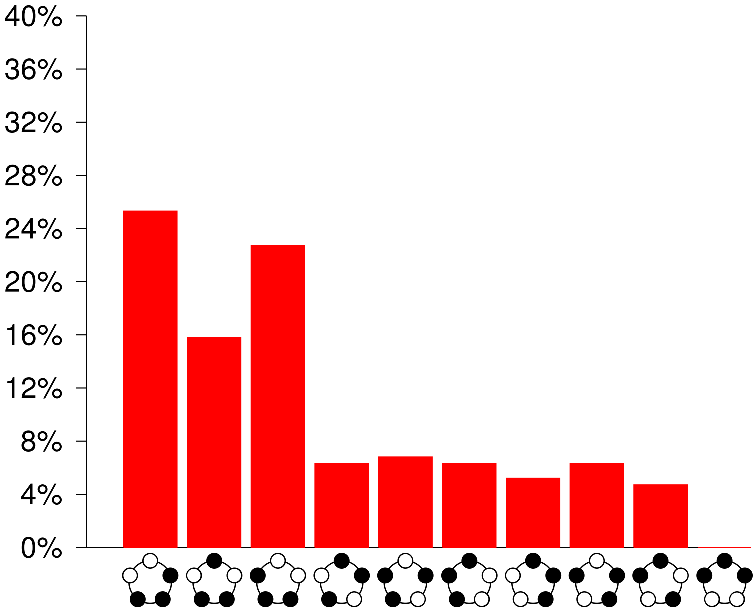

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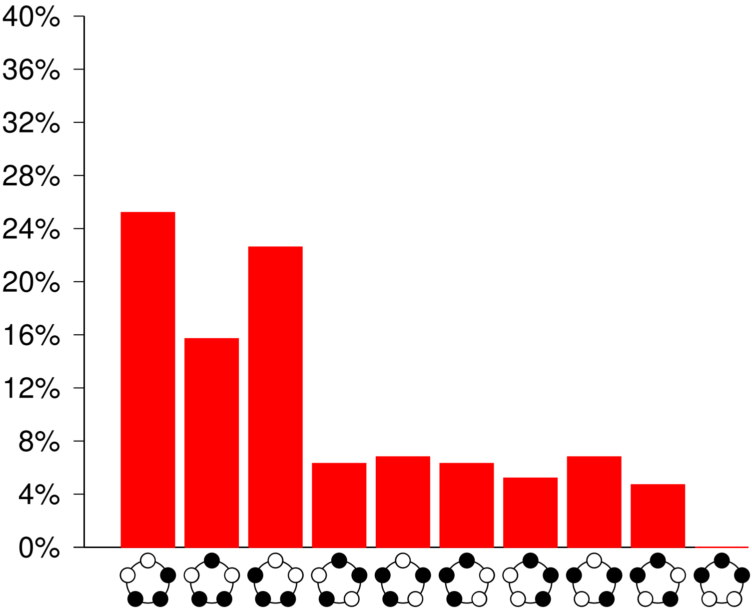

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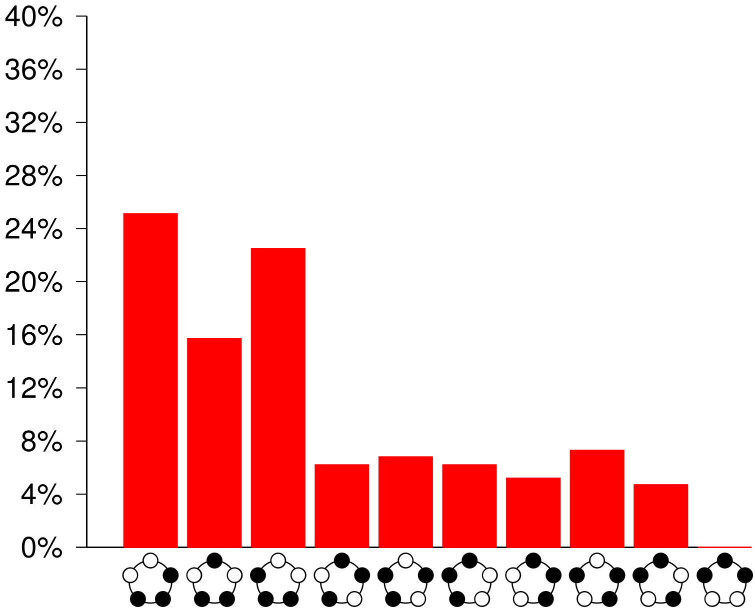

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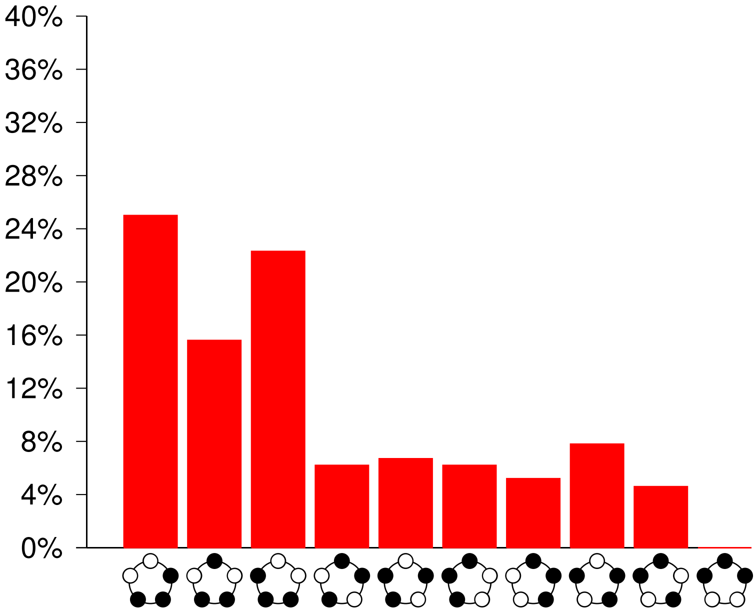

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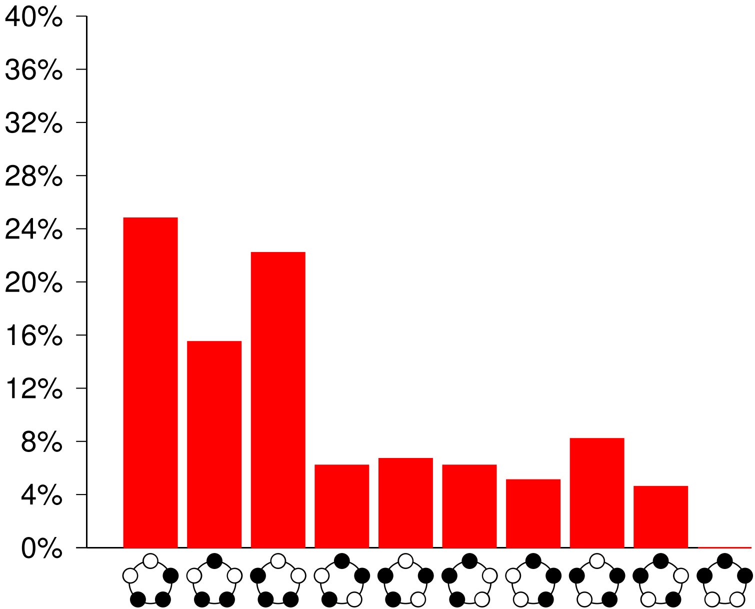

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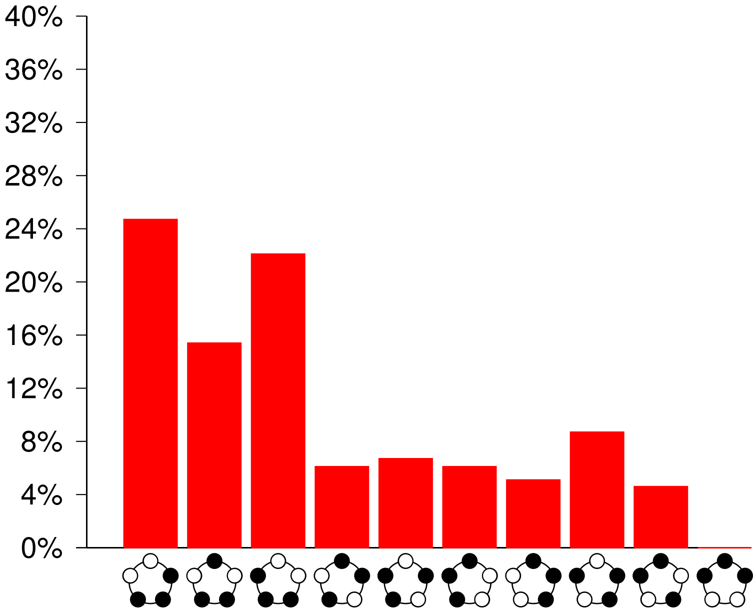

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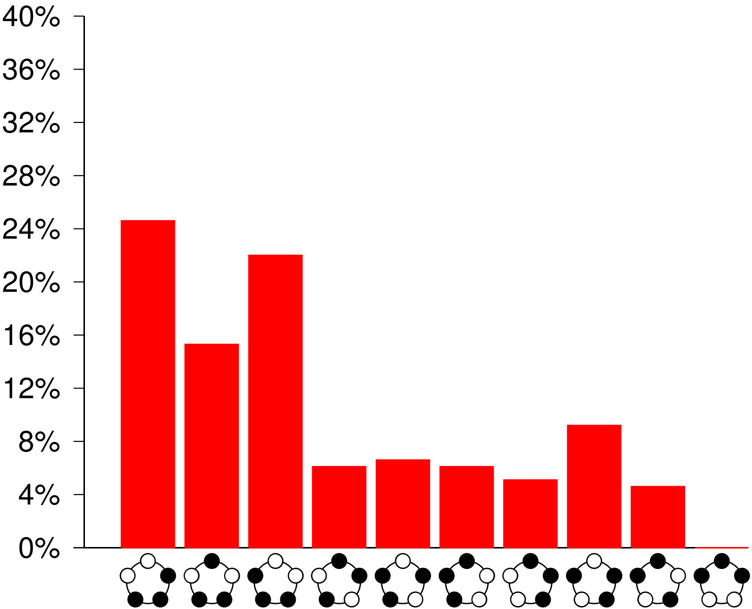

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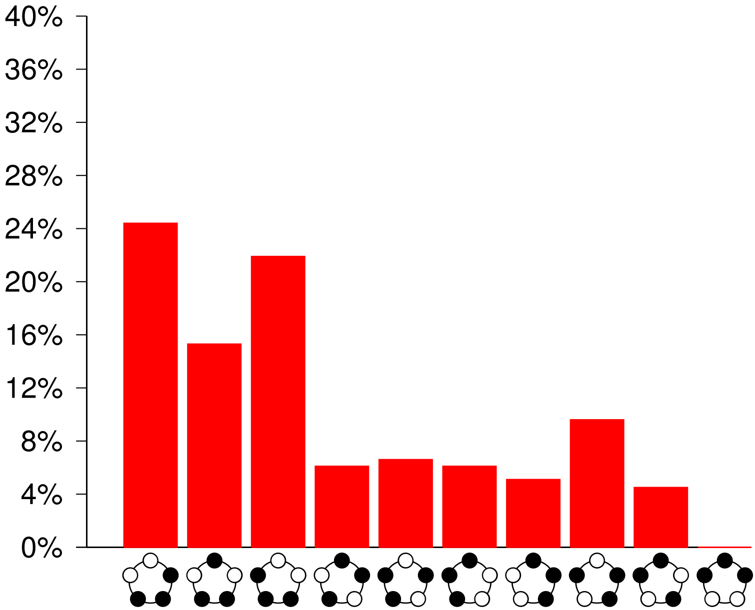

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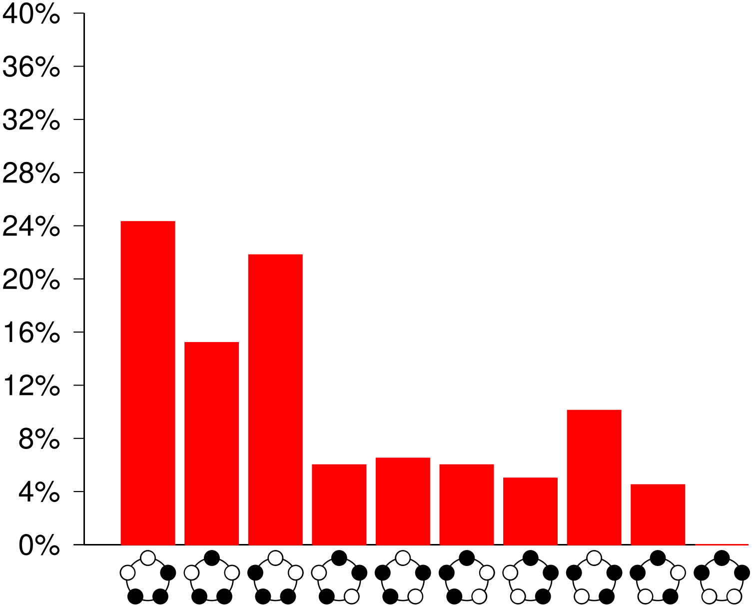

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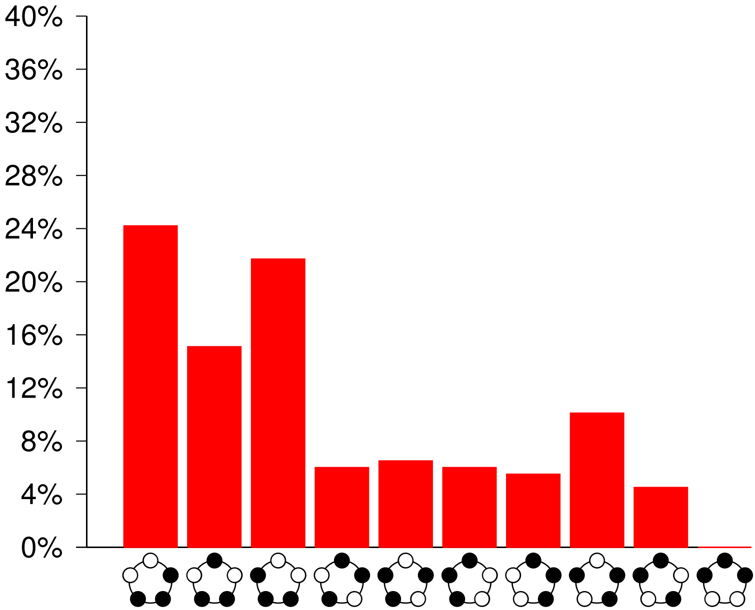

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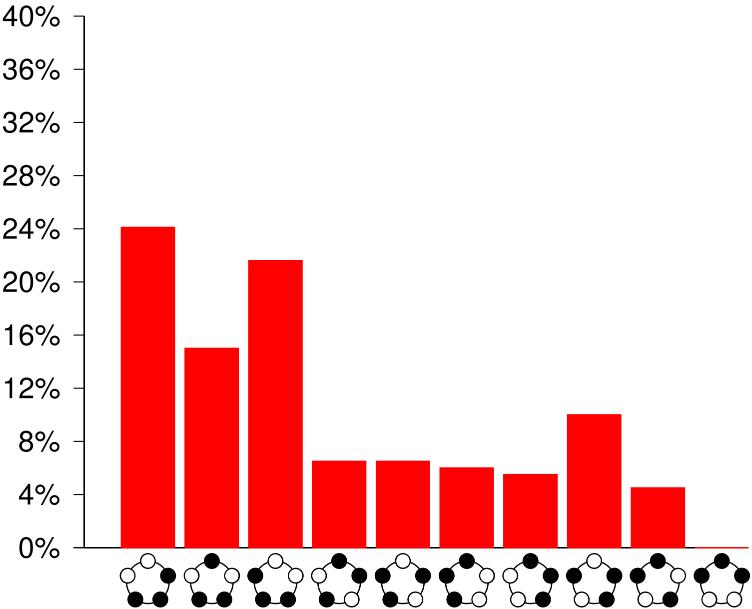

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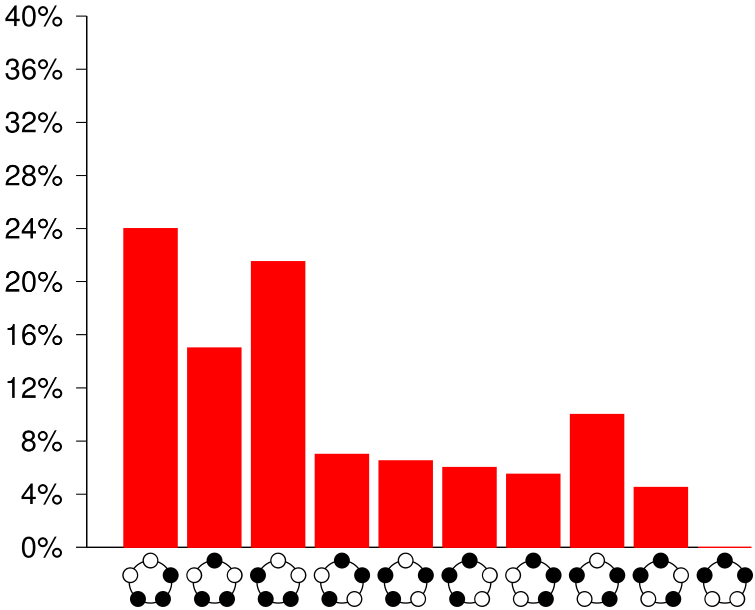

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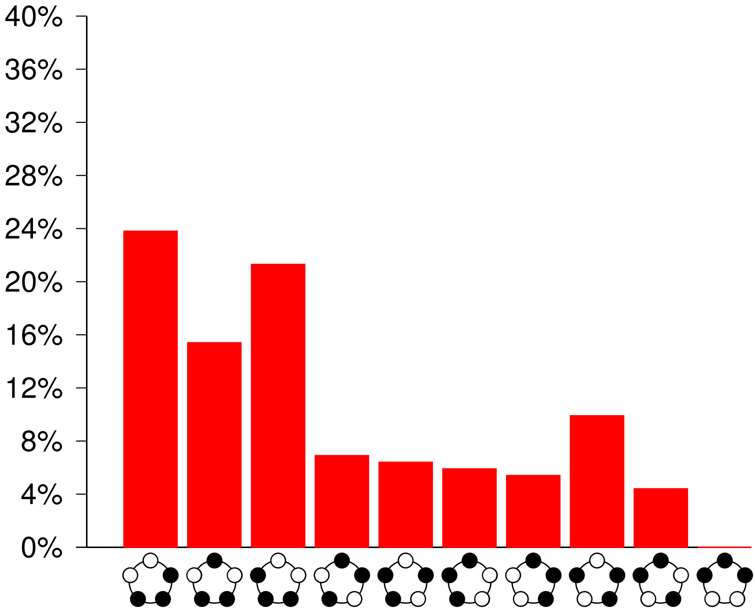

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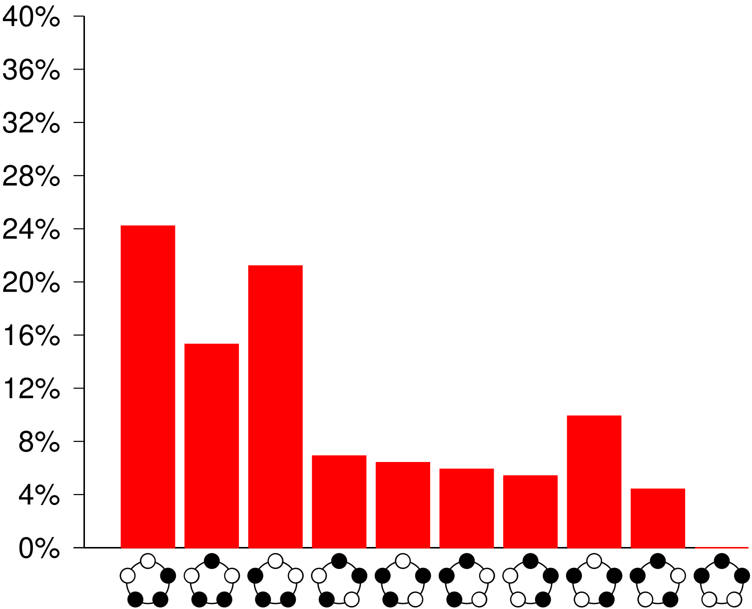

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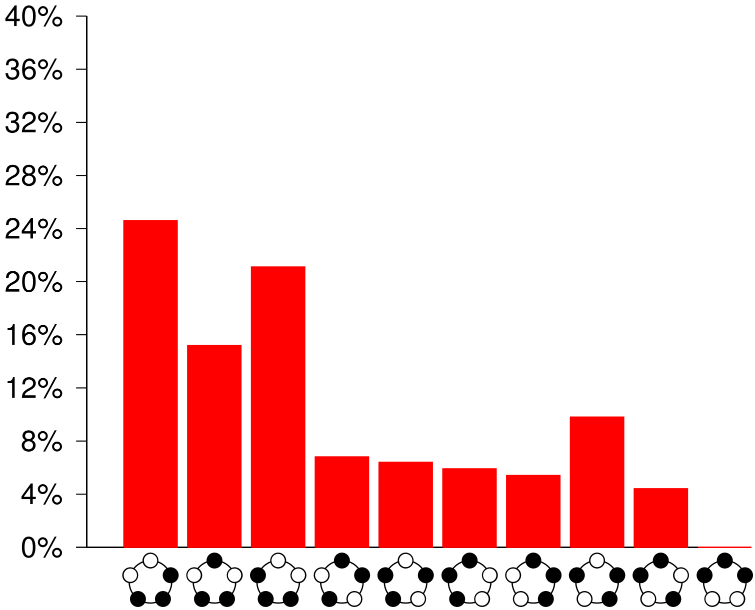

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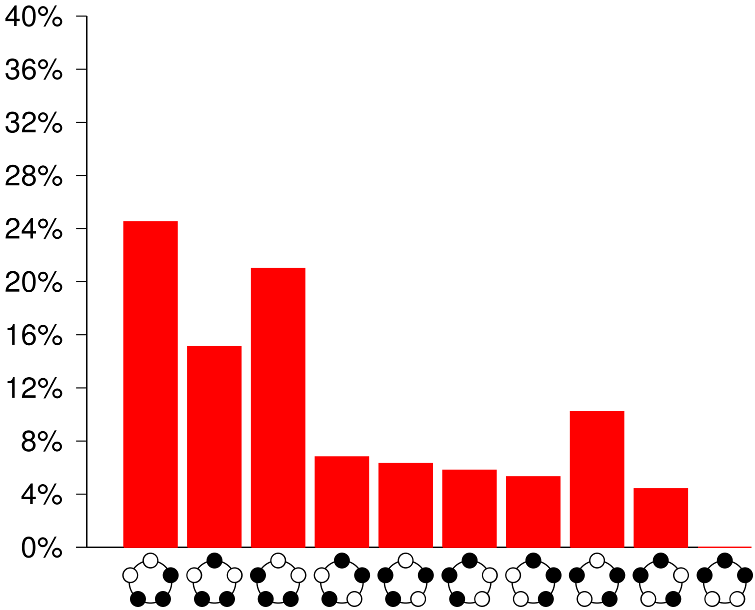

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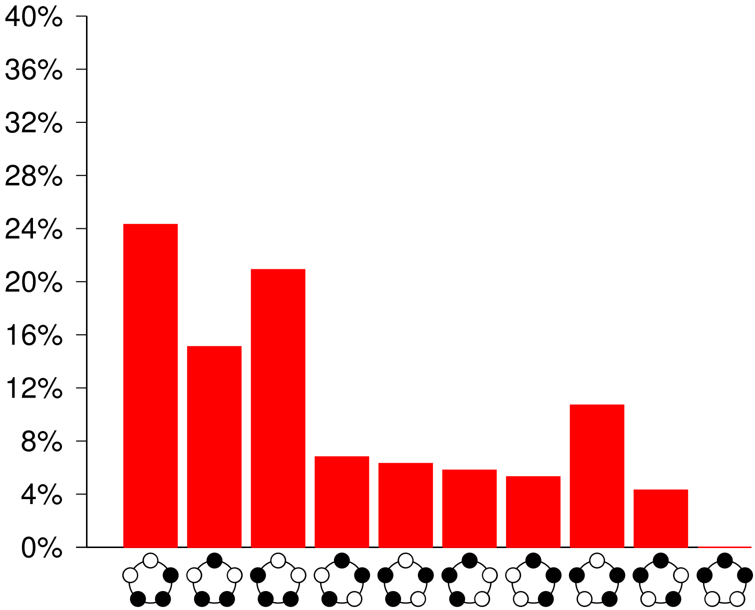

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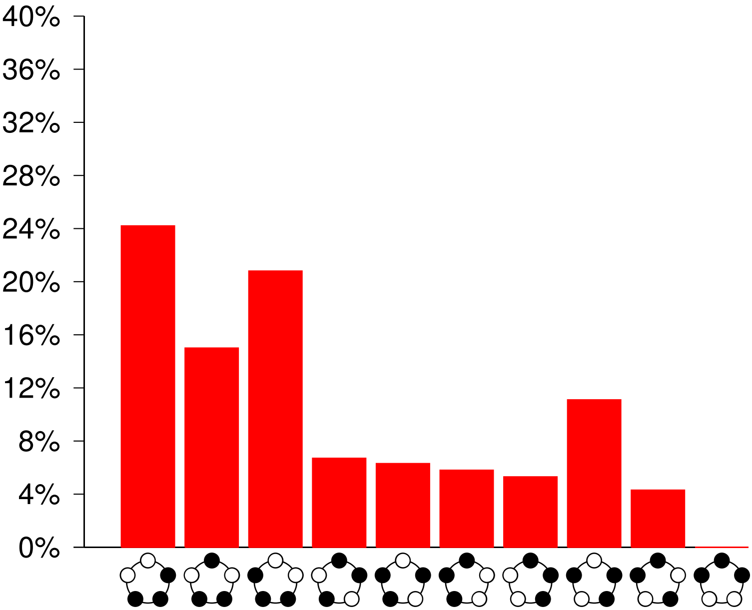

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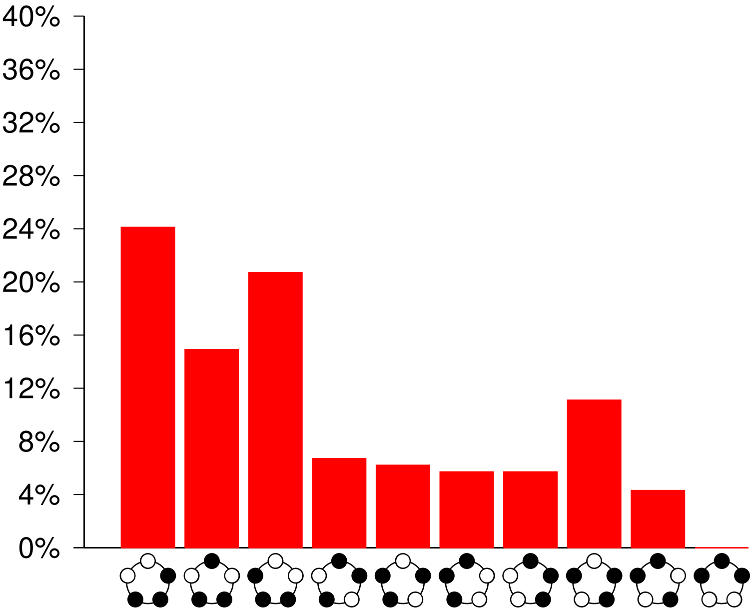

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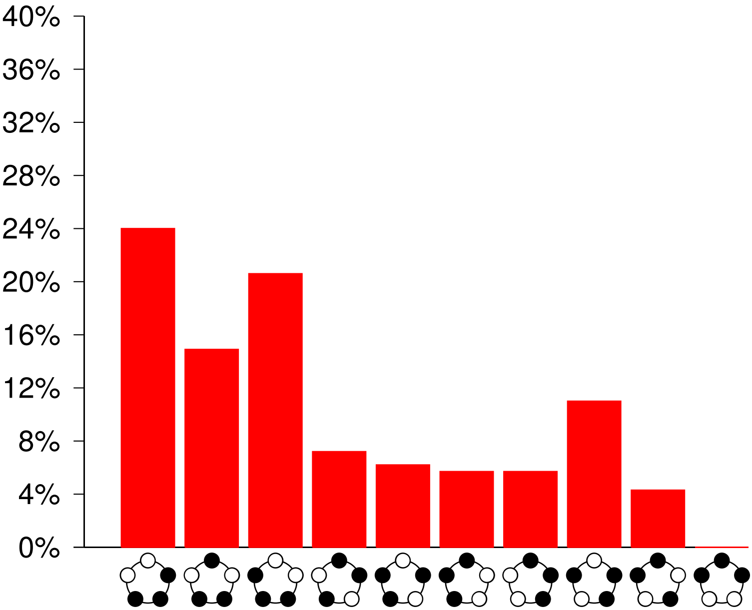

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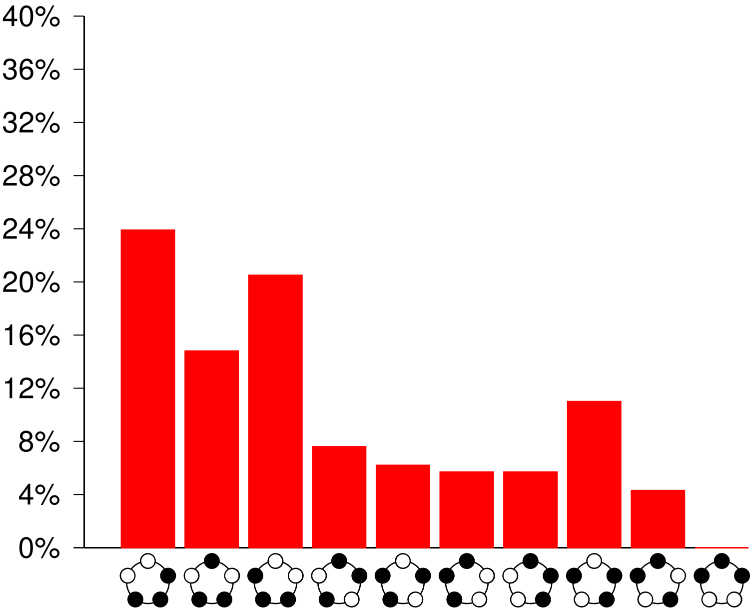

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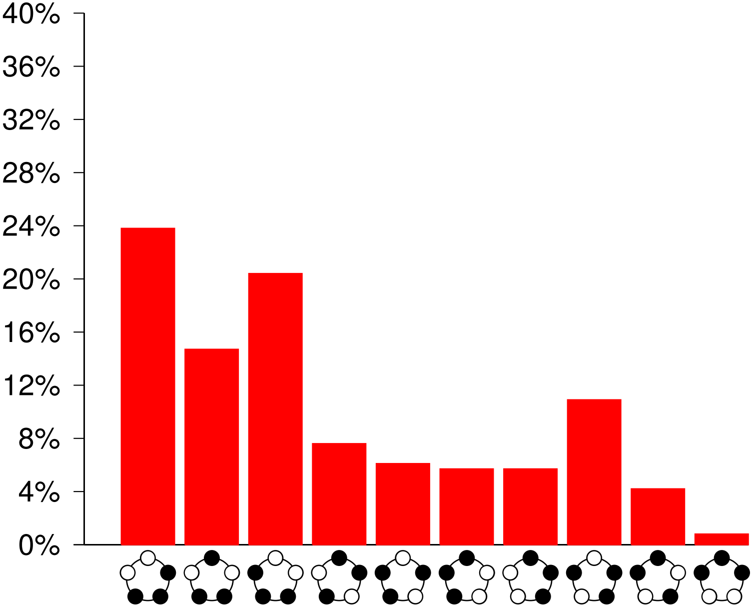

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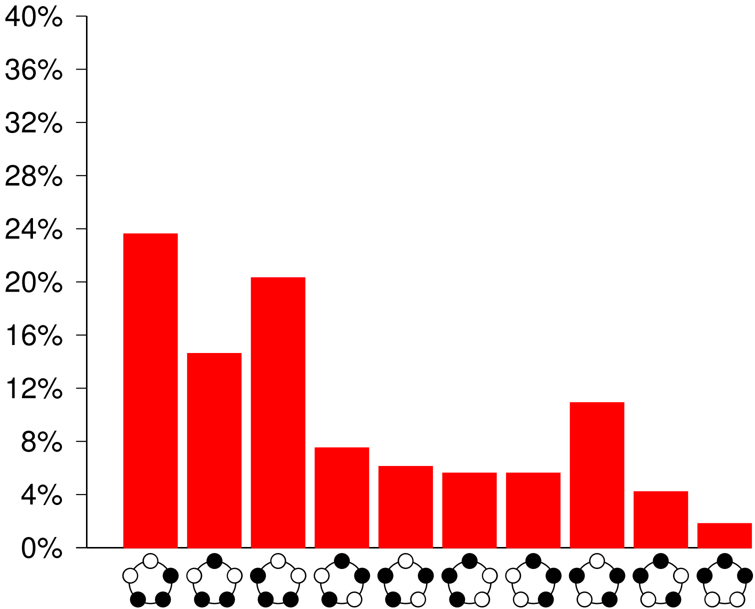

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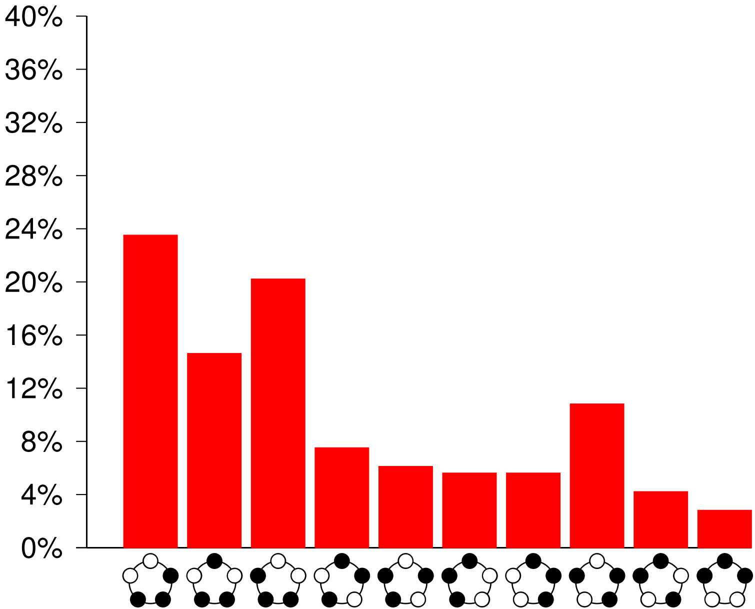

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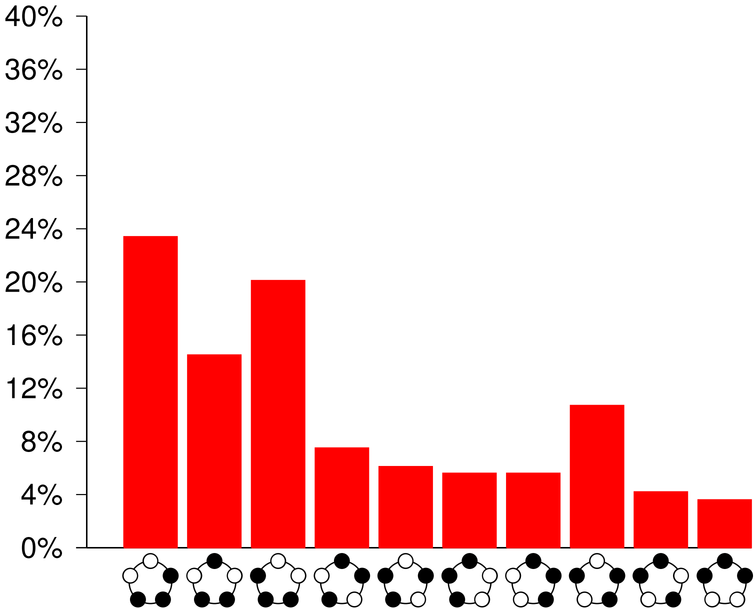

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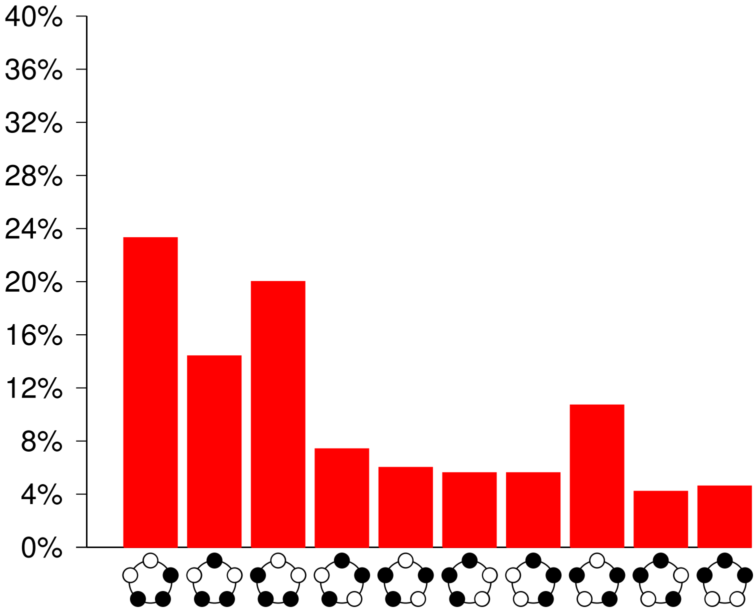

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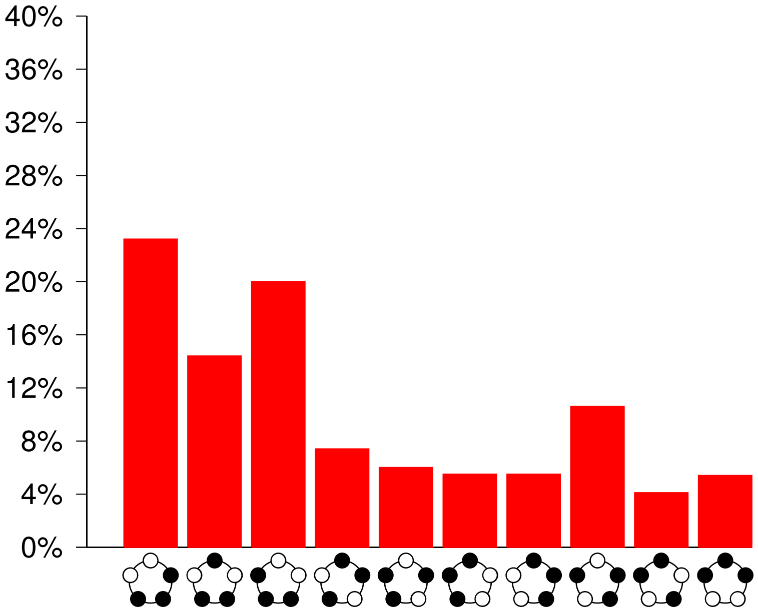

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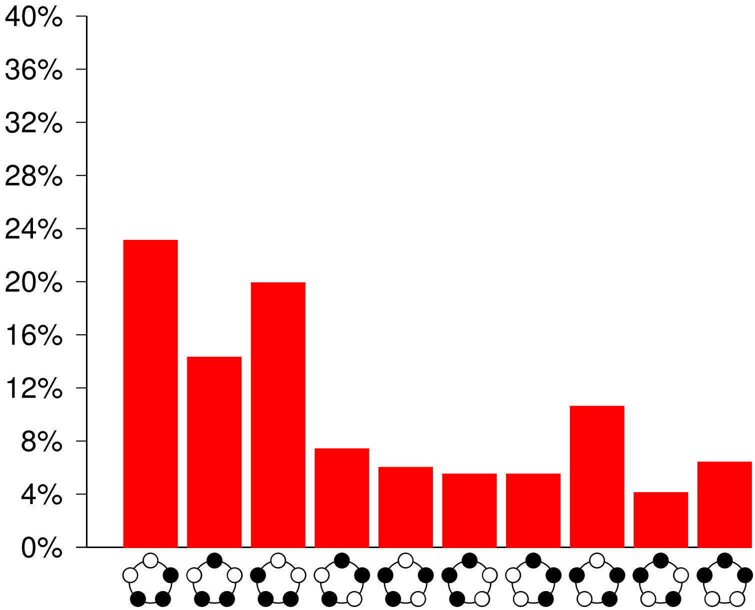

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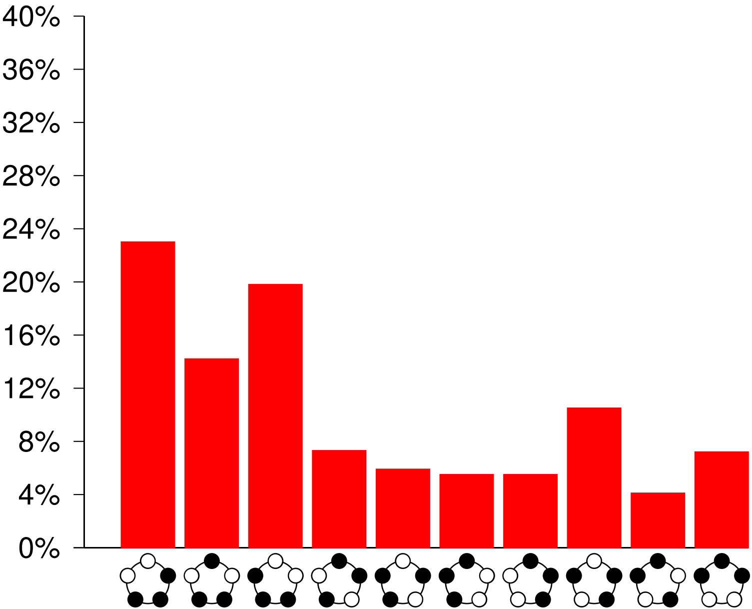

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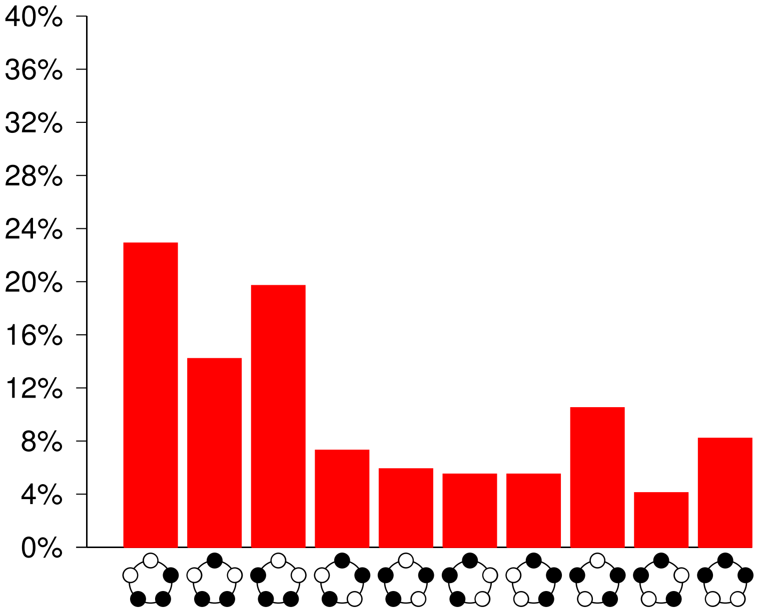

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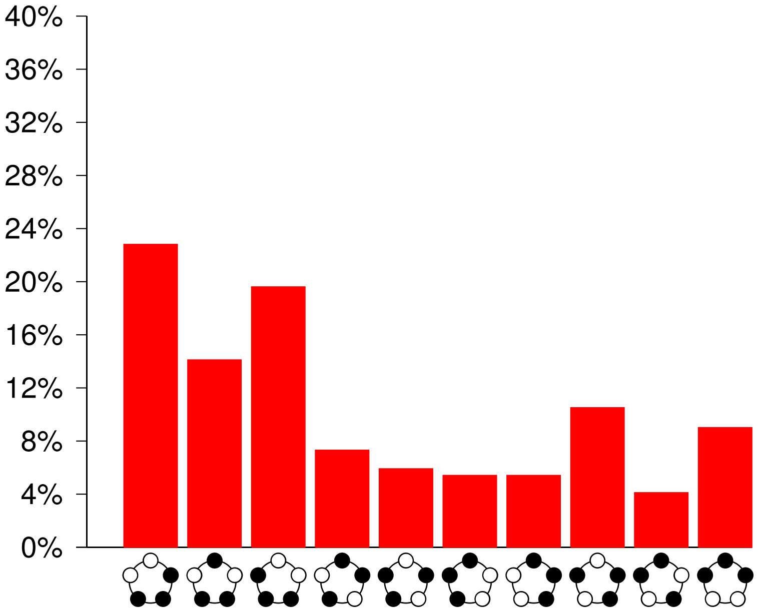

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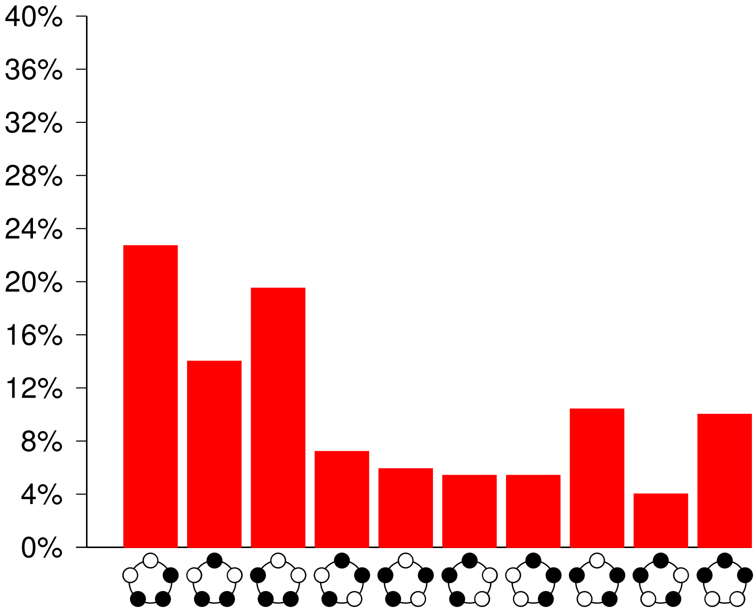

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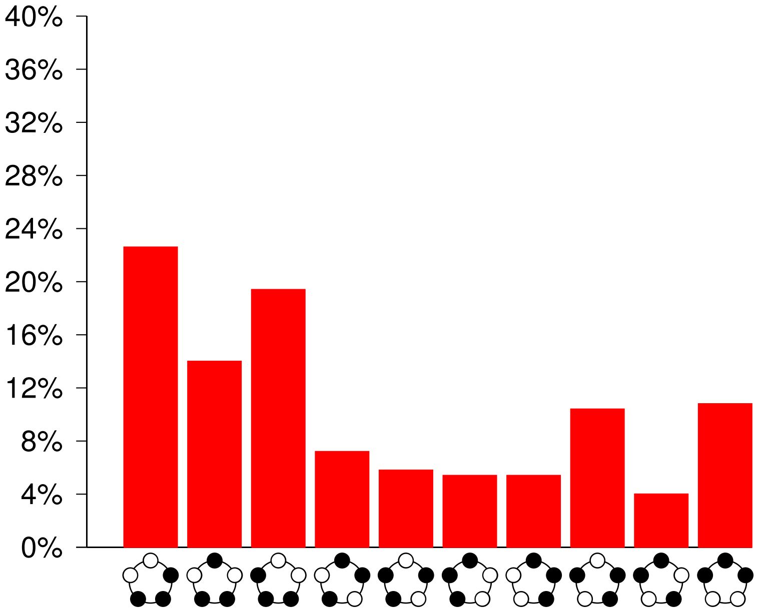

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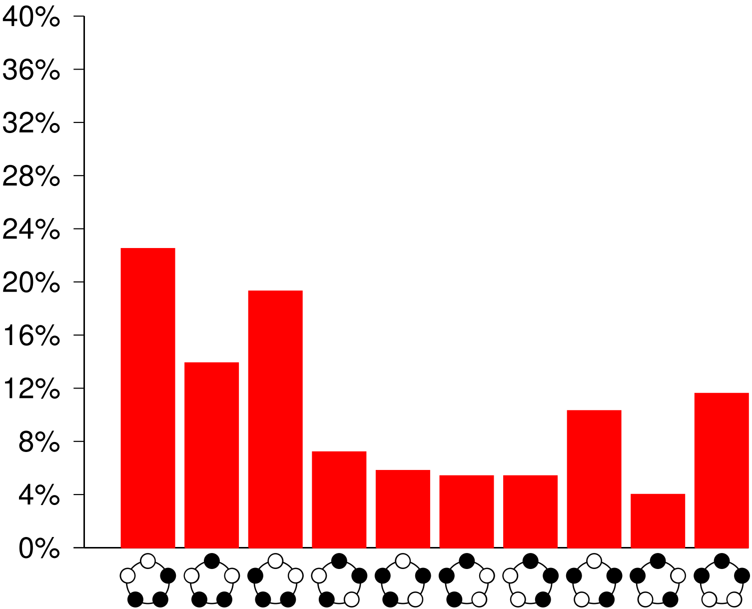

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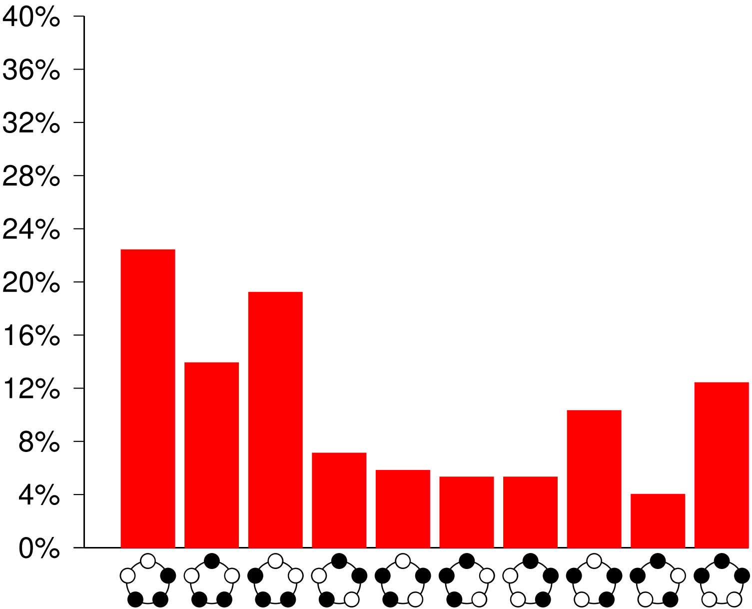

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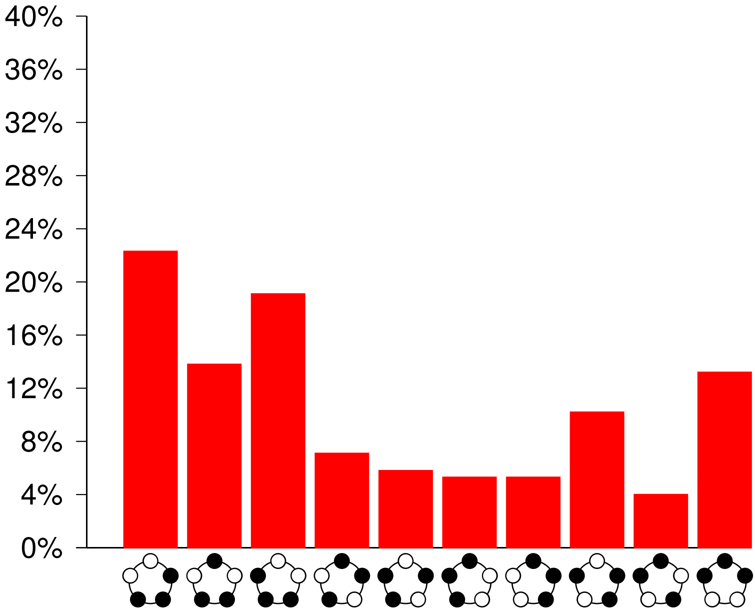

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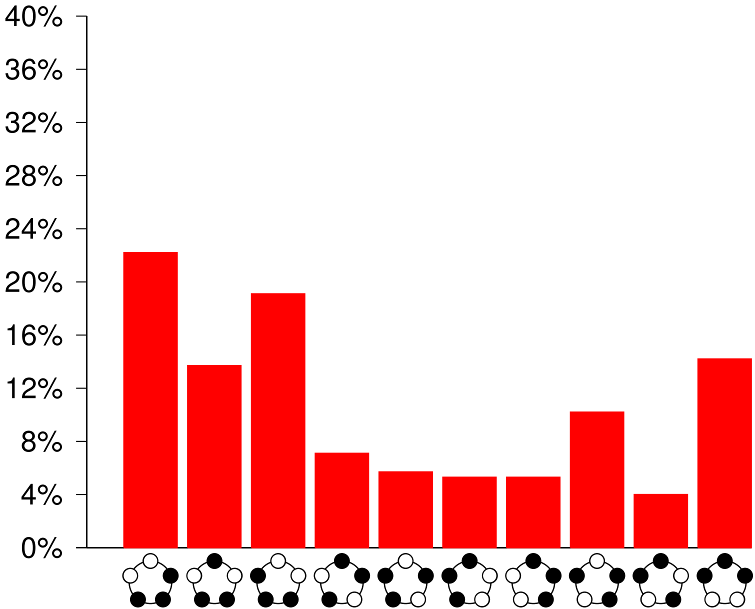

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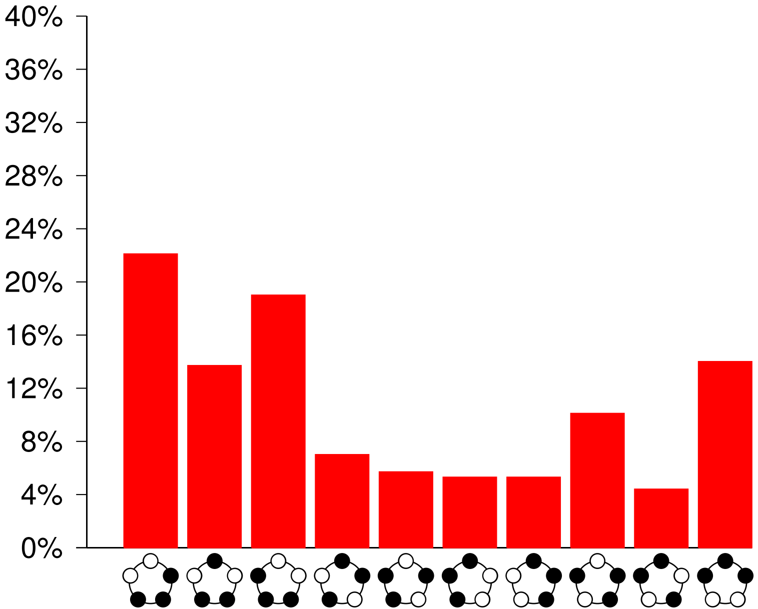

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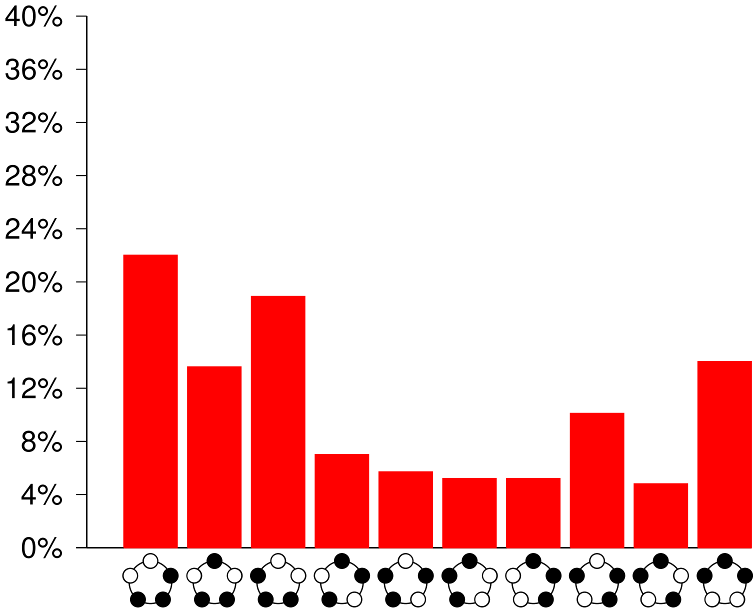

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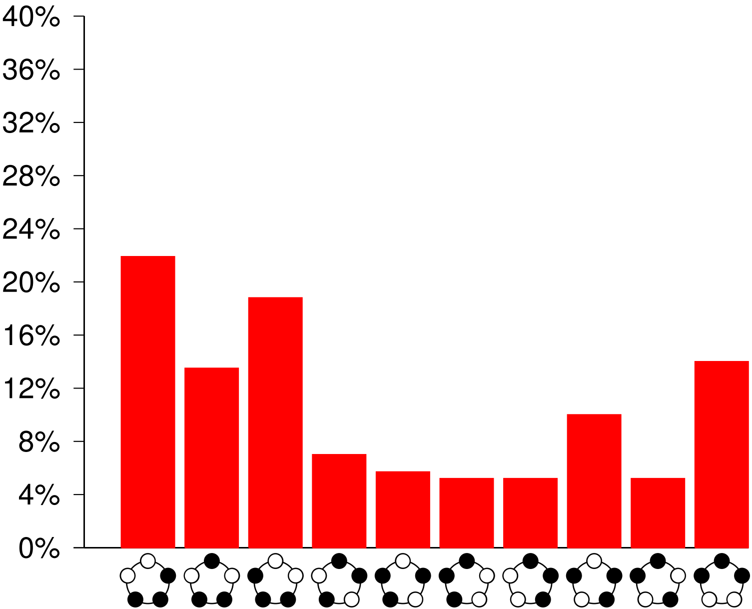

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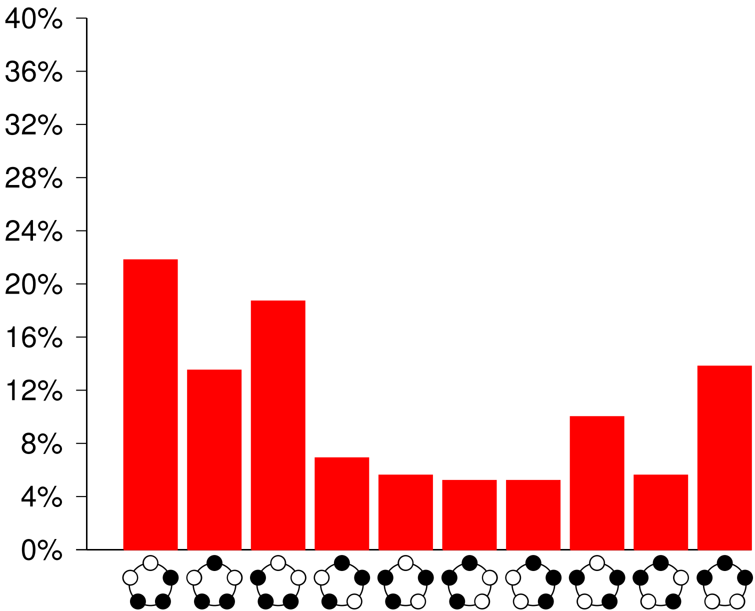

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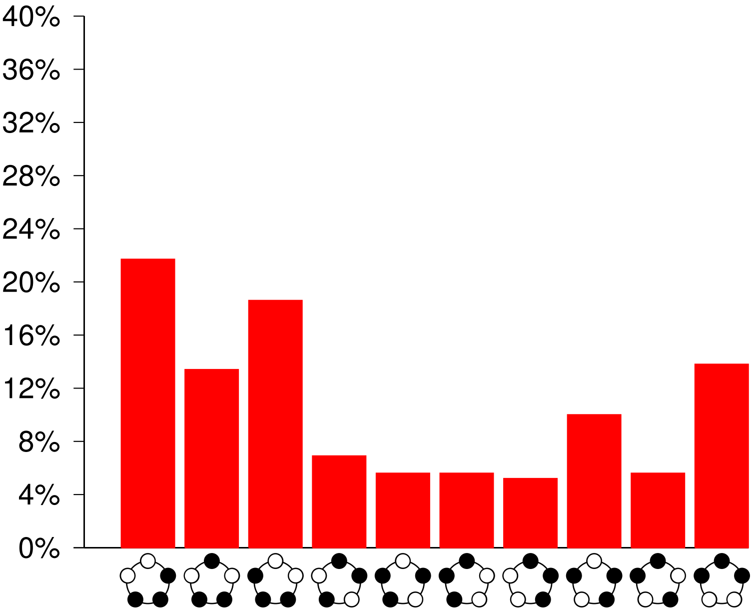

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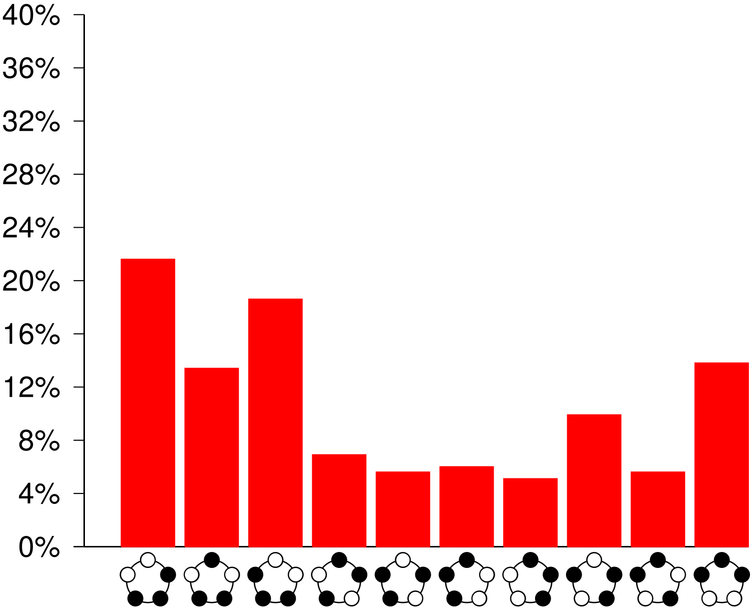

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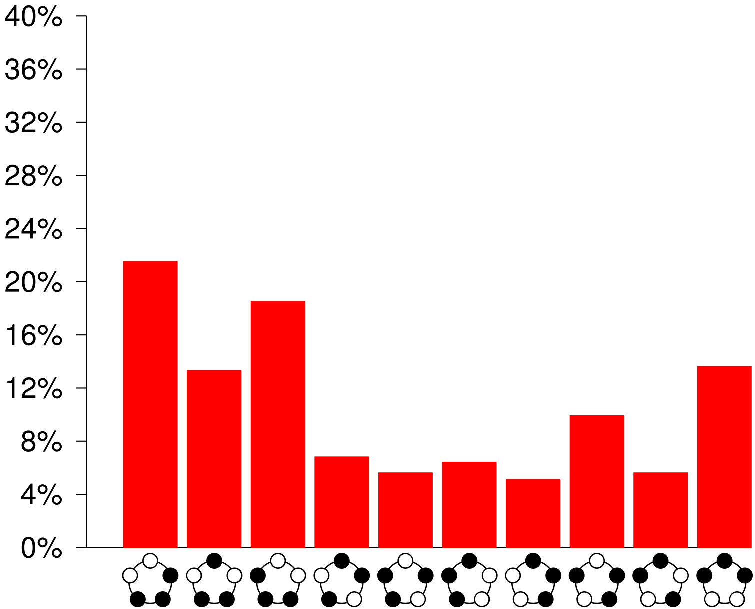

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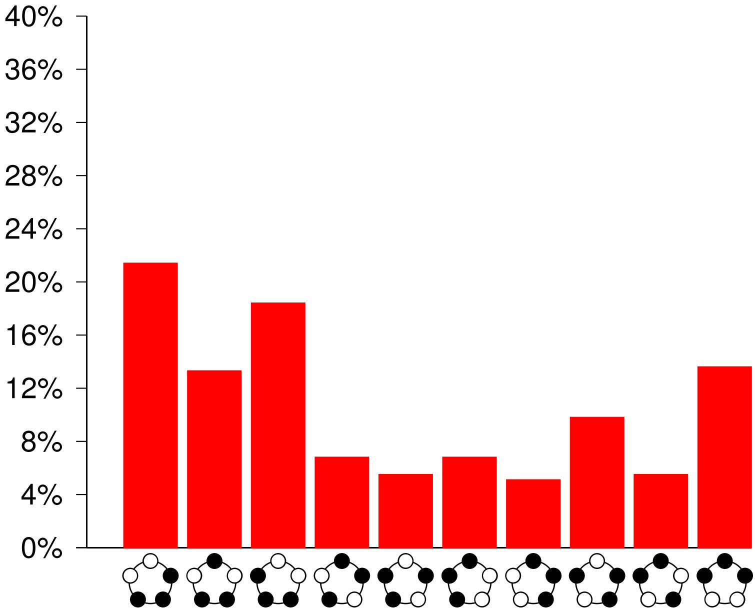

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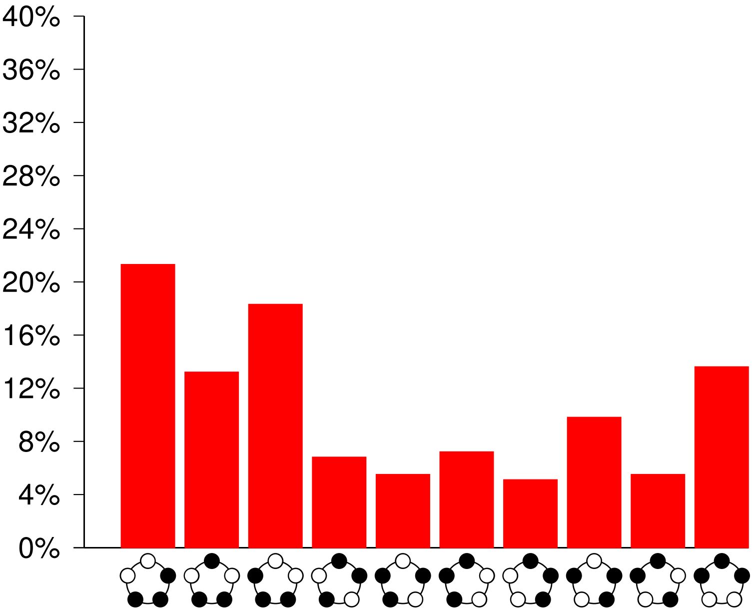

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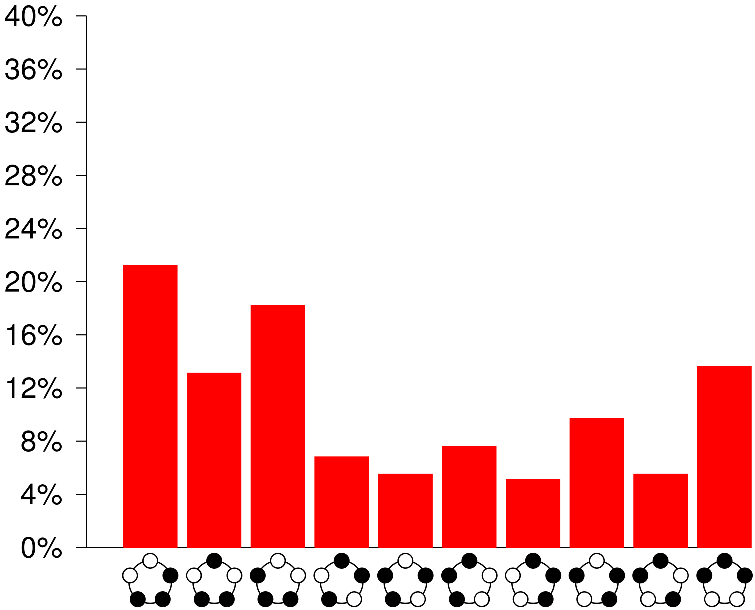

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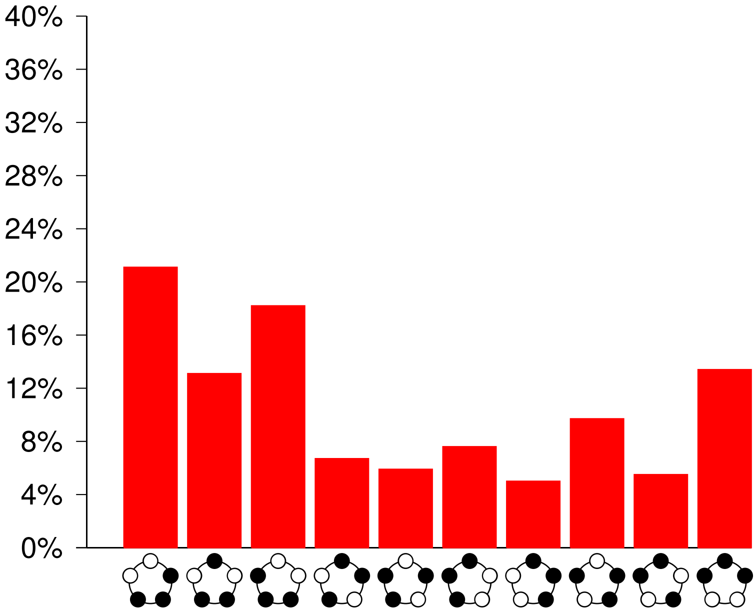

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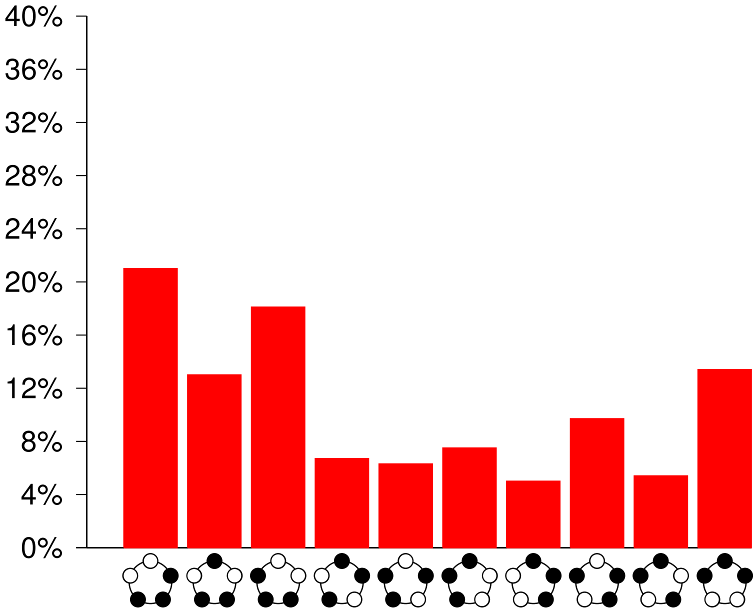

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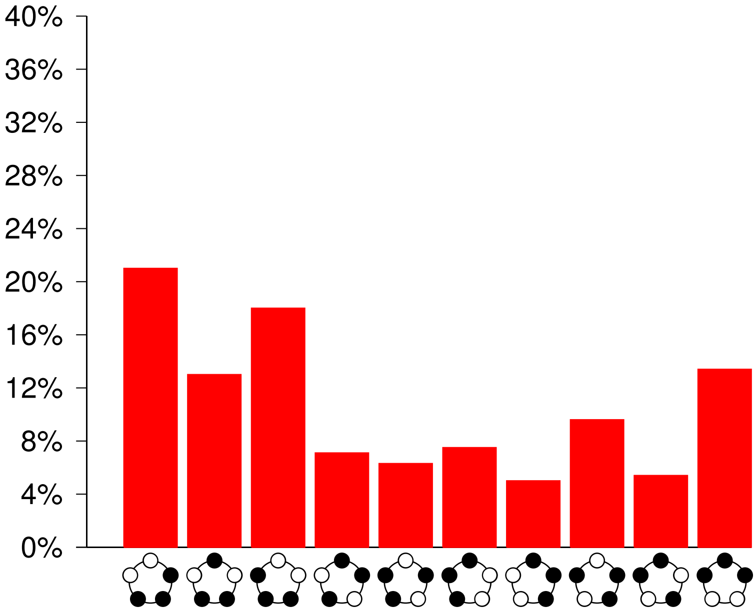

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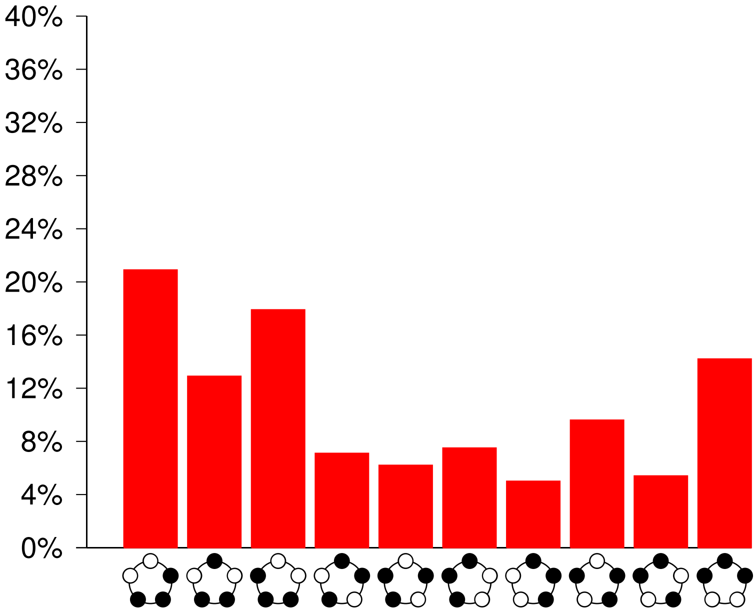

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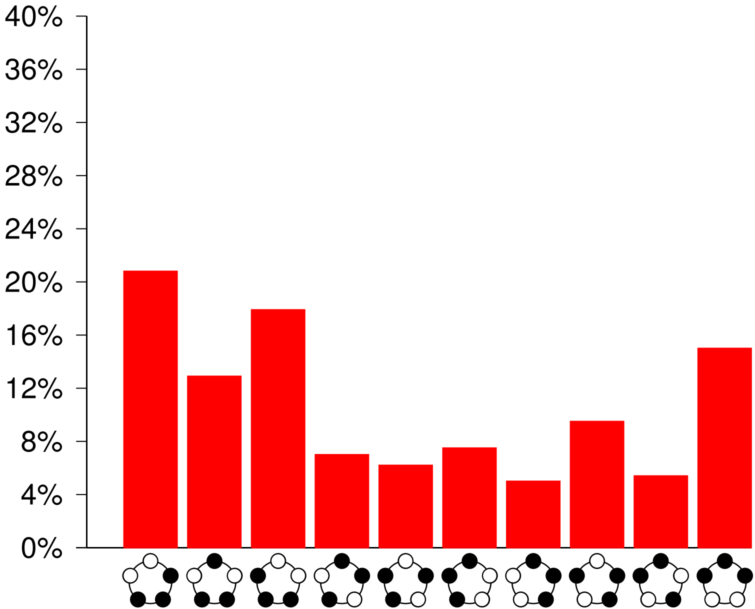

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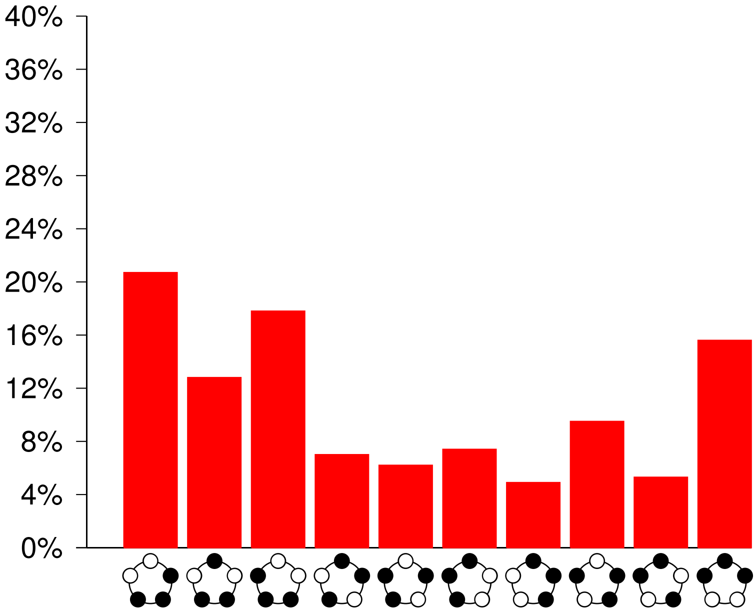

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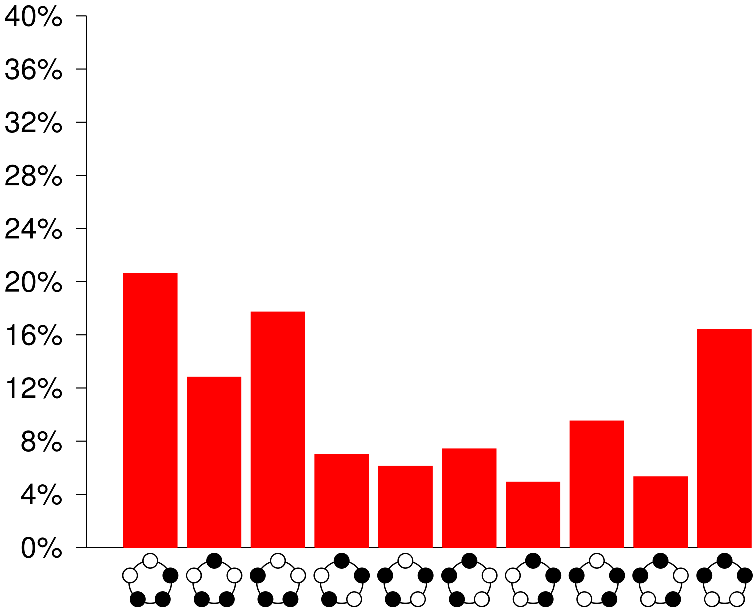

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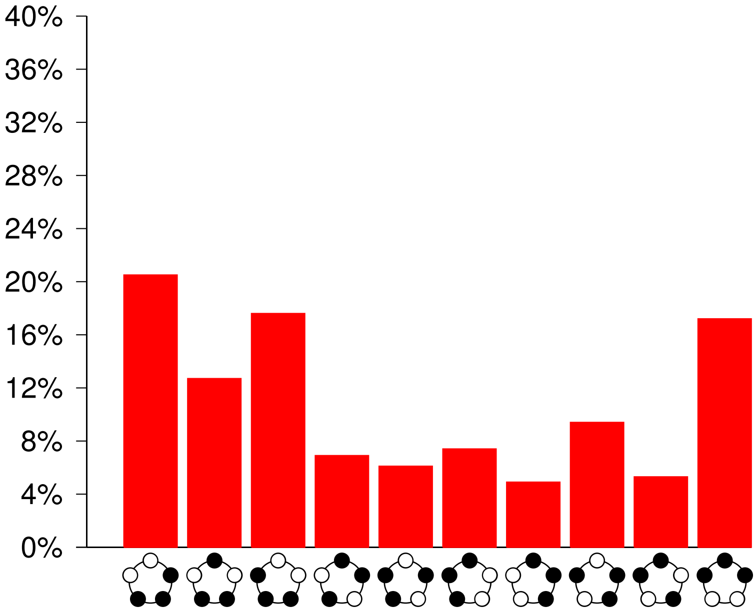

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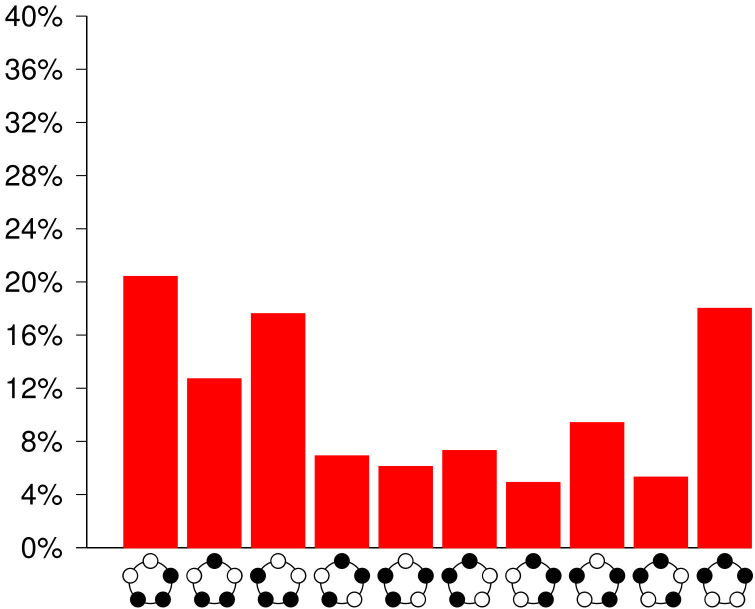

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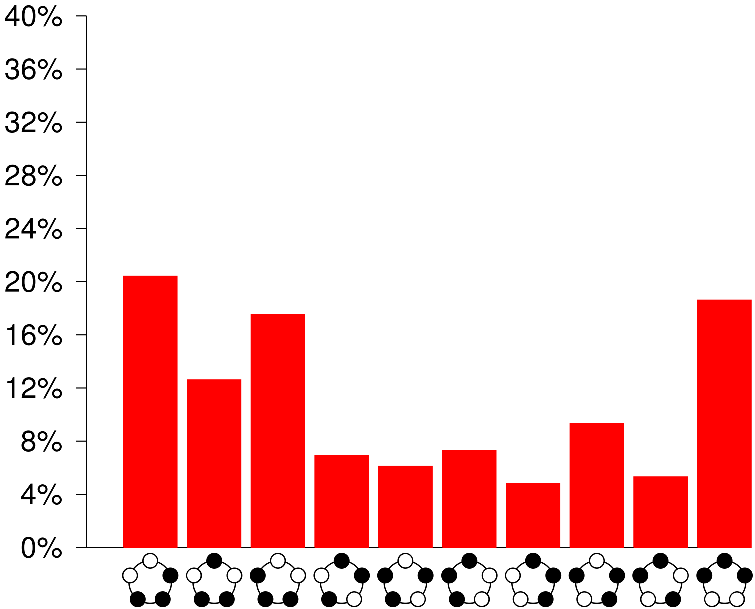

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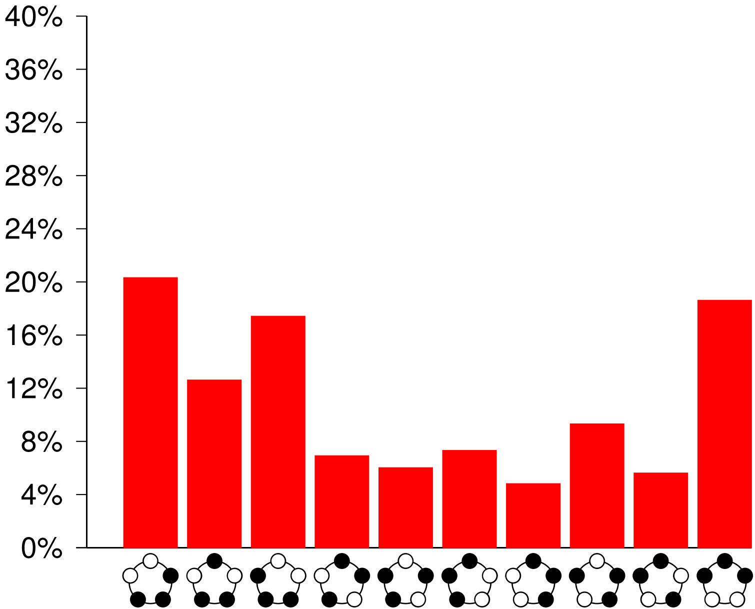

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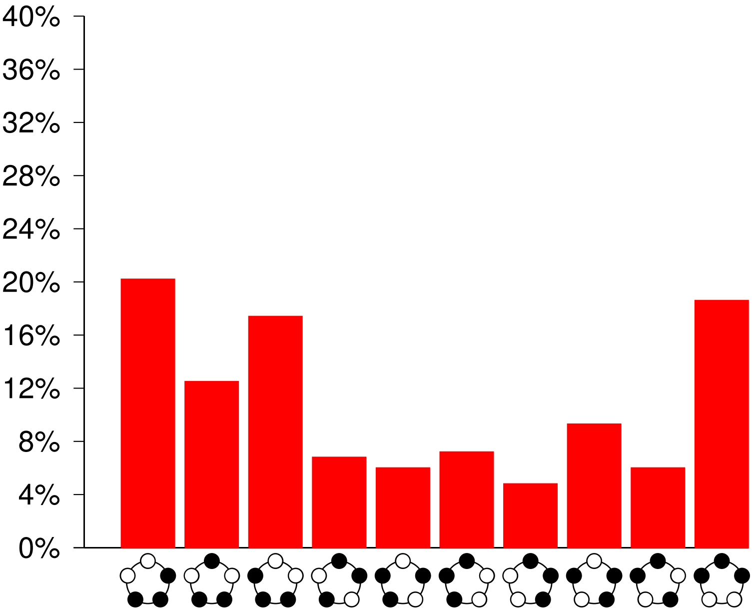

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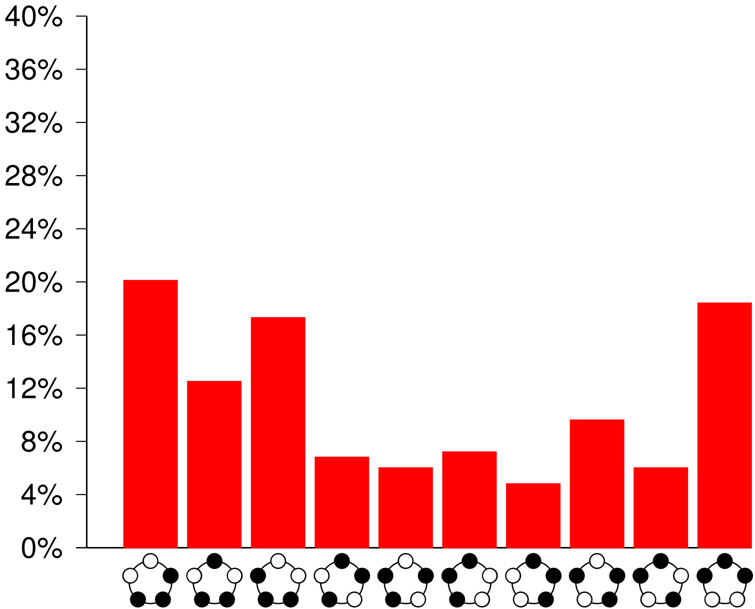

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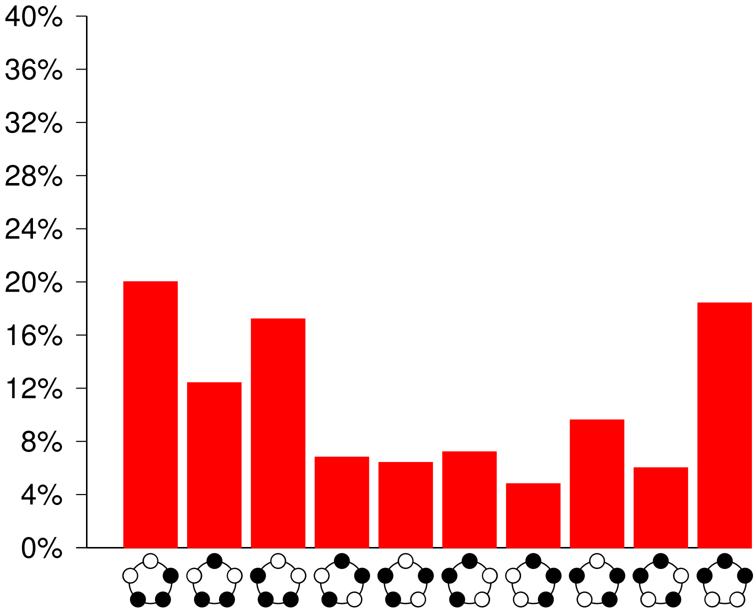

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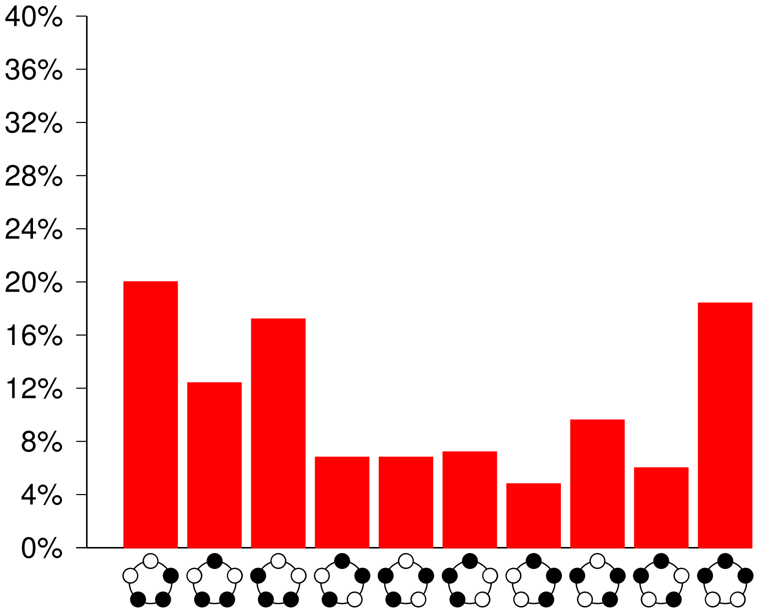

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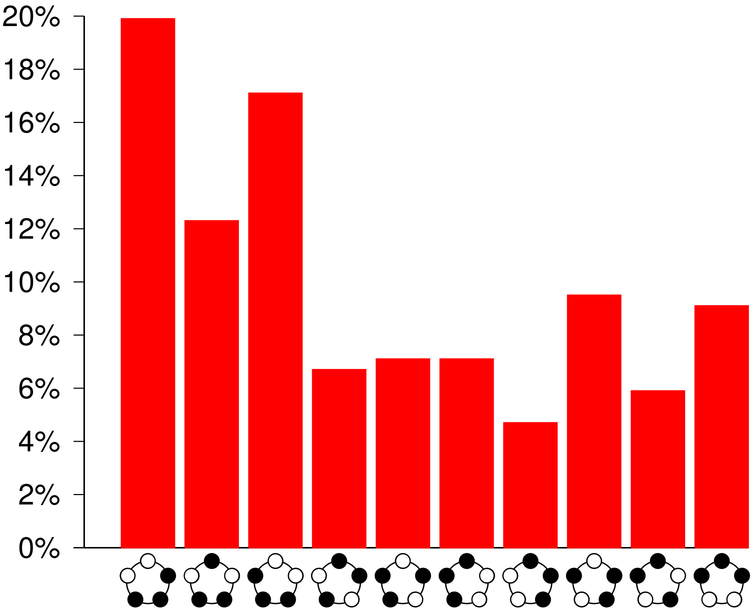

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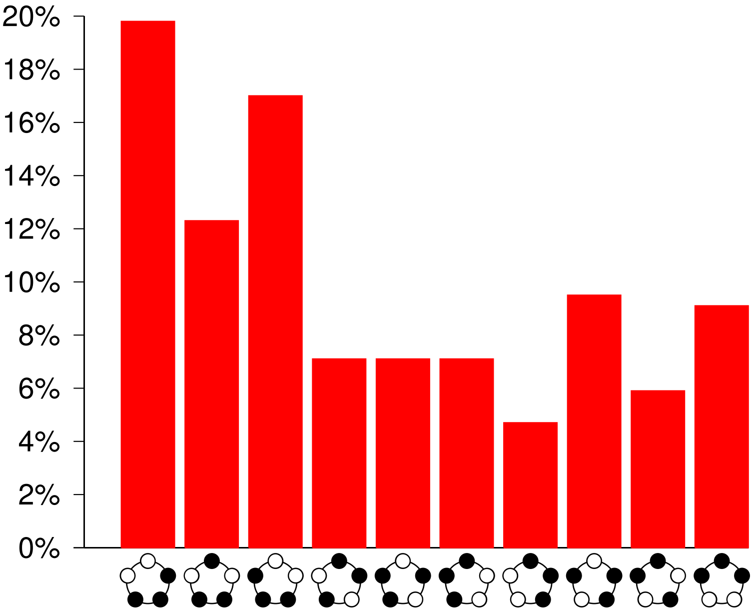

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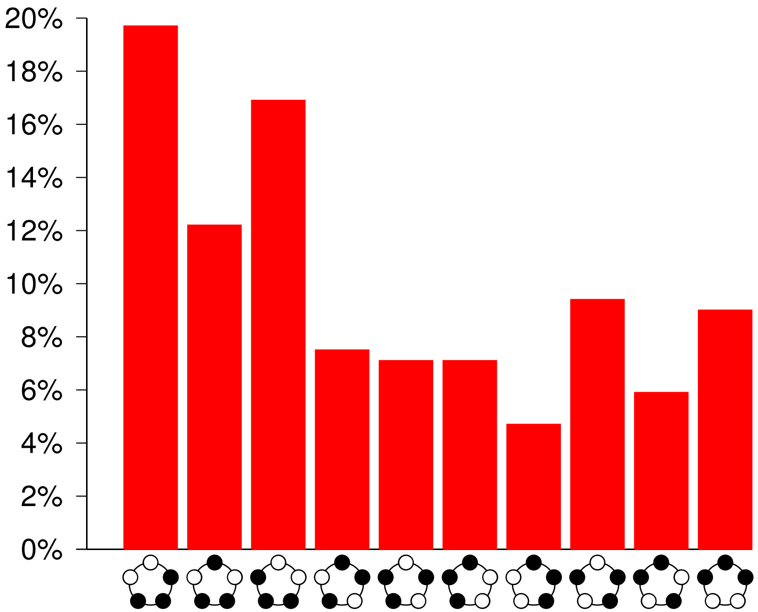

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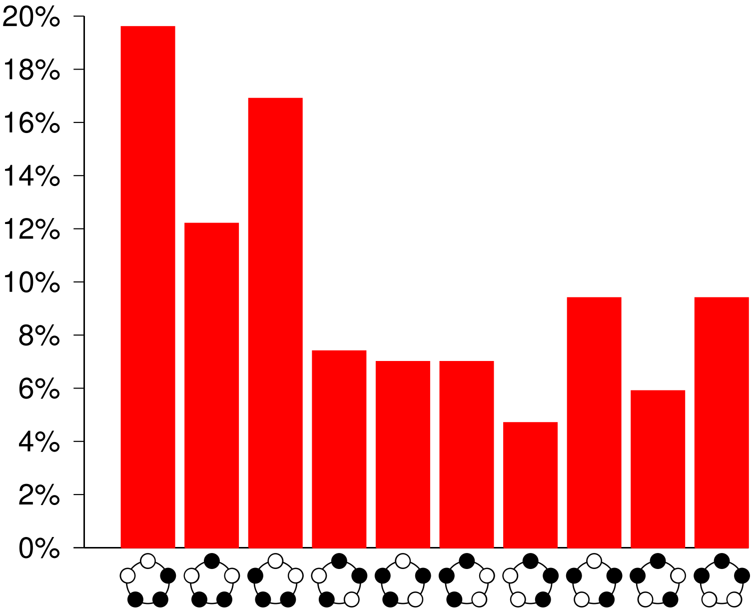

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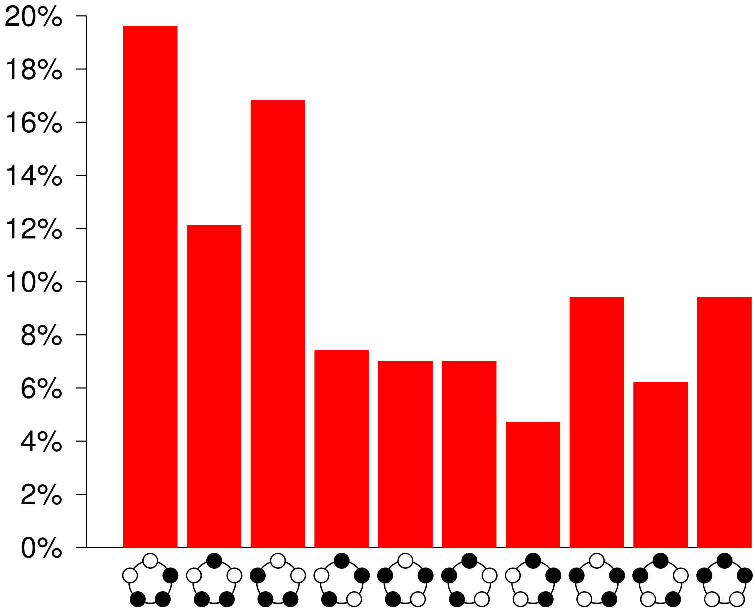

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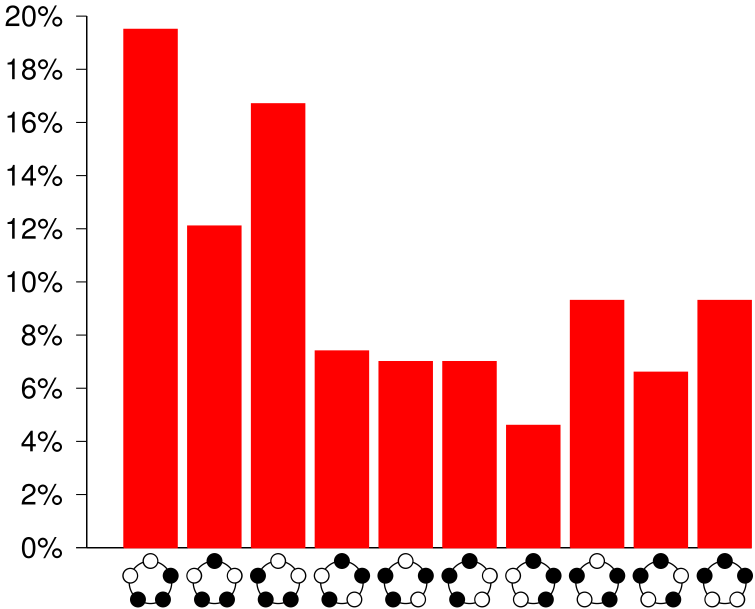

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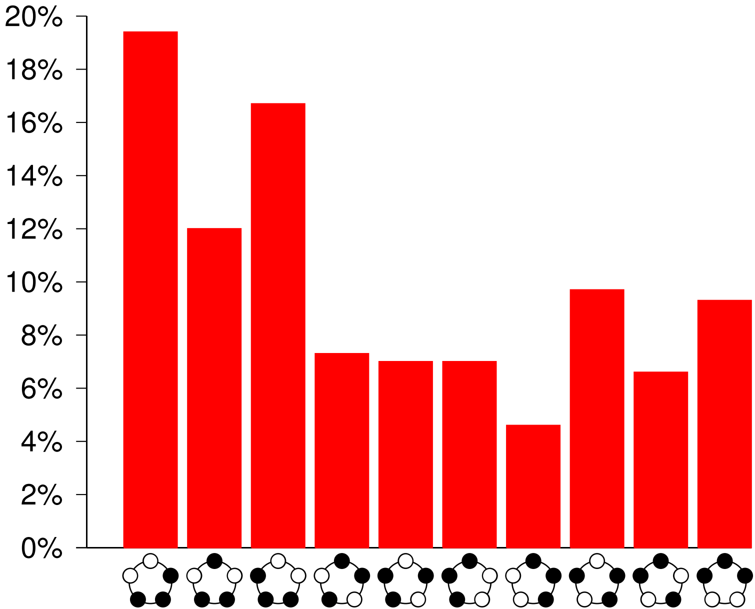

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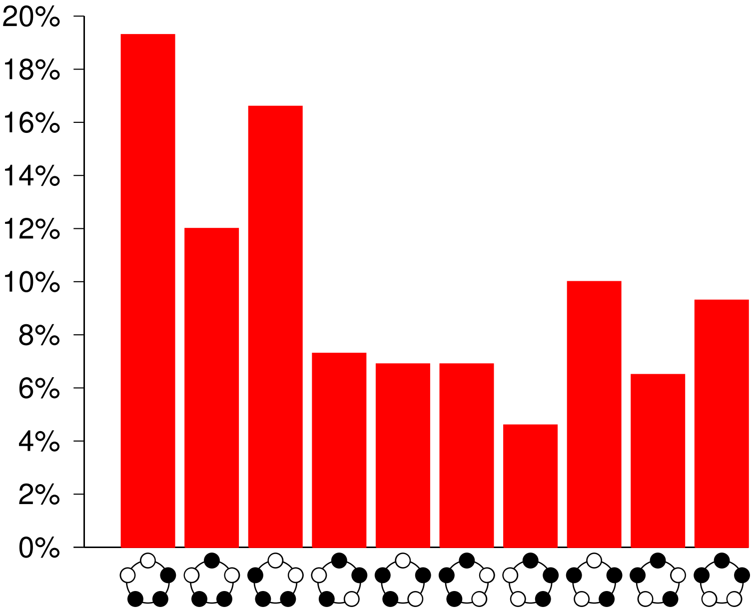

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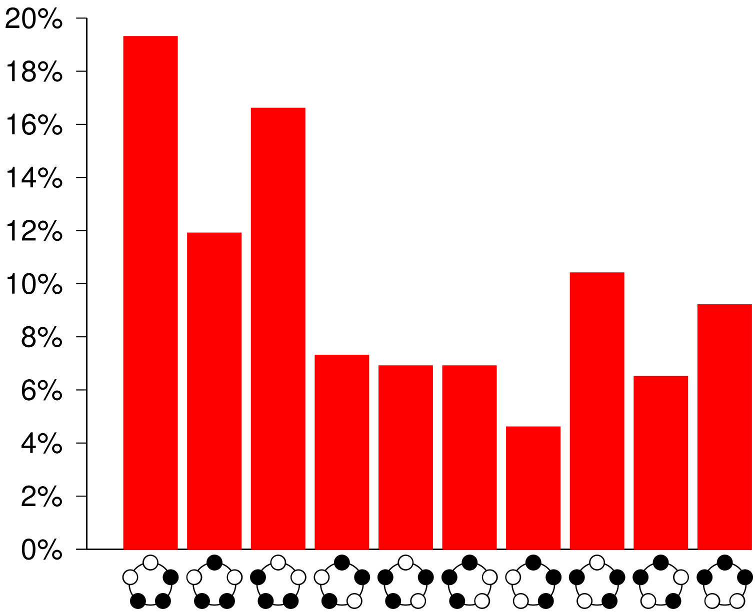

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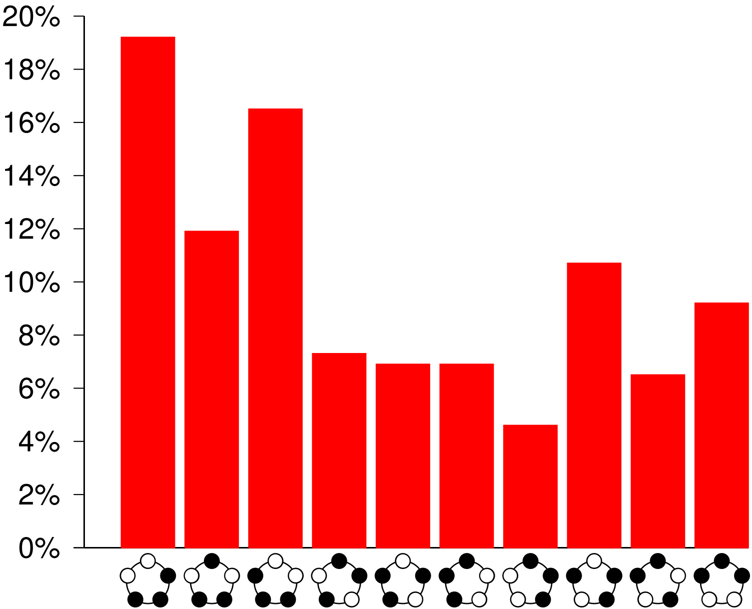

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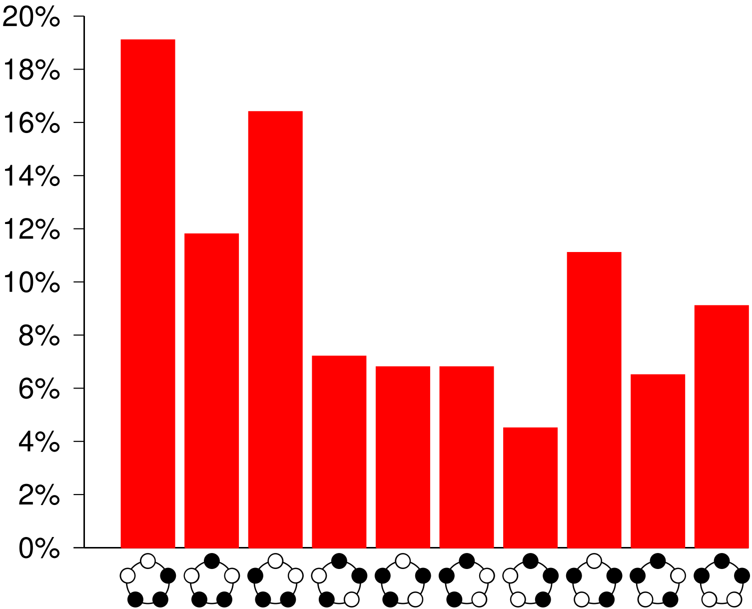

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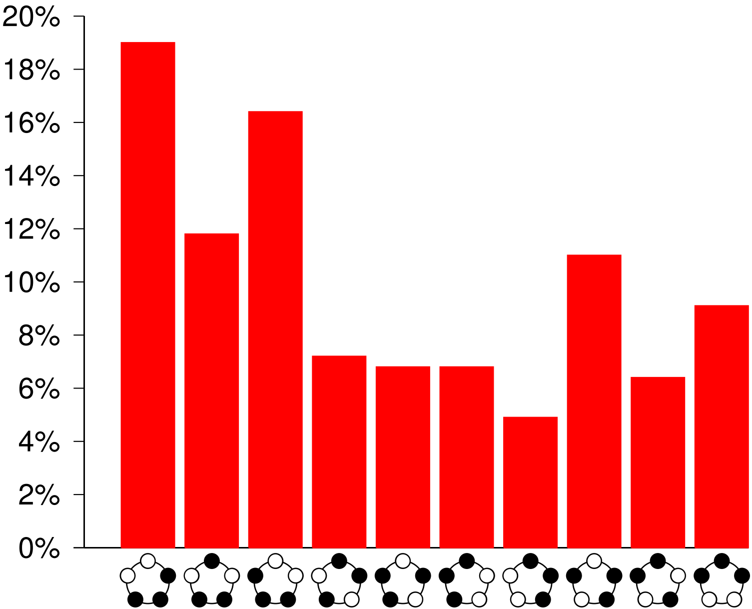

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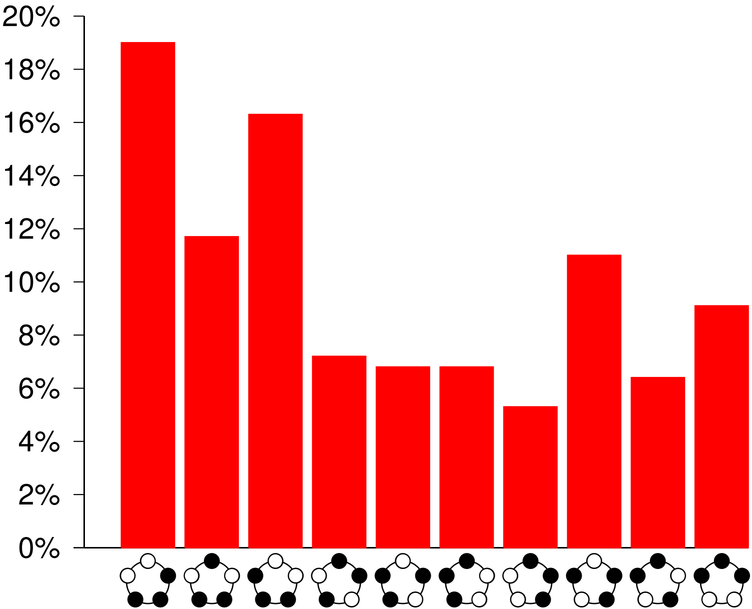

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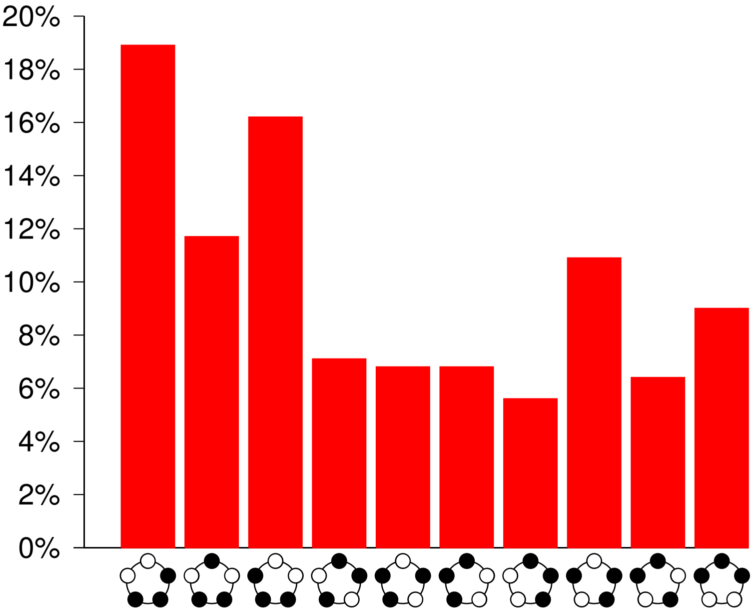

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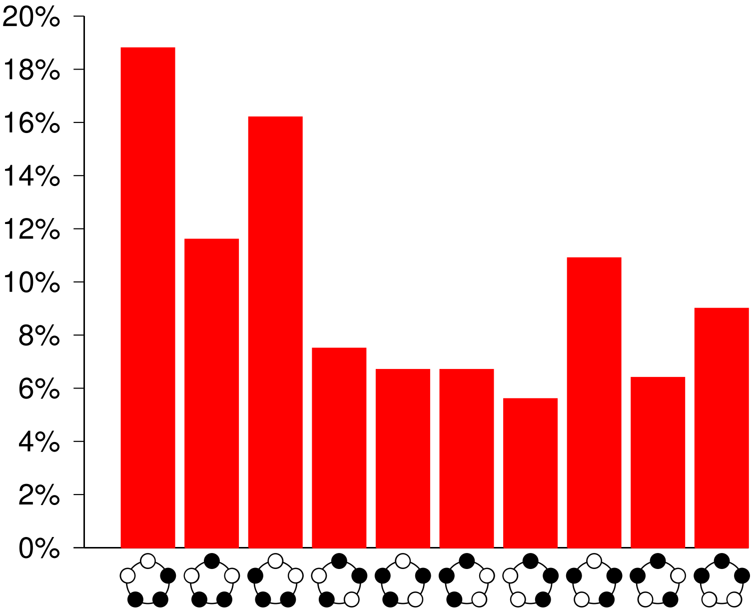

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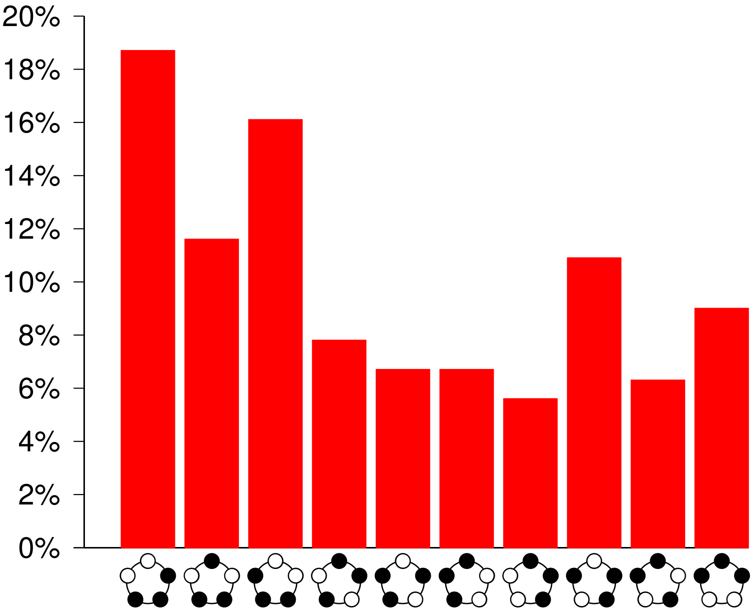

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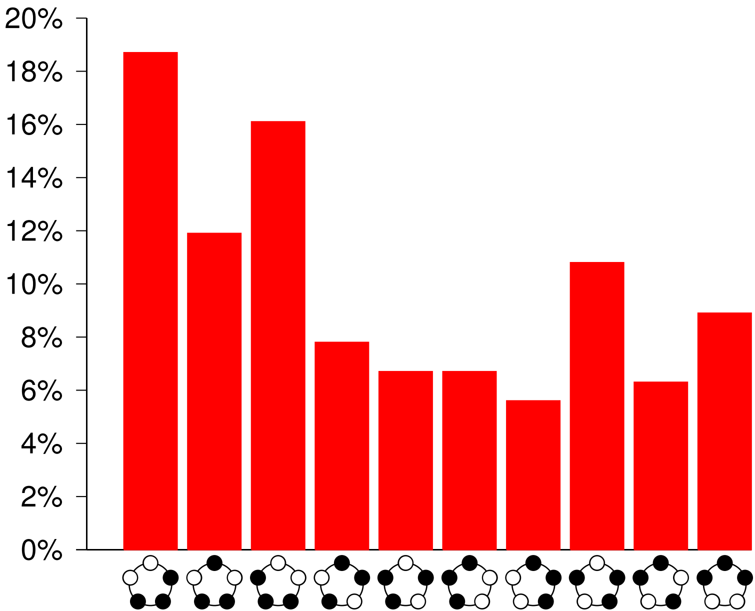

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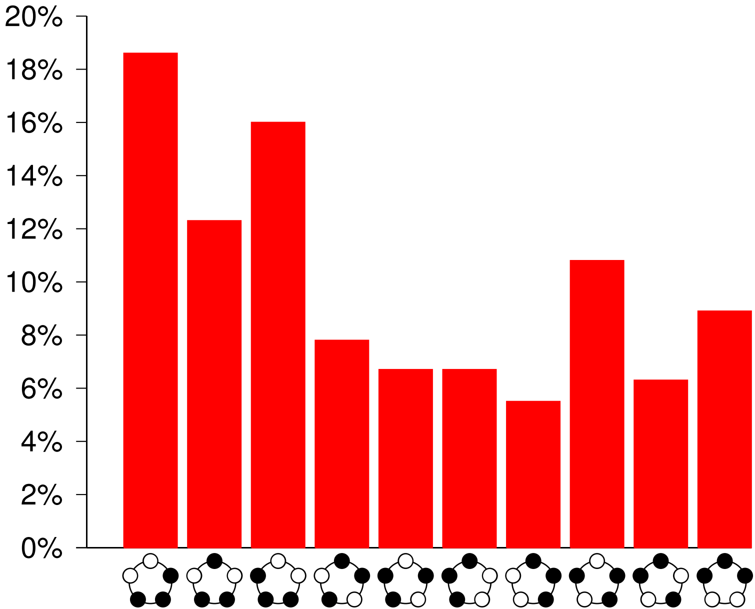

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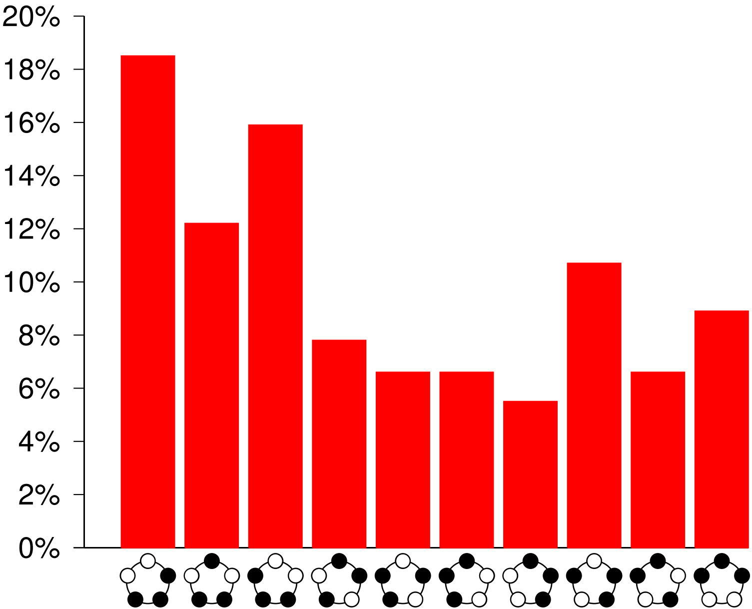

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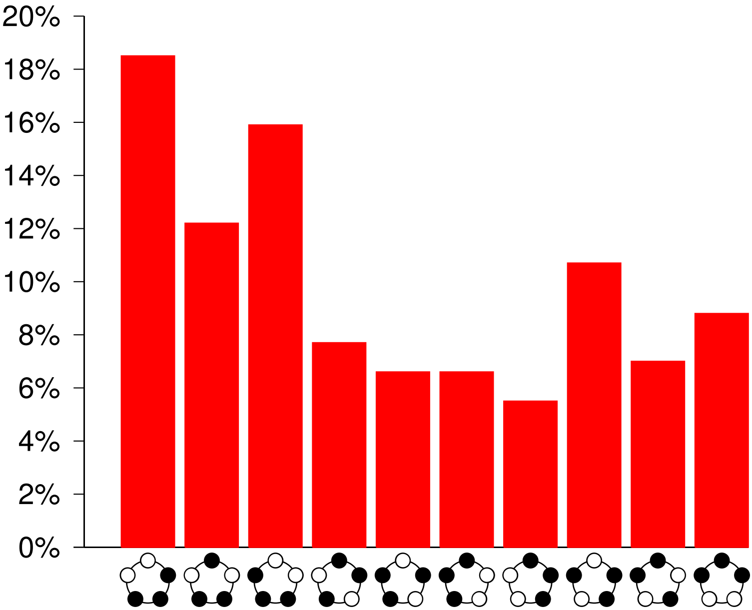

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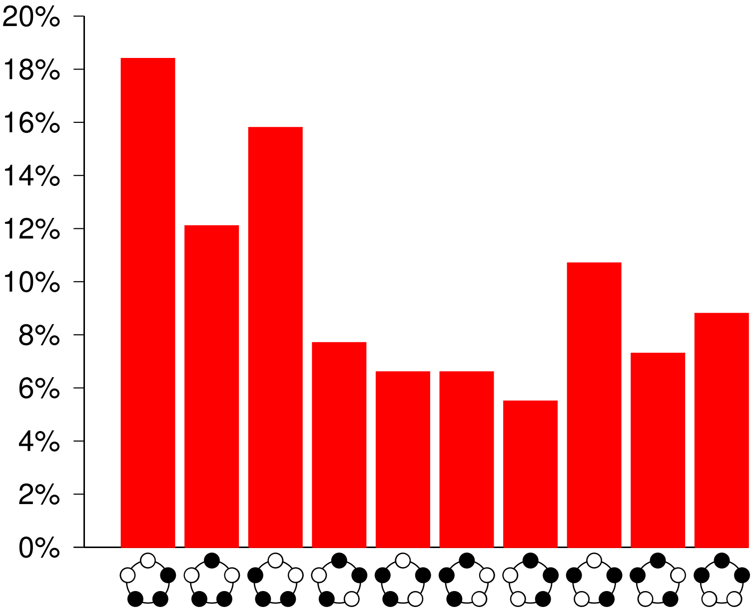

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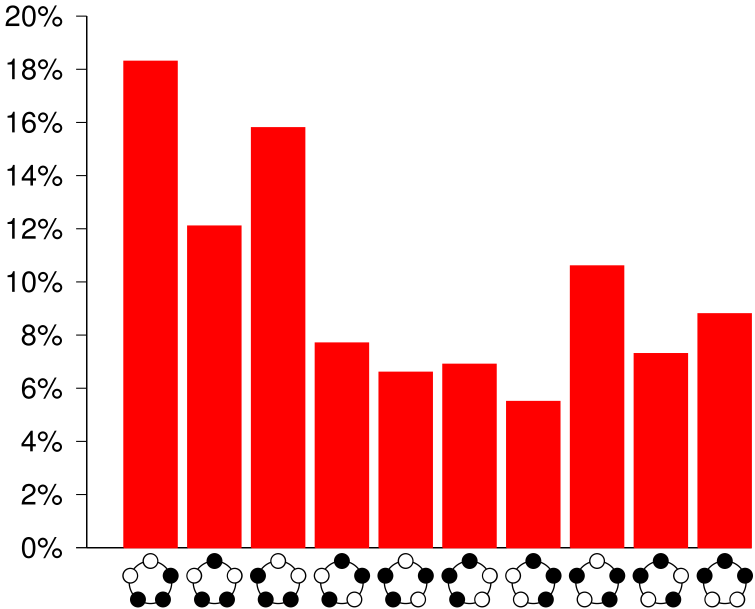

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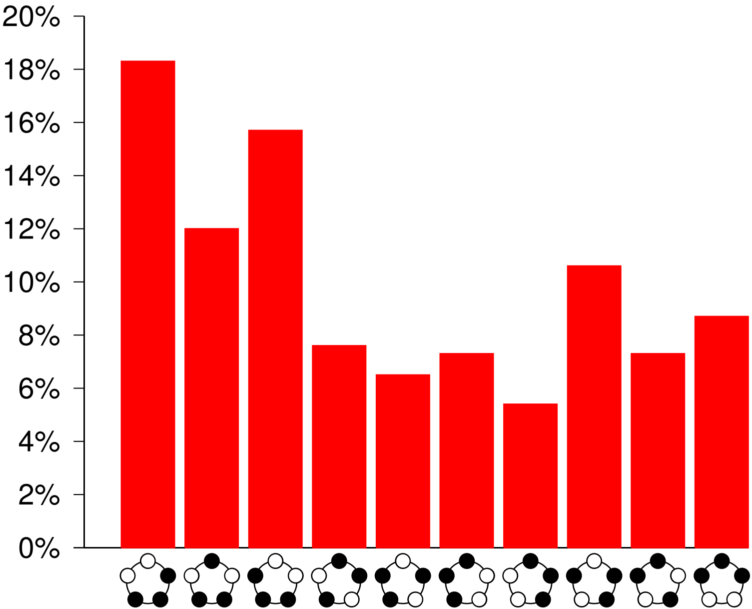

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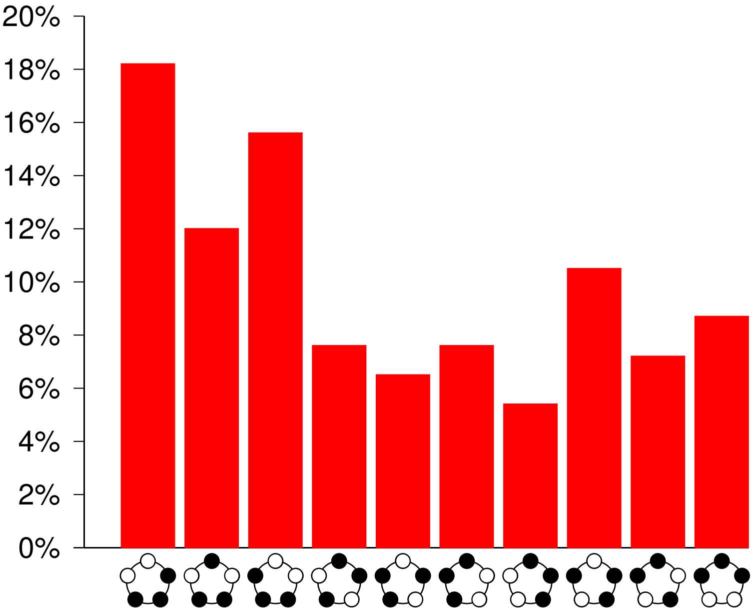

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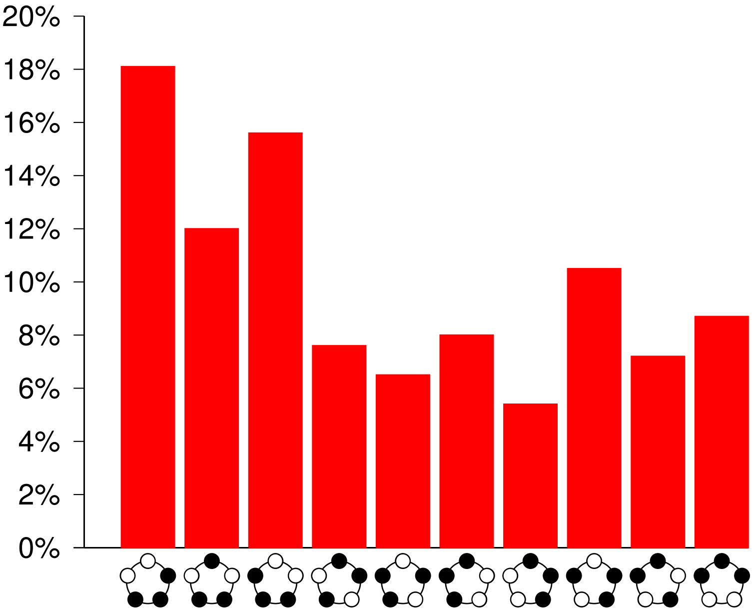

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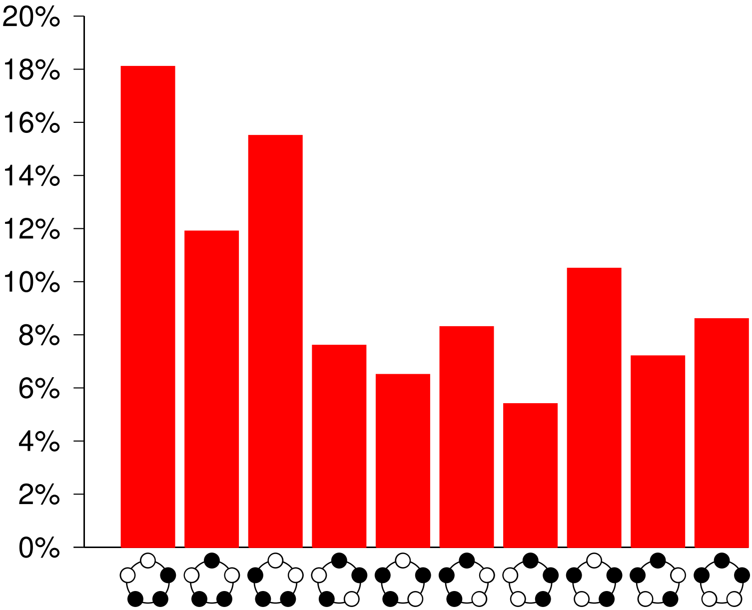

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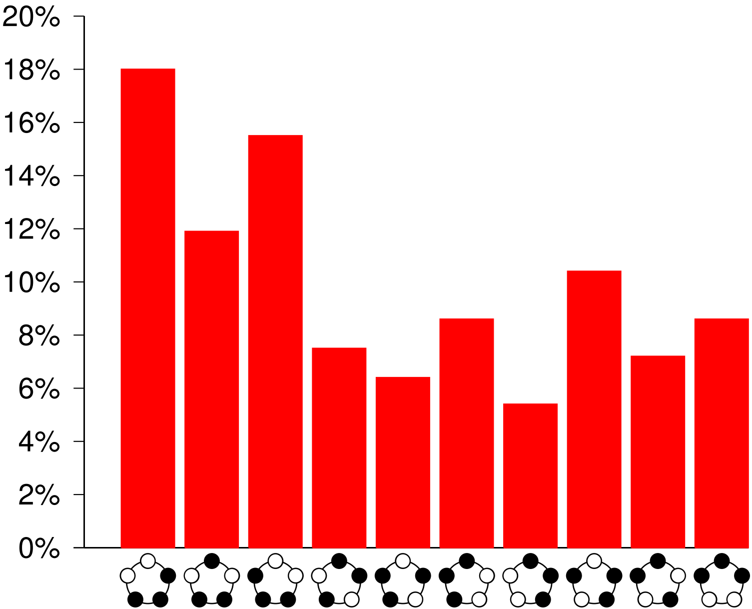

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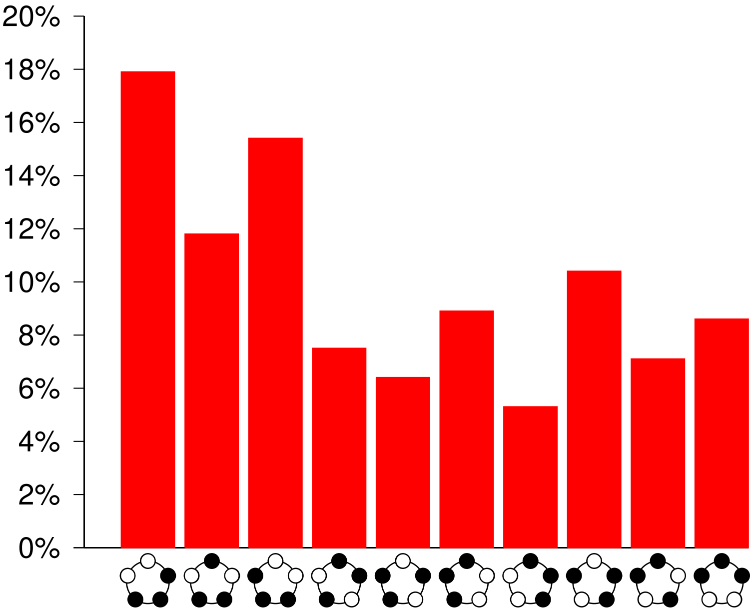

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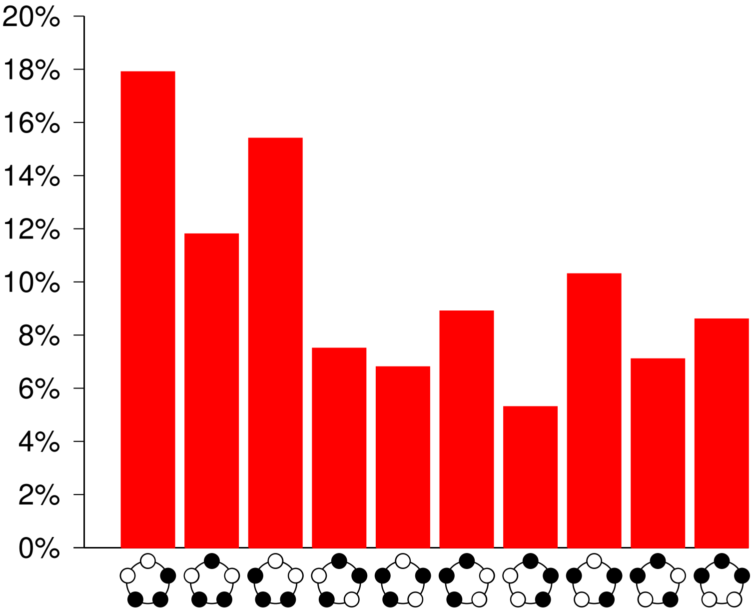

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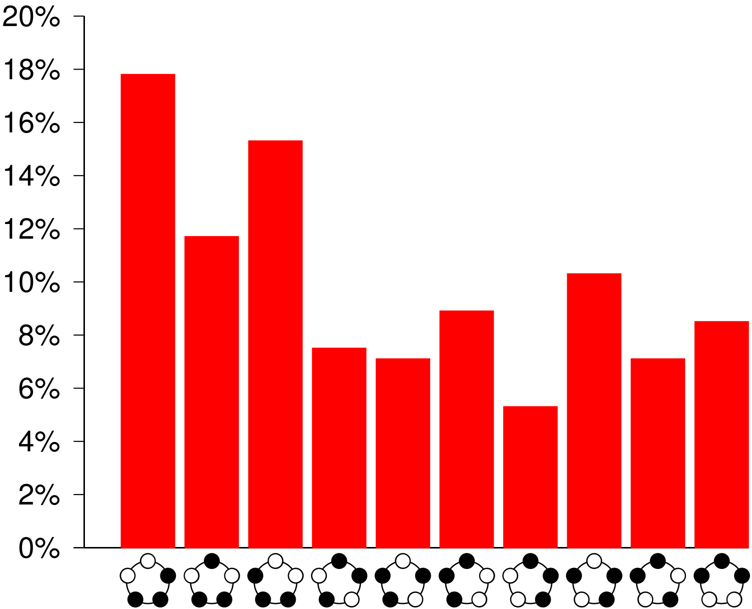

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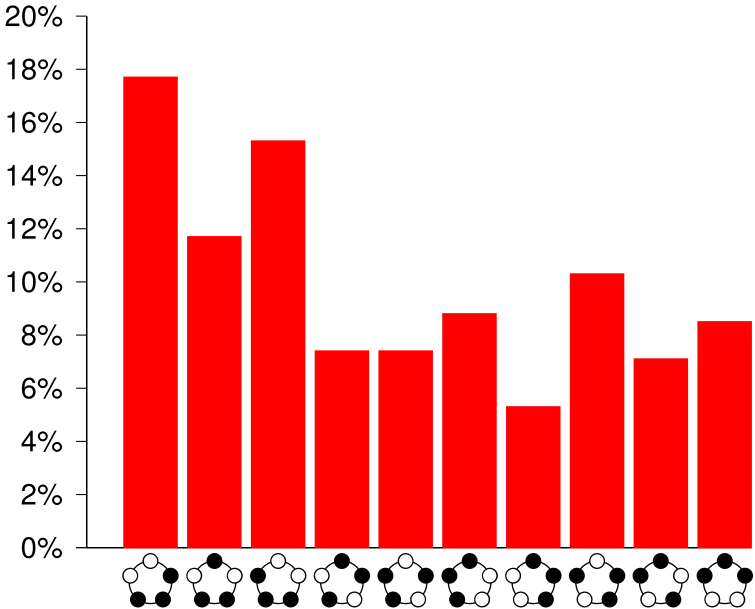

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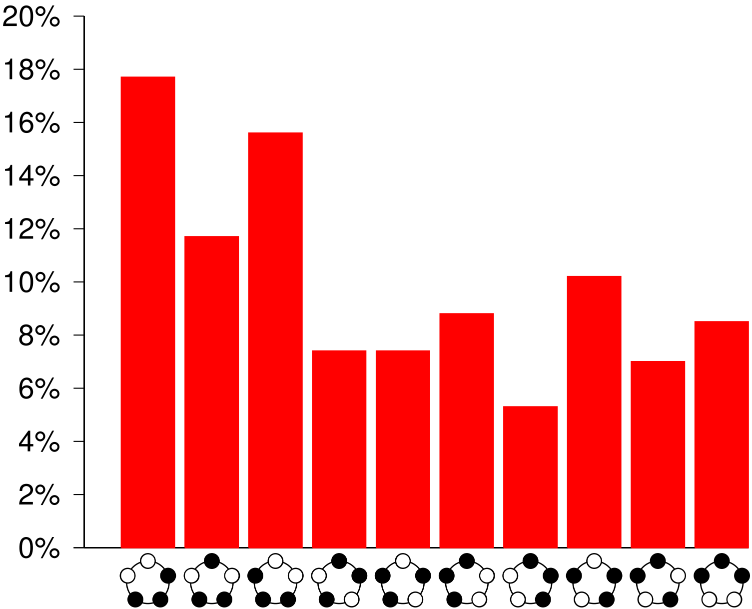

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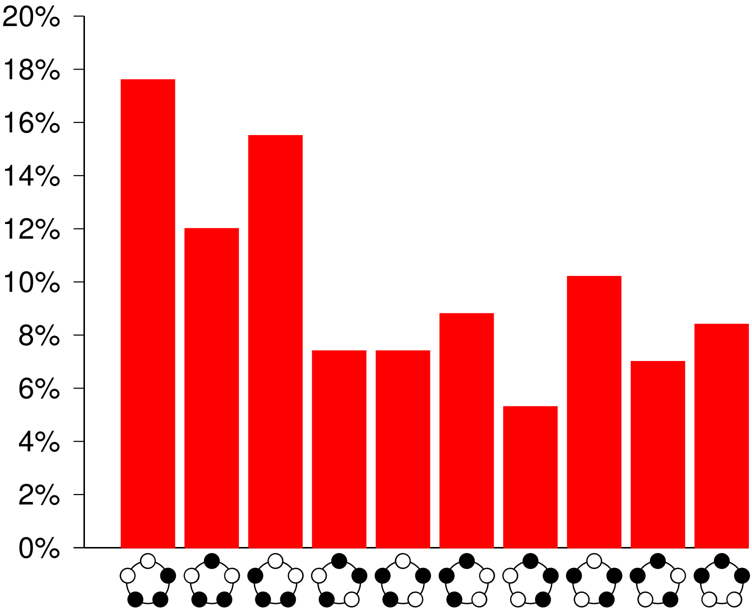

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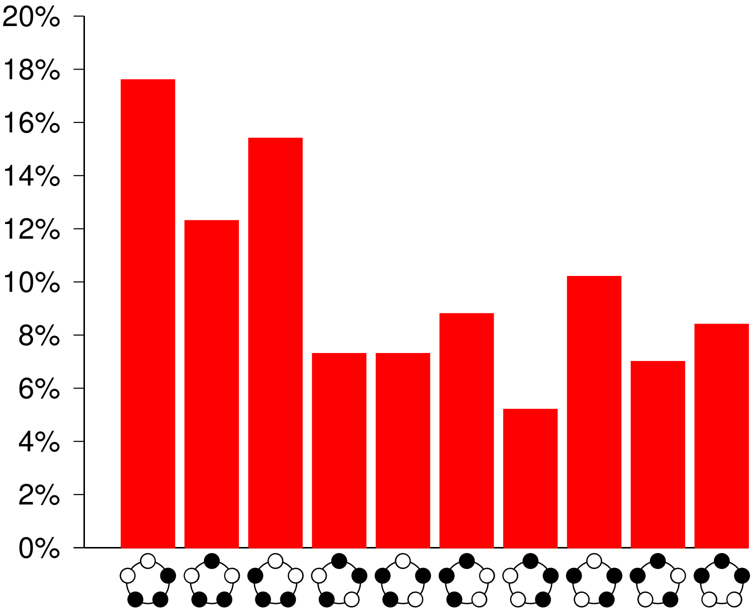

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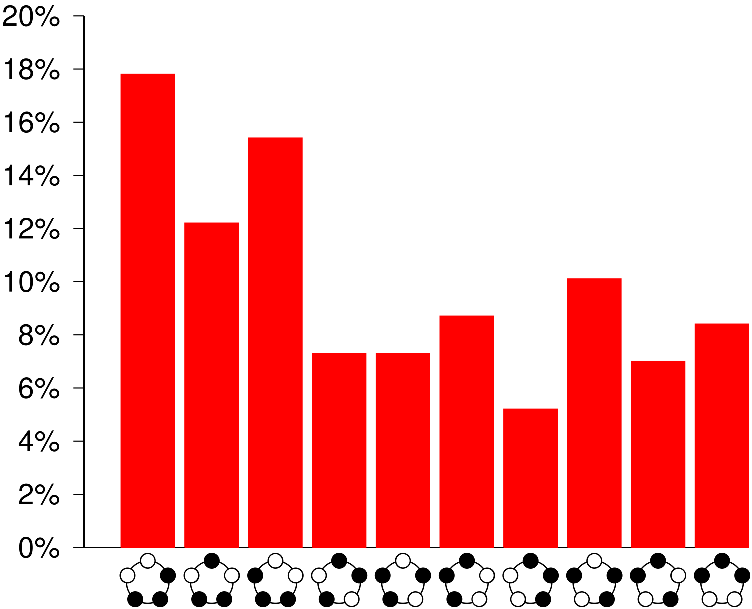

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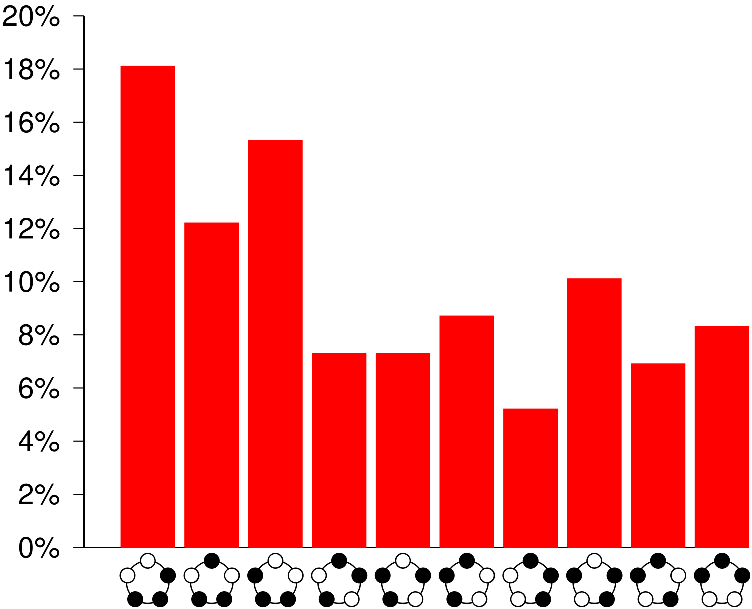

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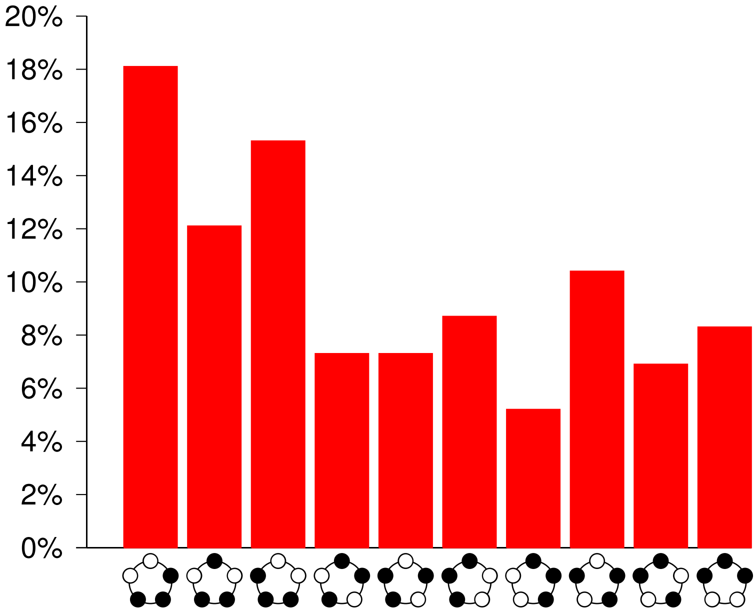

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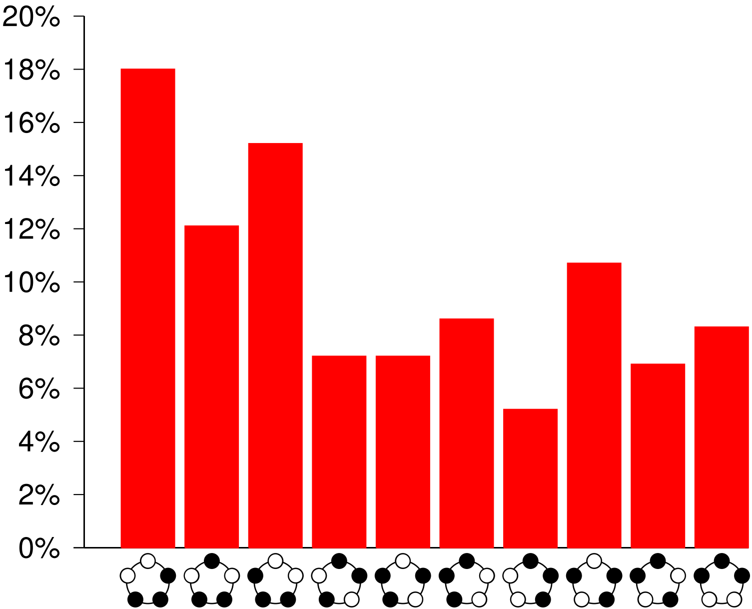

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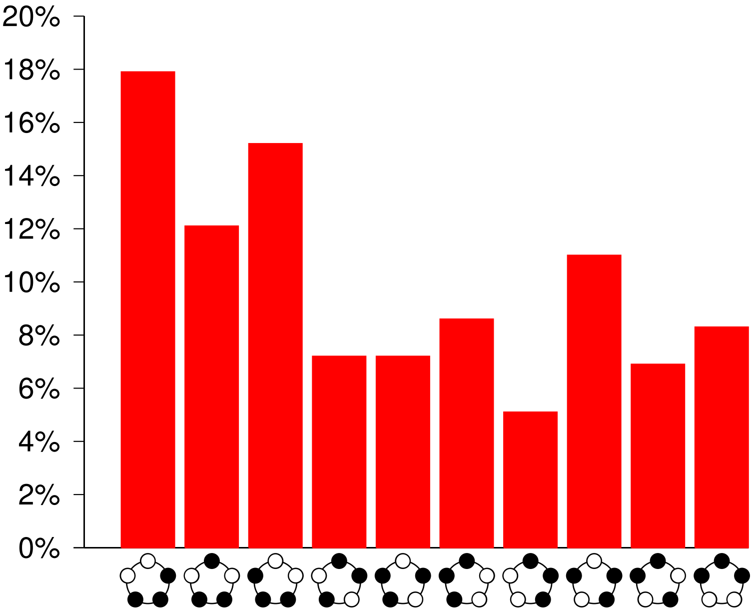

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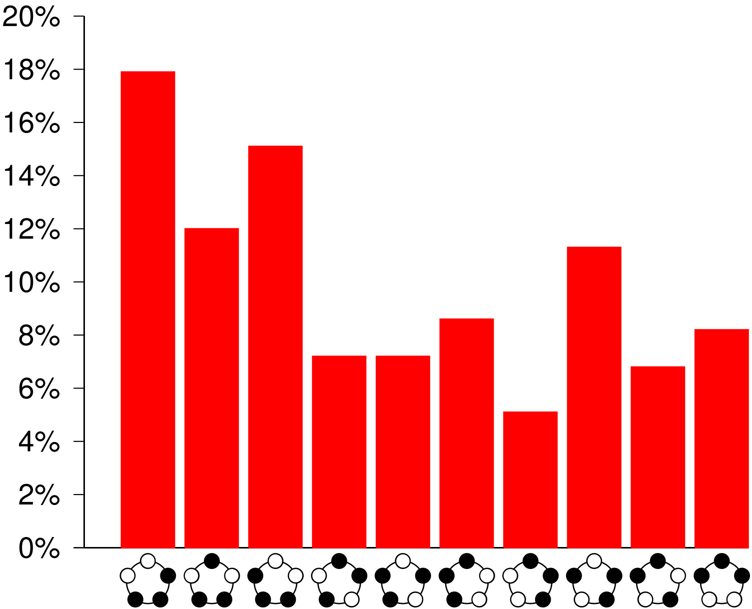

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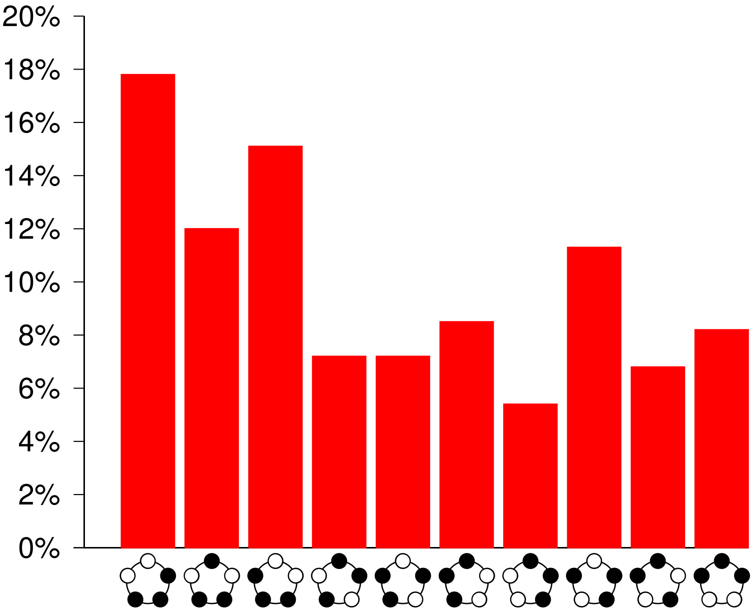

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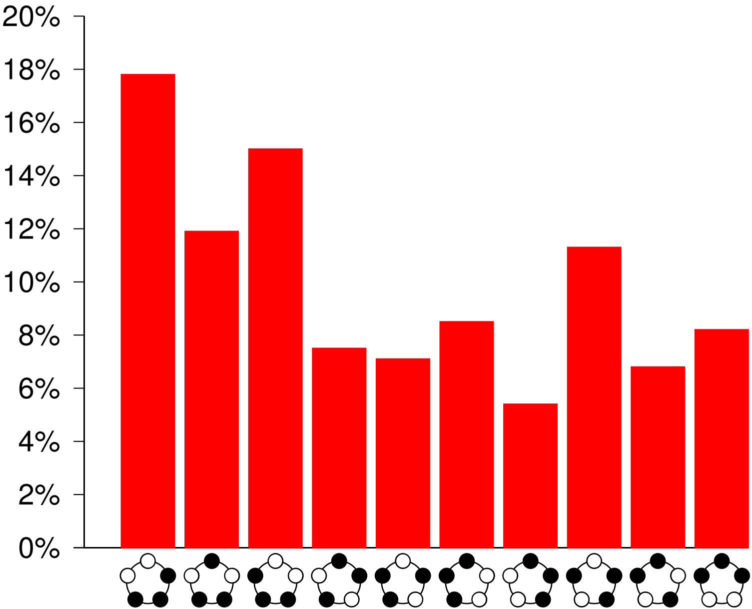

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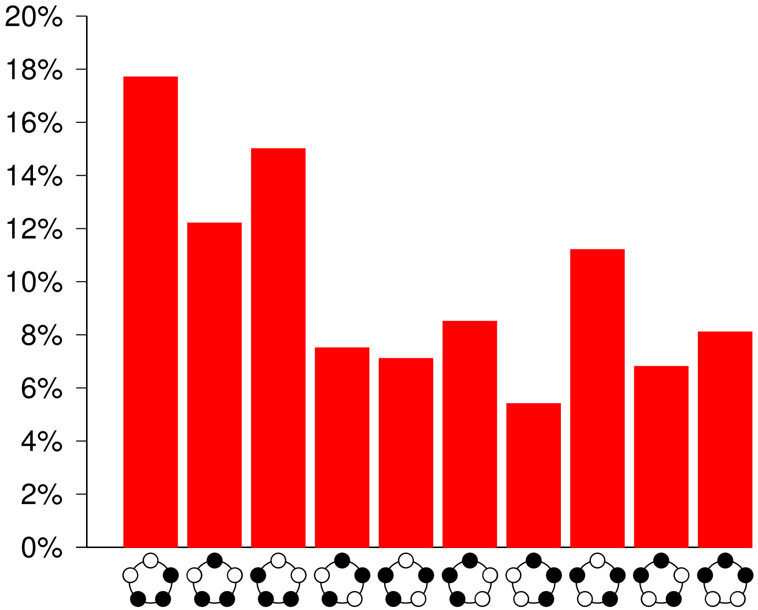

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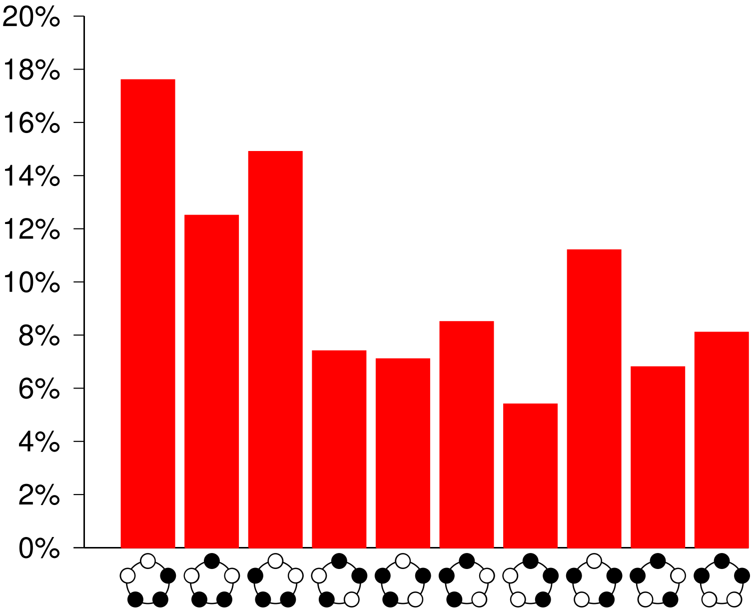

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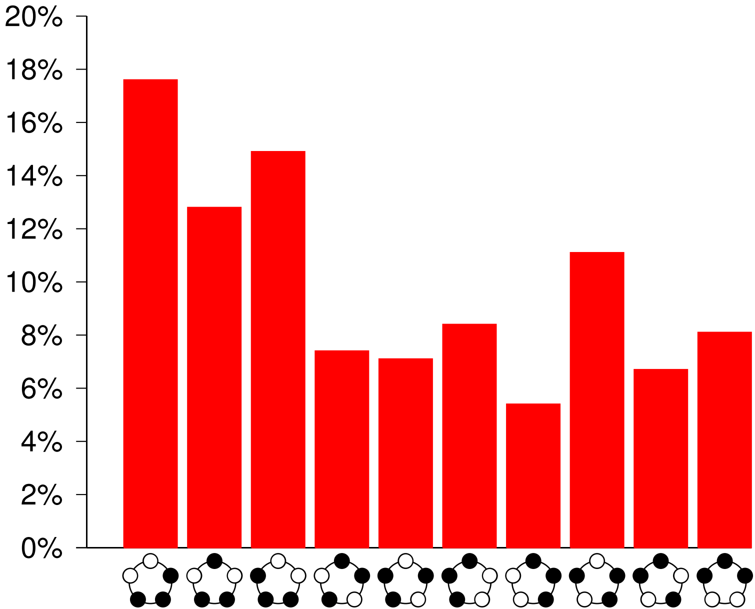

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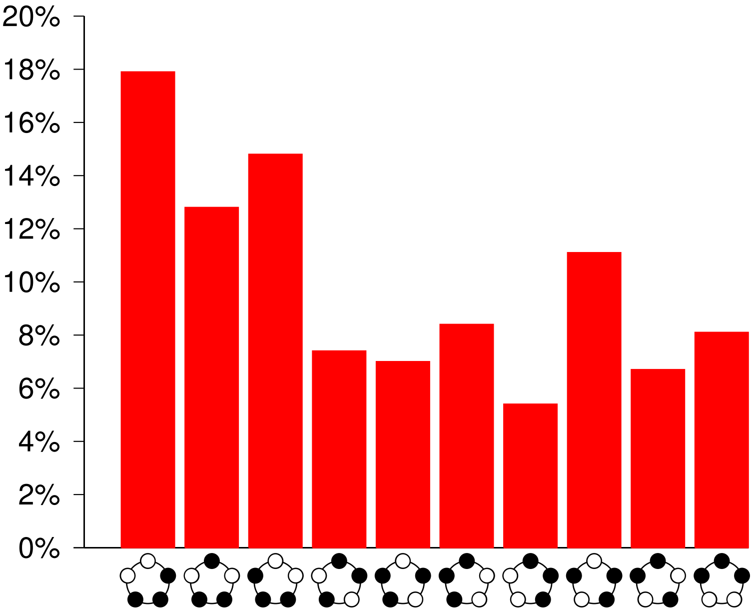

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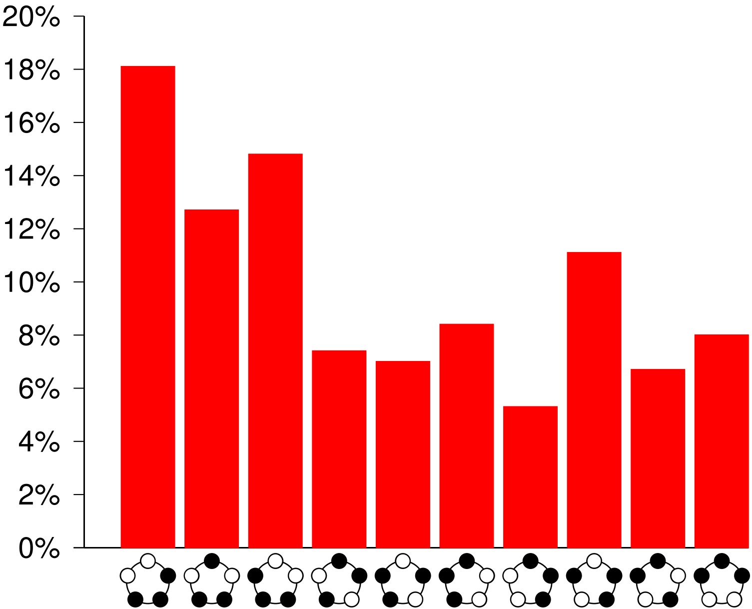

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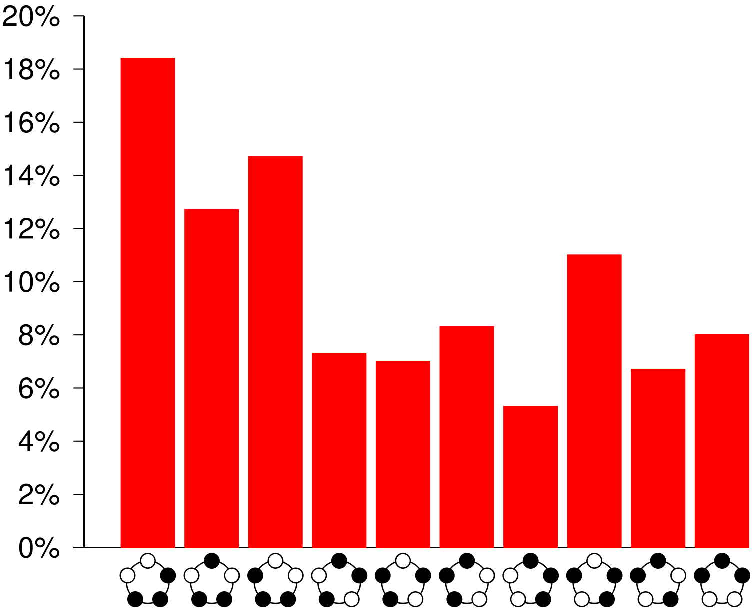

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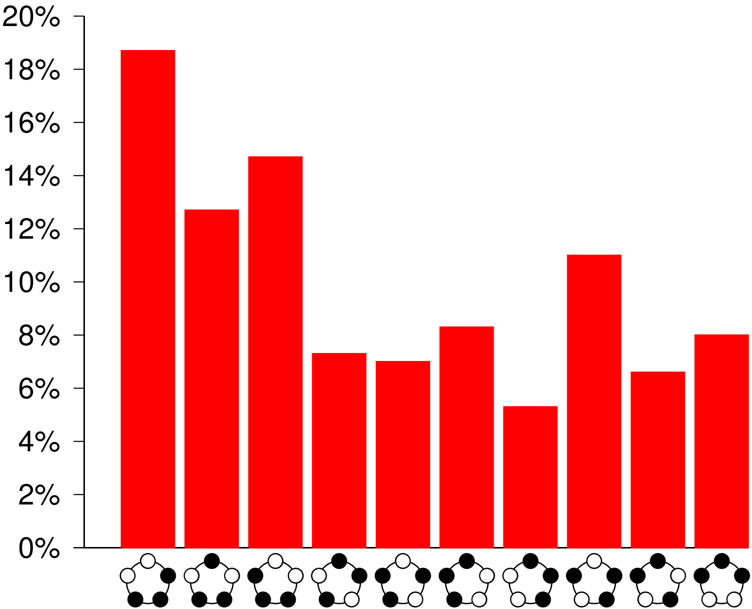

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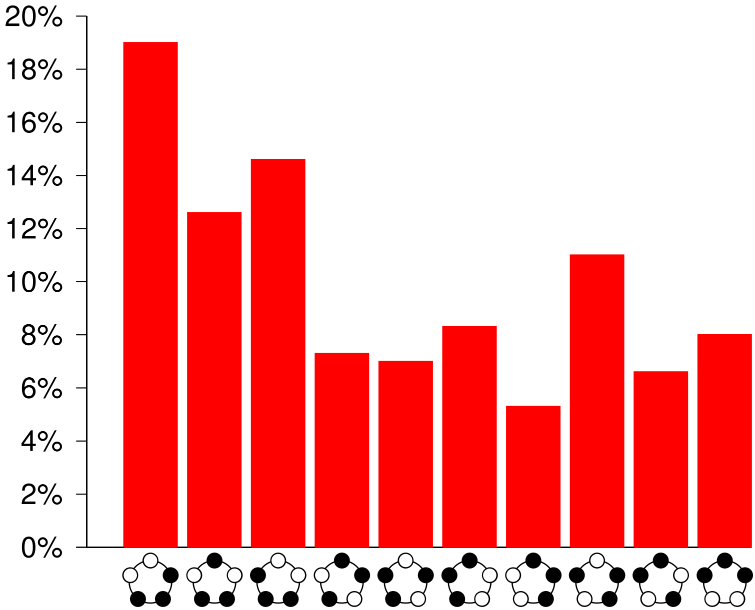

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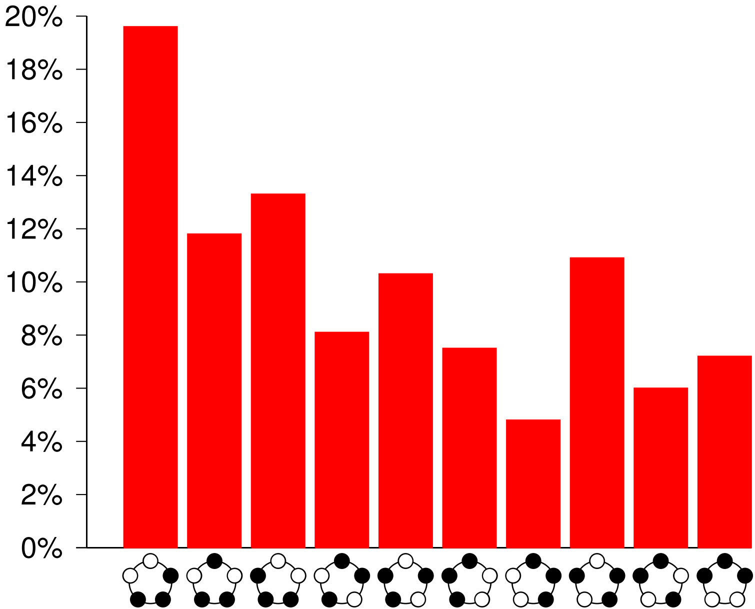

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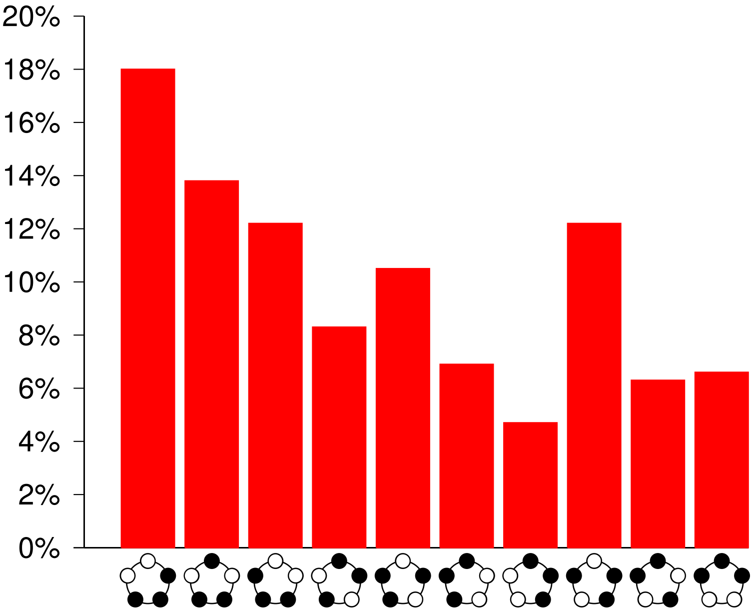

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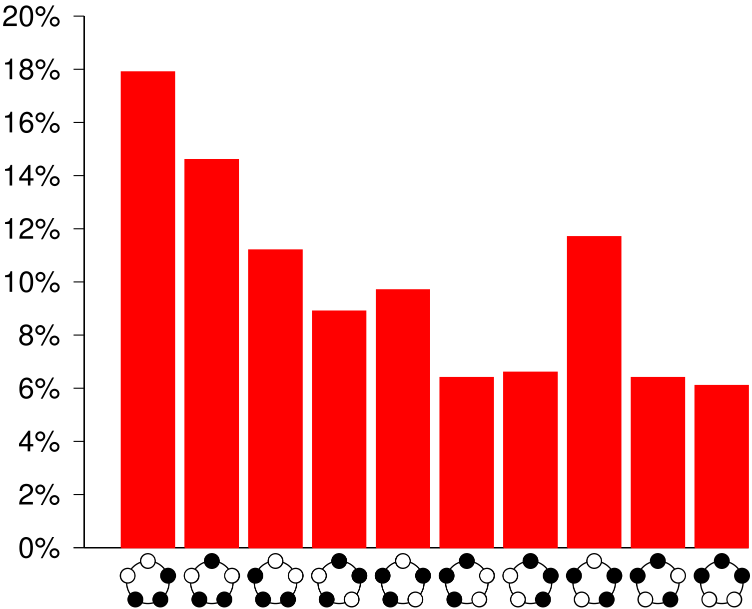

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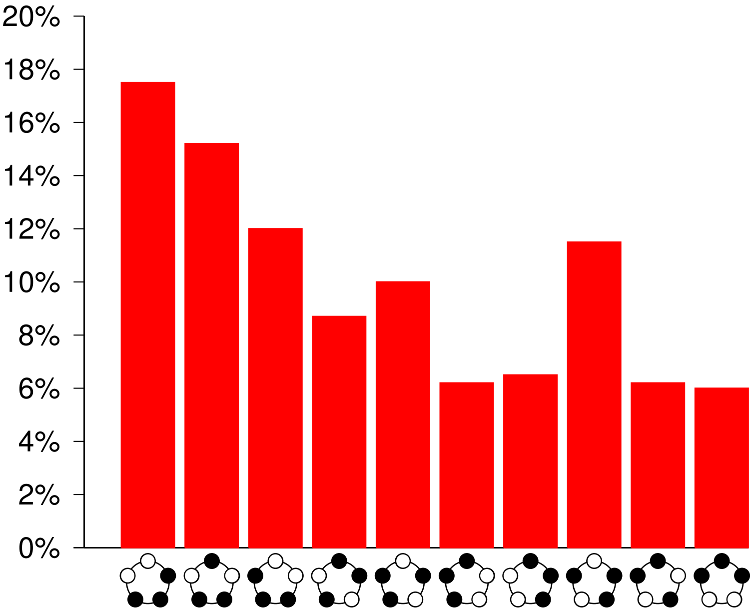

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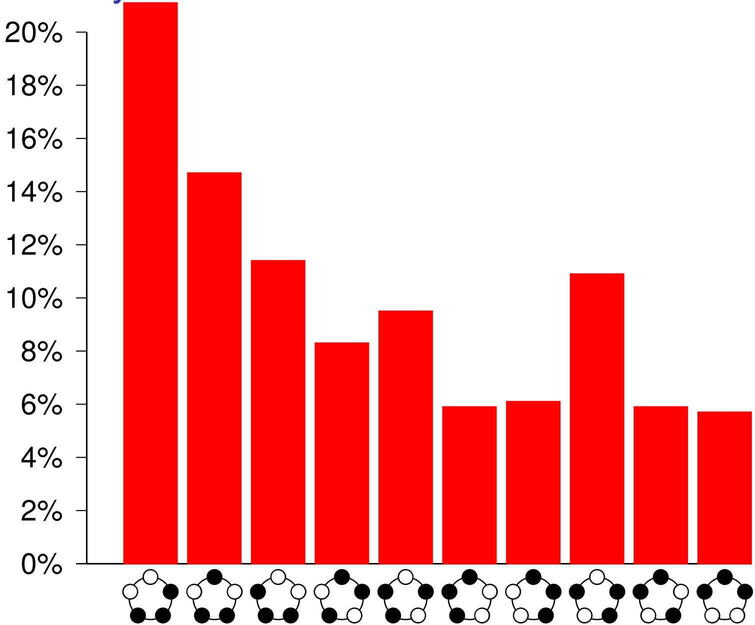

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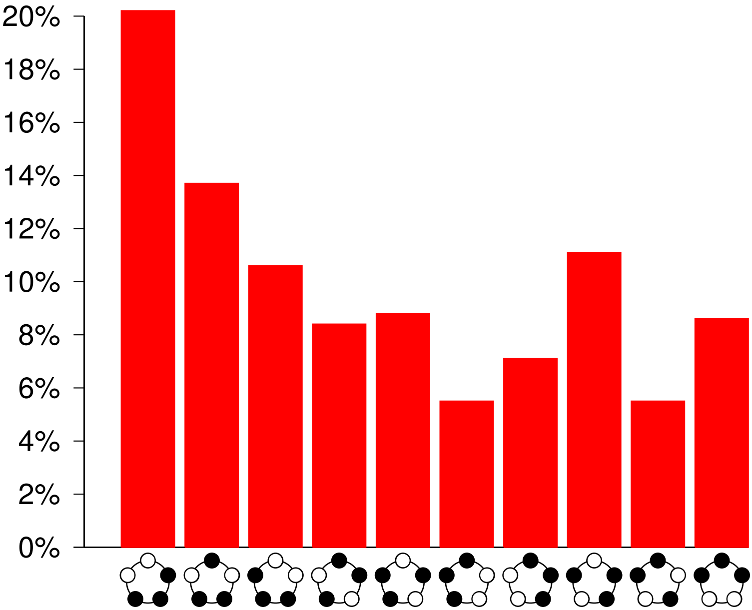

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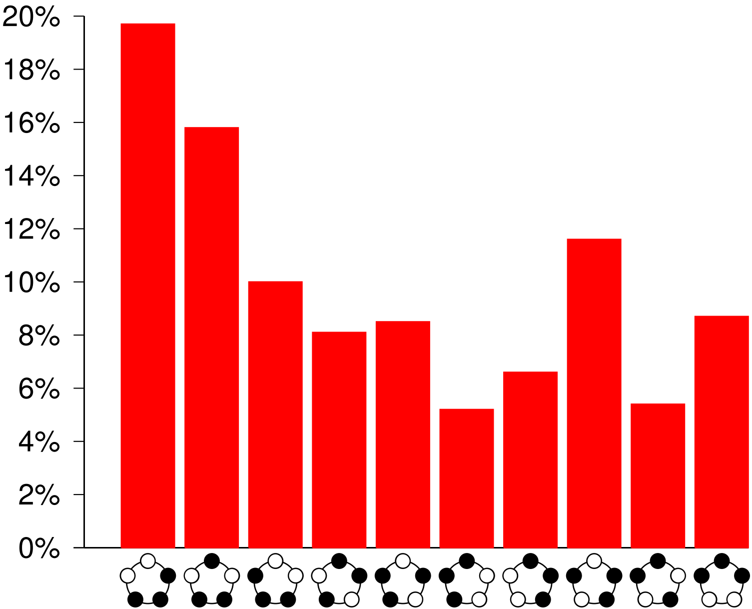

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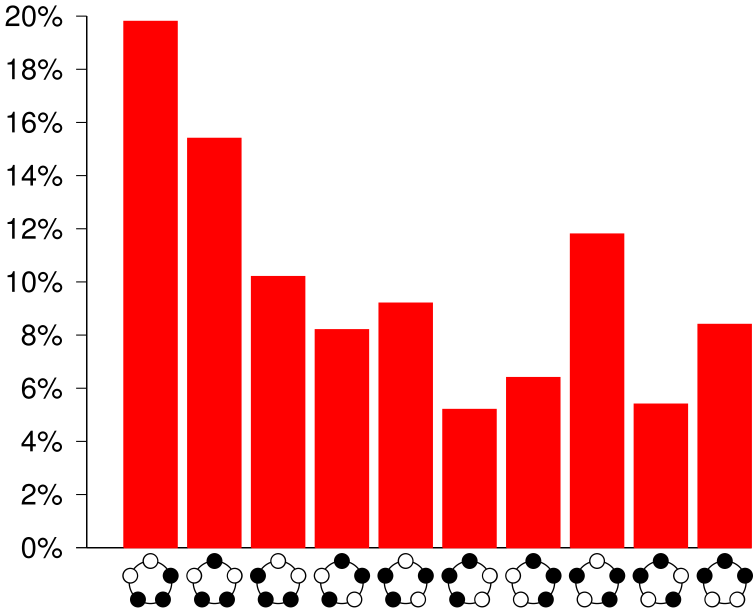

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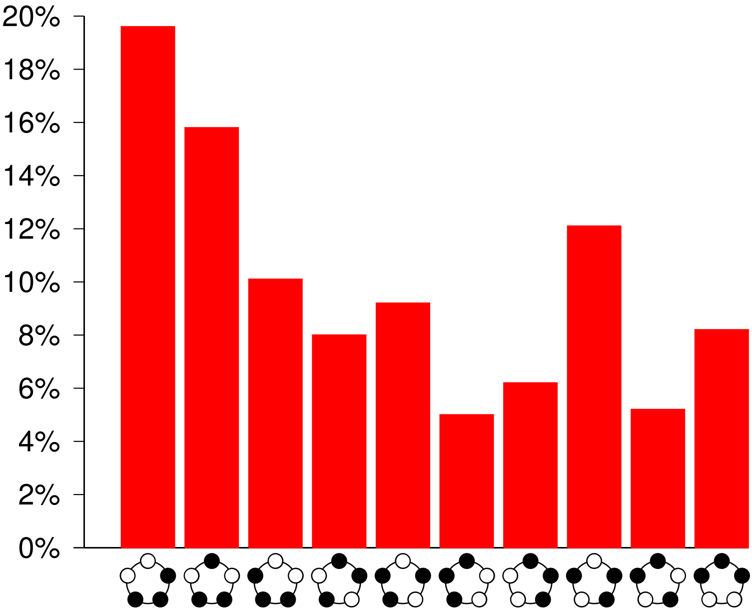

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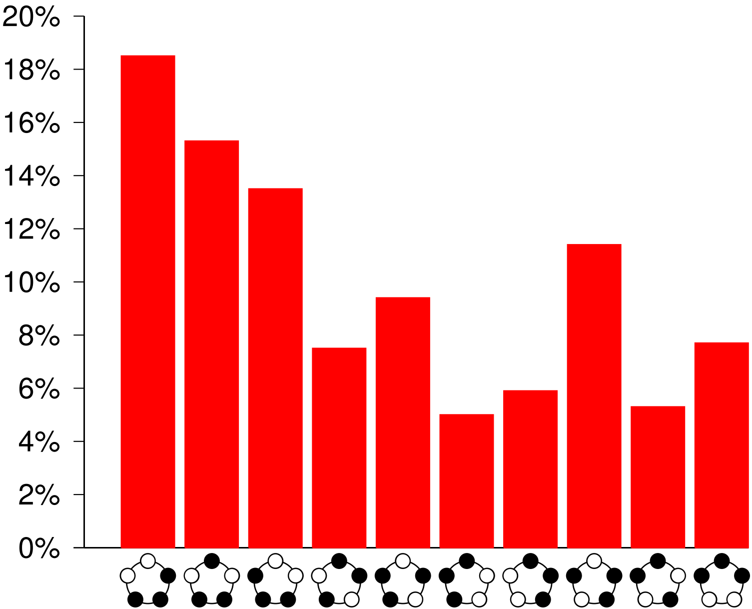

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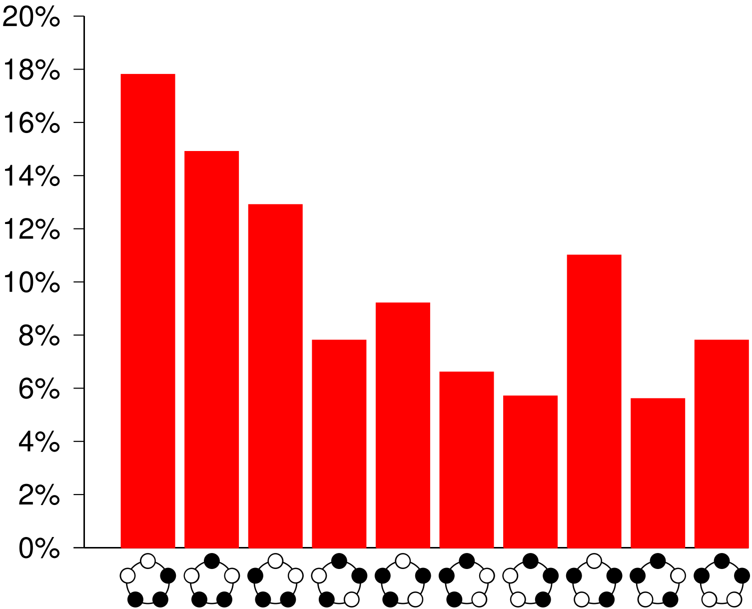

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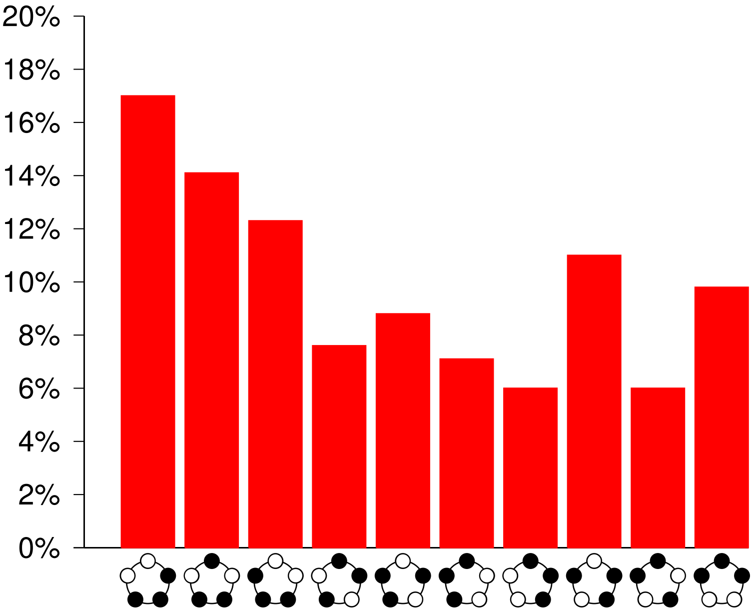

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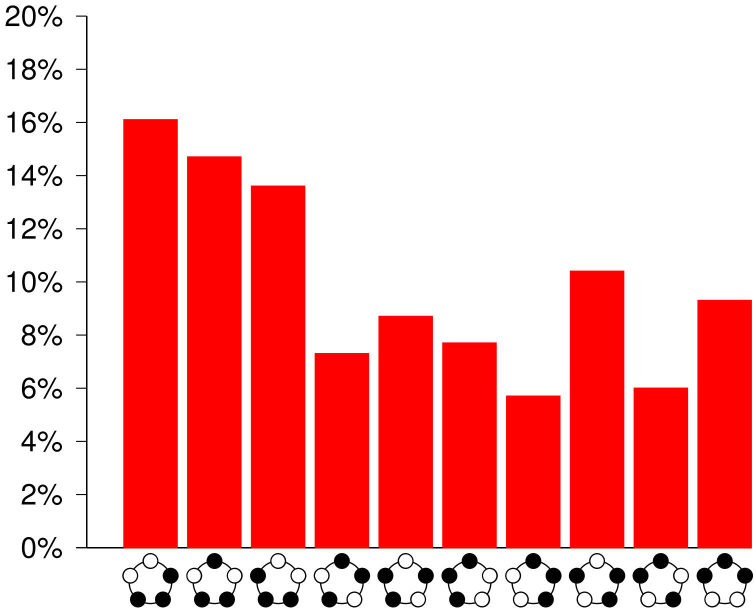

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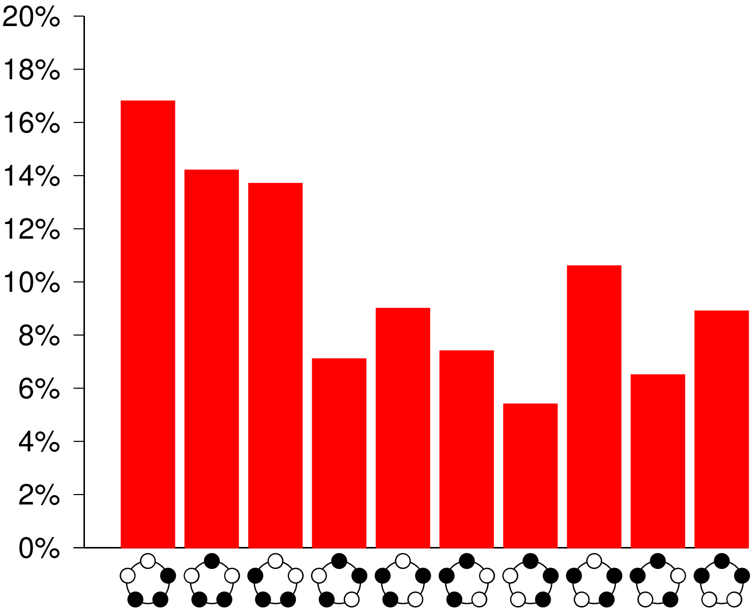

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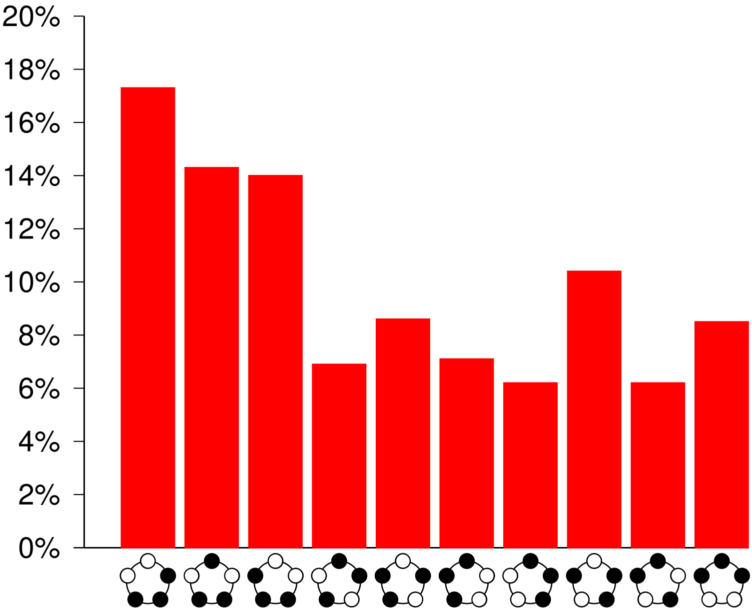

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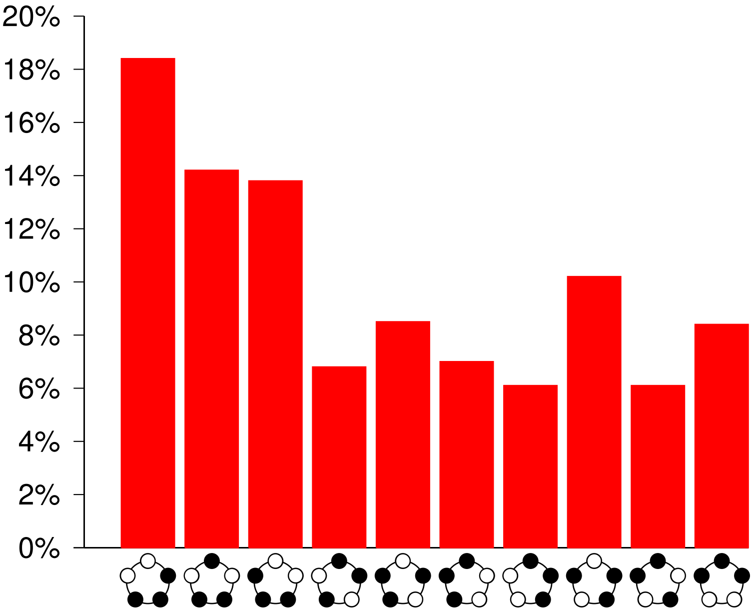

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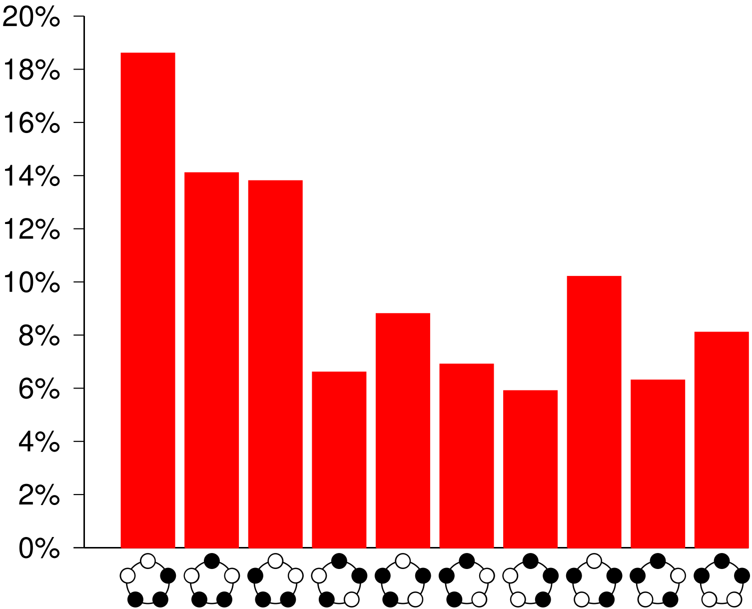

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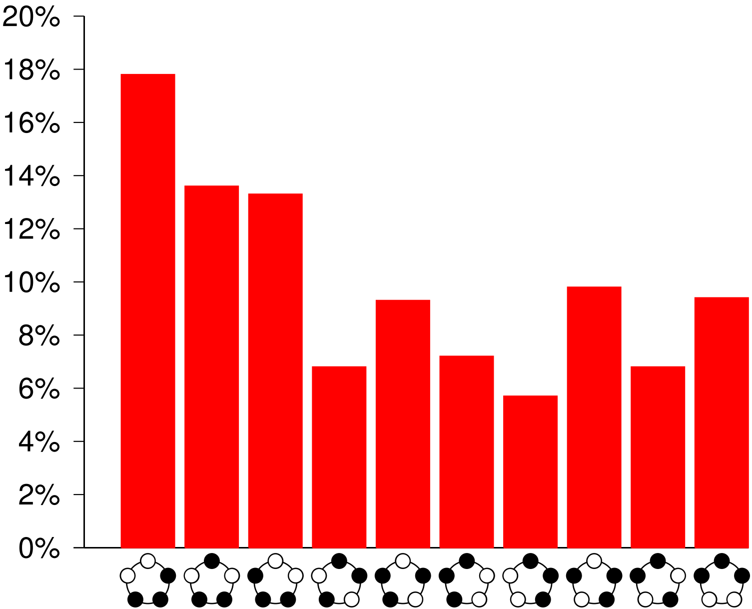

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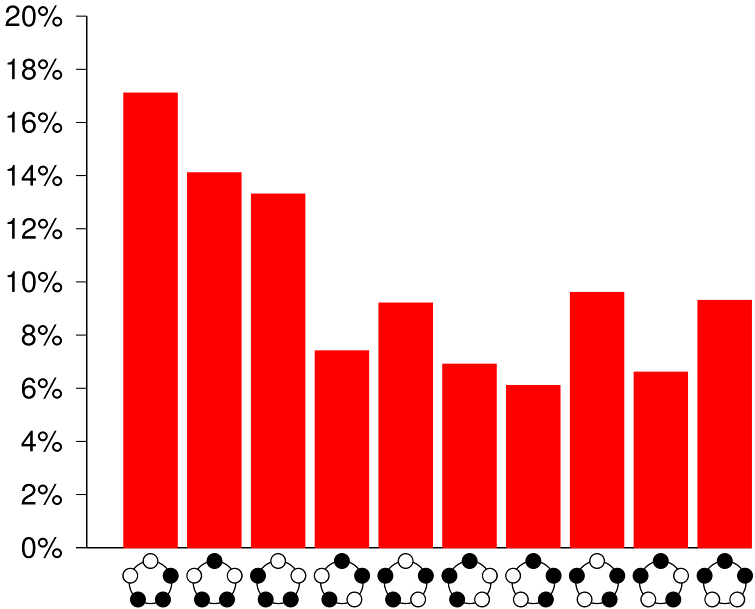

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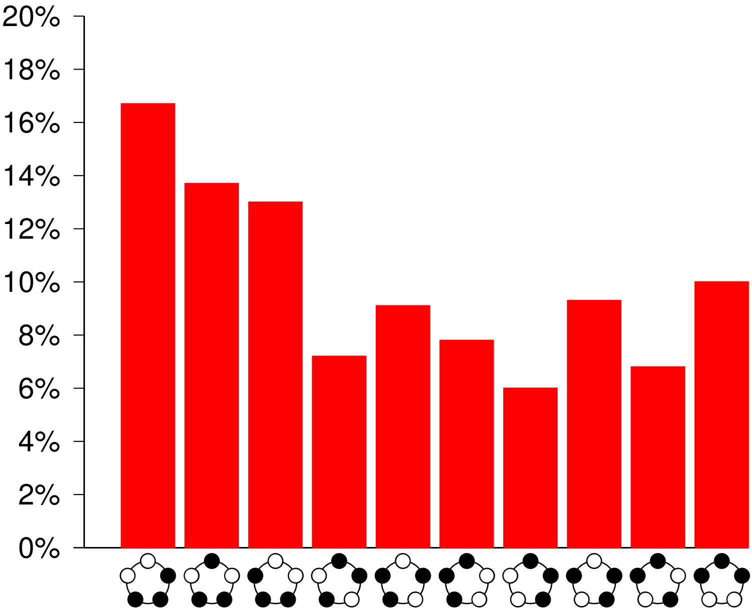

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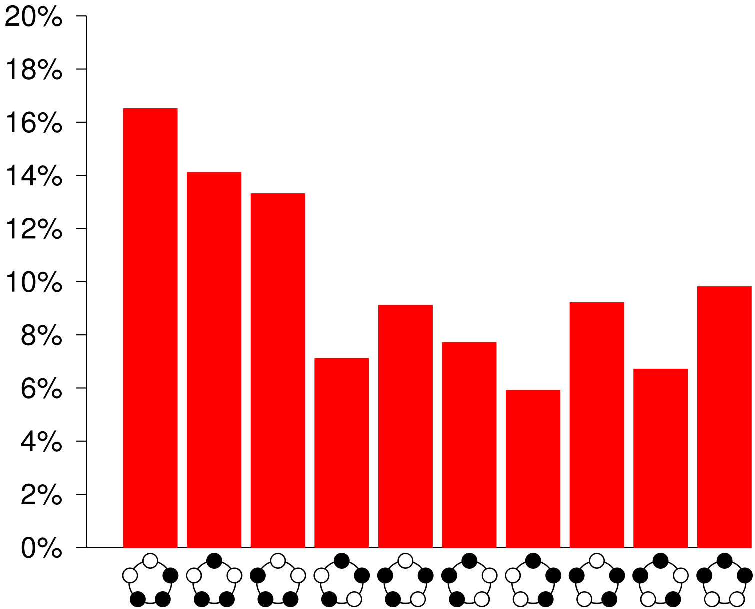

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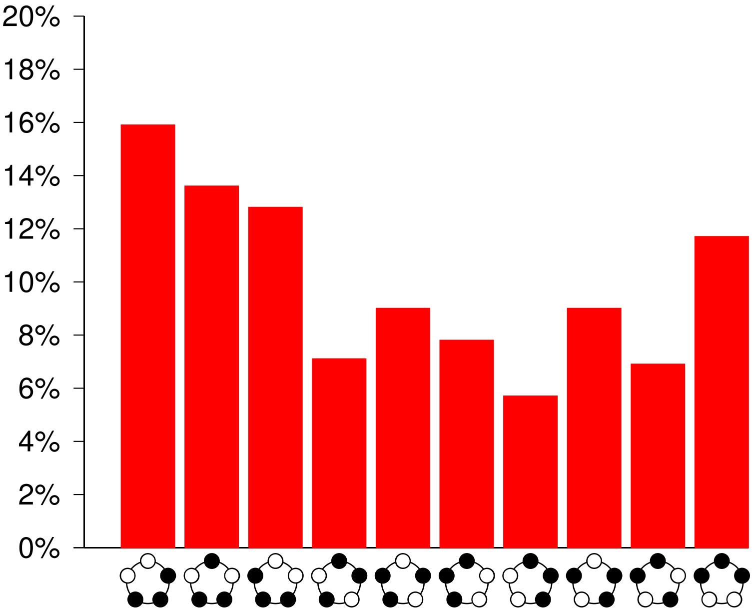

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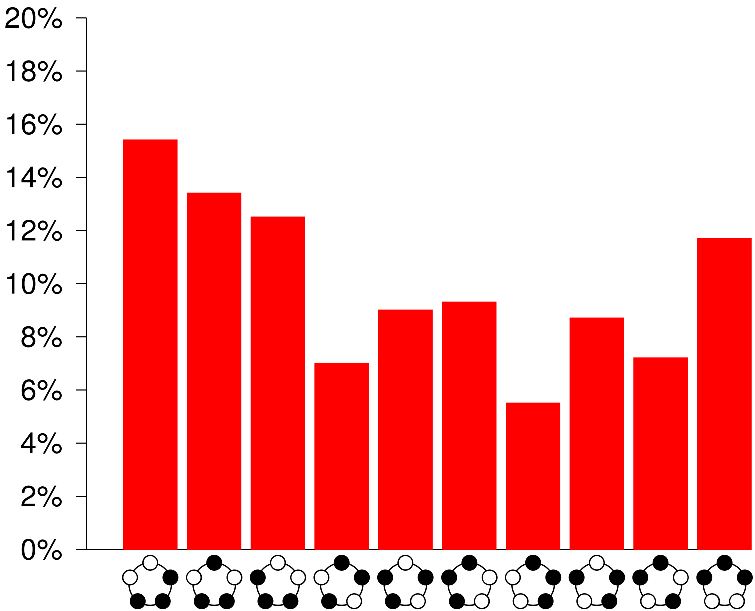

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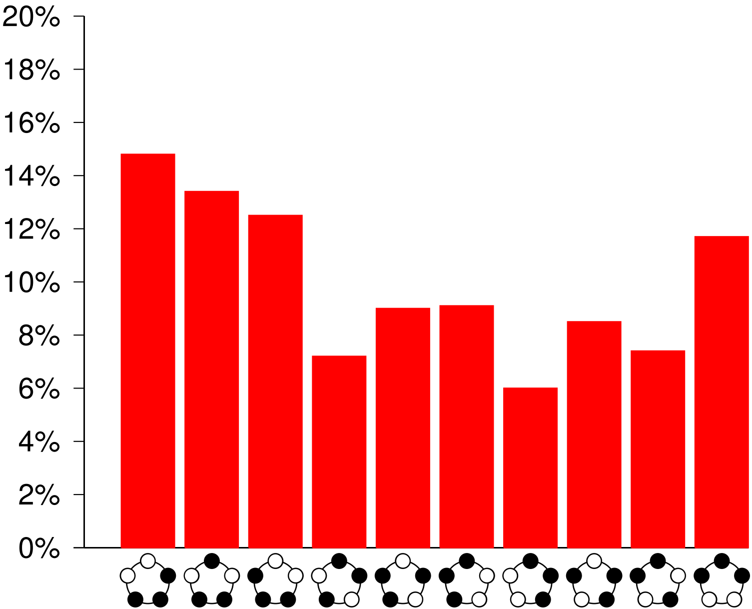

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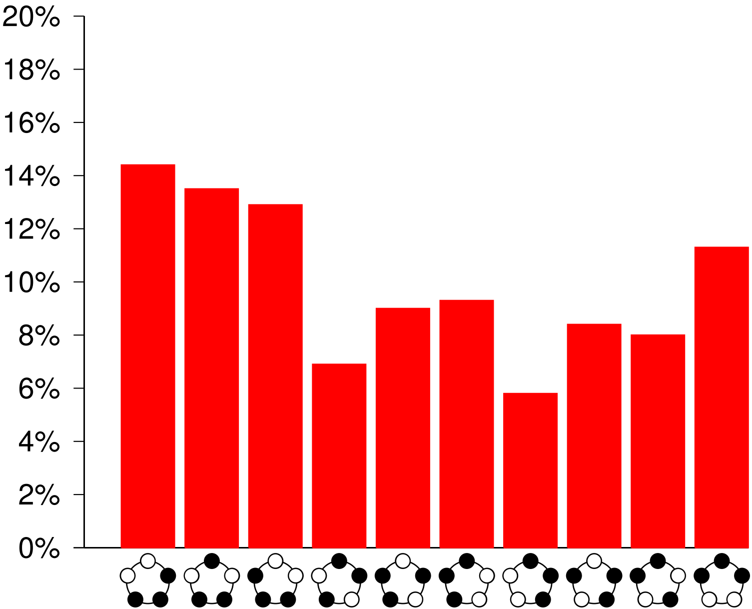

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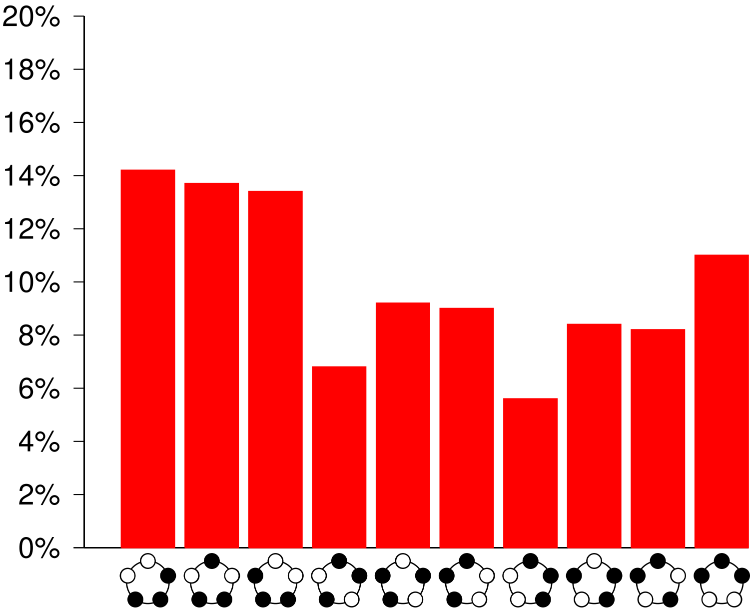

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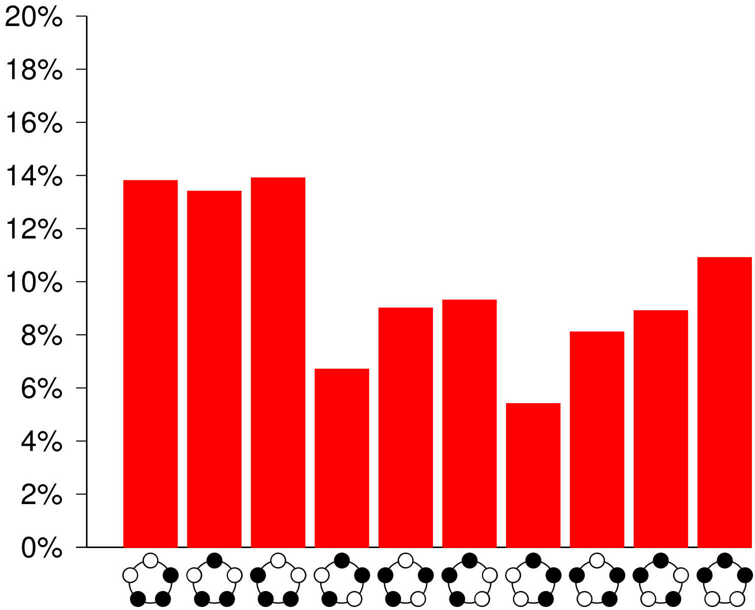

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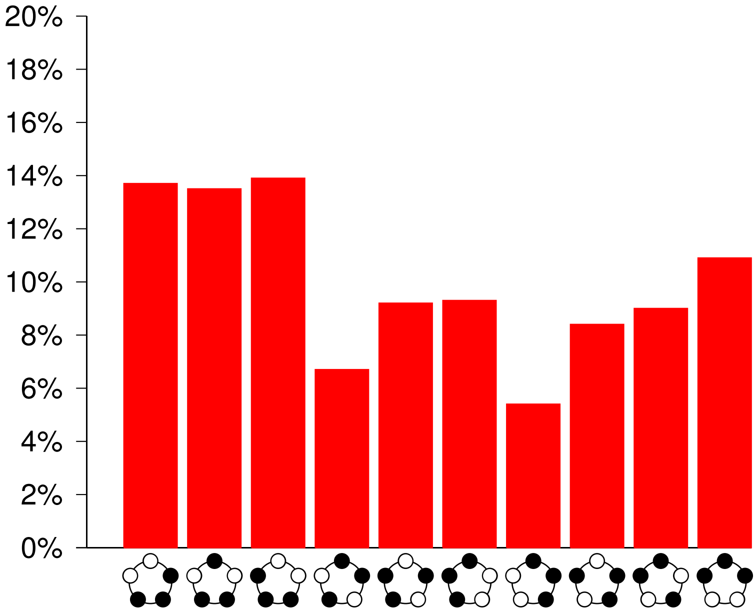

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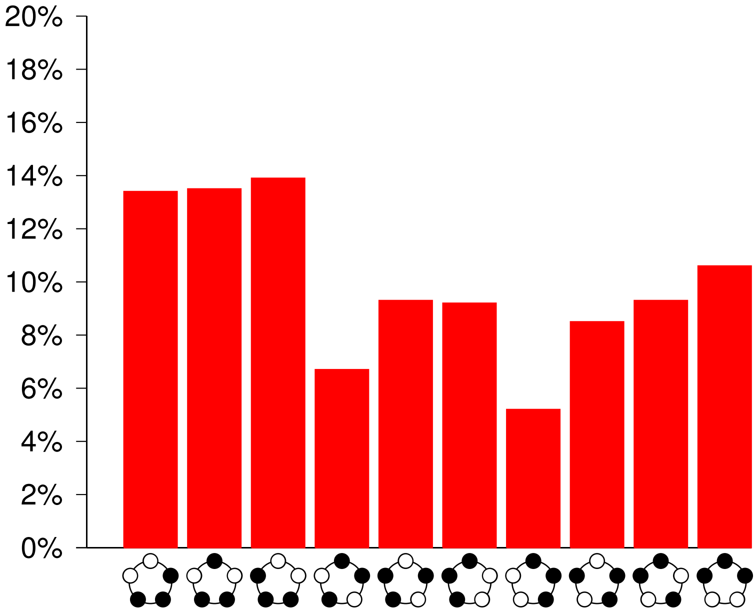

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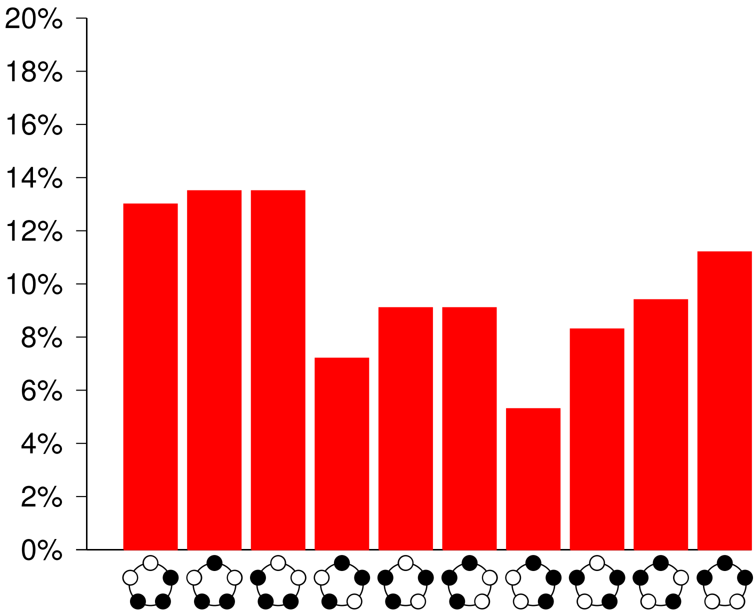

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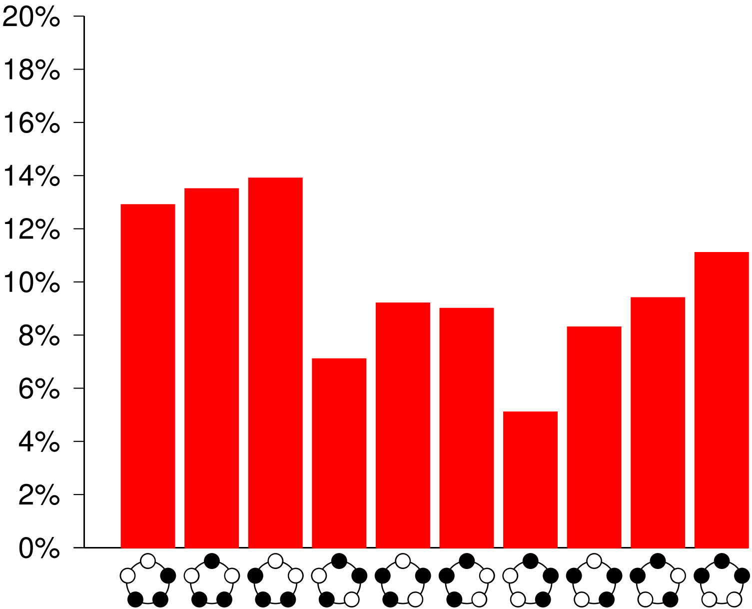

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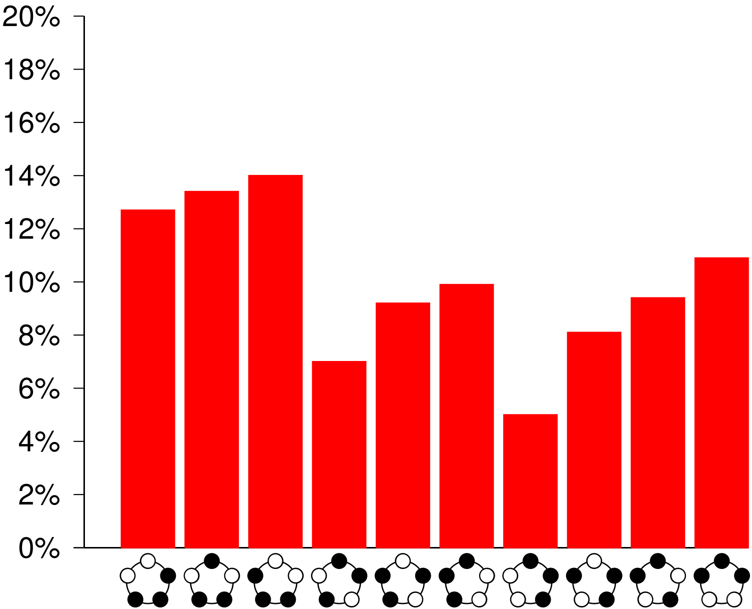

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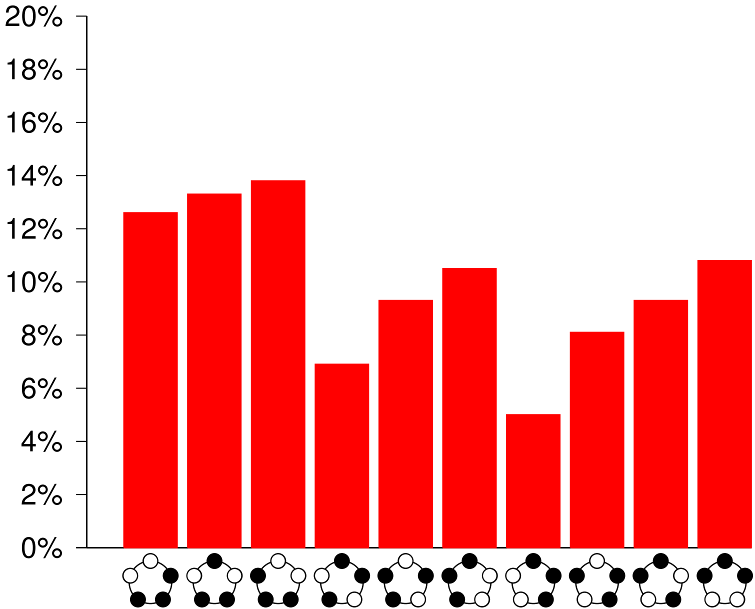

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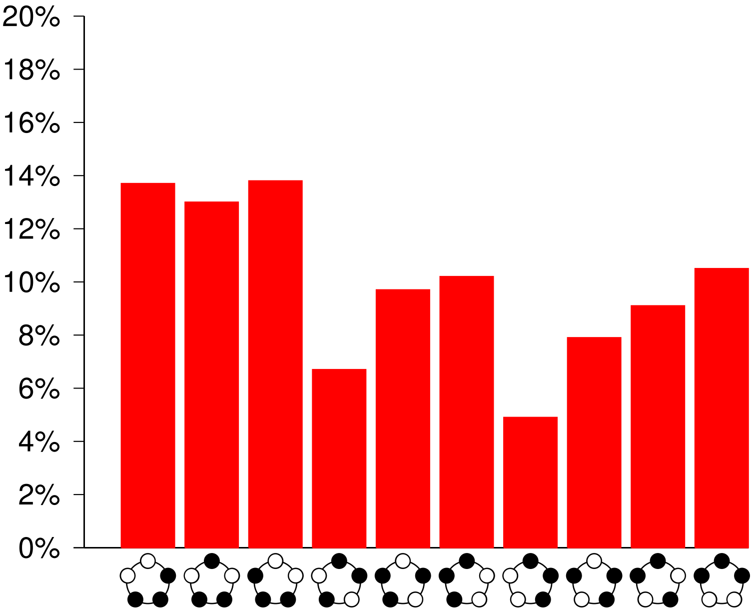

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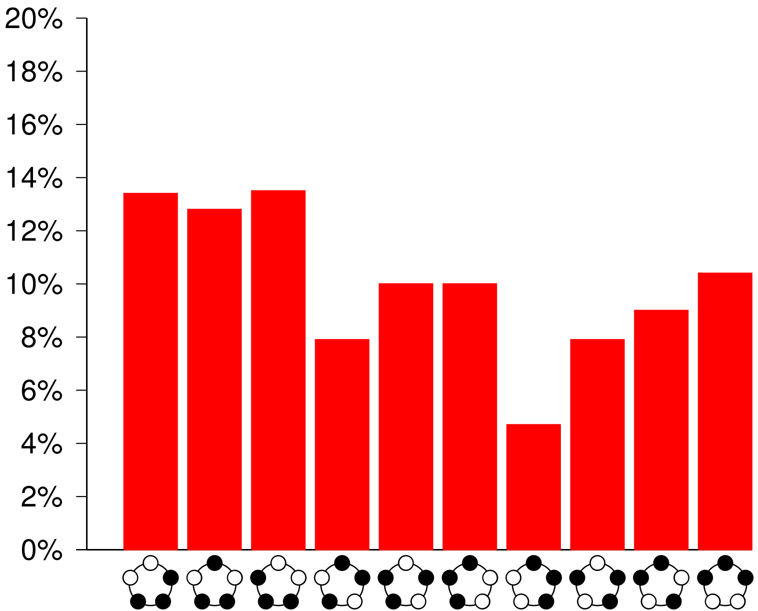

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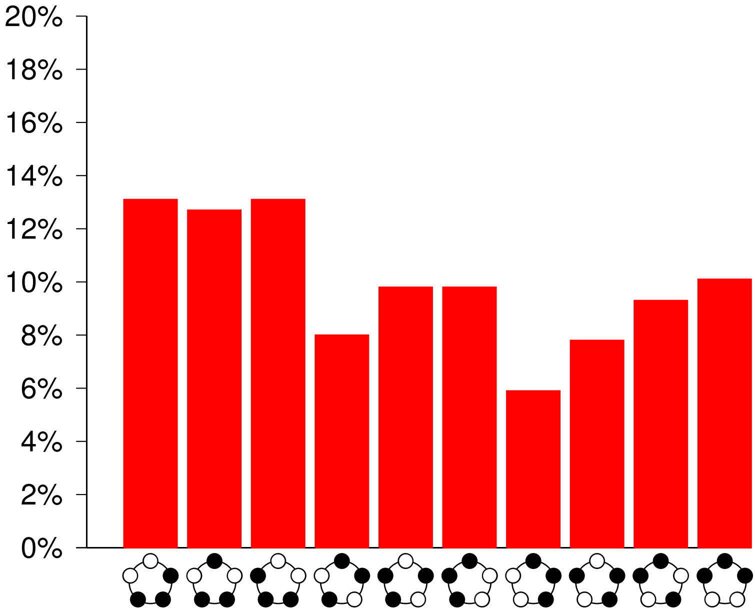

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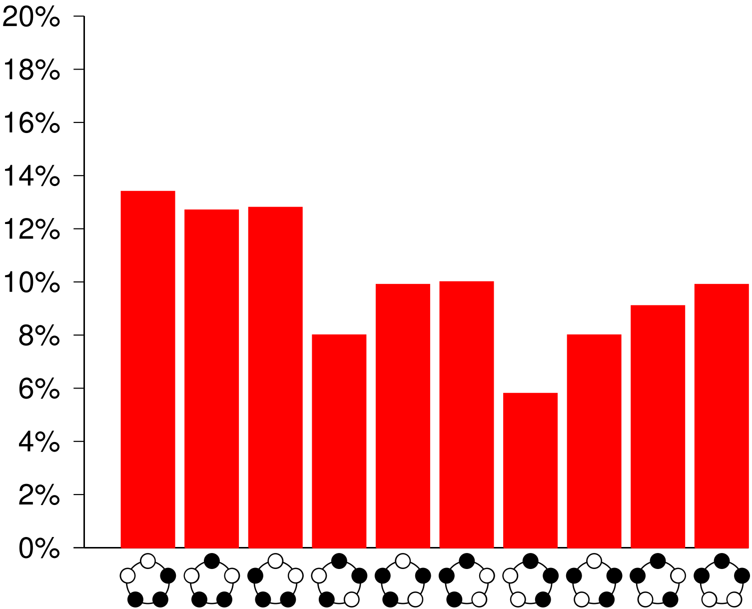

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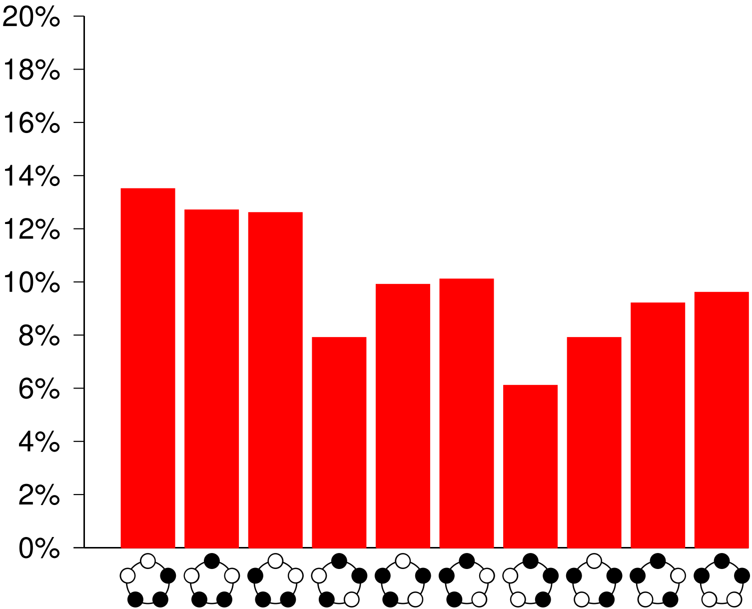

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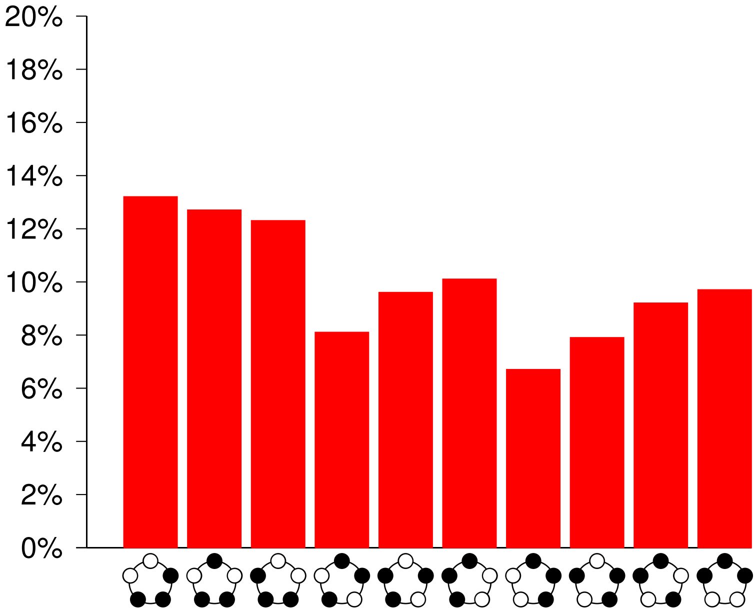

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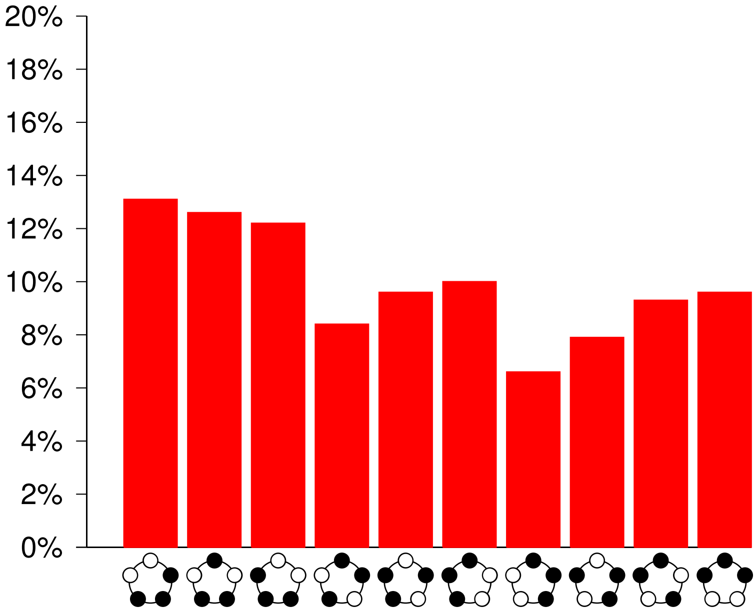

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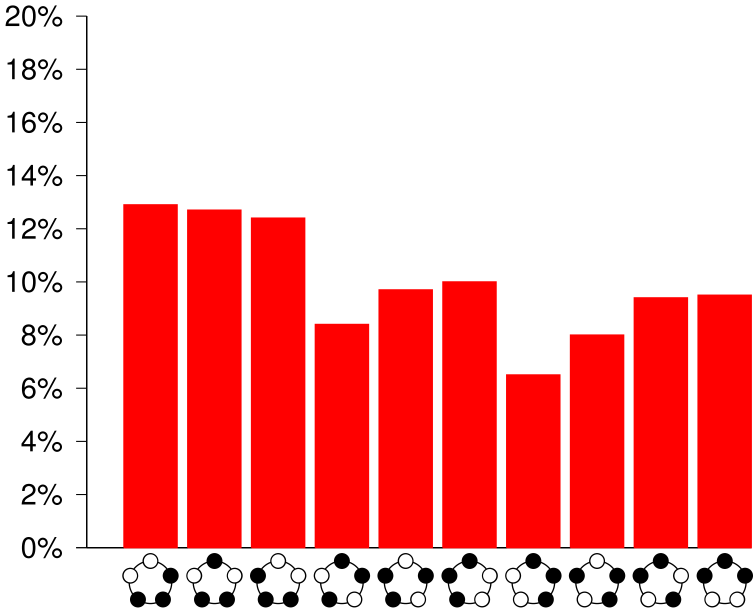

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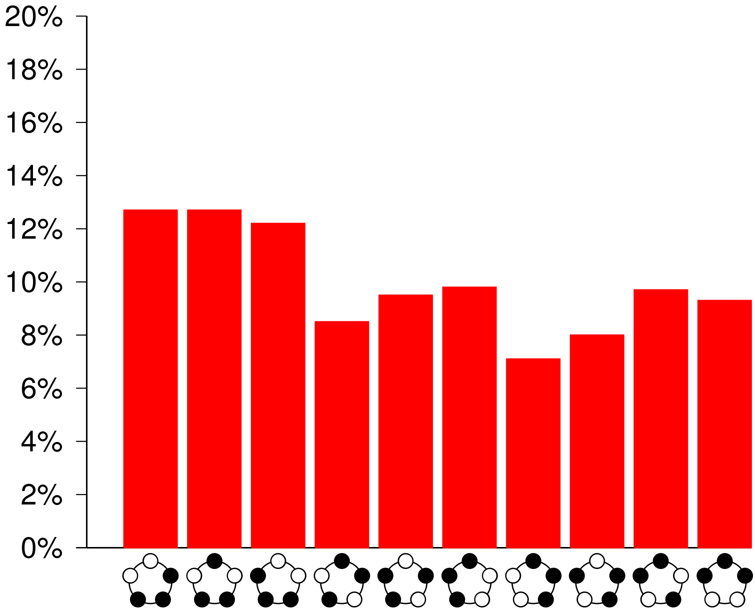

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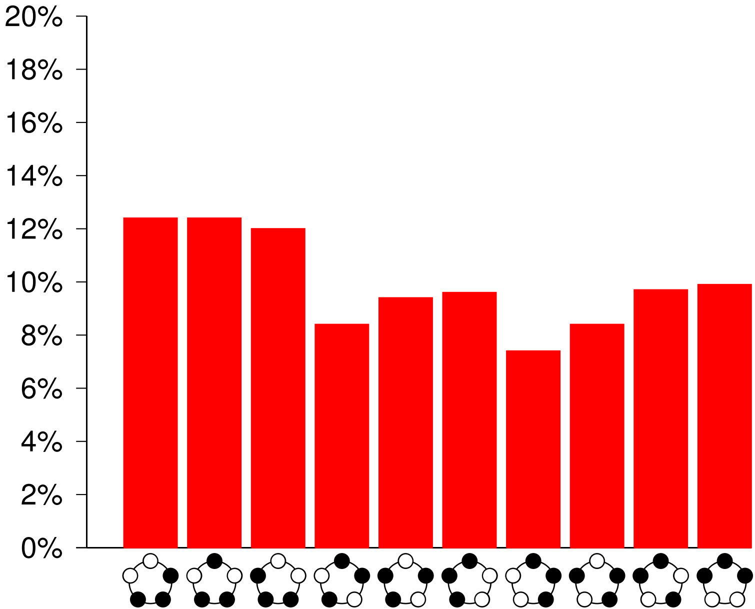

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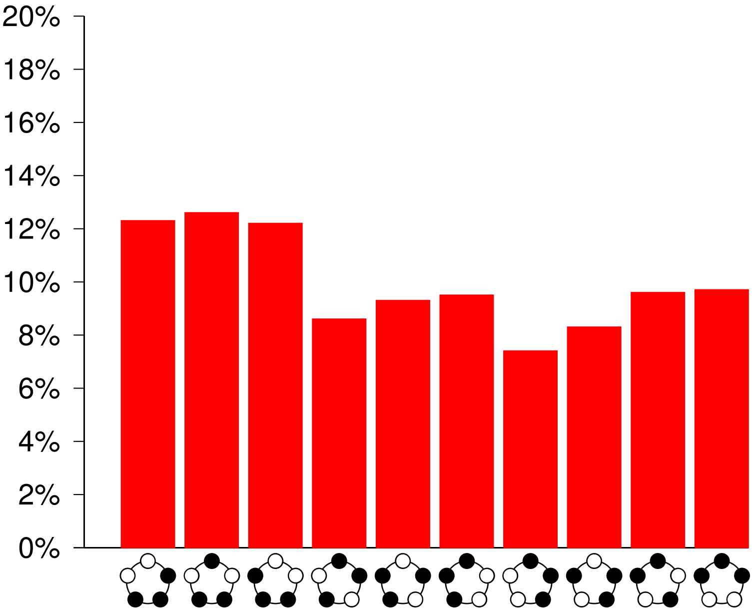

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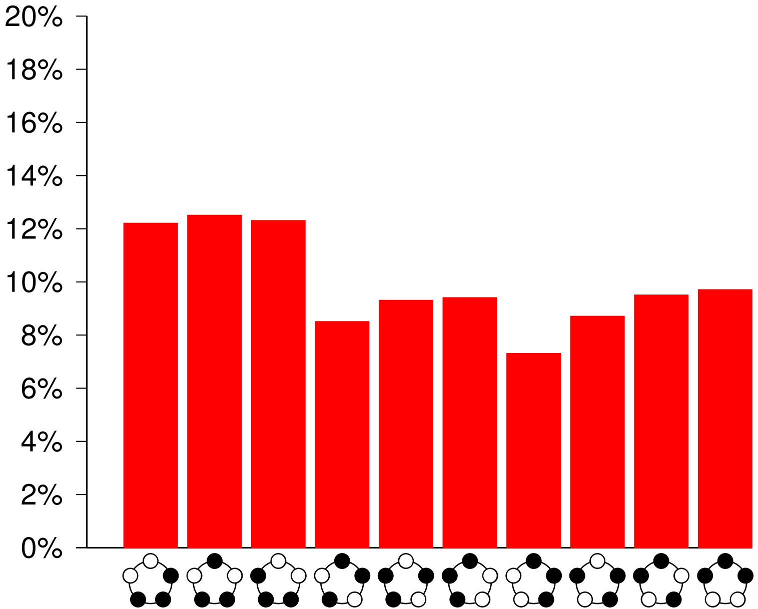

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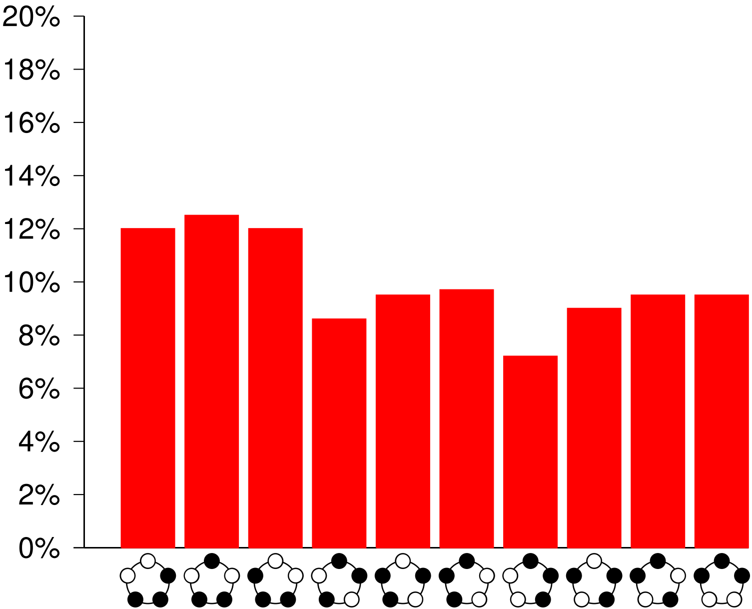

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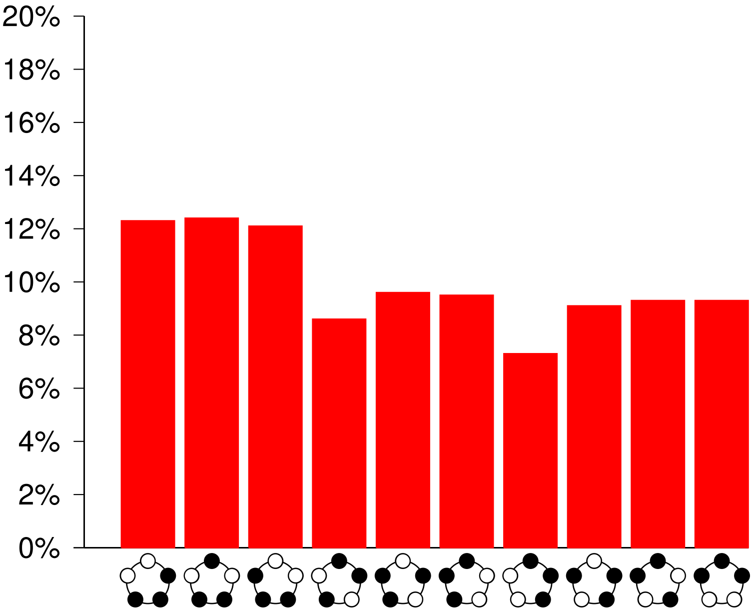

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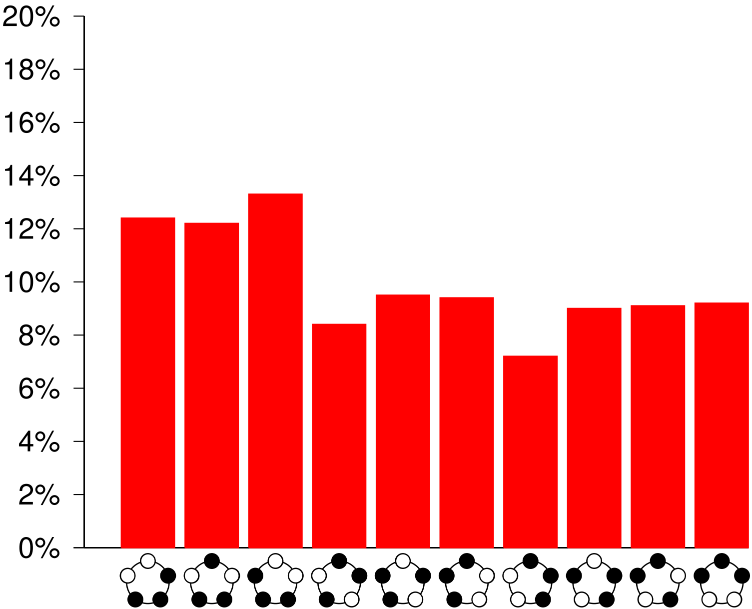

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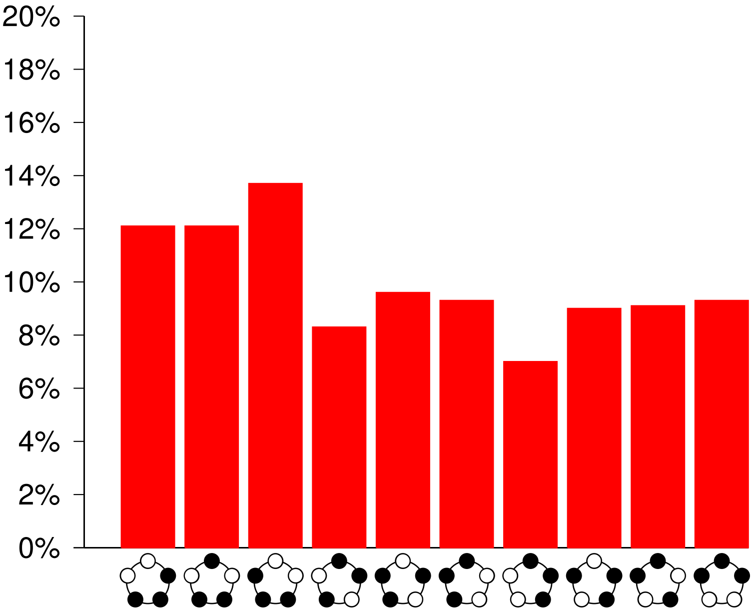

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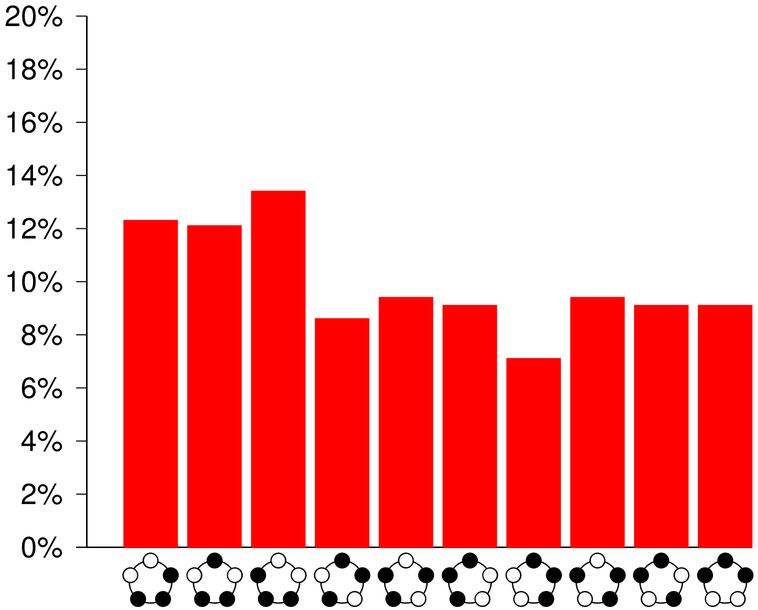

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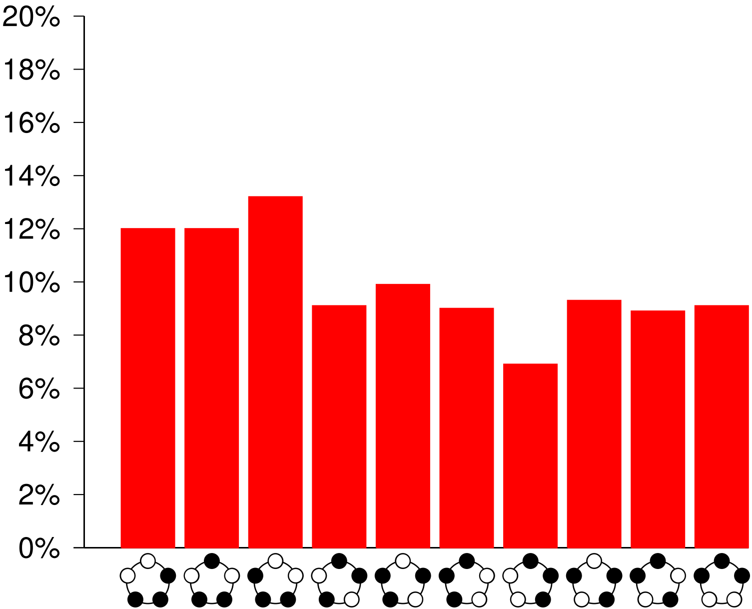

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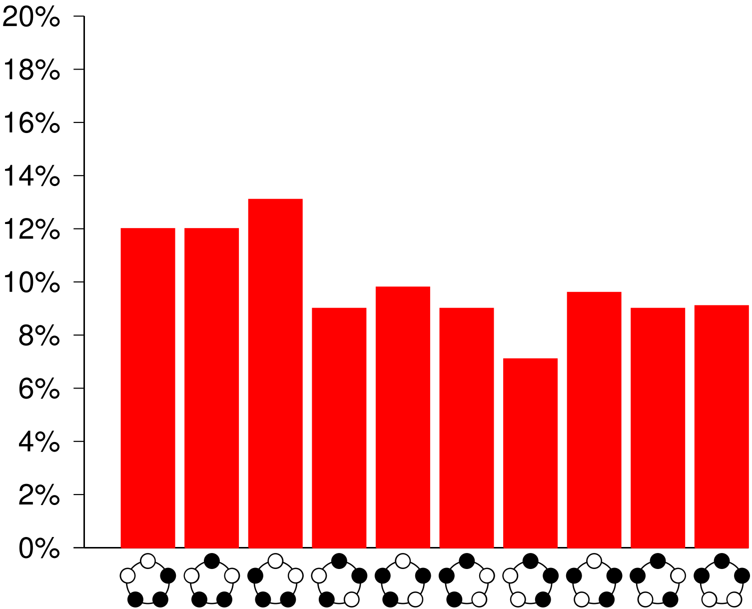

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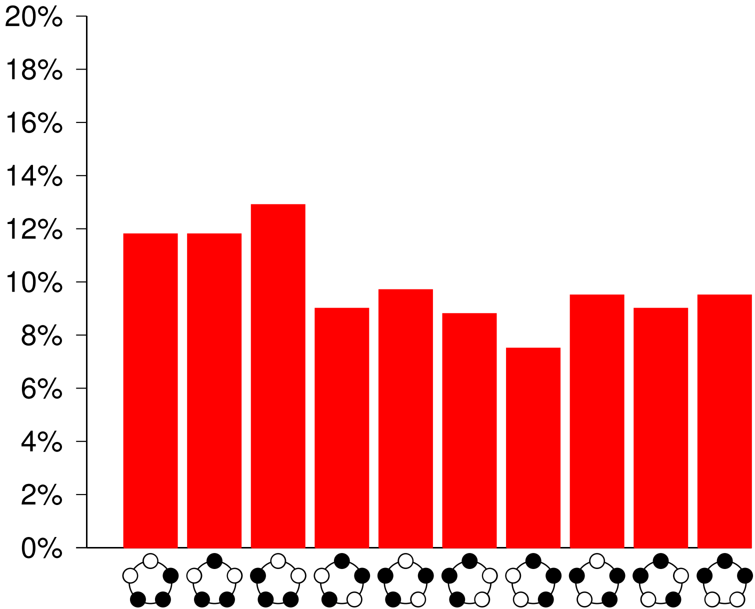

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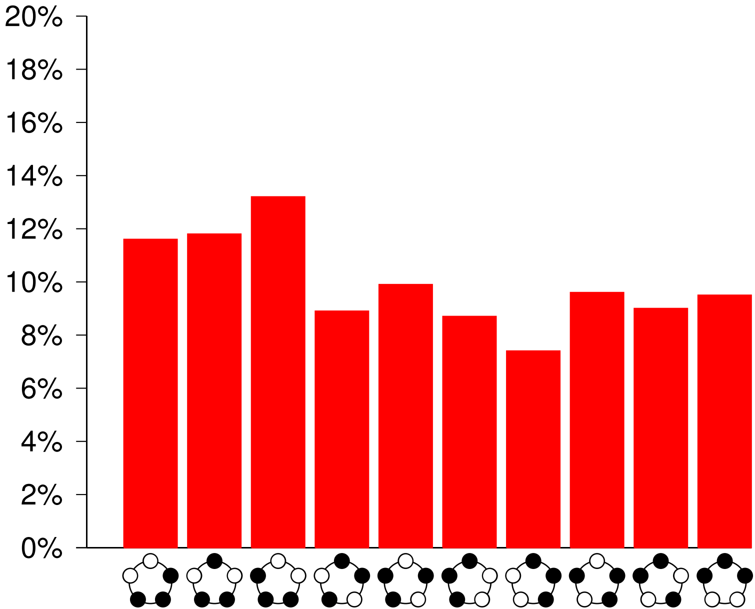

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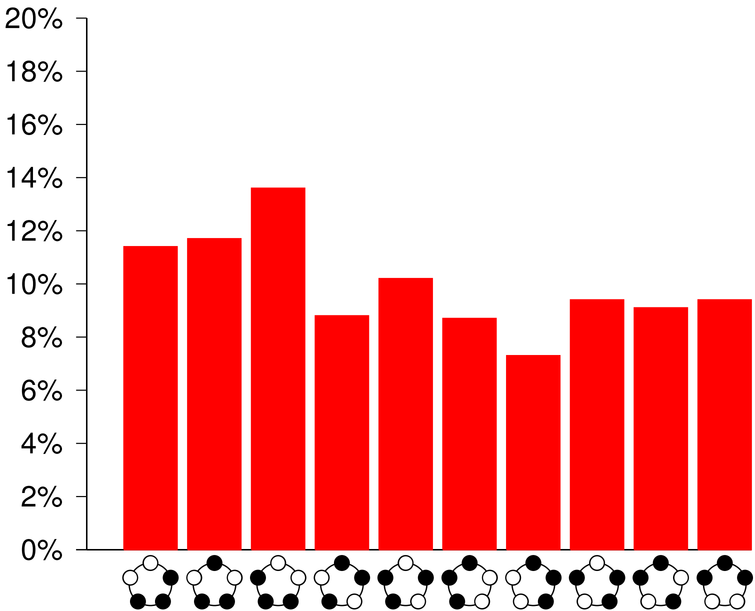

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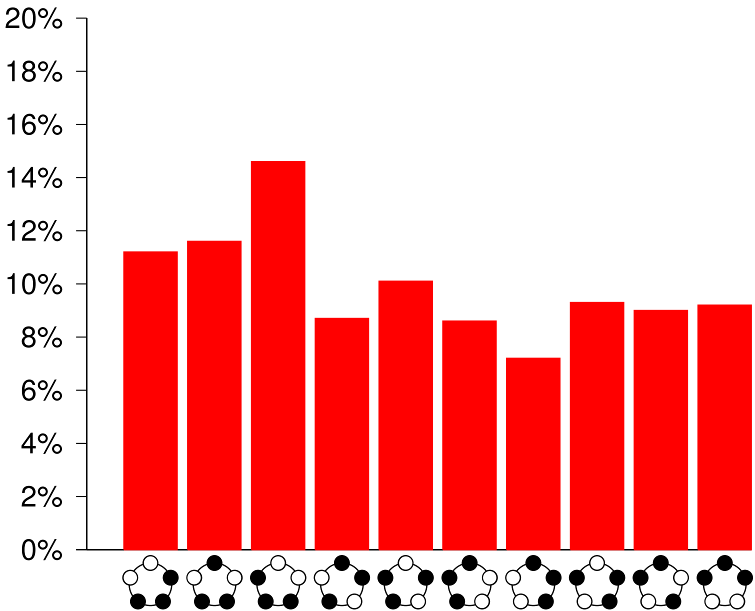

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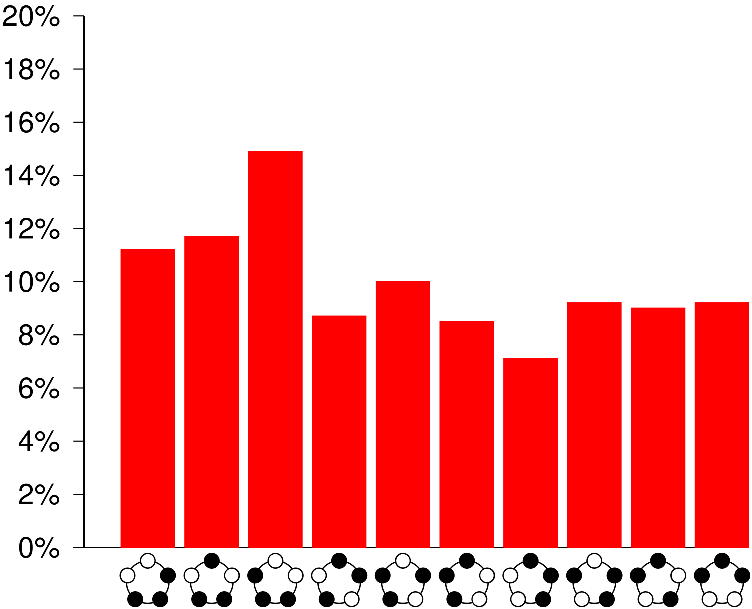

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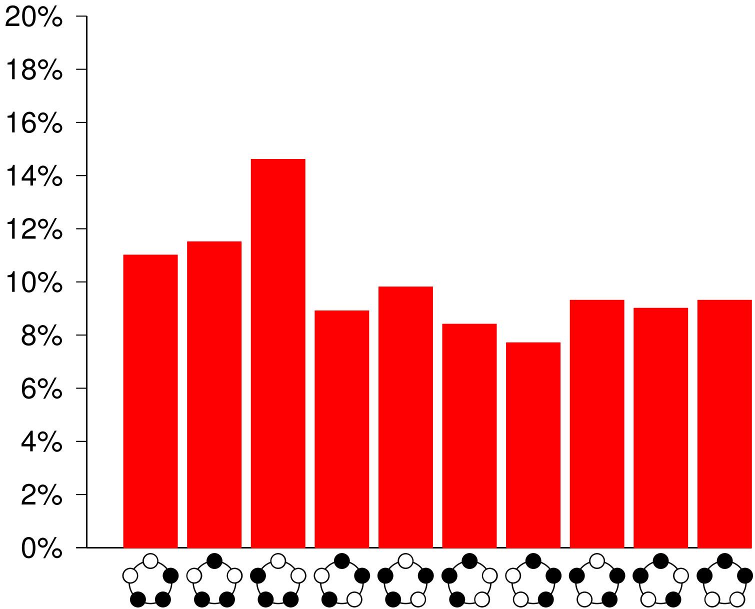

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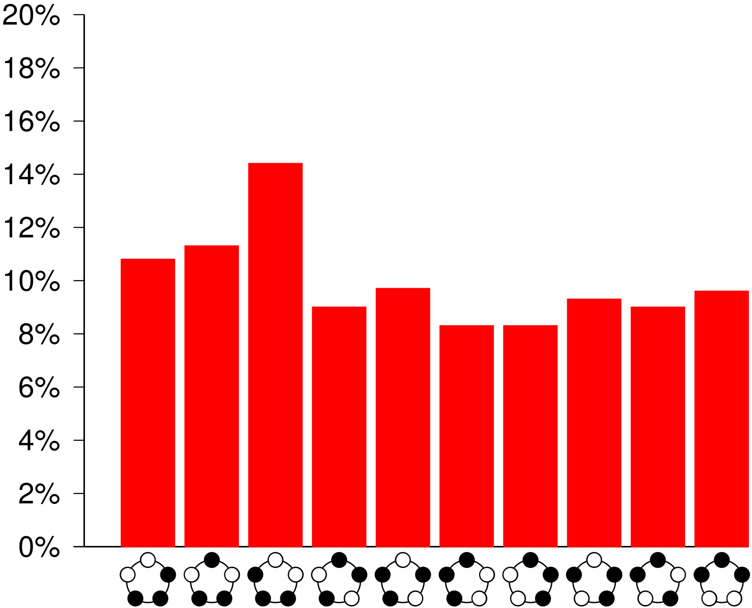

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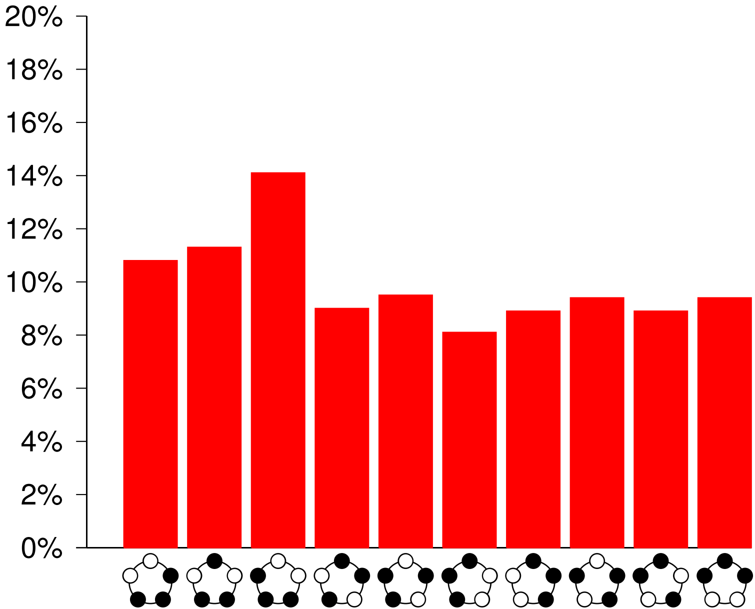

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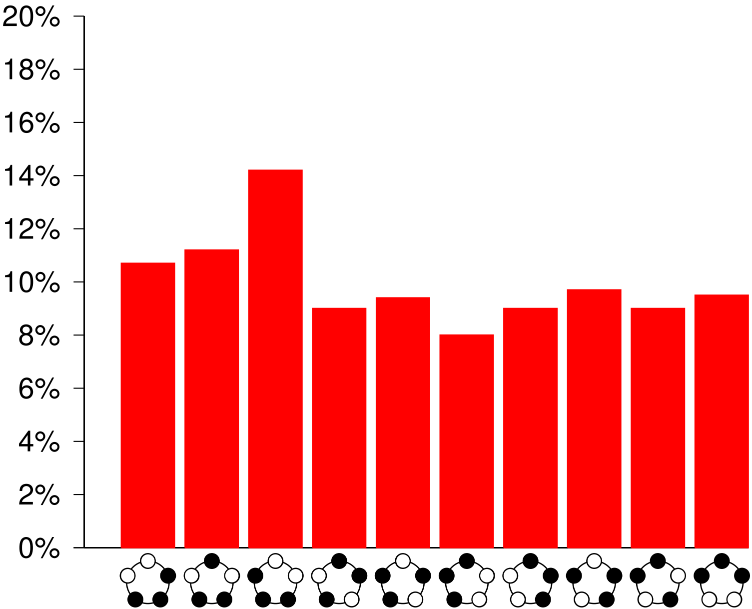

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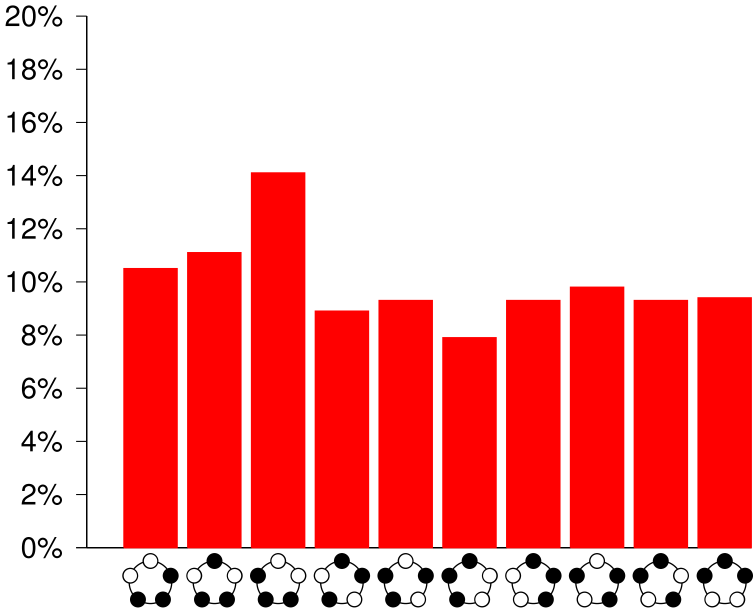

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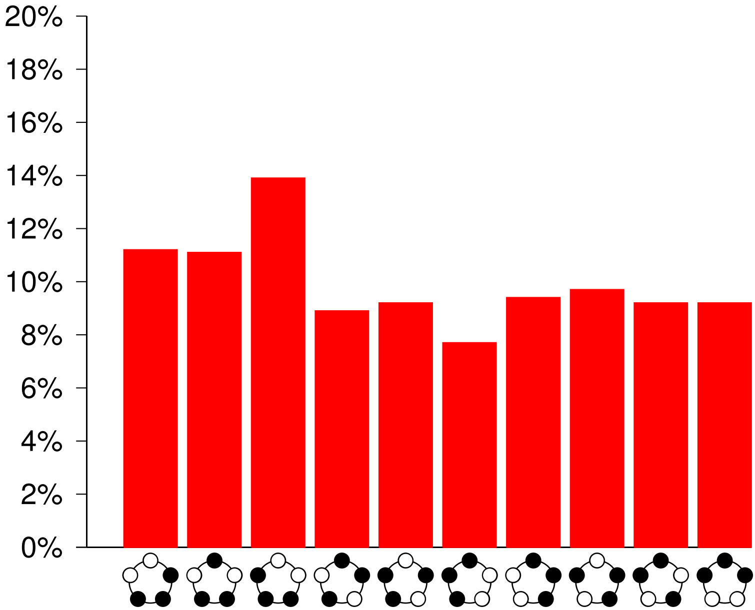

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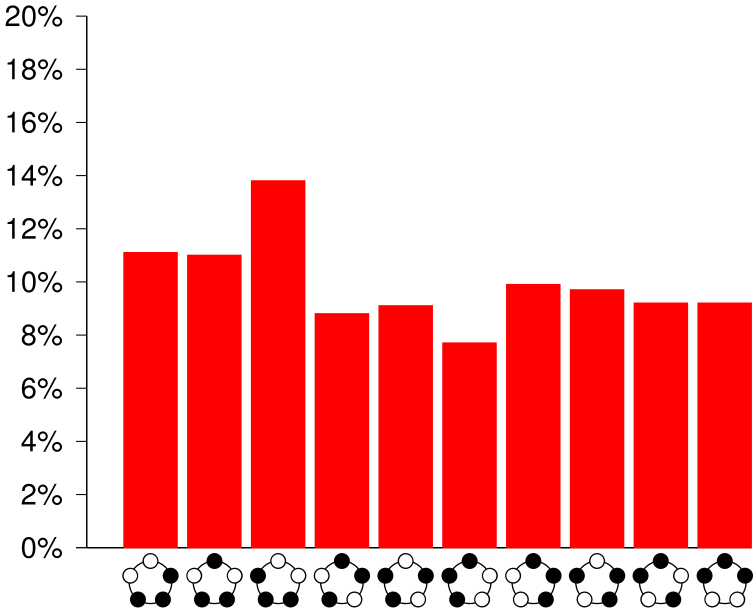

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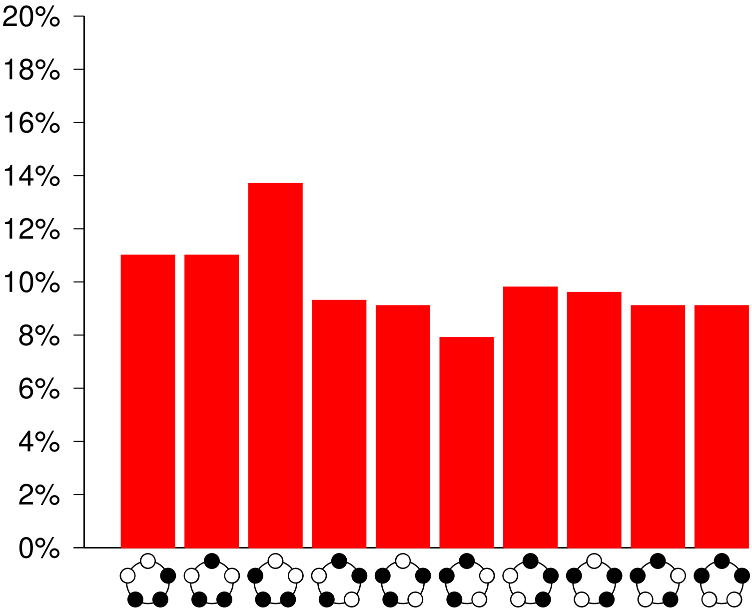

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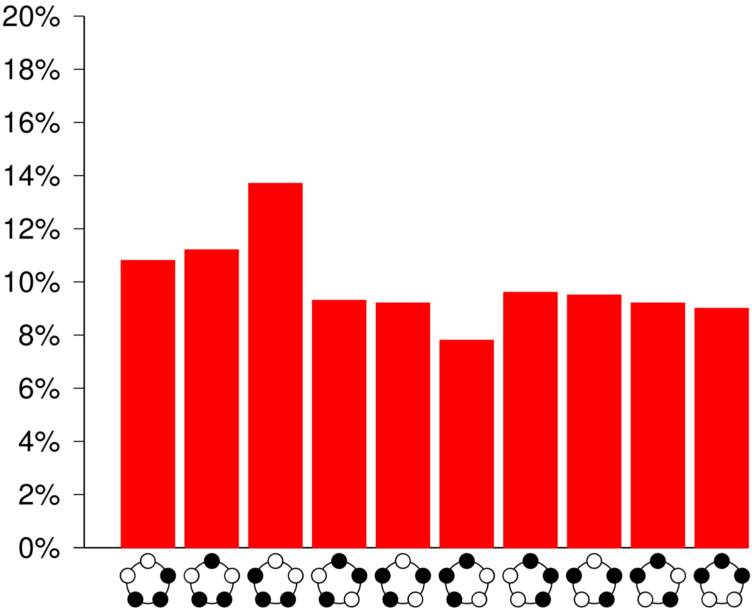

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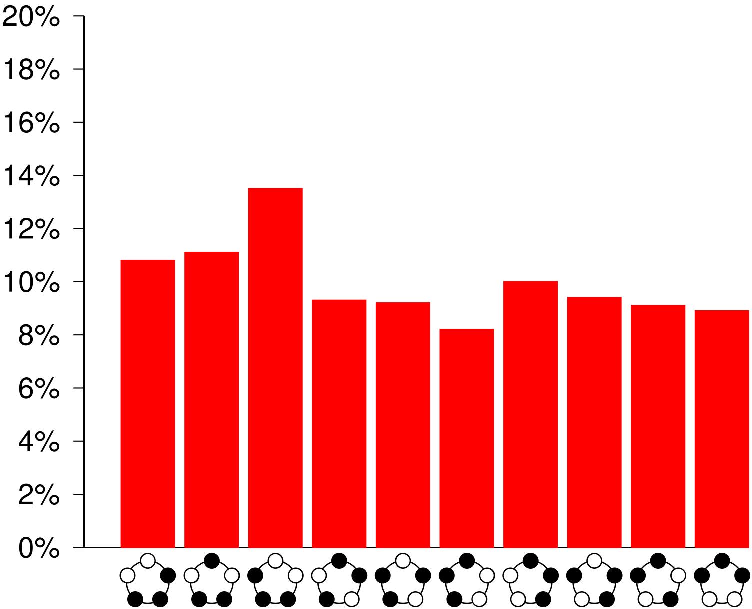

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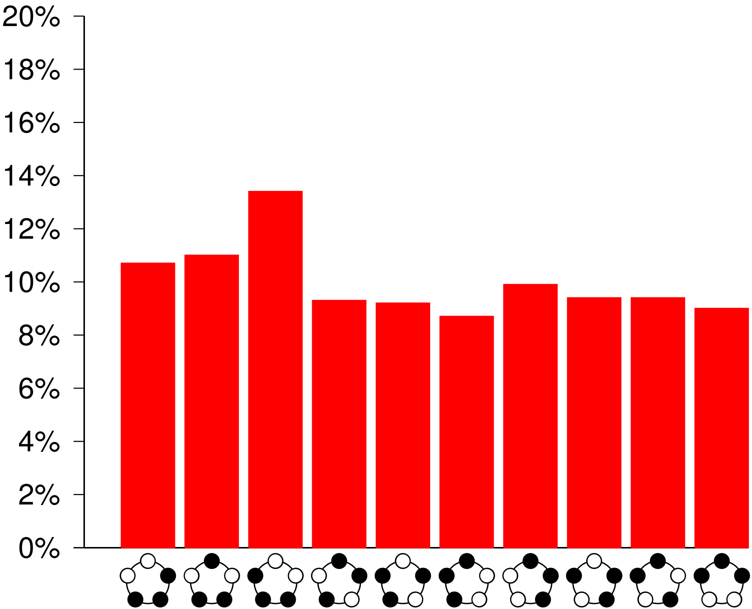

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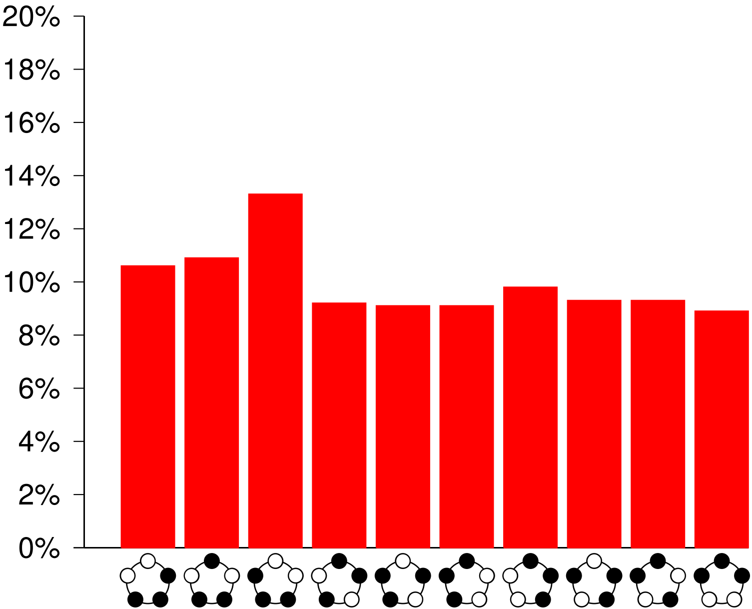

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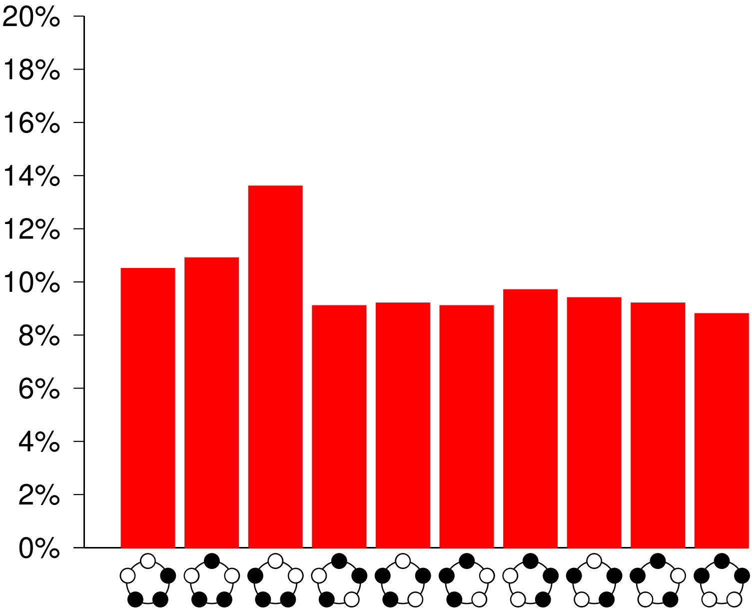

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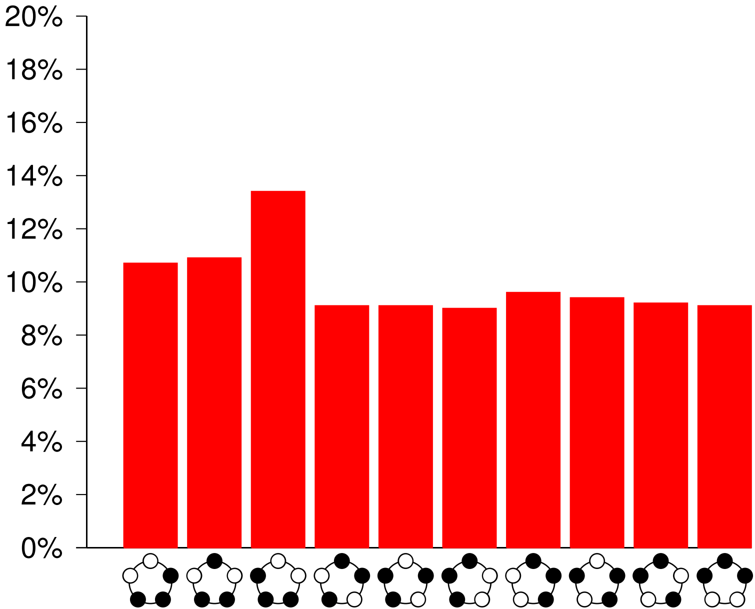

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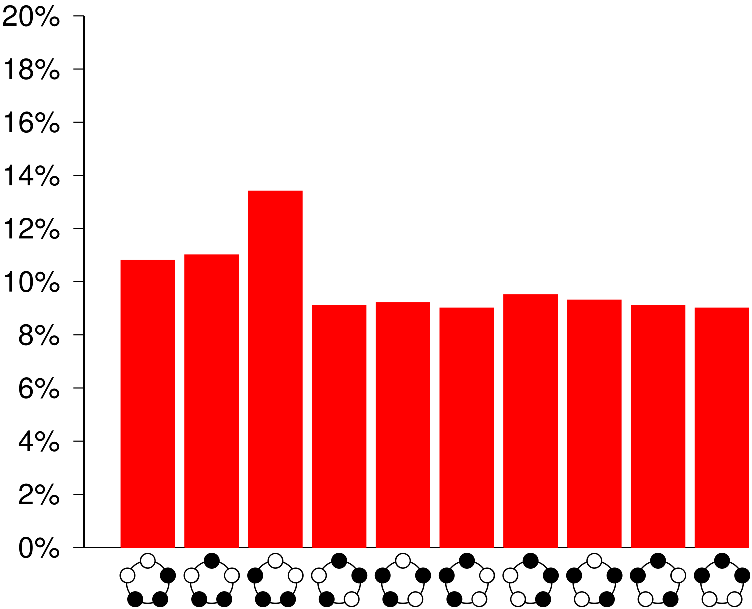

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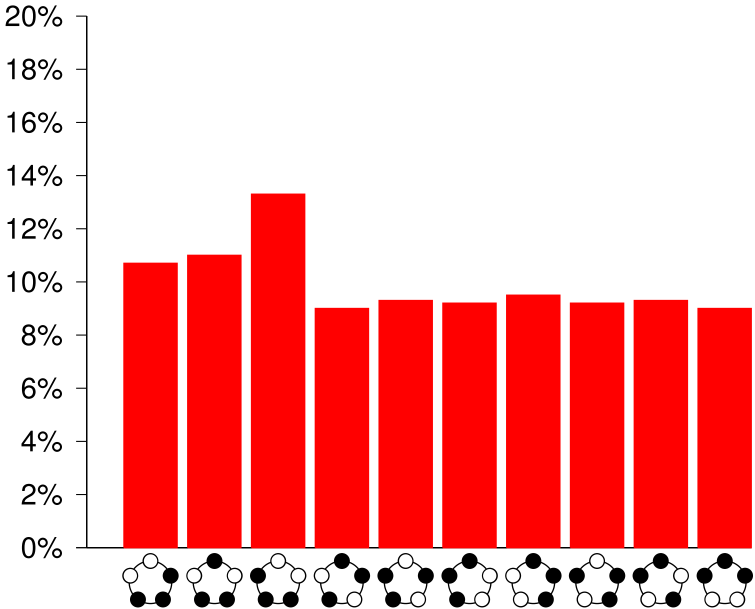

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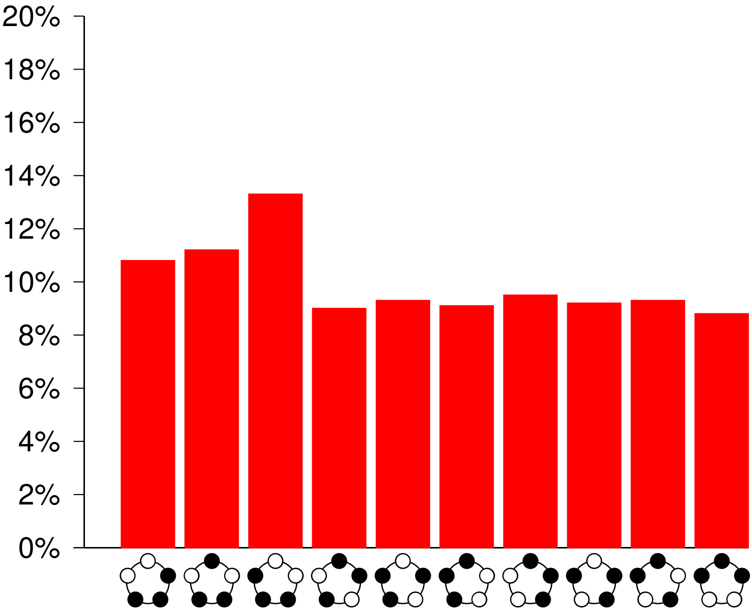

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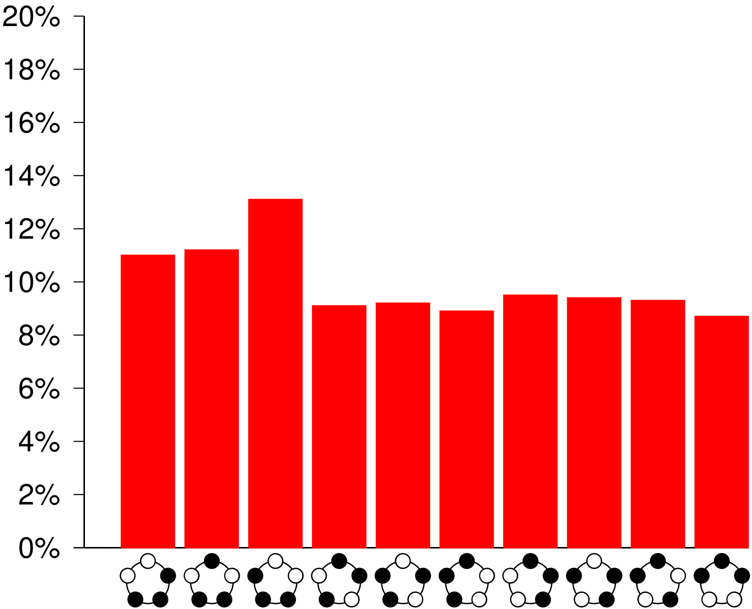

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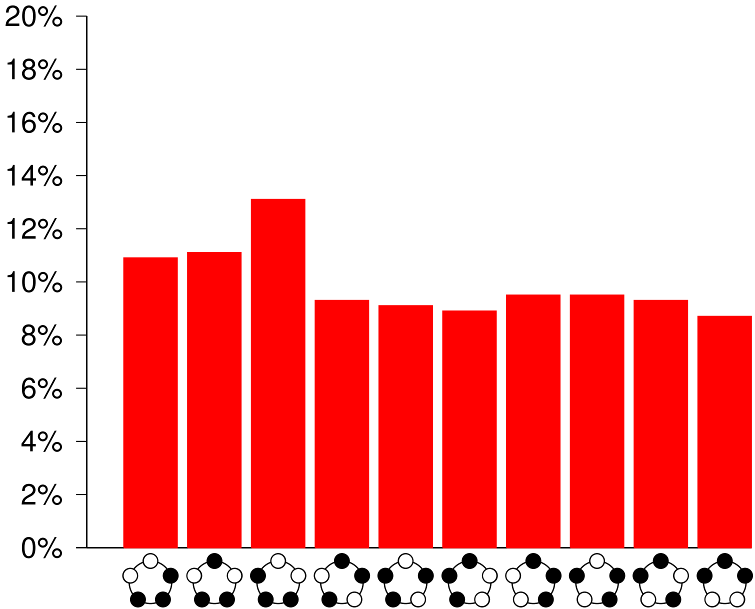

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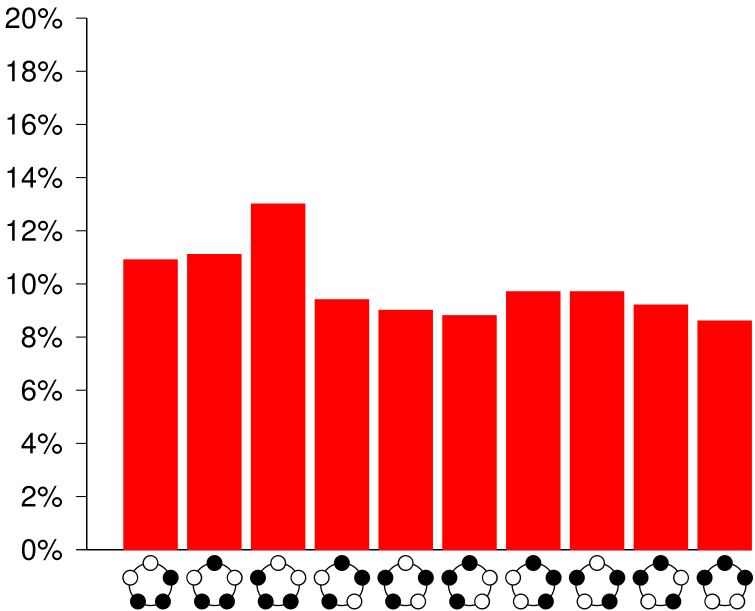

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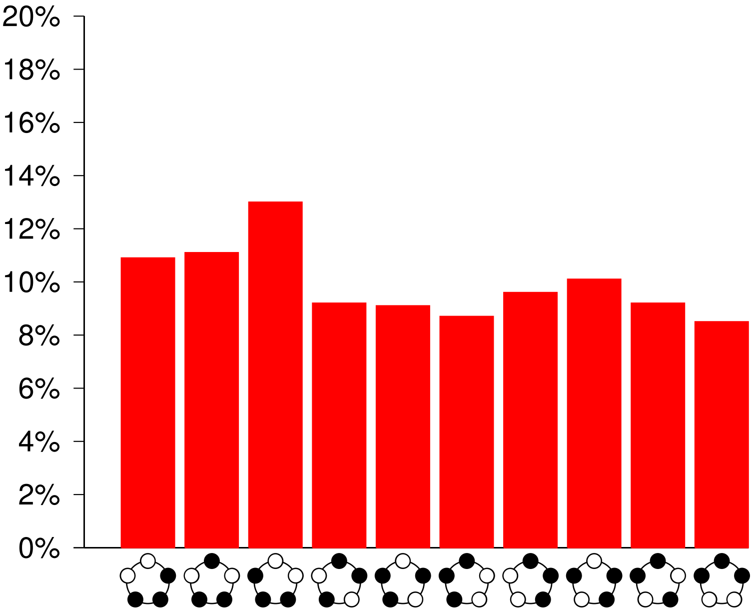

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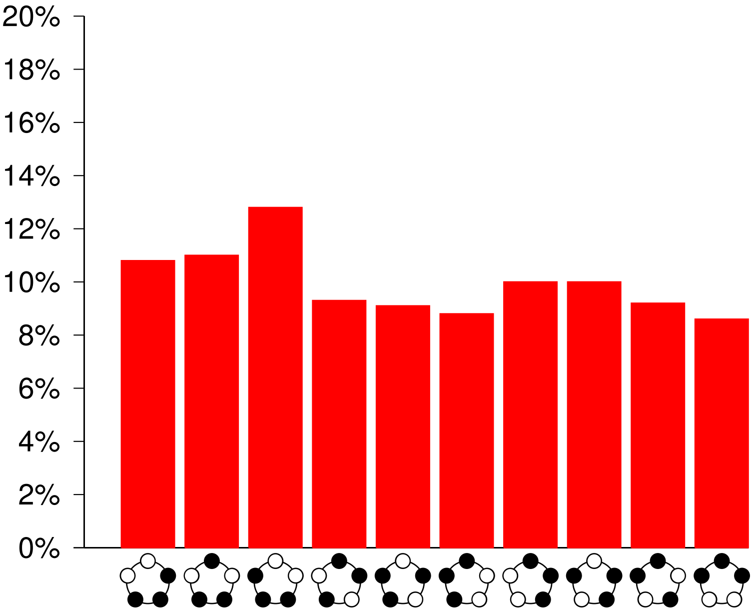

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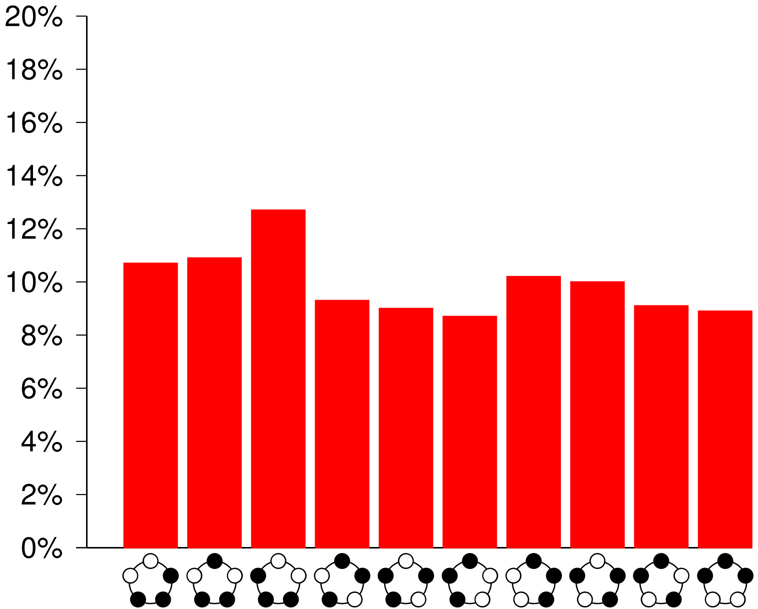

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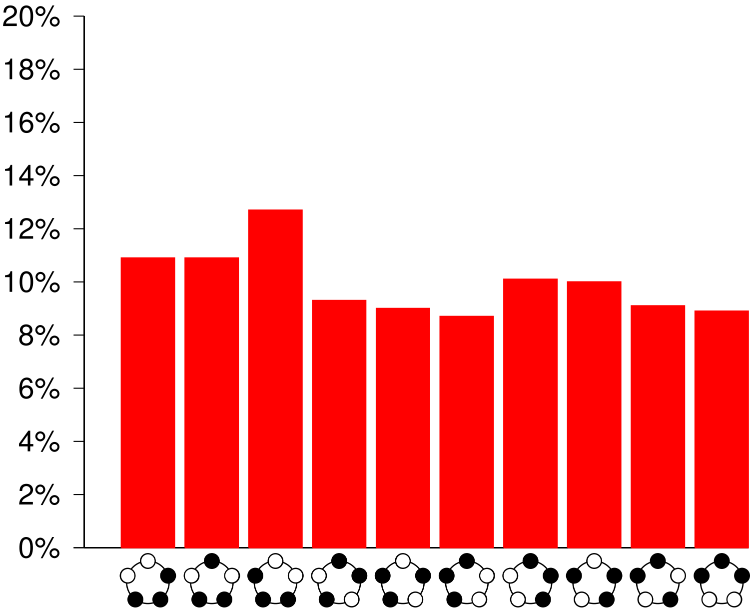

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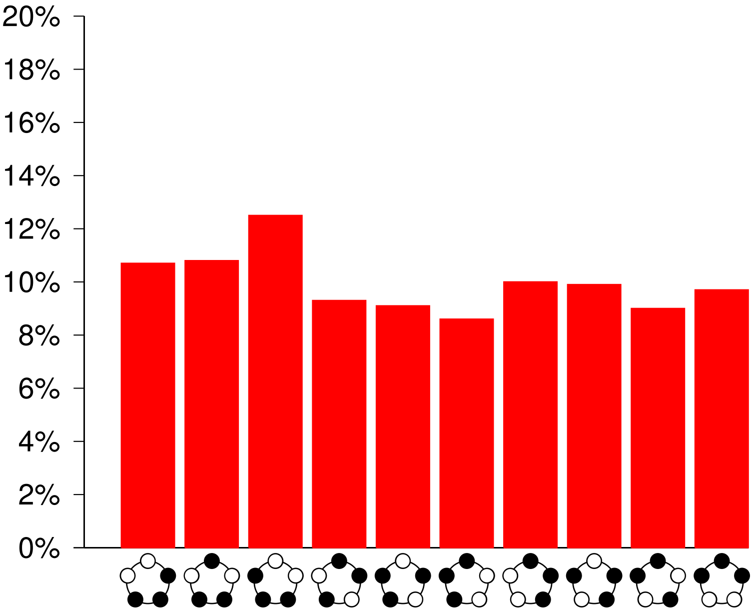

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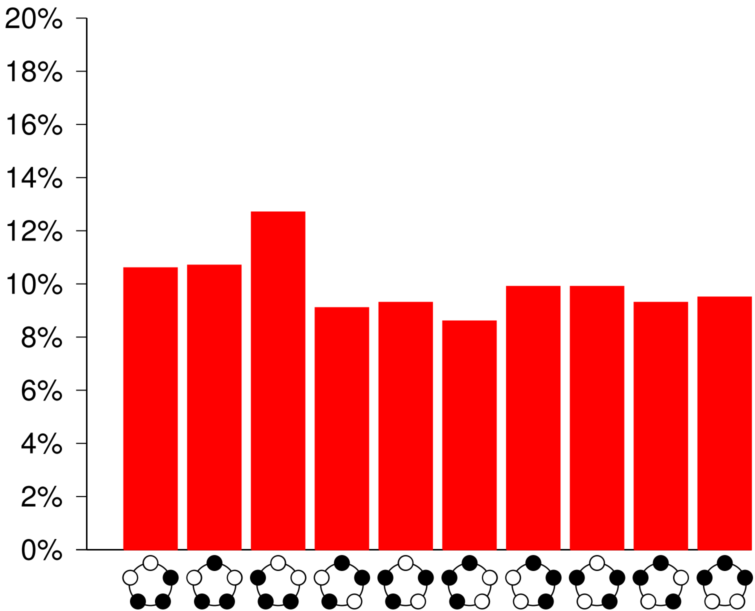

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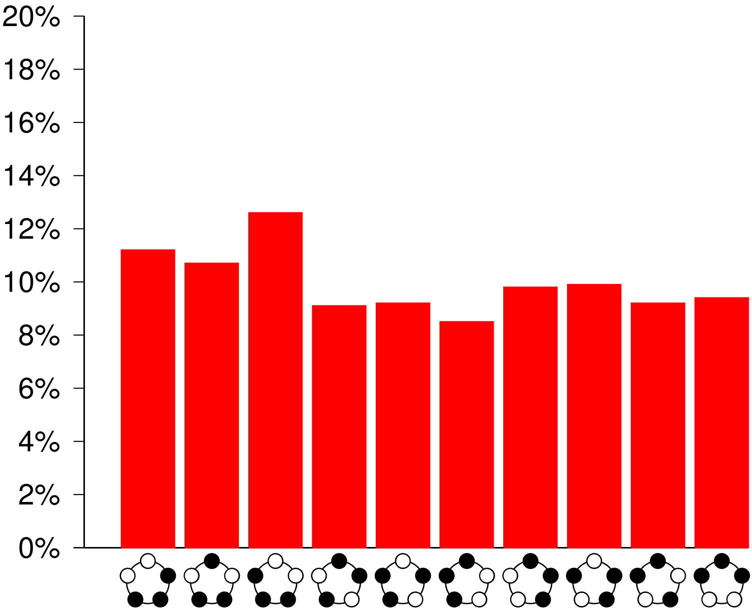

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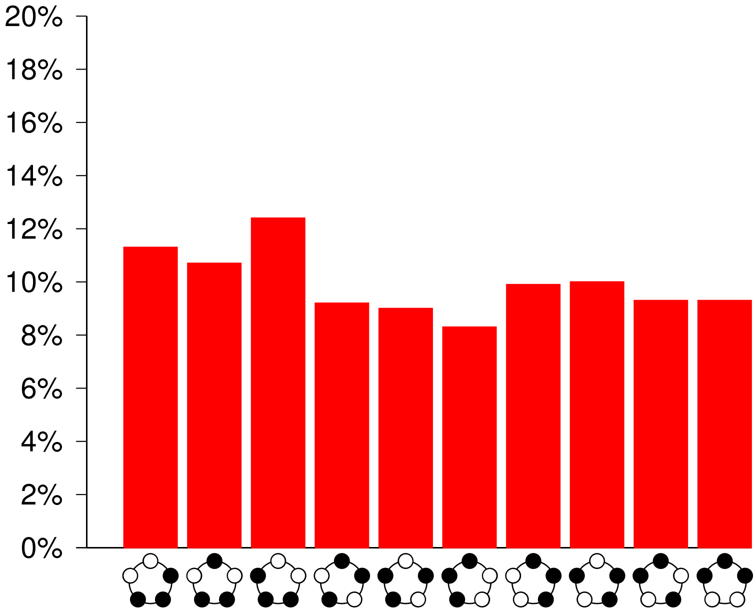

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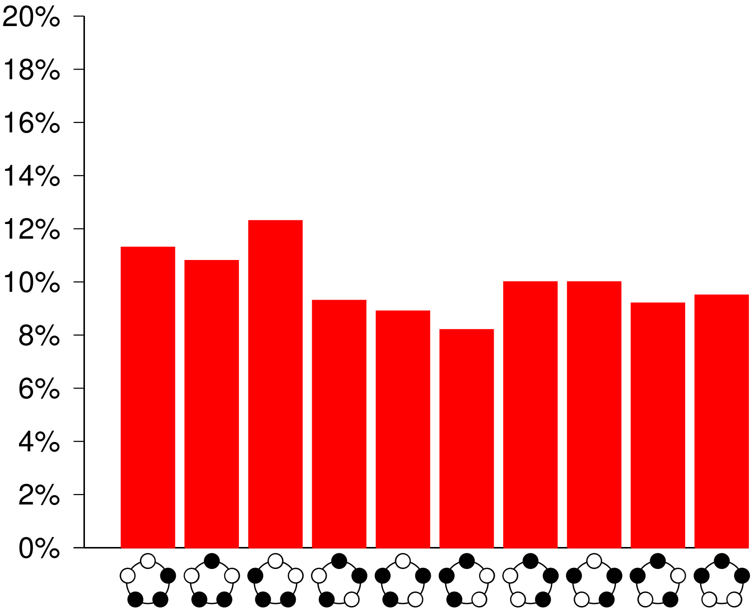

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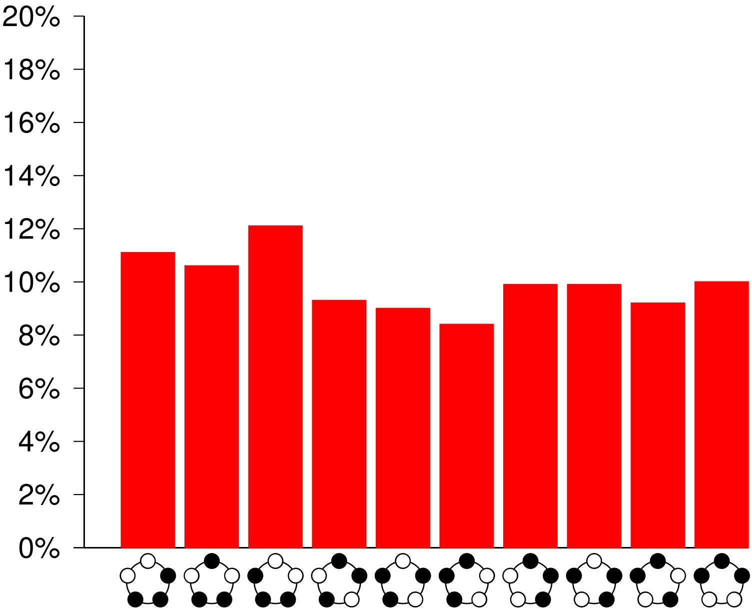

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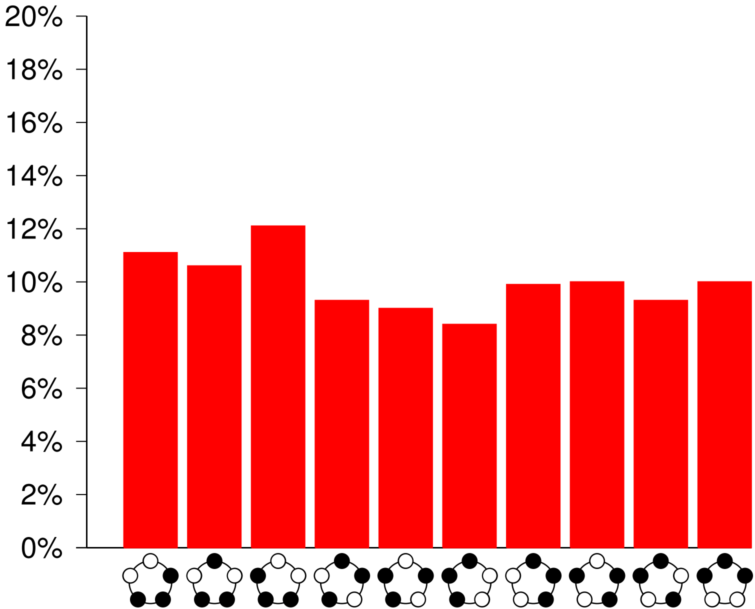

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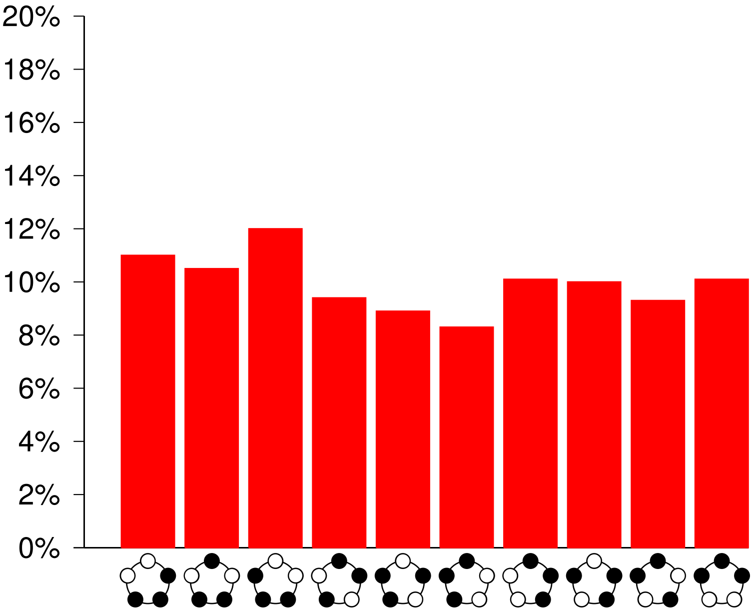

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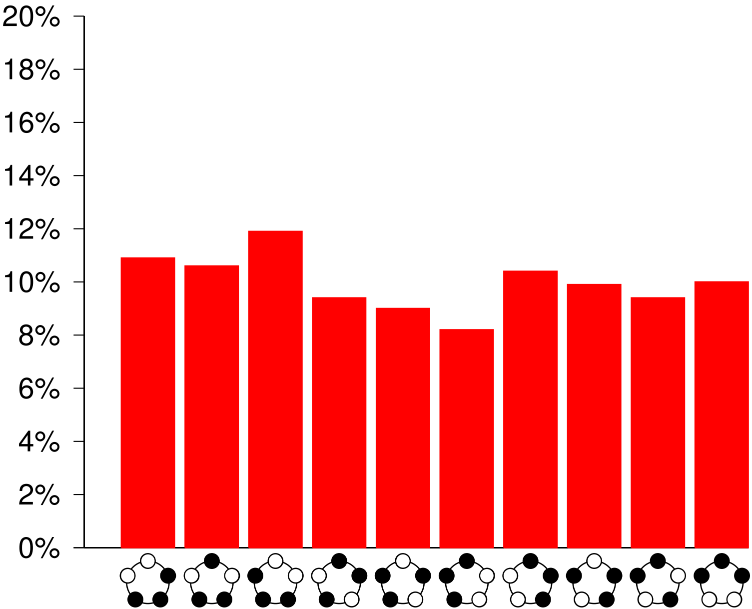

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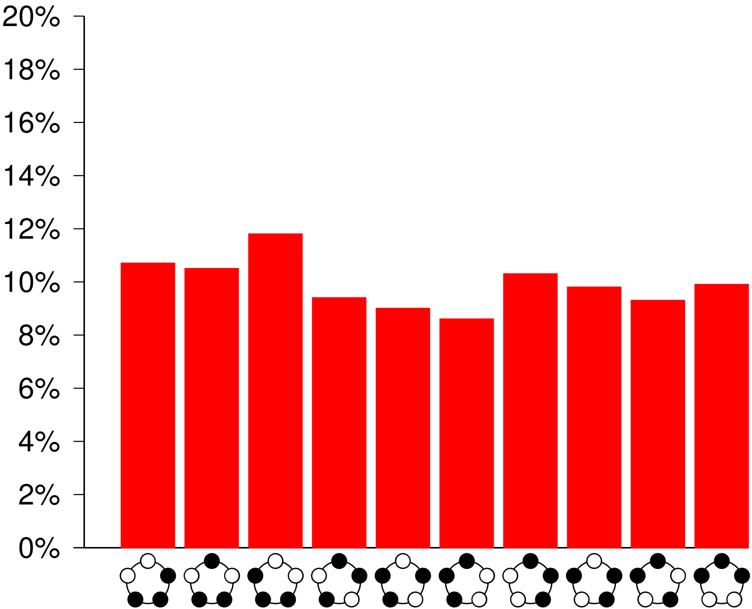

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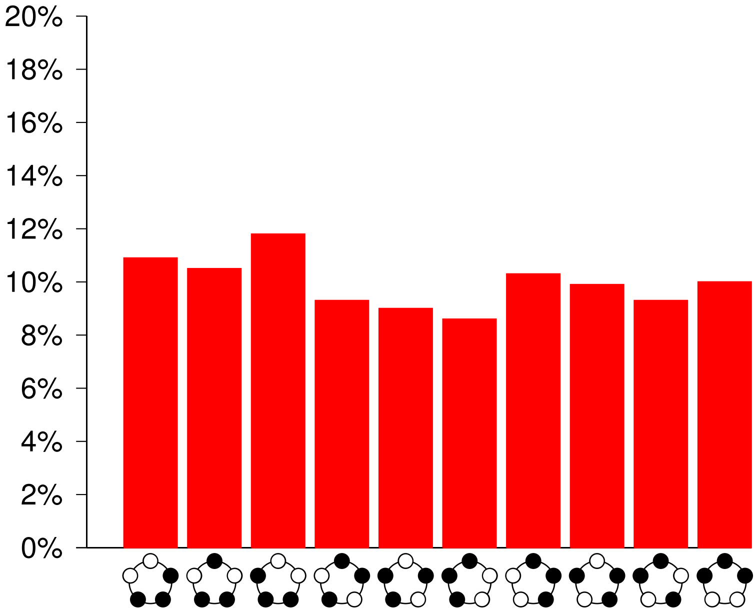

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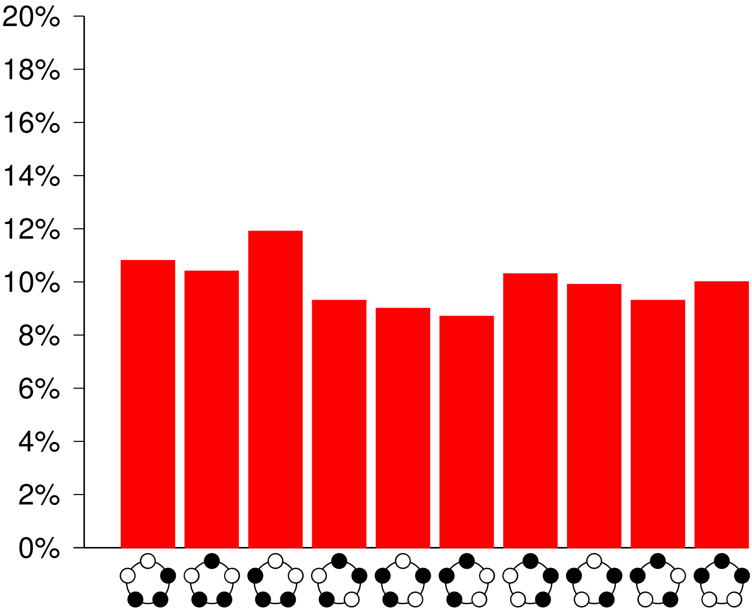

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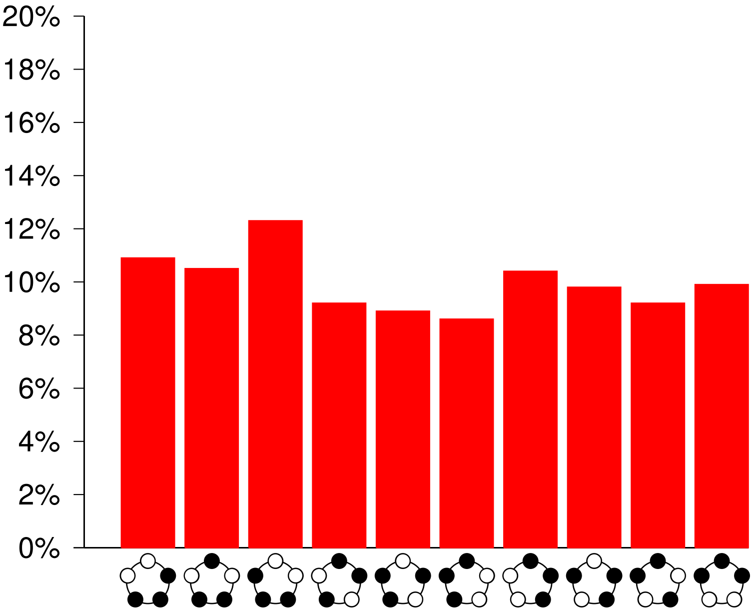

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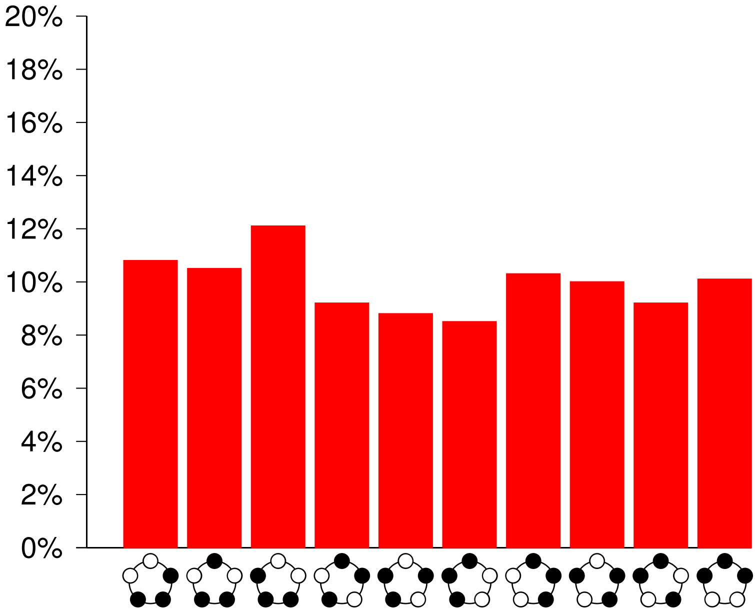

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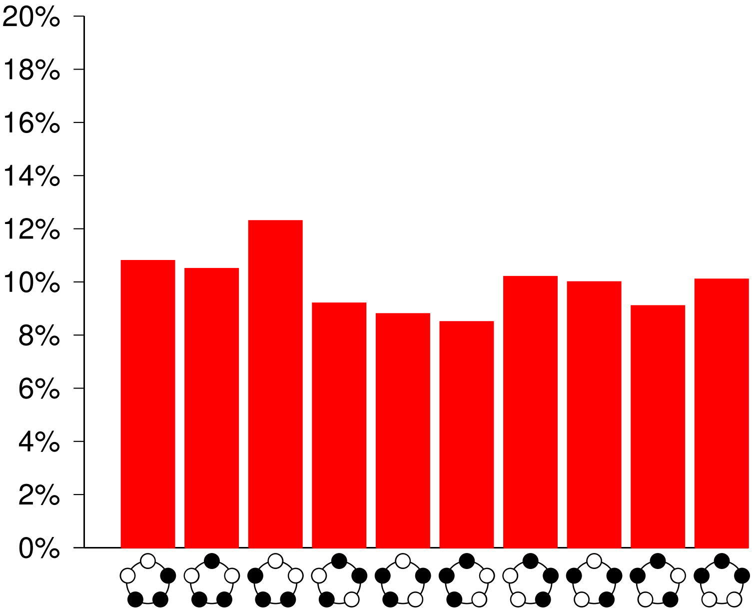

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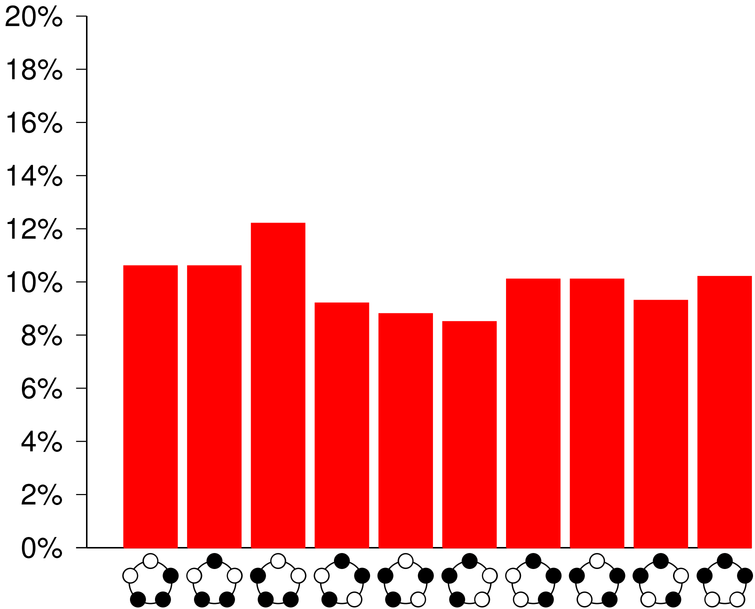

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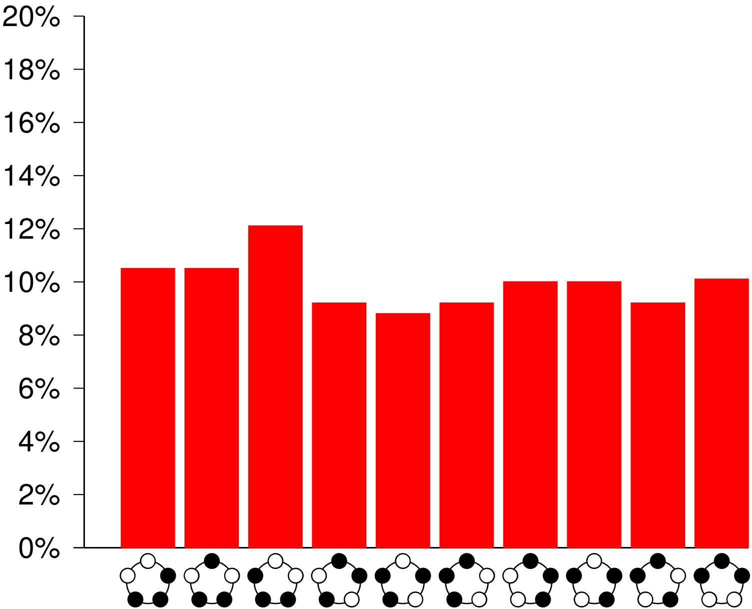

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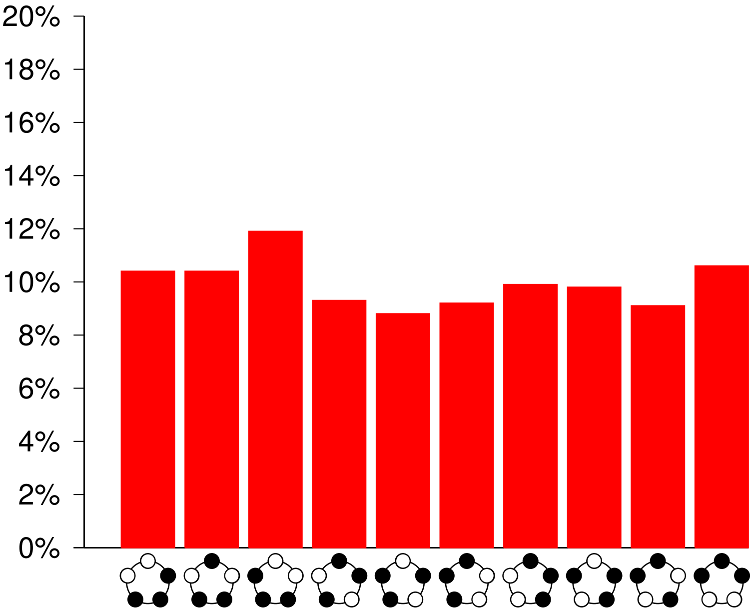

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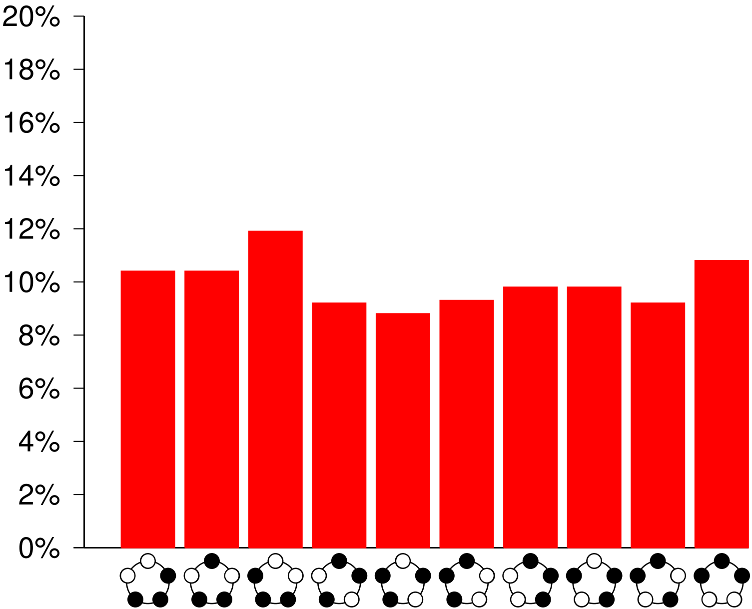

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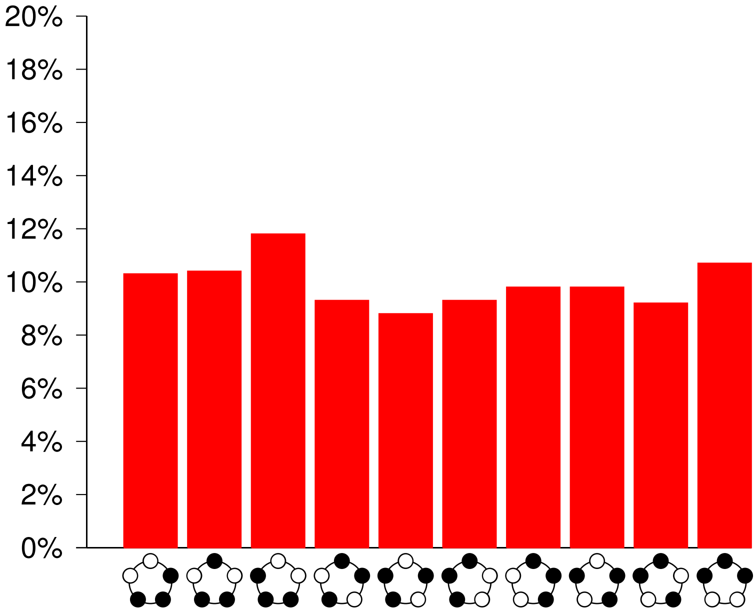

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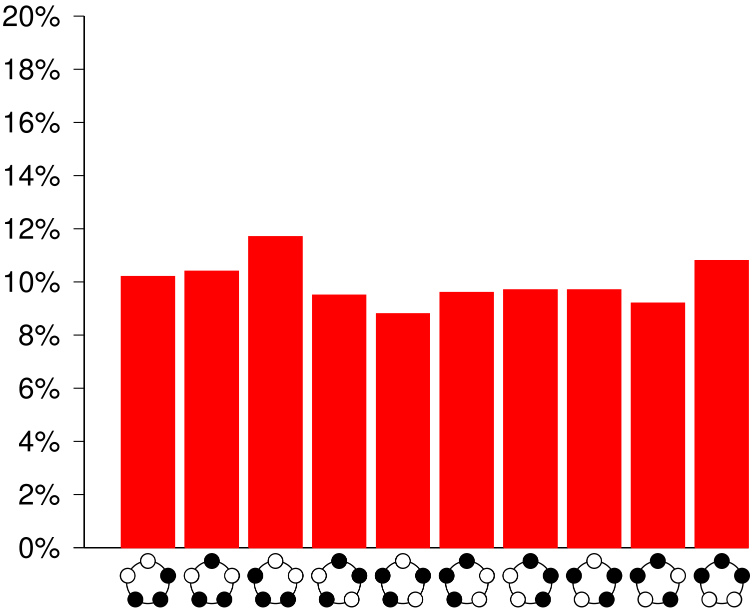

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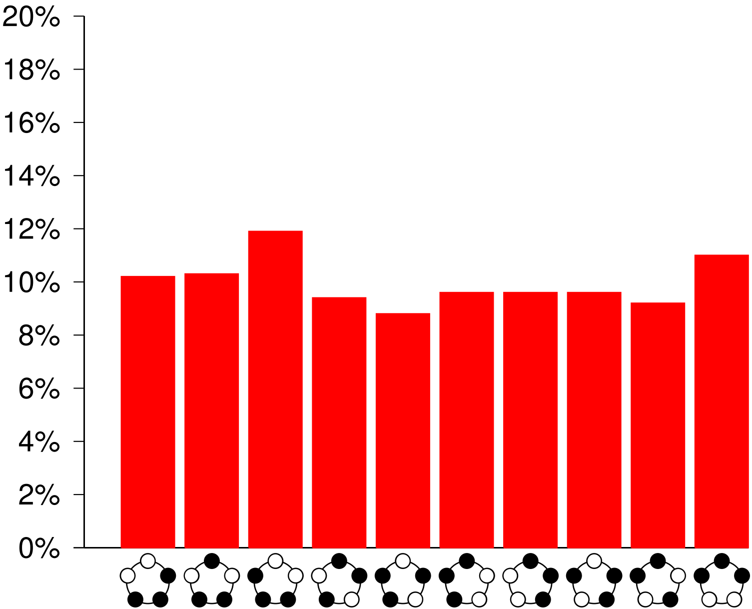

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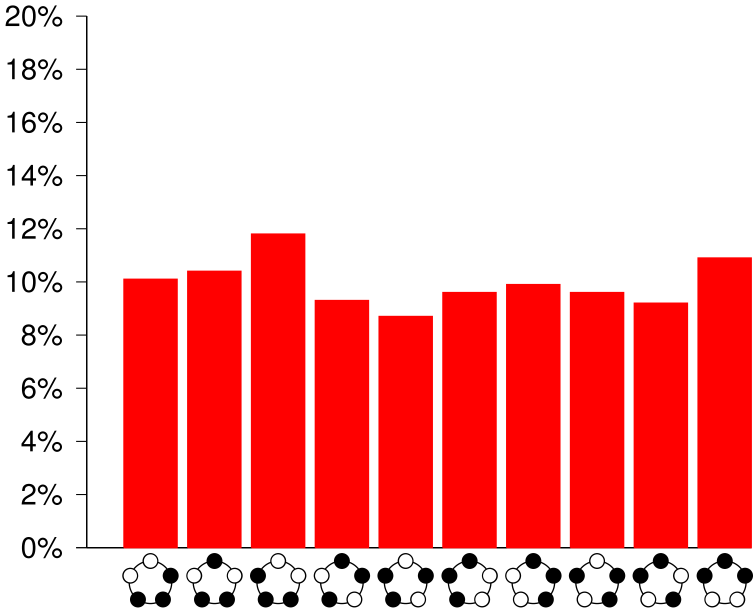

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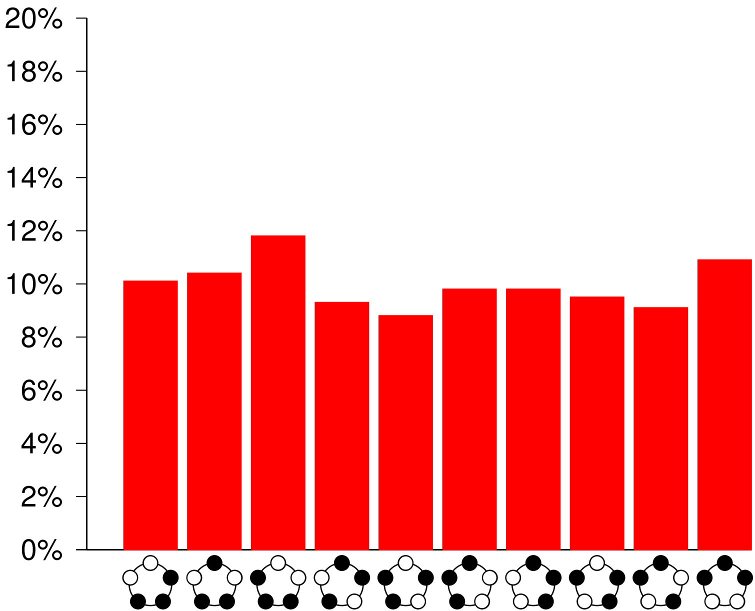

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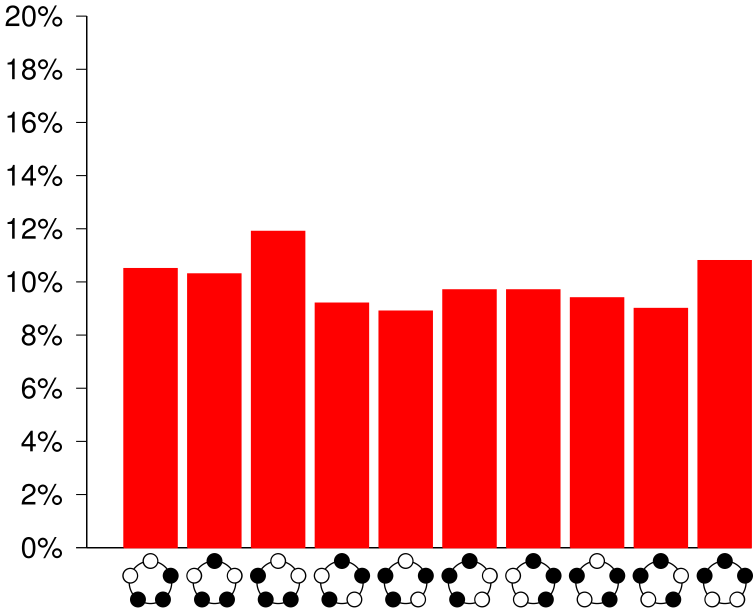

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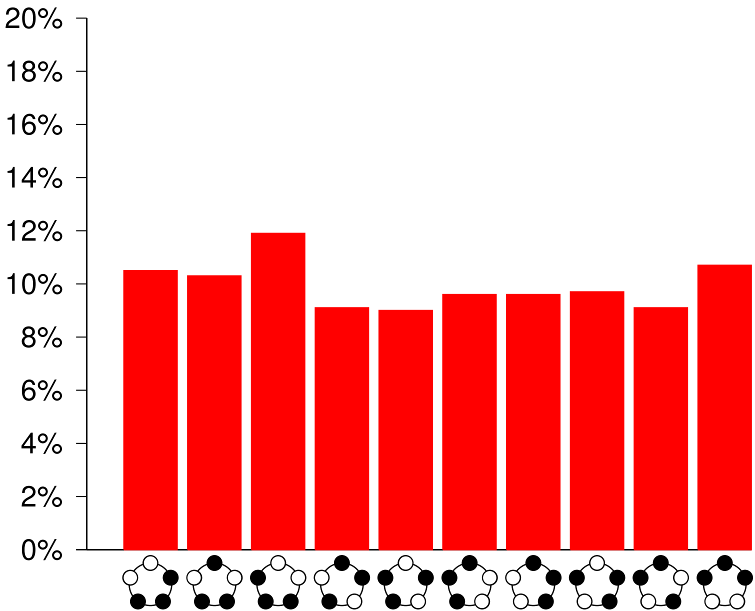

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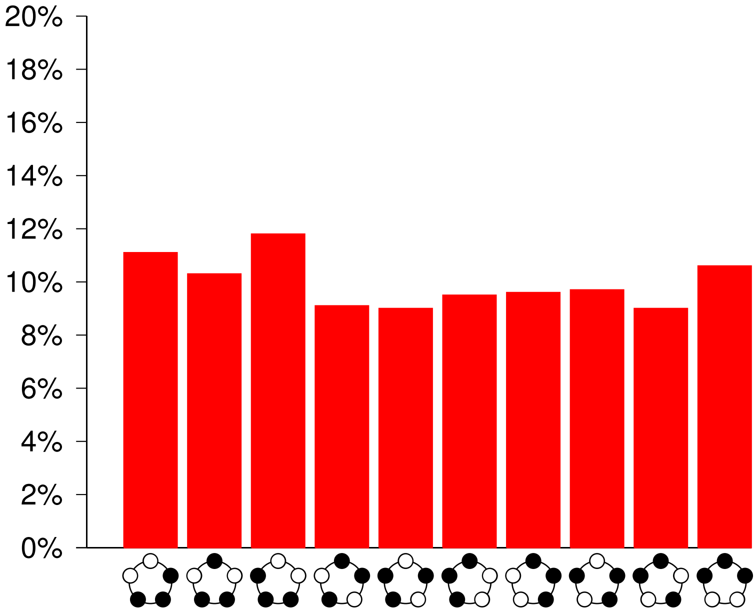

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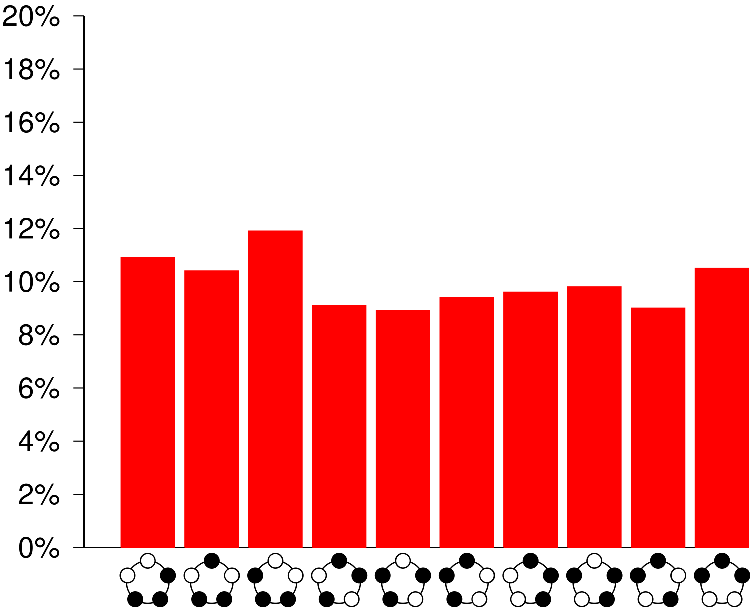

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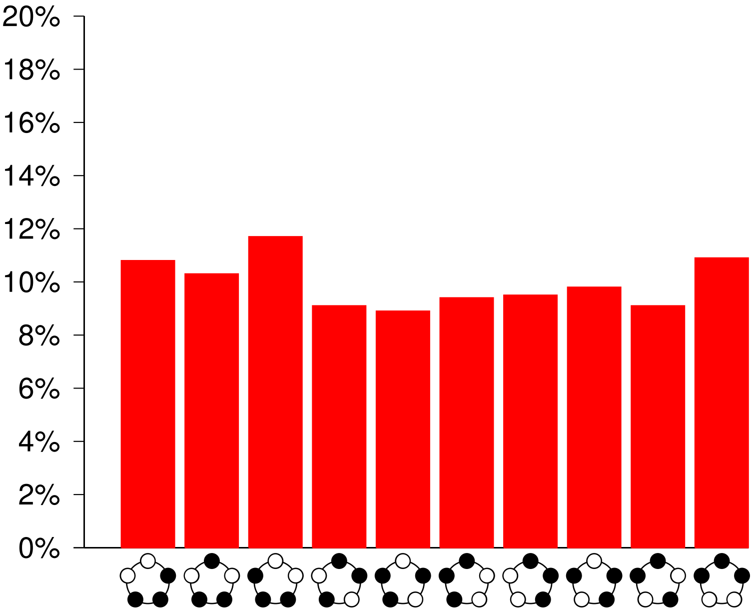

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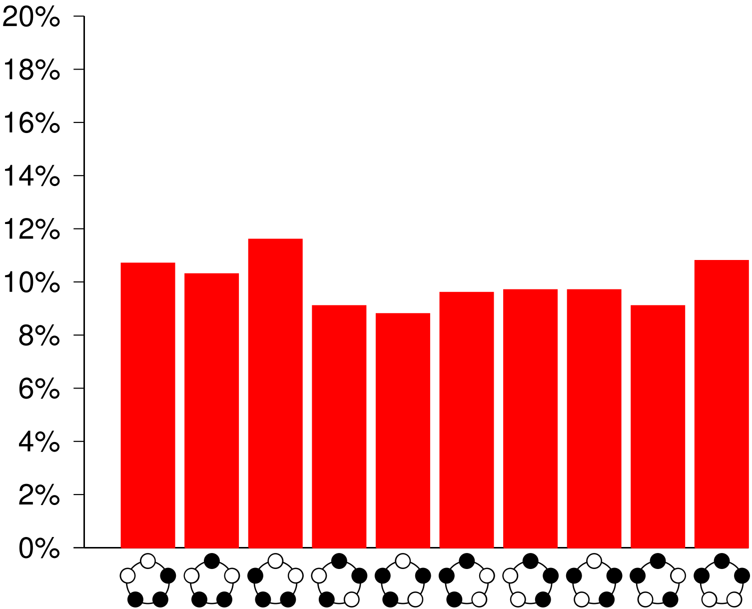

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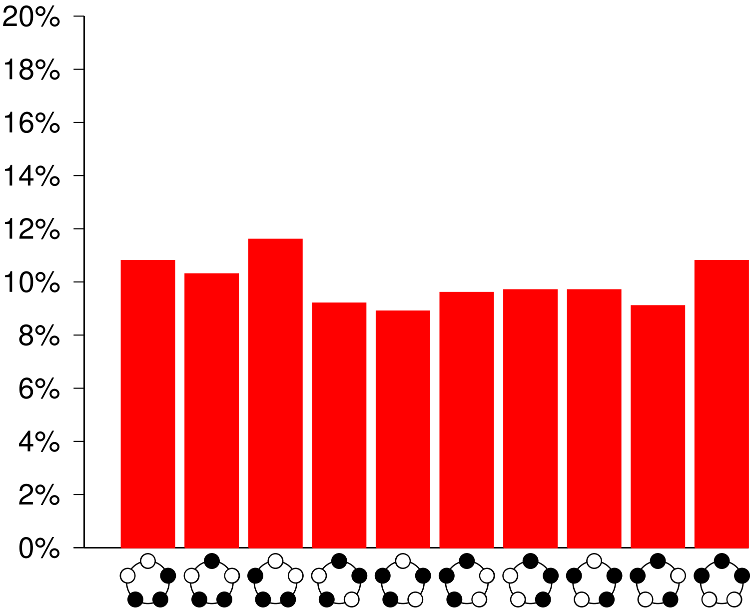

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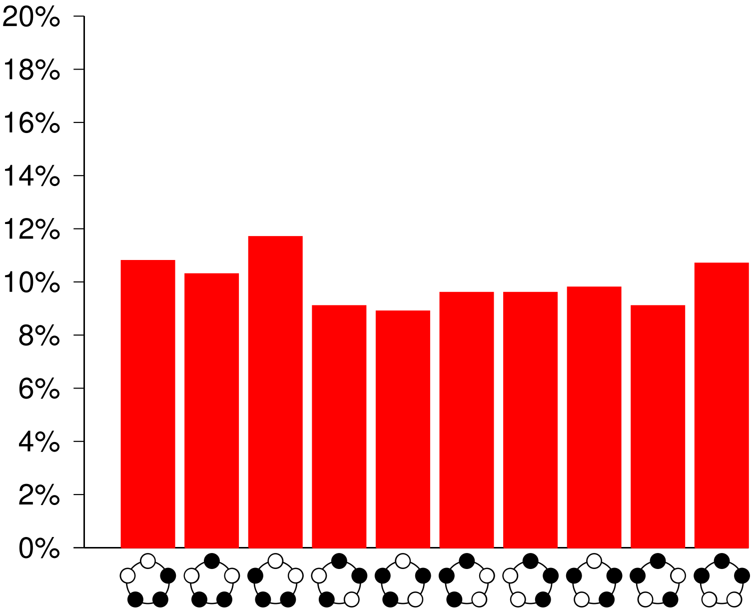

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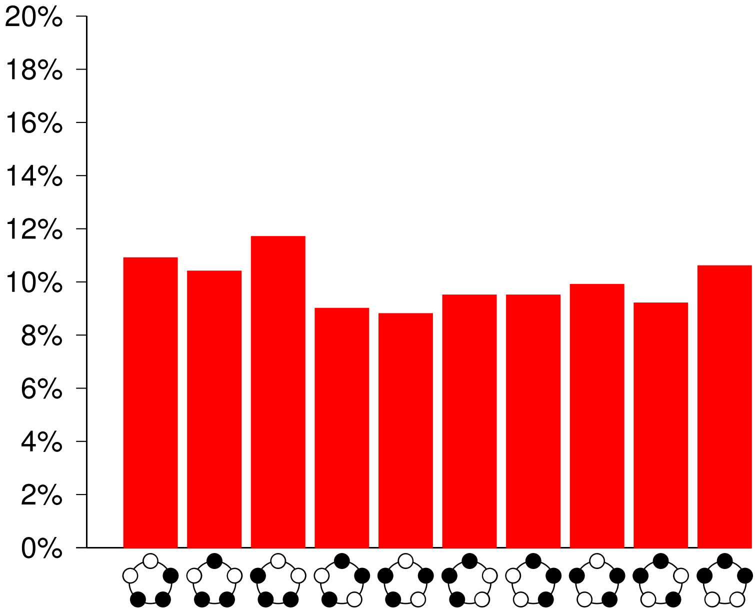

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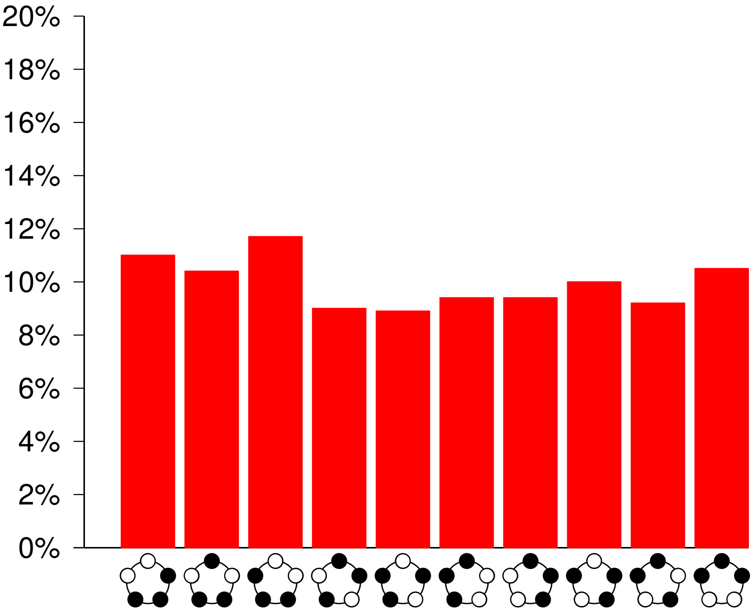

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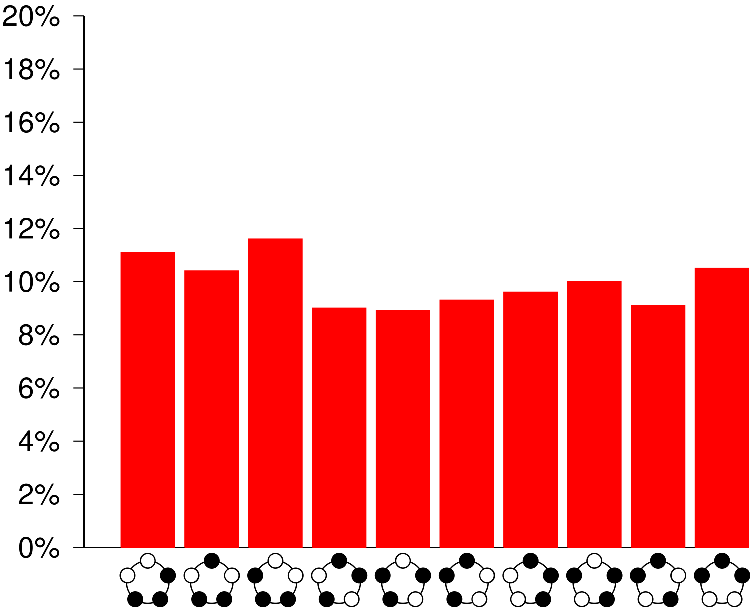

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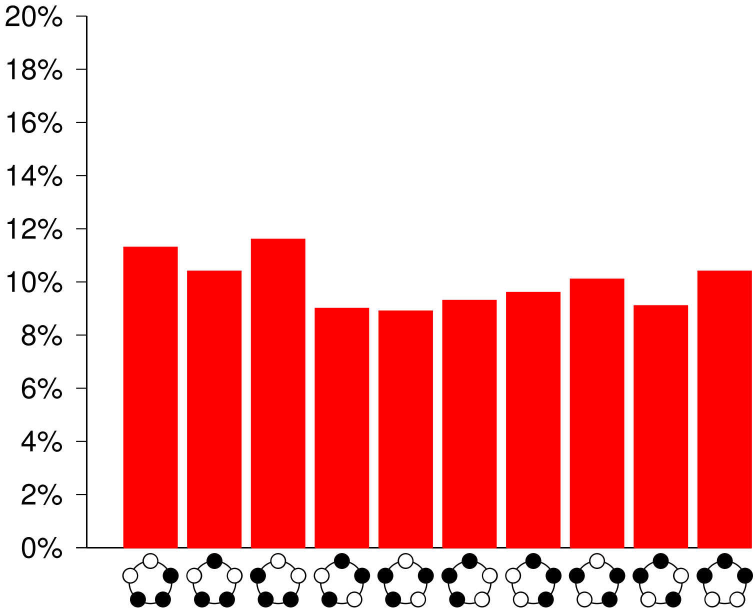

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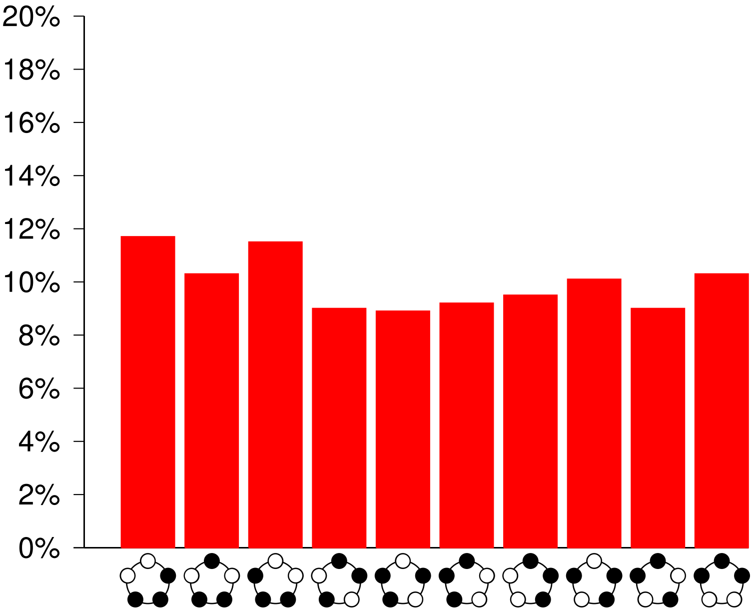

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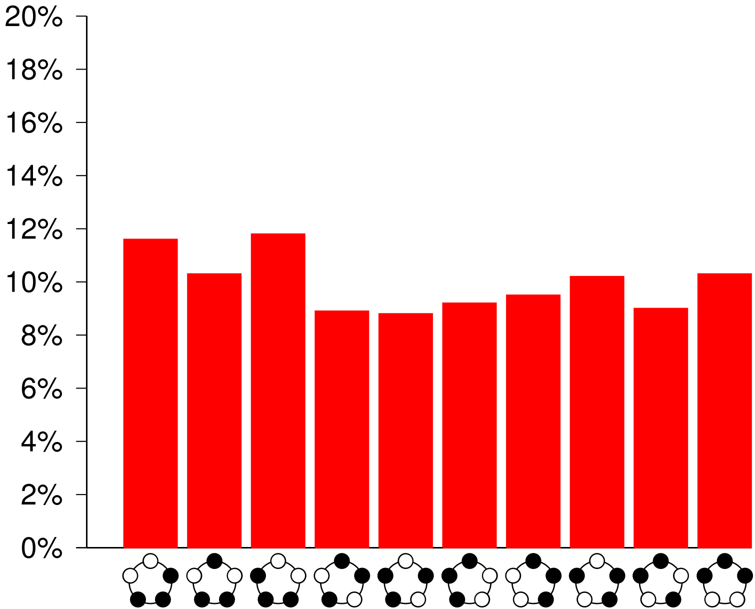

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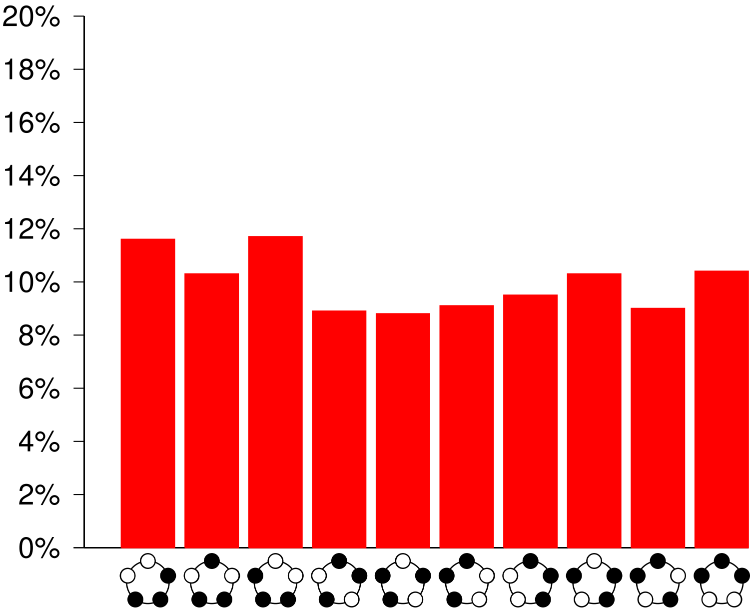

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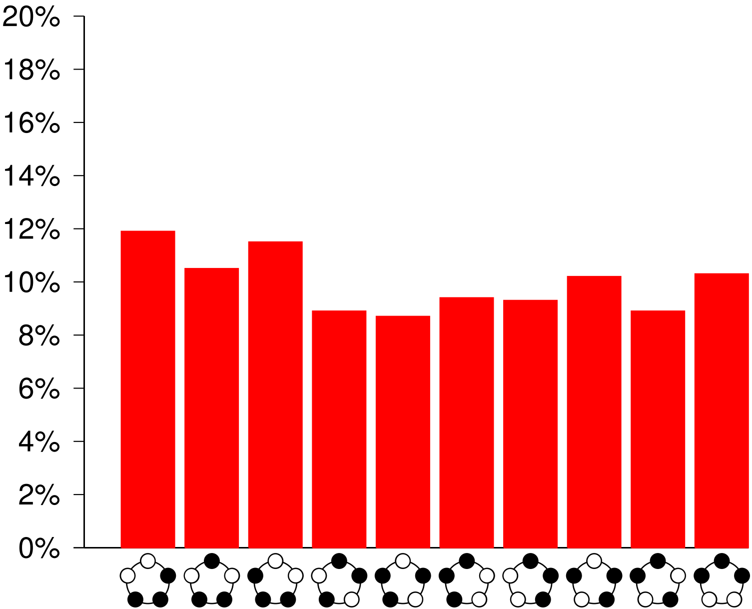

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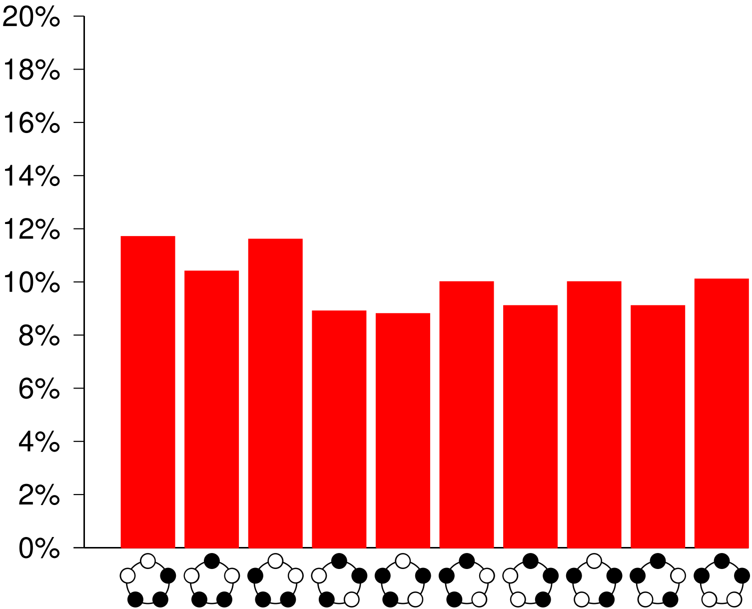

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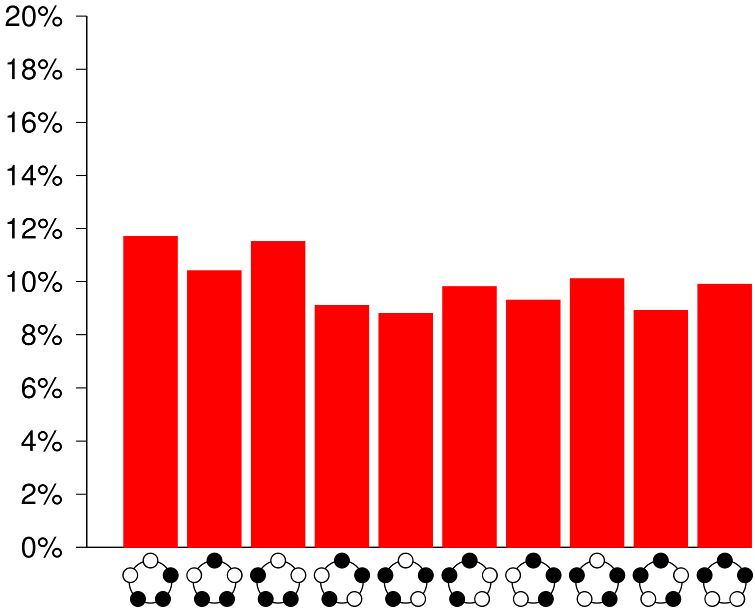

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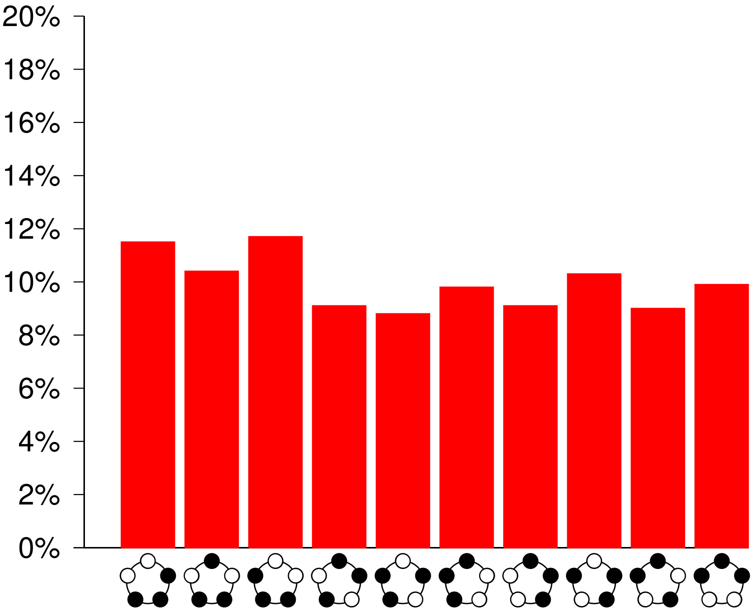

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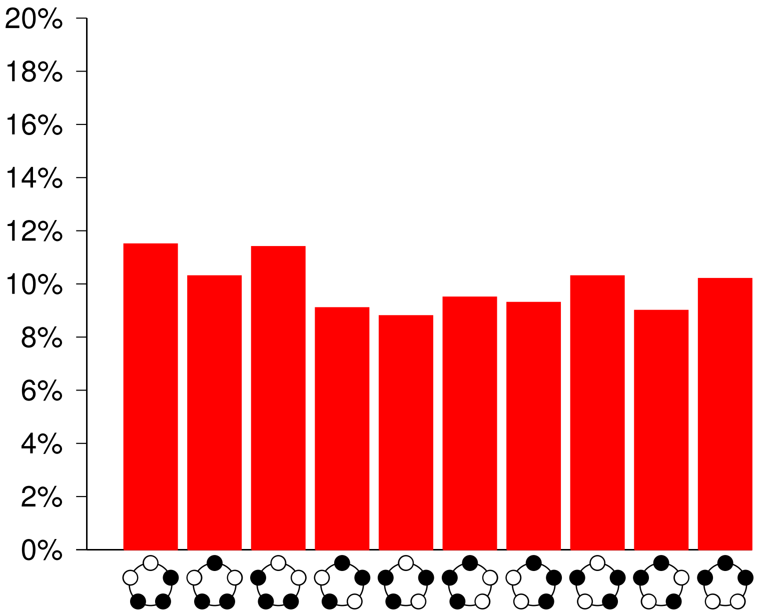

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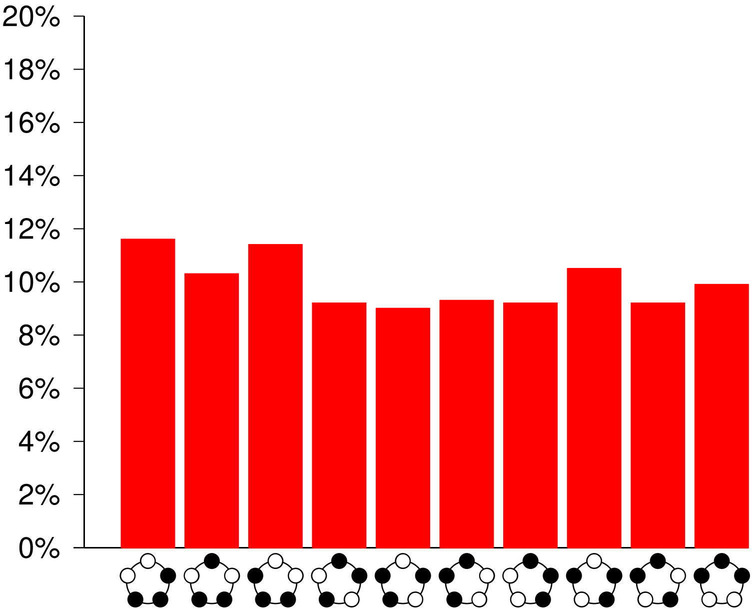

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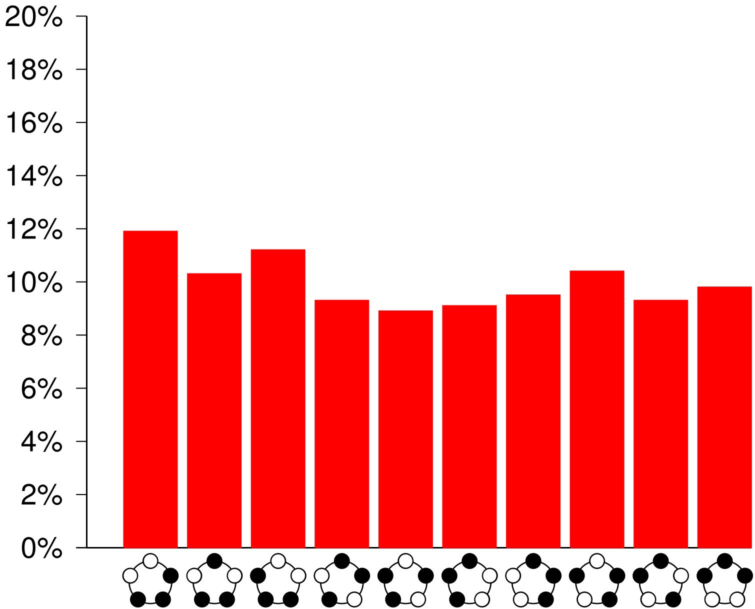

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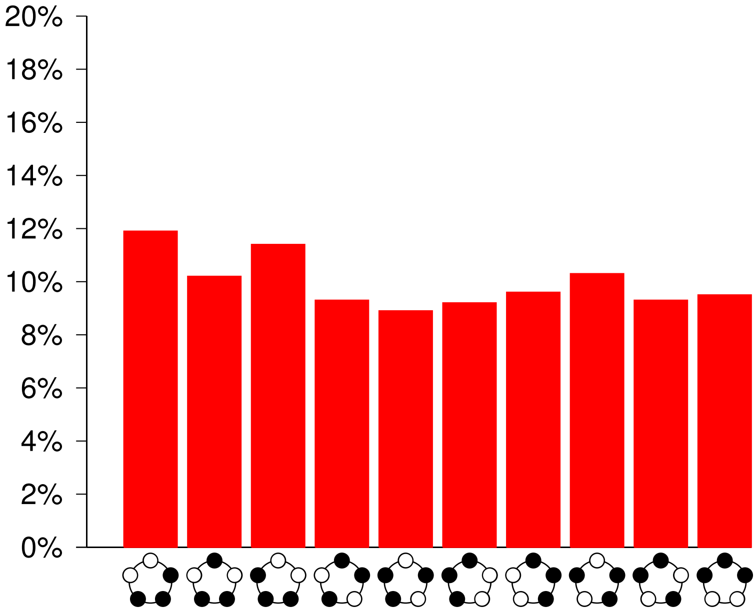

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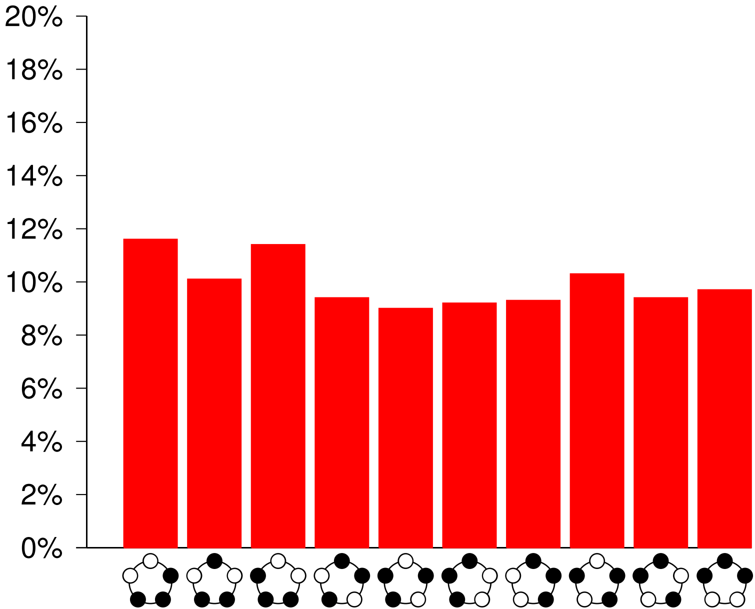

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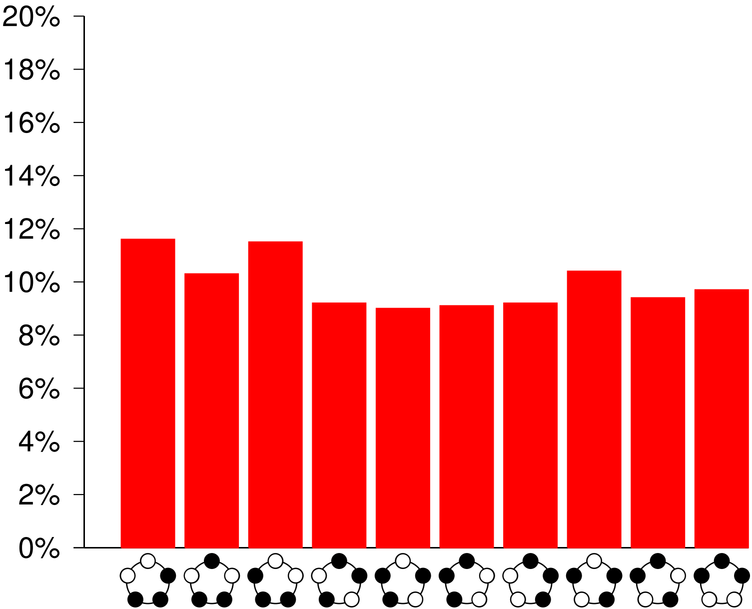

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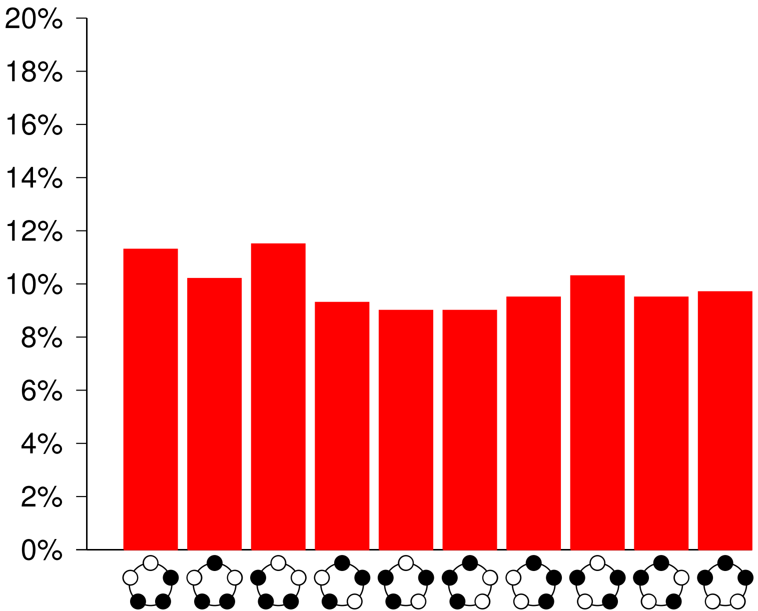

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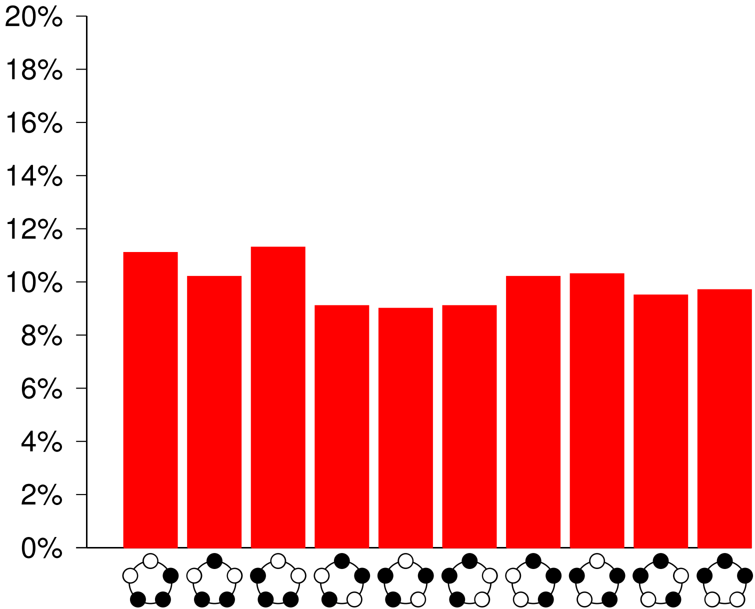

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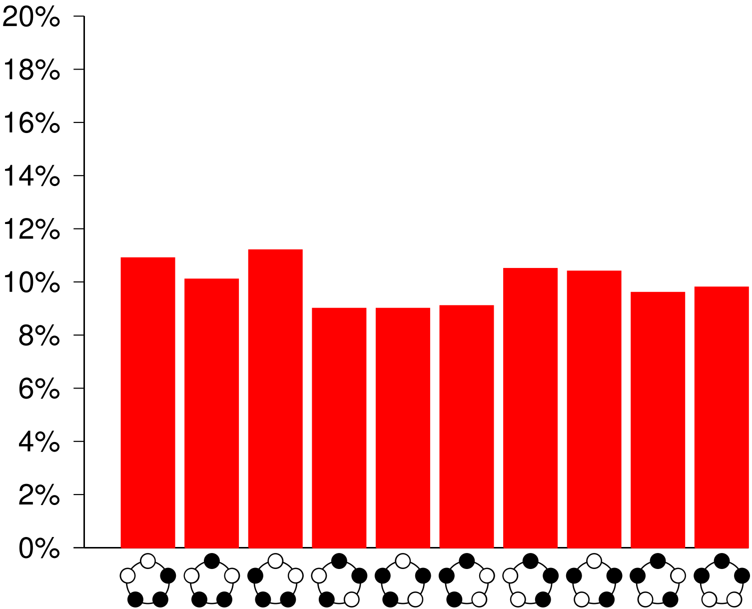

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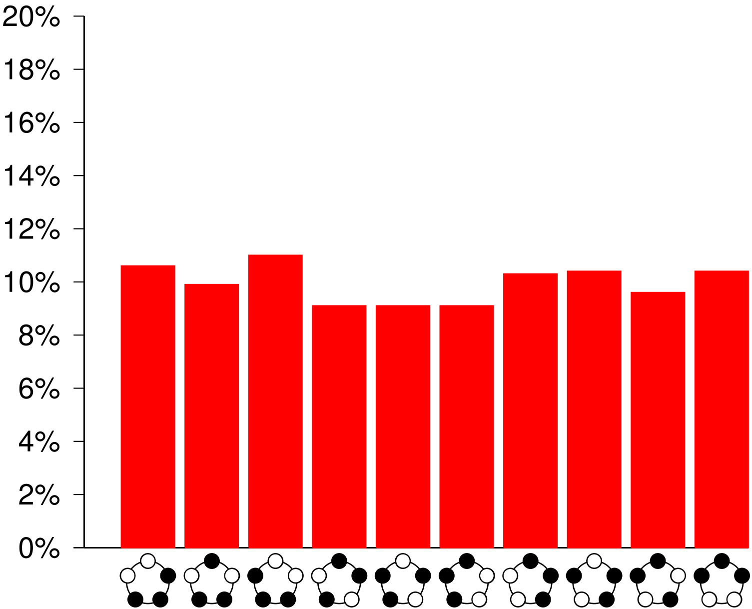

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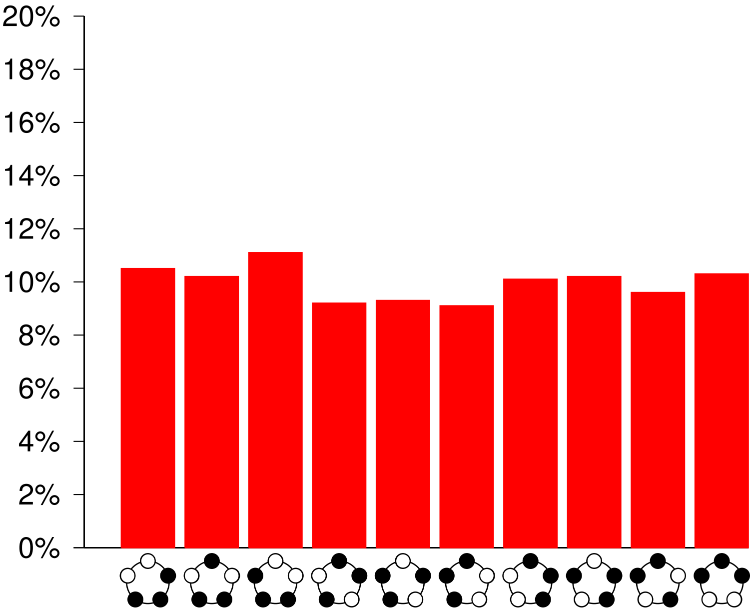

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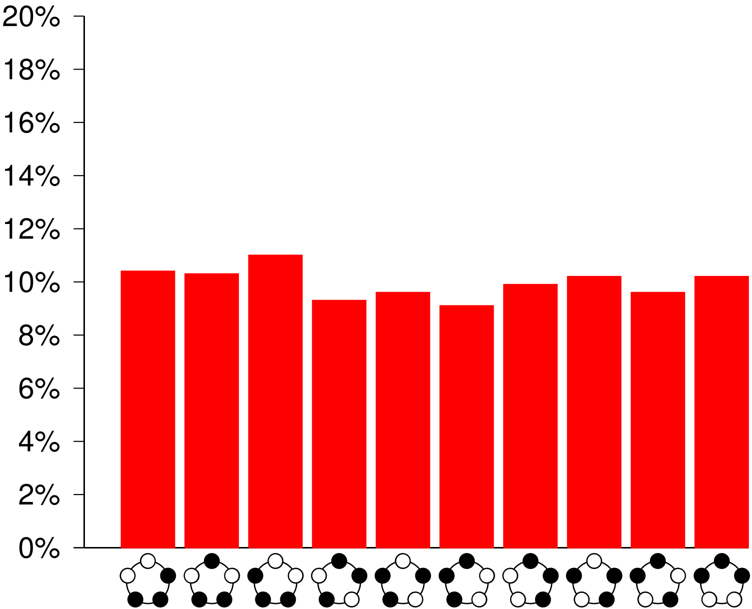

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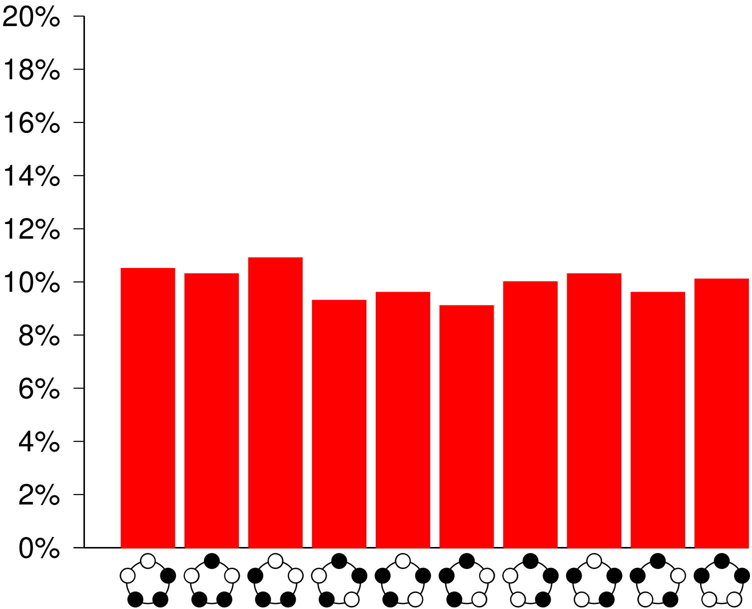

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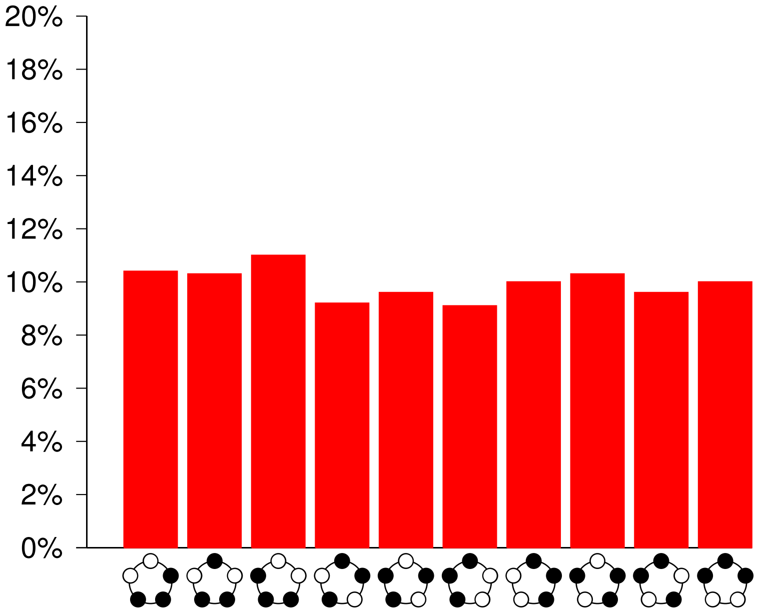

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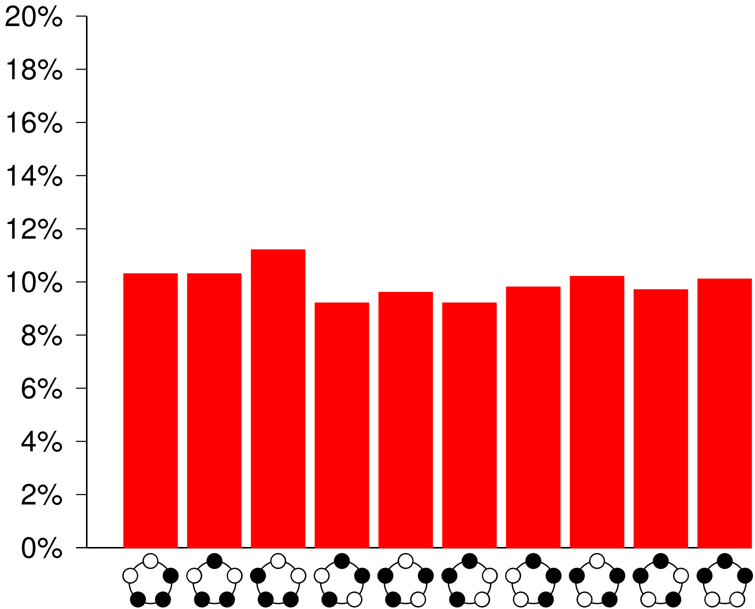

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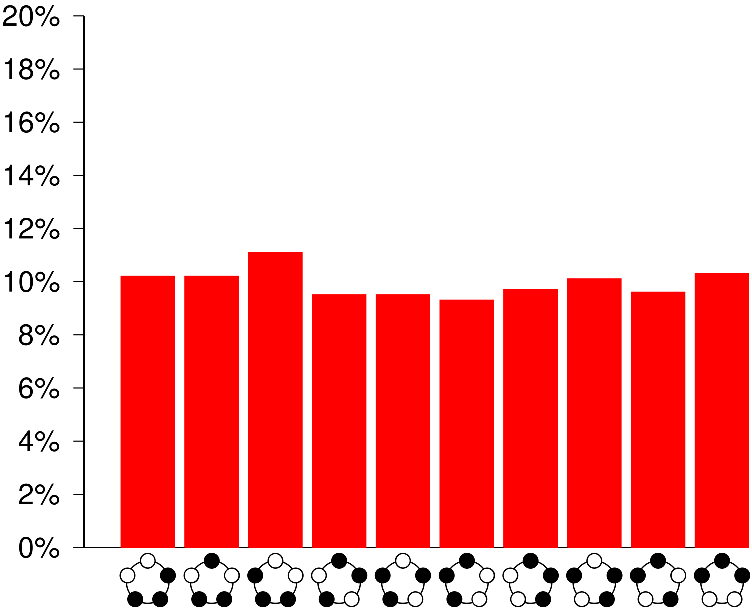

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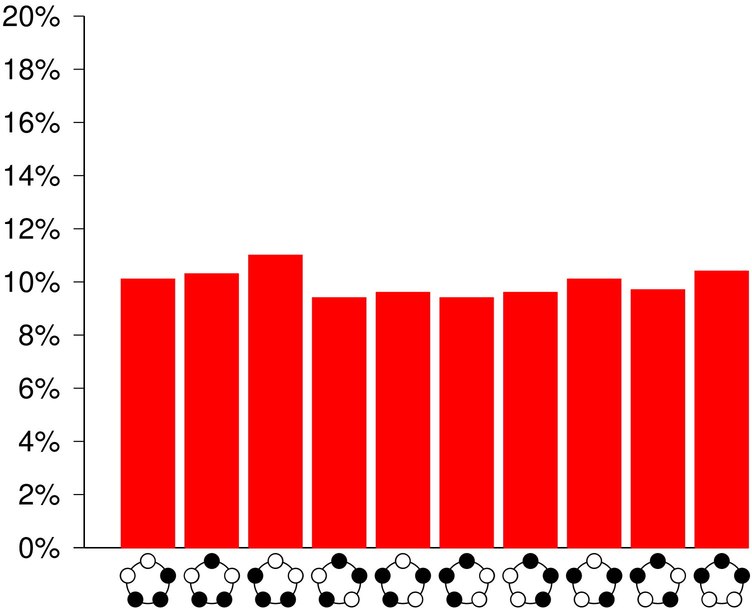

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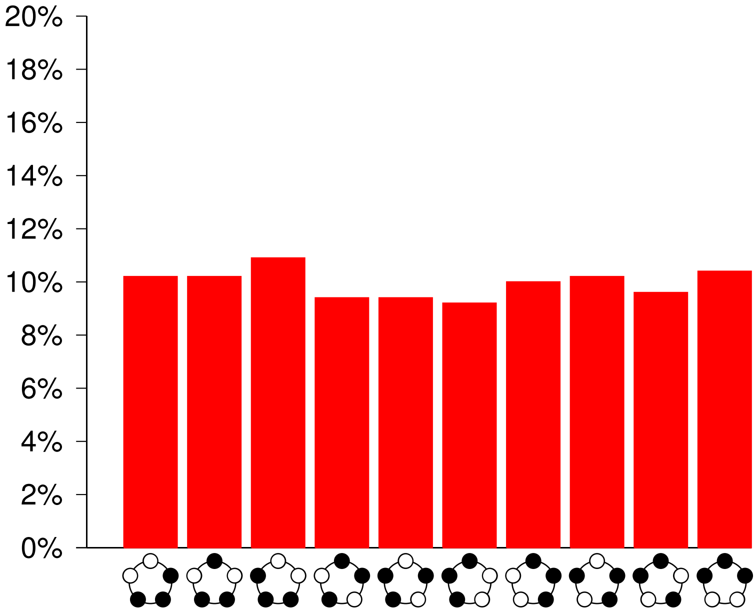

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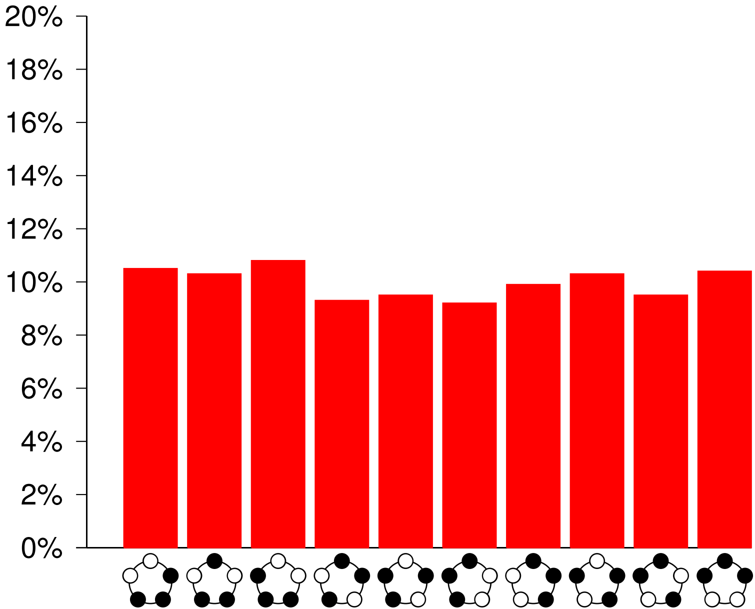

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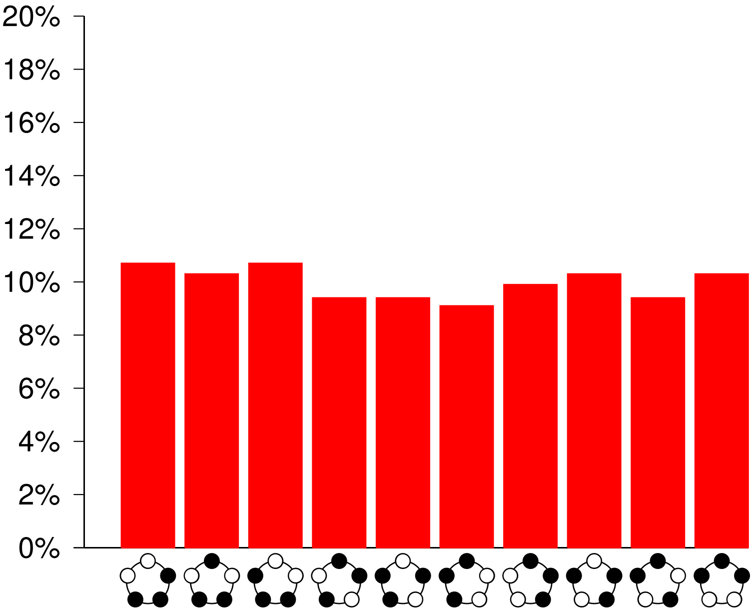

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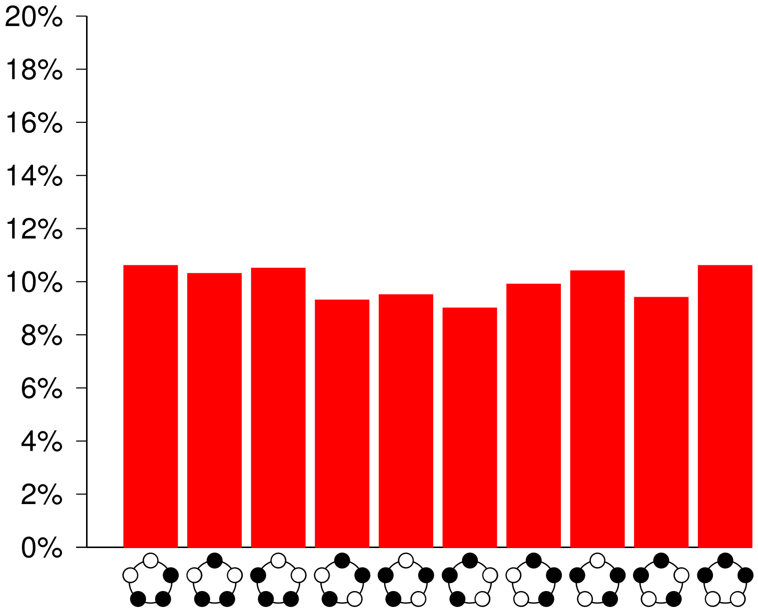

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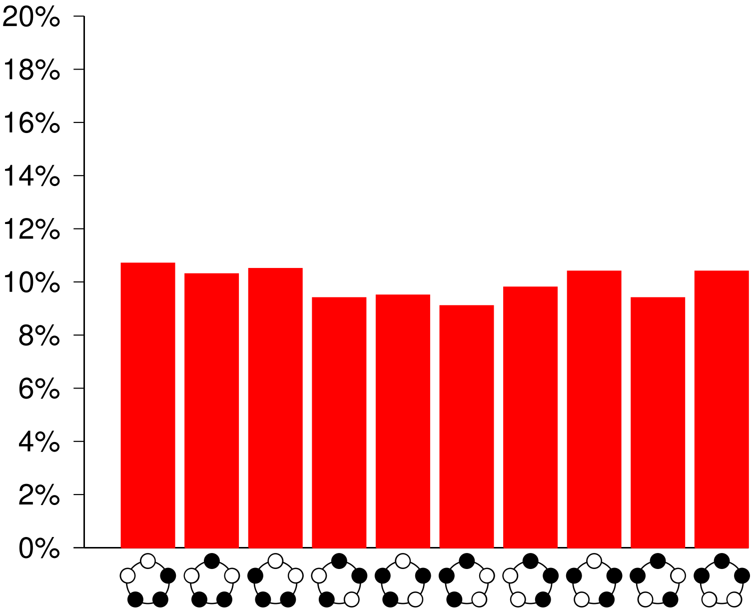

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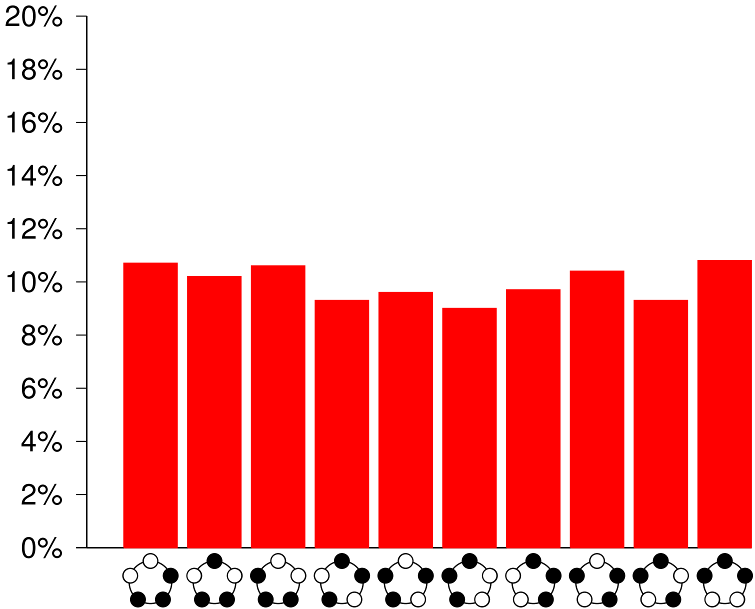

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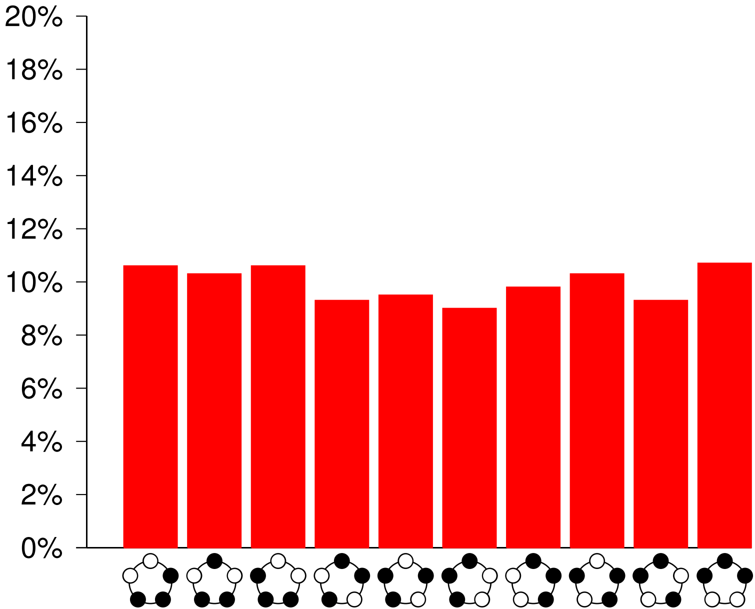

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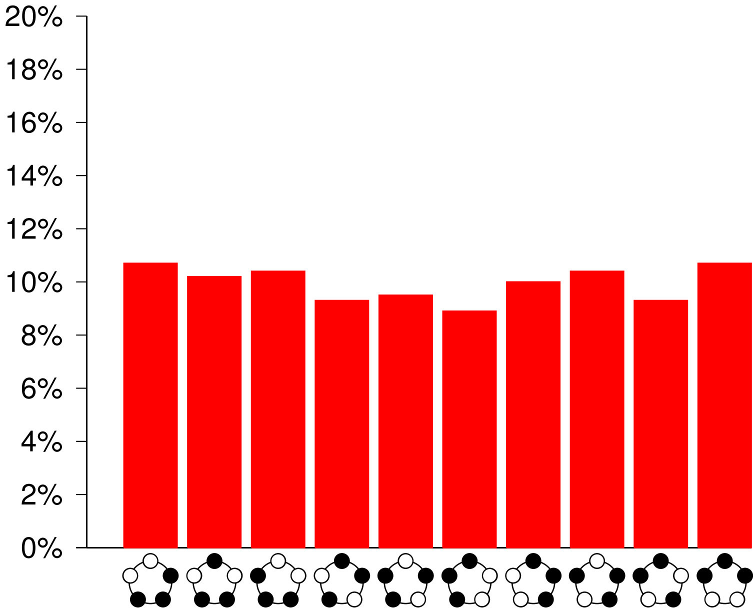

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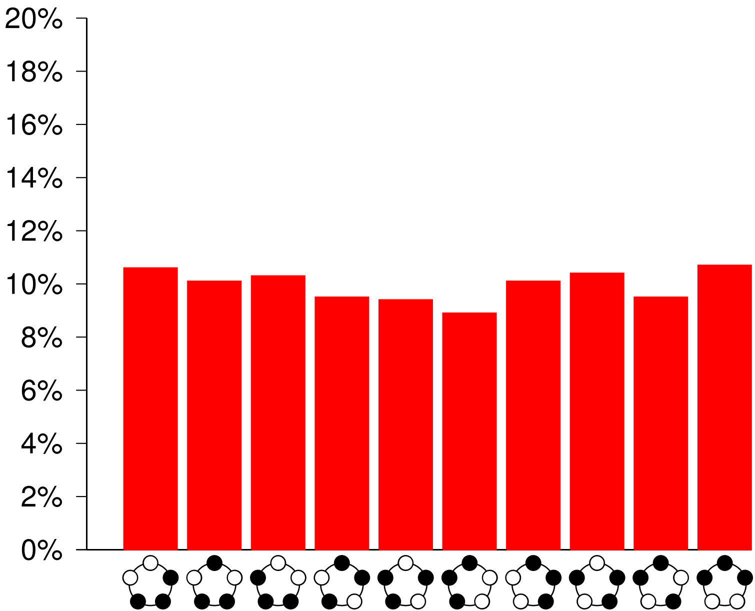

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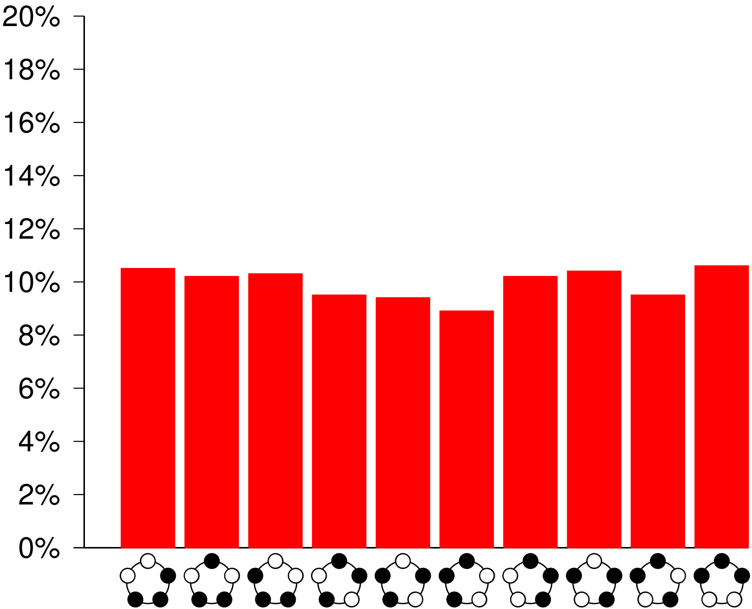

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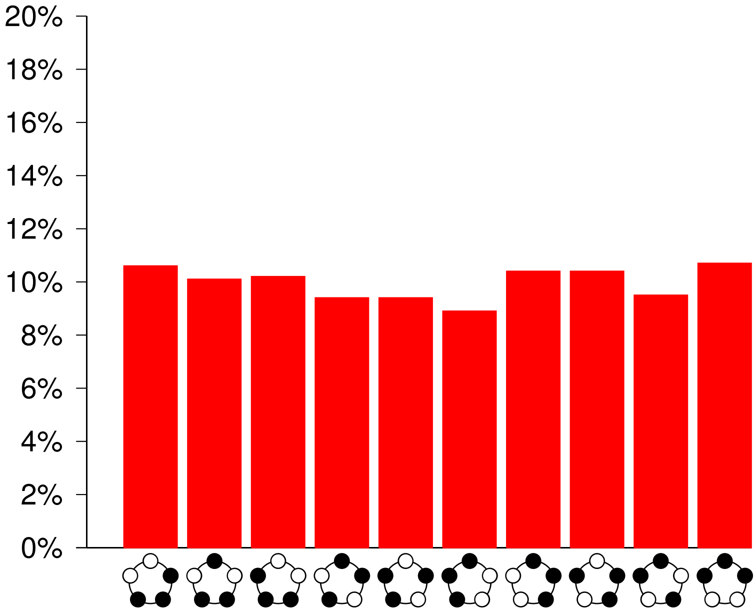

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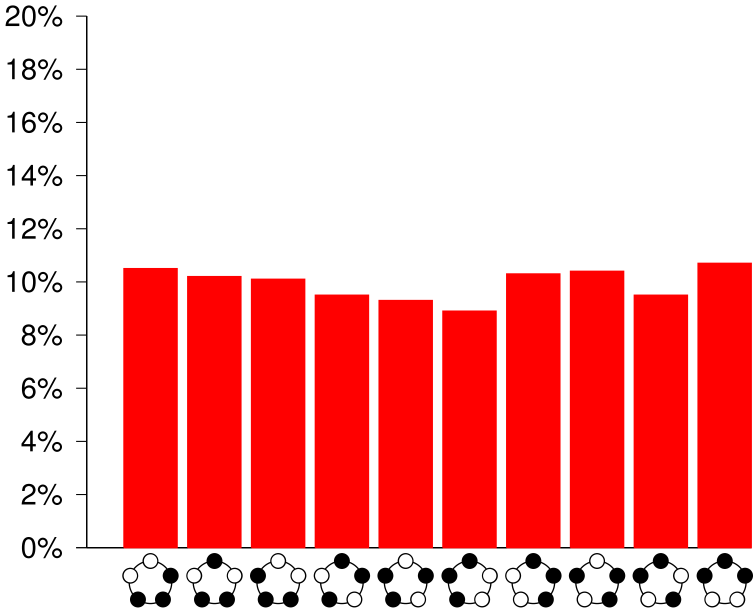

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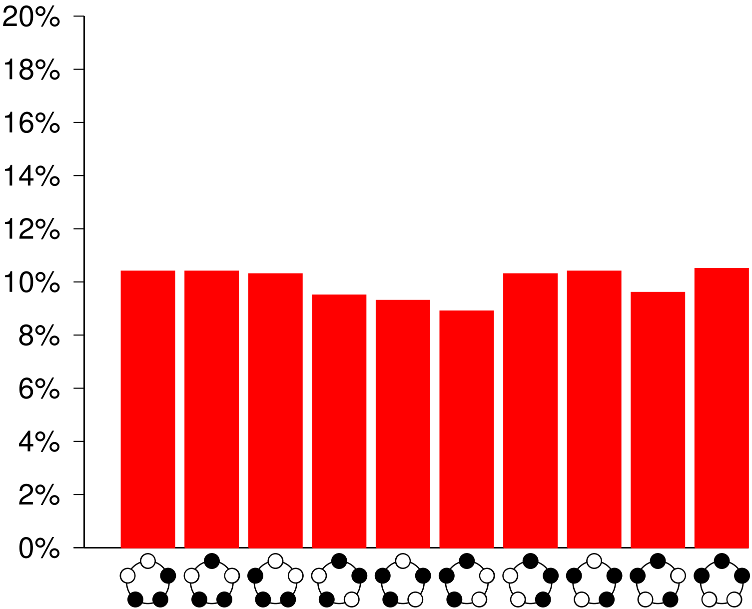

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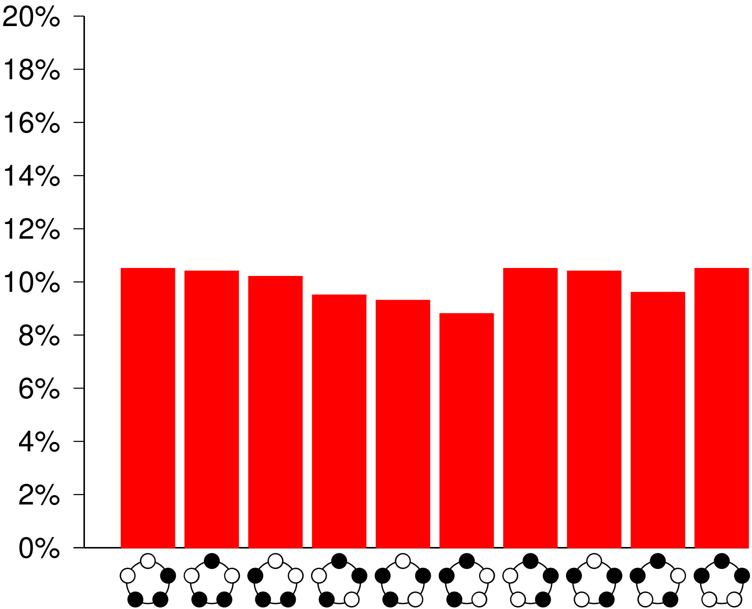

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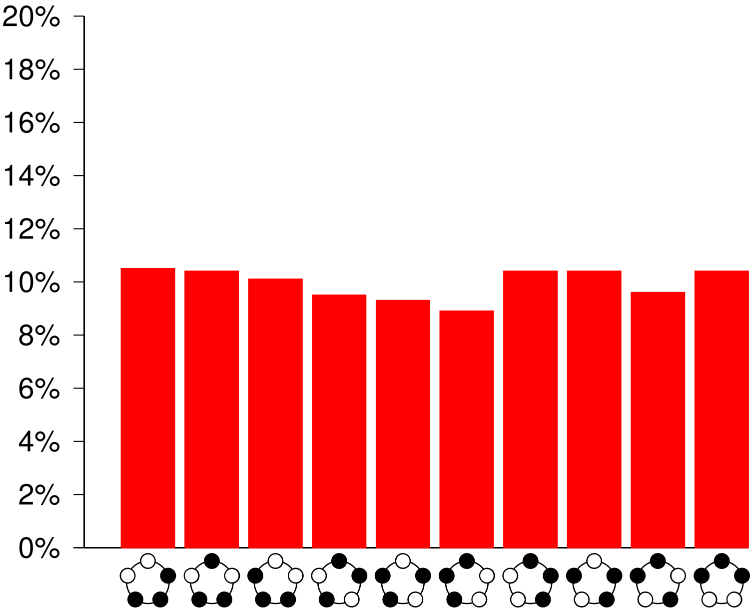

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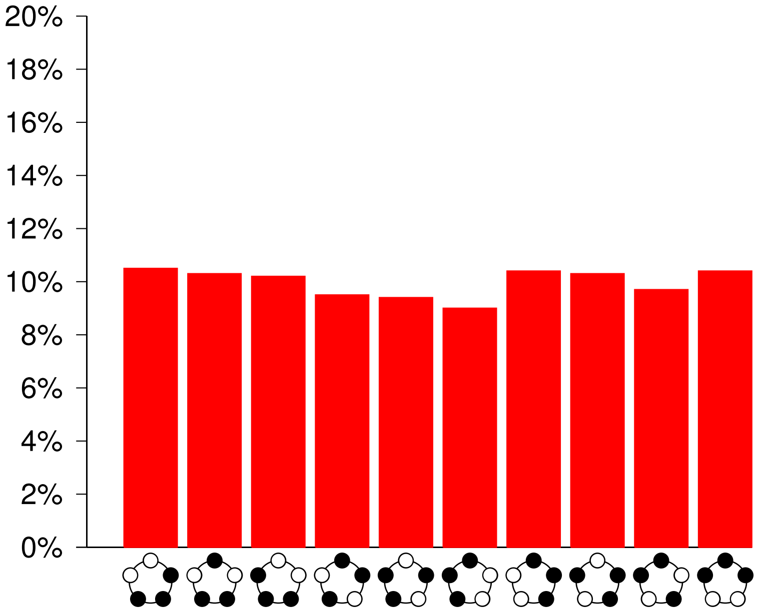

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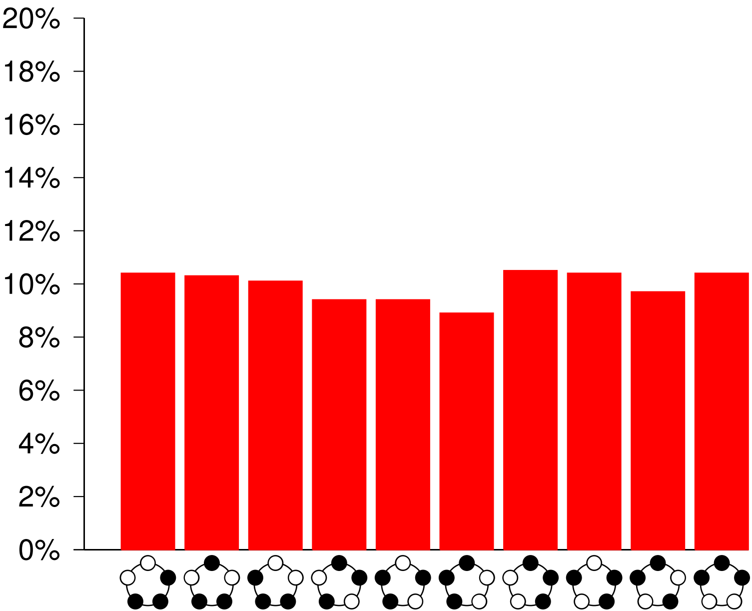

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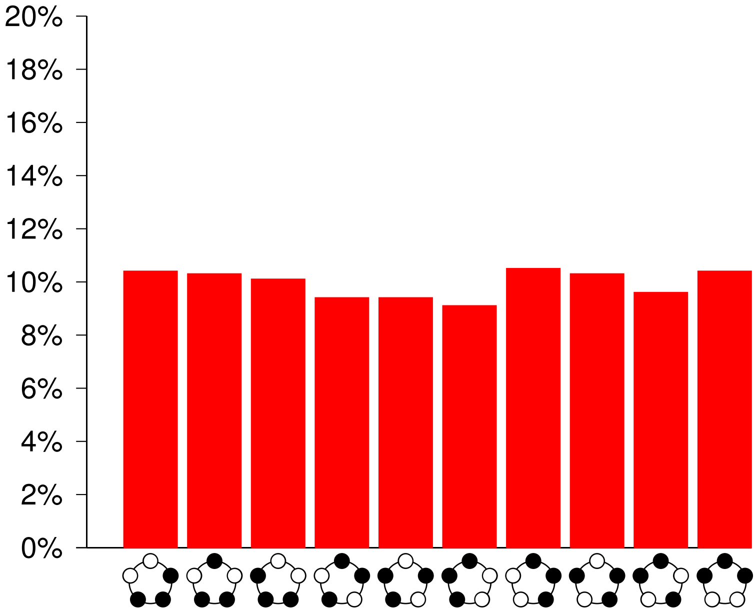

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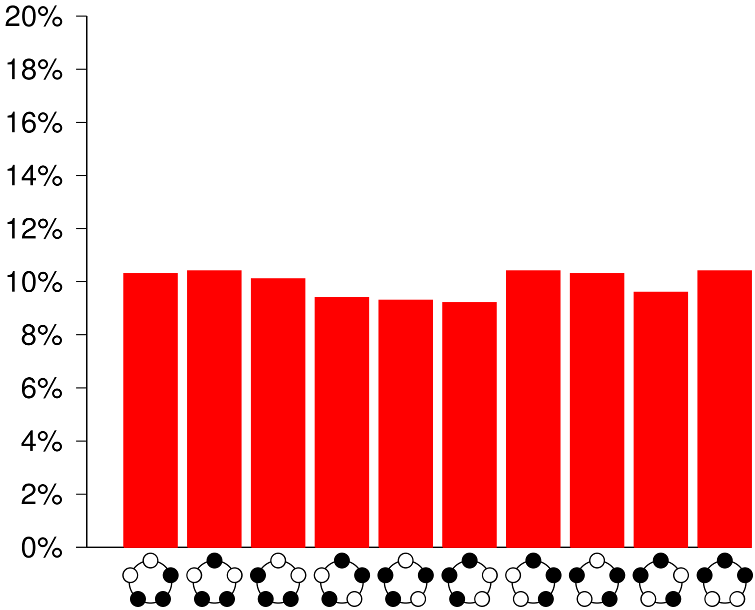

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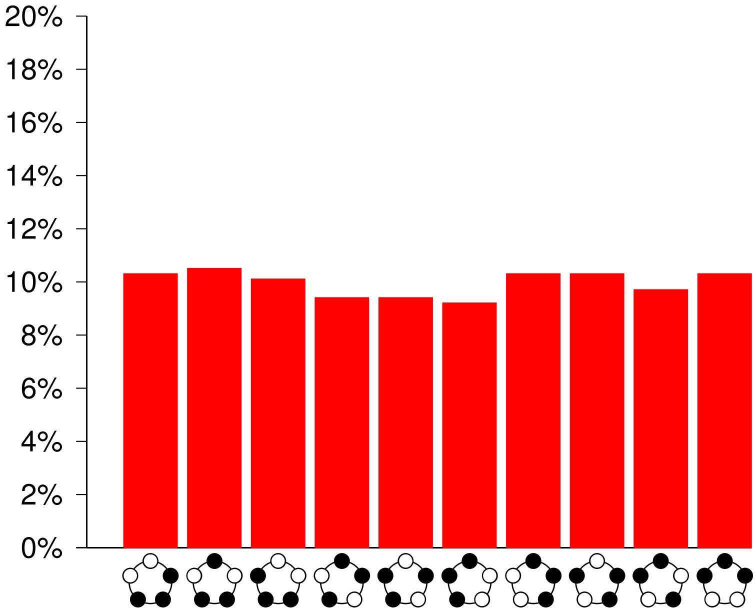

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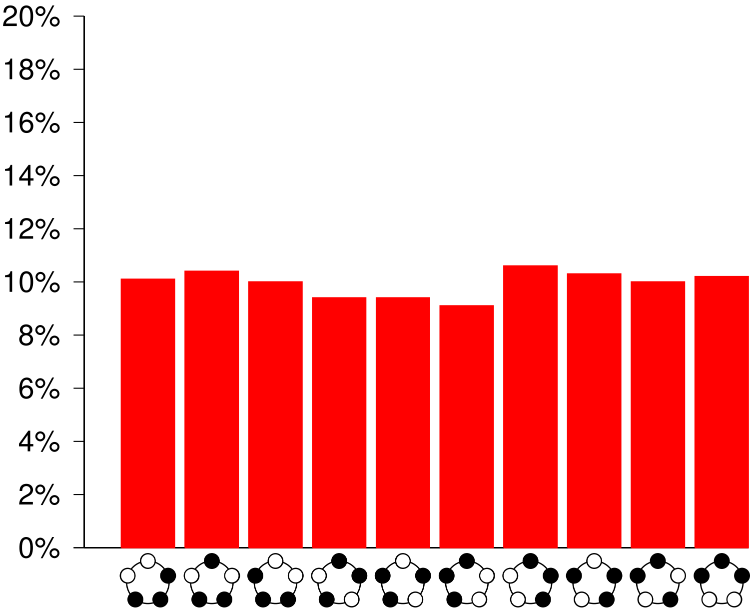

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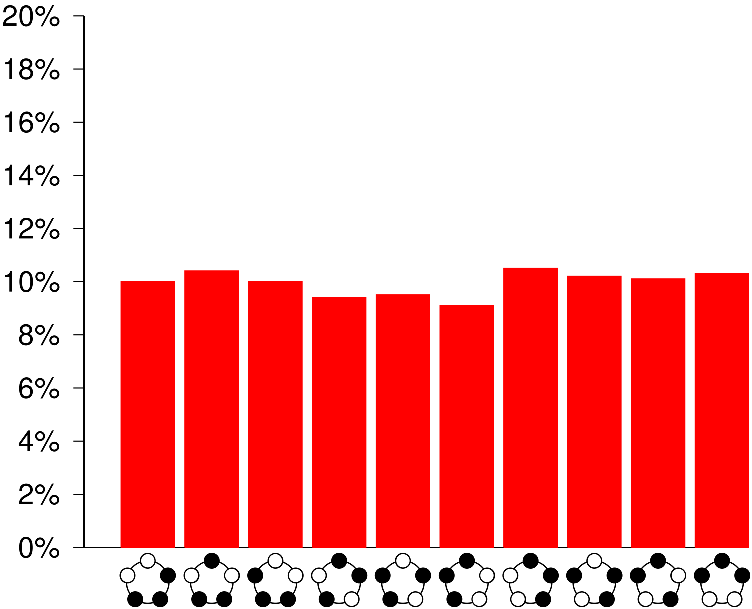

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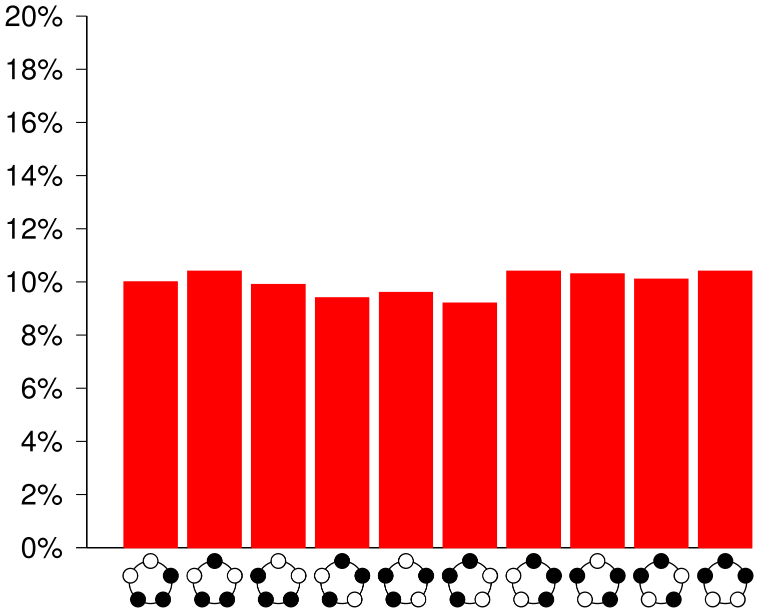

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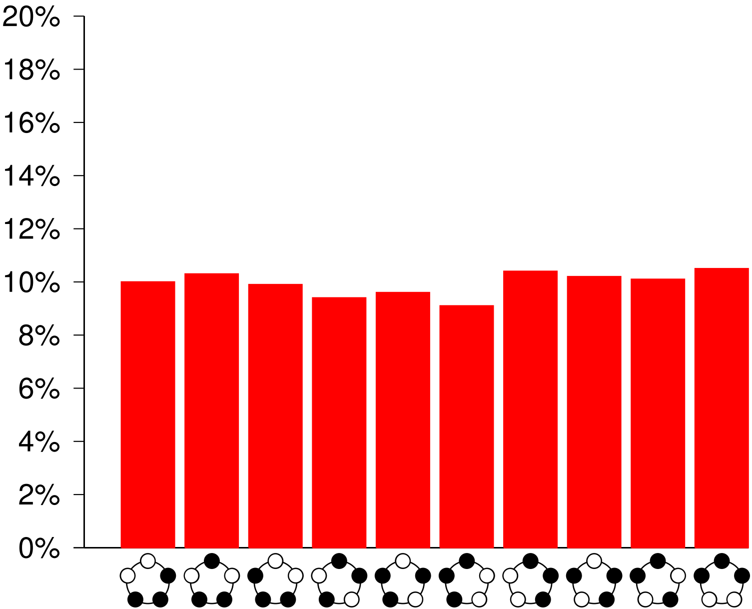

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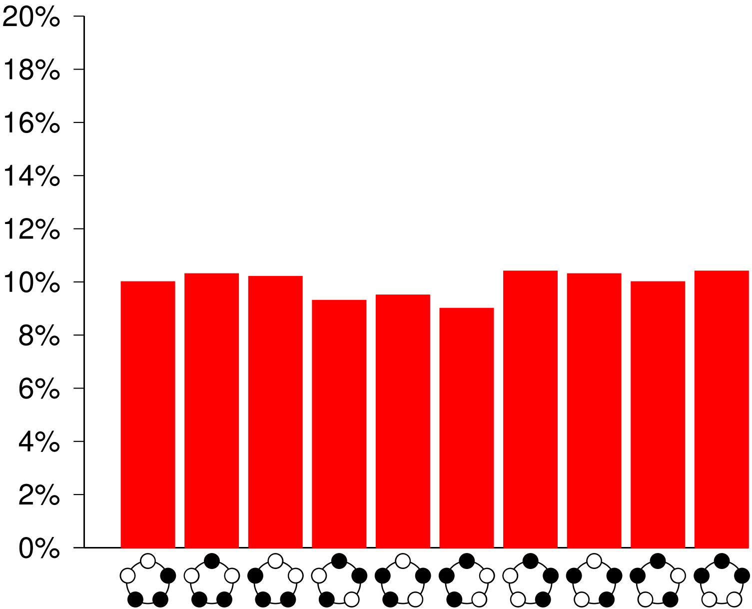

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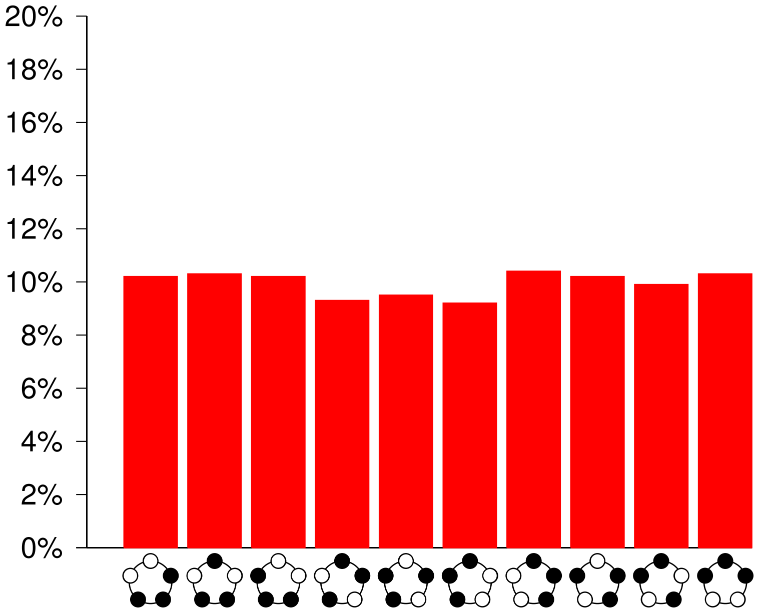

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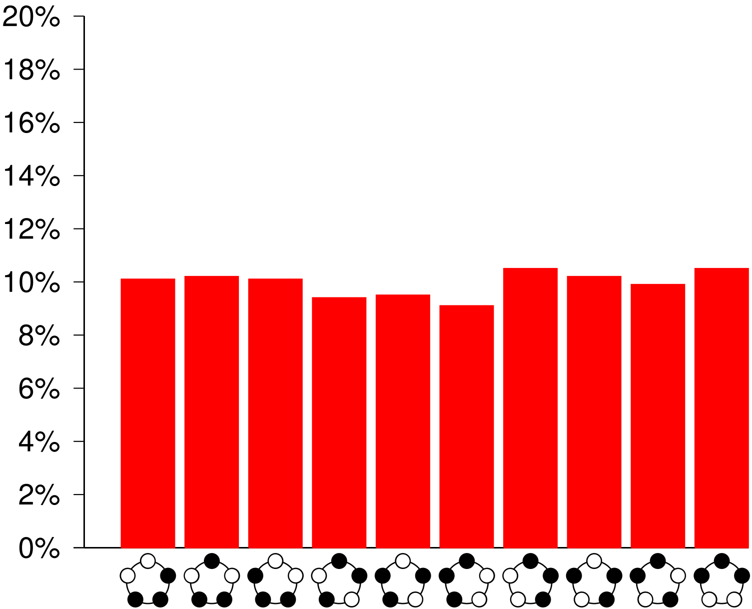

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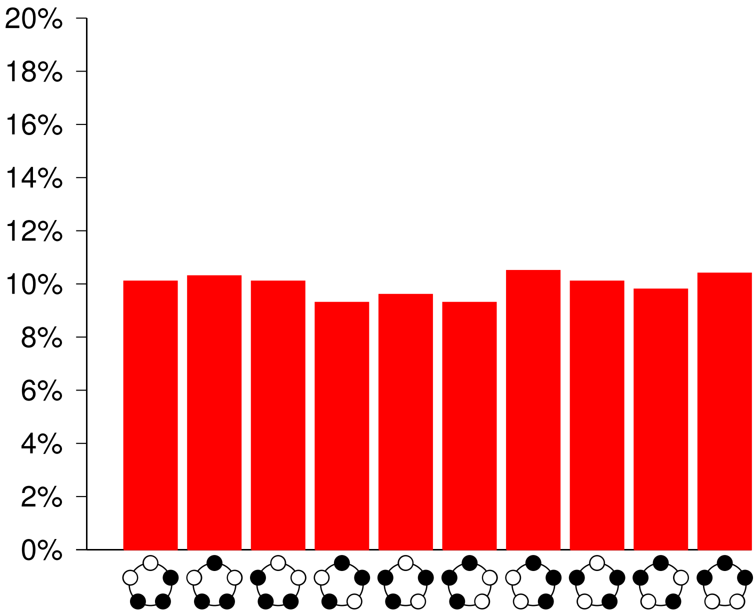

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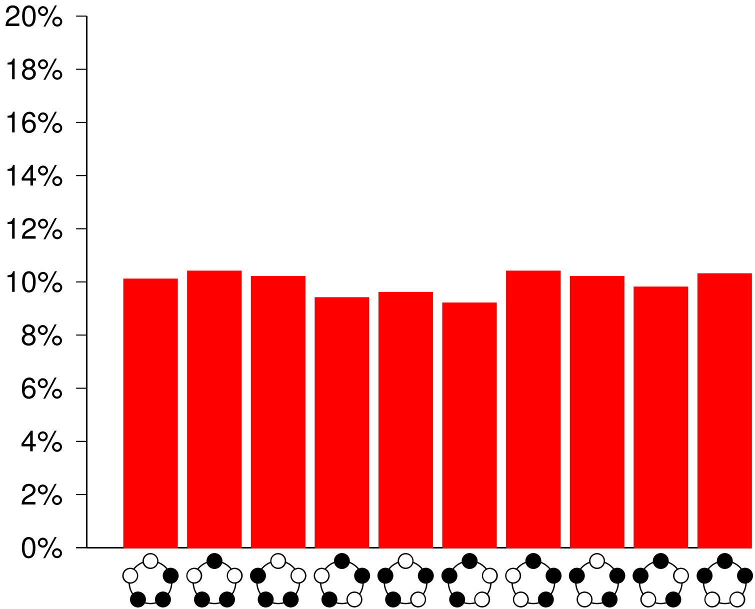

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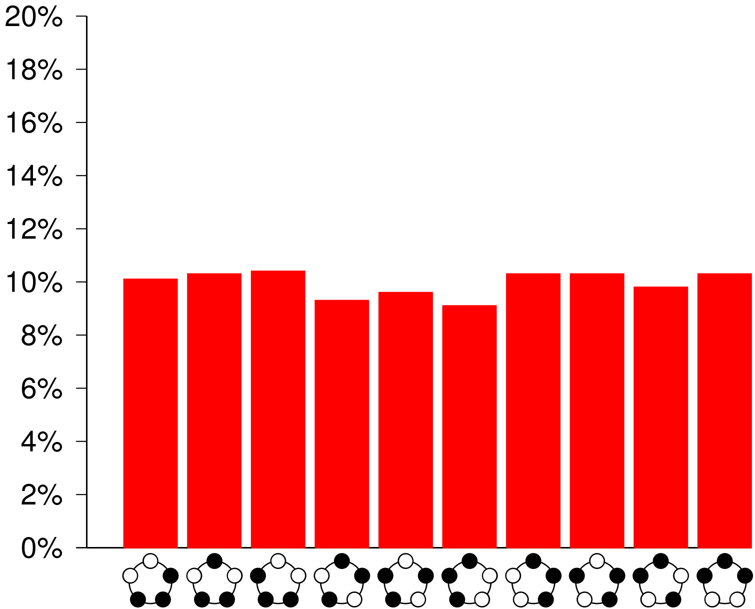

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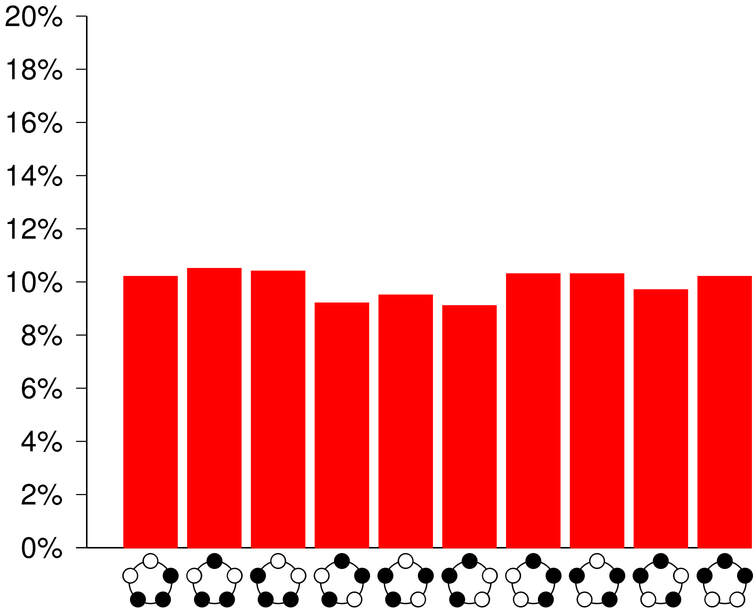

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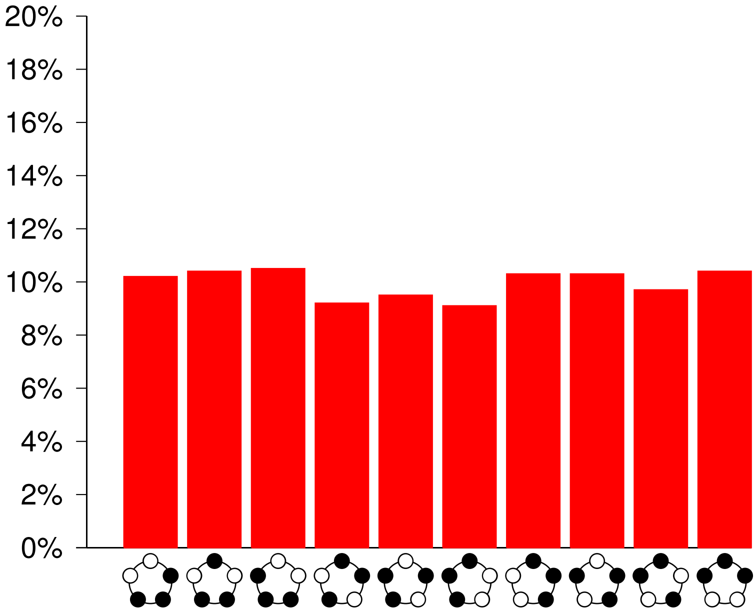

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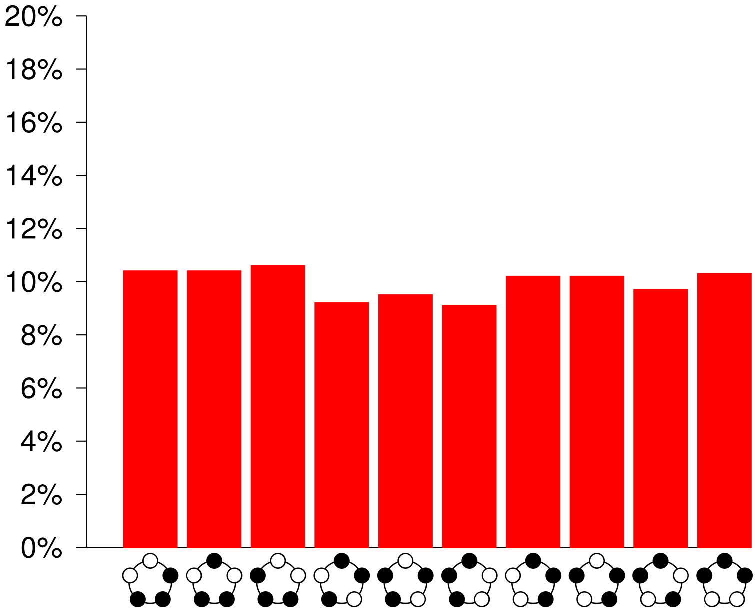

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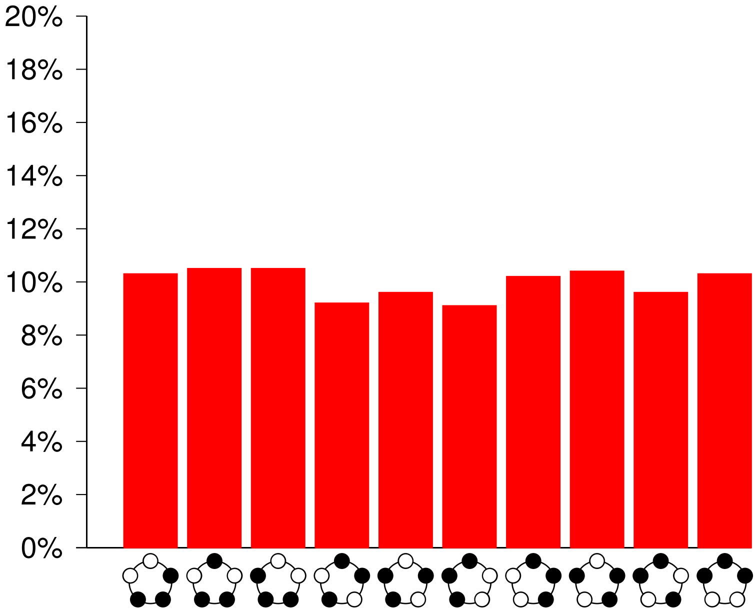

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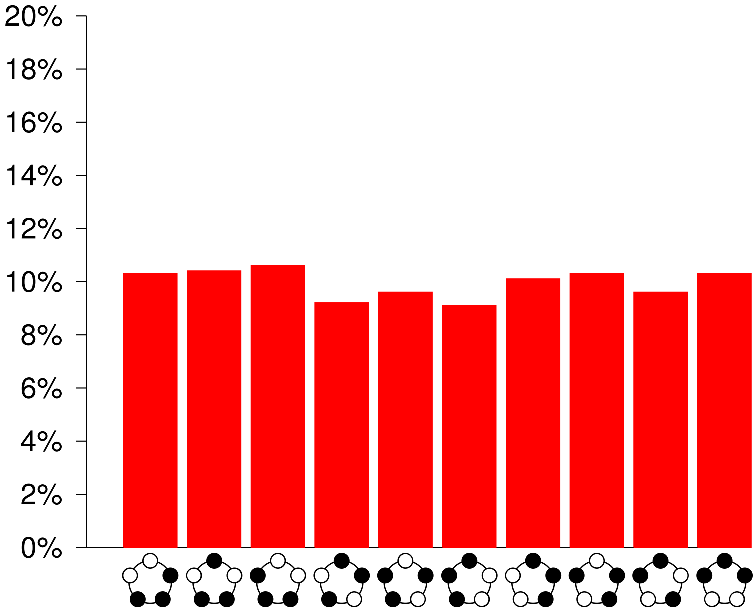

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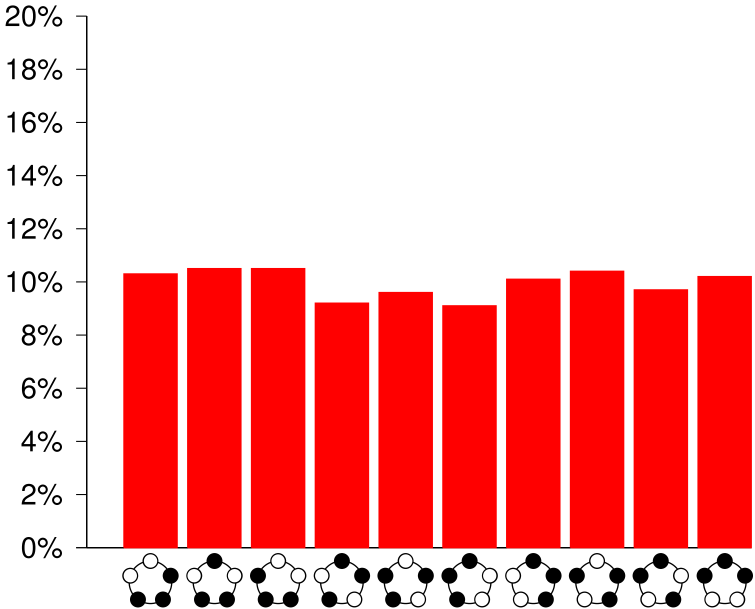

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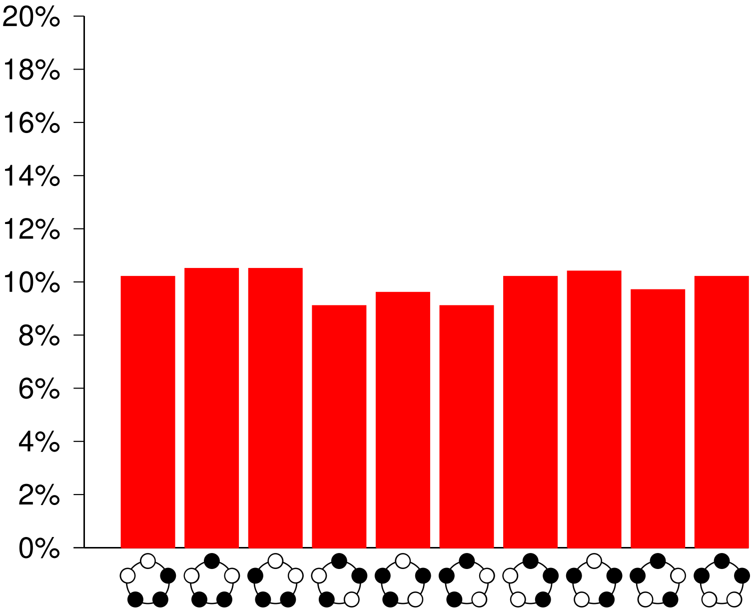

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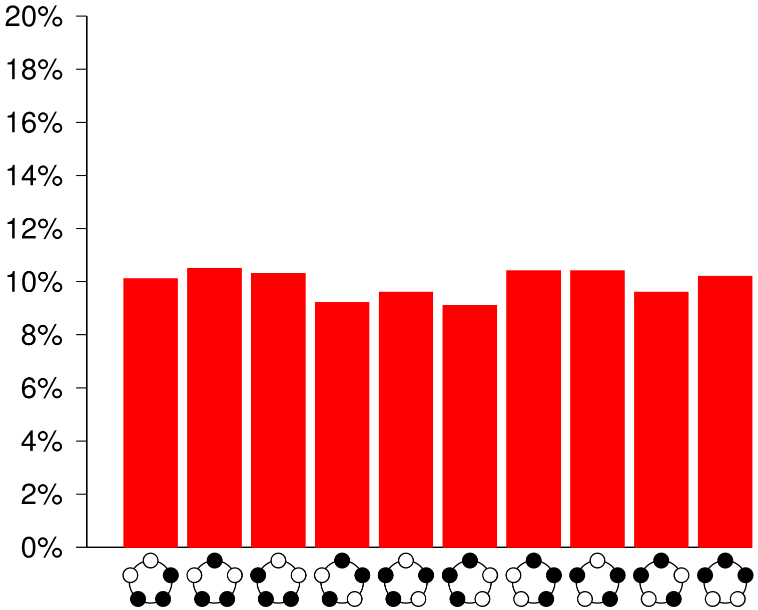

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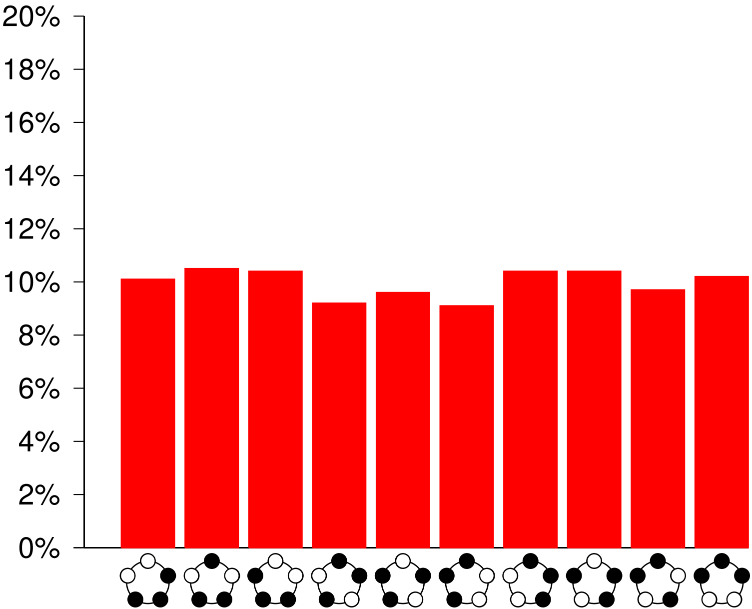

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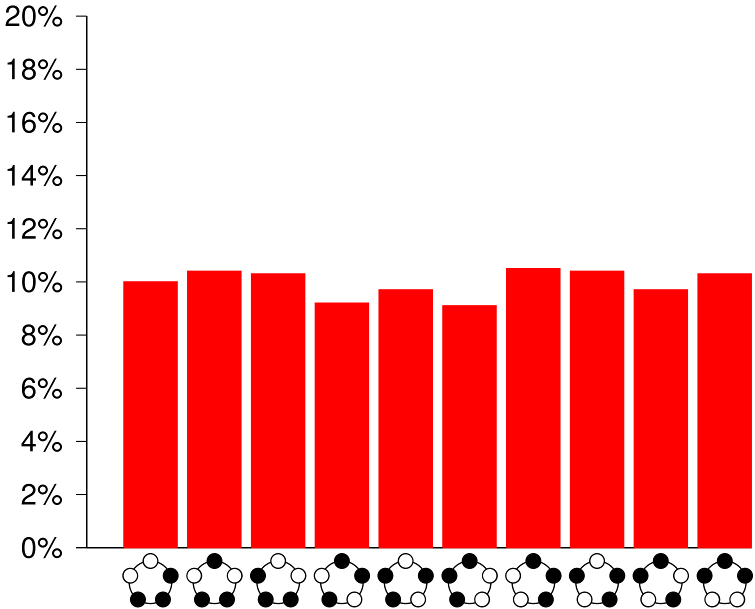

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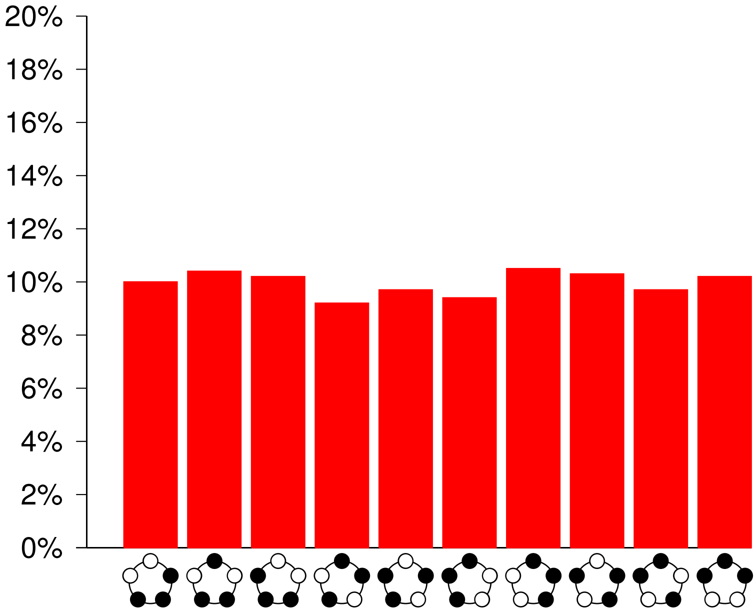

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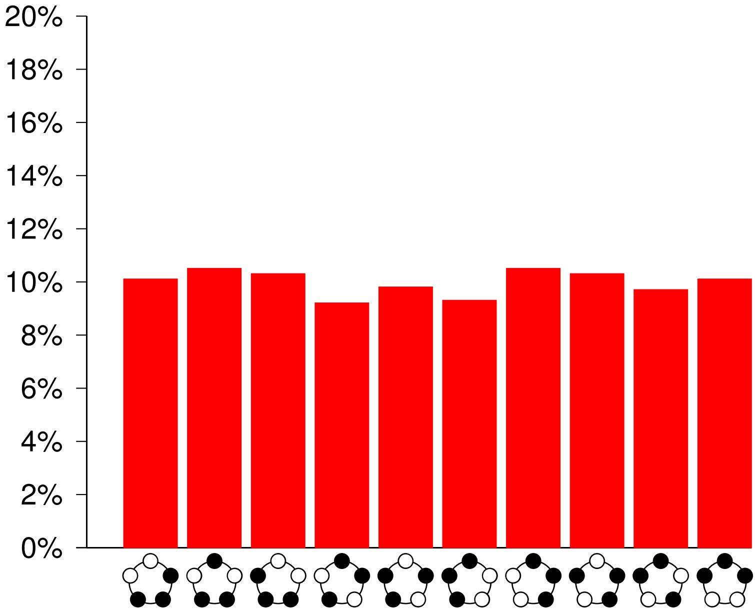

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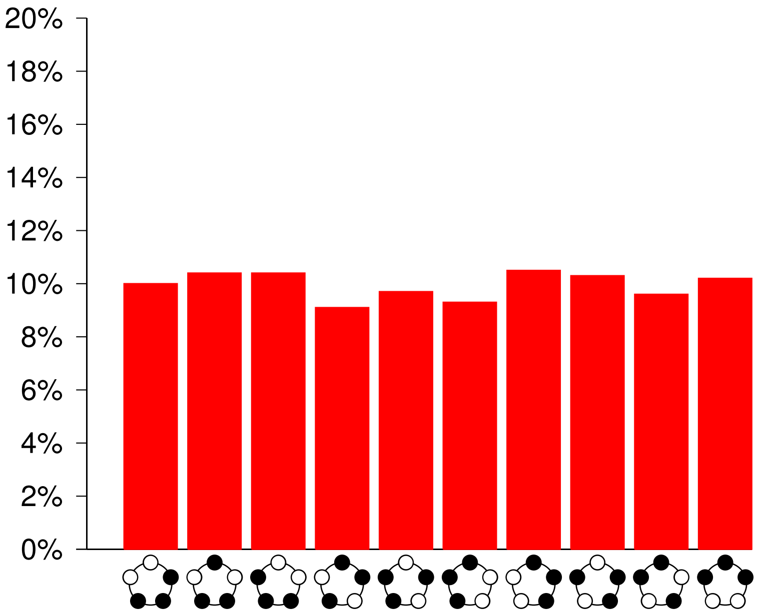

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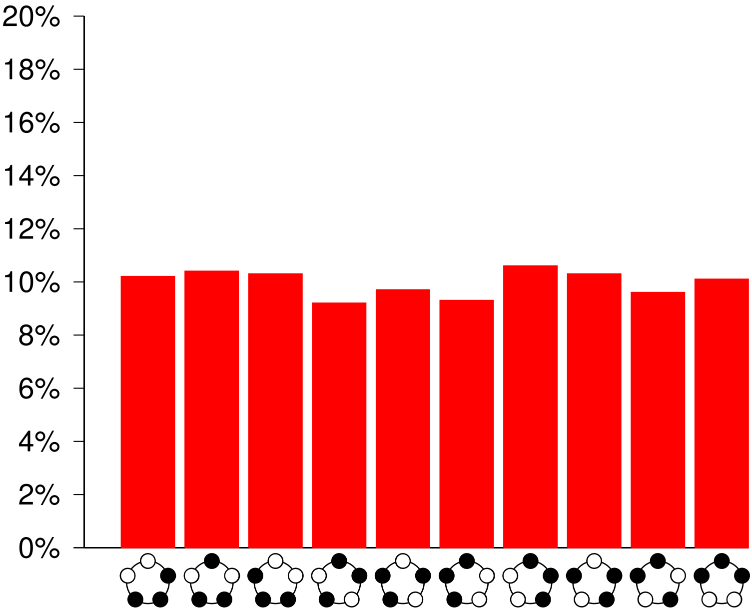

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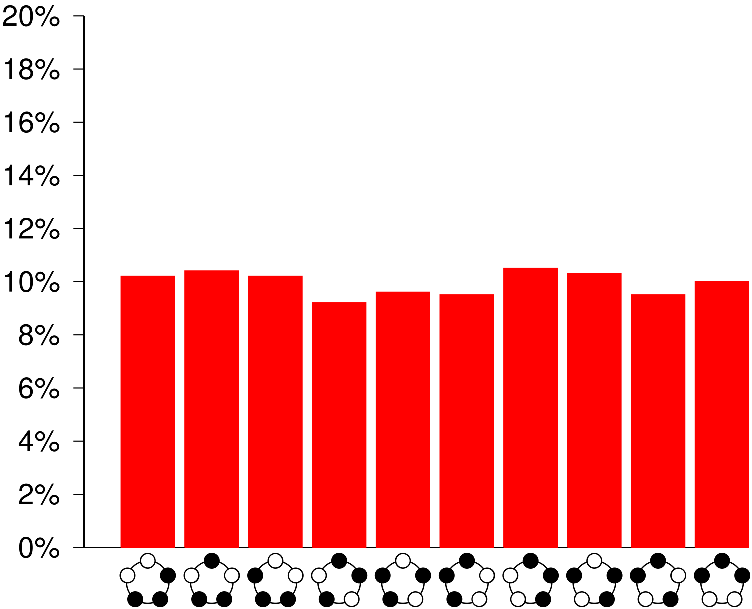

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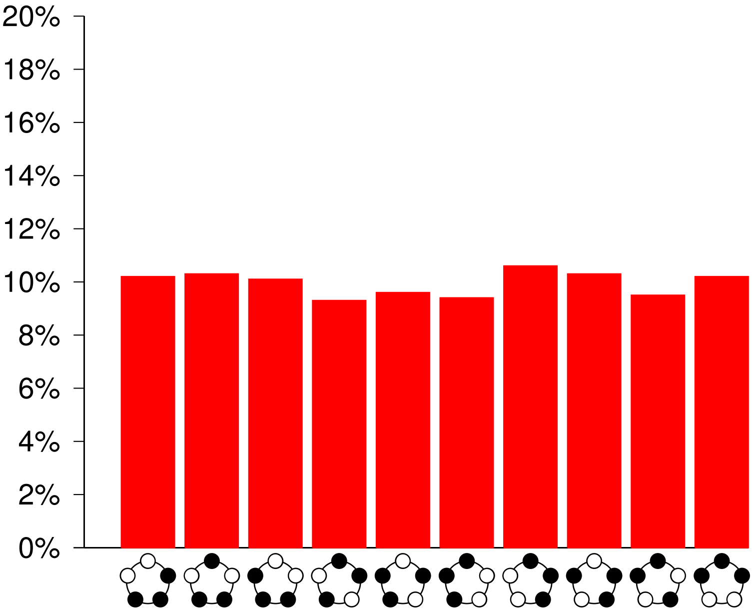

The infinite model

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In the limit we obtain a model on $\mathbb{Z}$. In its stationary distribution we have a ball with probability ϱ, and don't have one with probability $1-\varrho$ independently for each slot.

On large scales: hydrodynamics

Let us now look at the infinite model on the large scale, and let it evolve for a long time. If we change the initial density ϱ on the large (X) scale, then the process will not be stationary anymore. Instead, its density will change on the large time scale (T).

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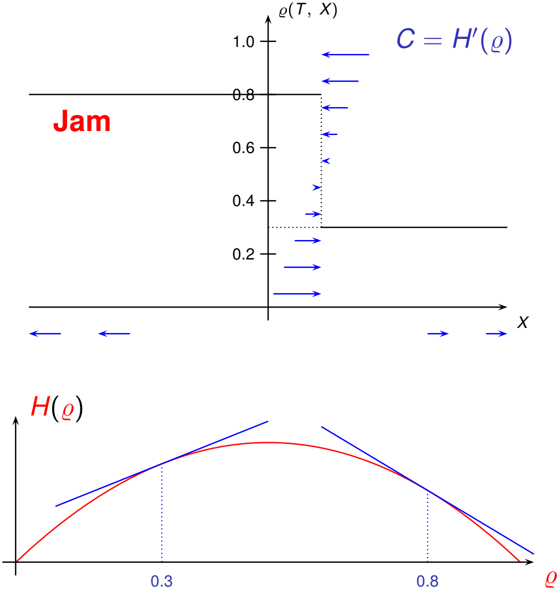
So, $\dot{X}(T) = H'(\varrho) =: C$ is the *characteristic speed*.

Rescaled version: rarefaction fan



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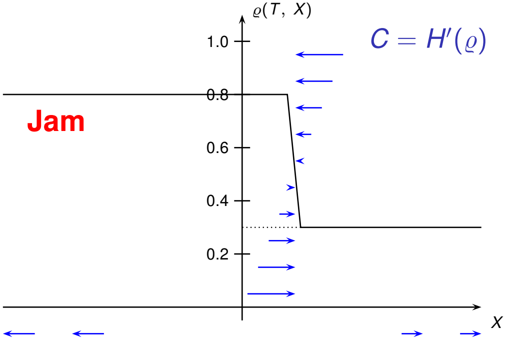
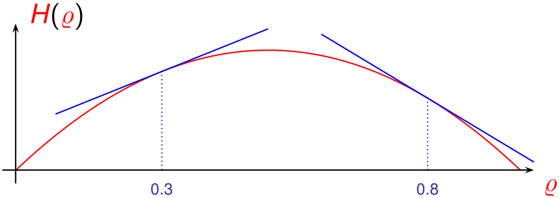
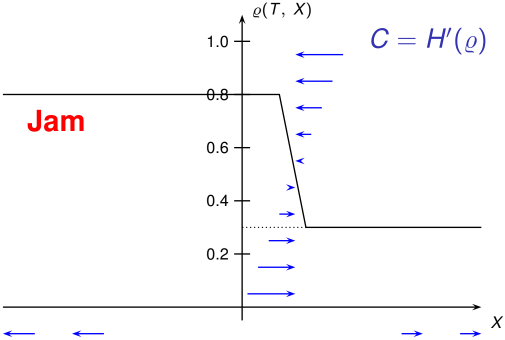
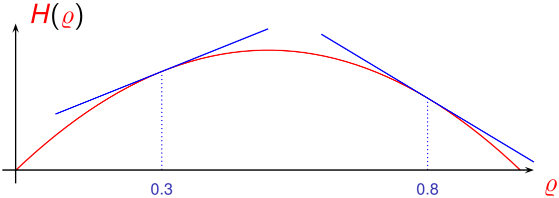




Rescaled version: rarefaction fan





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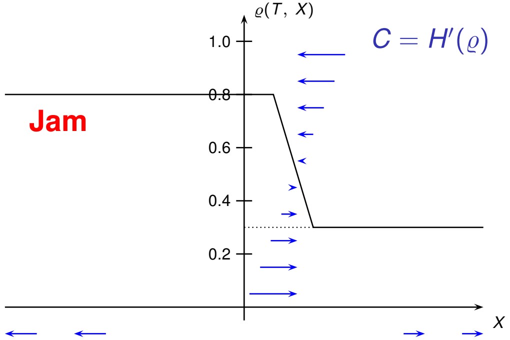

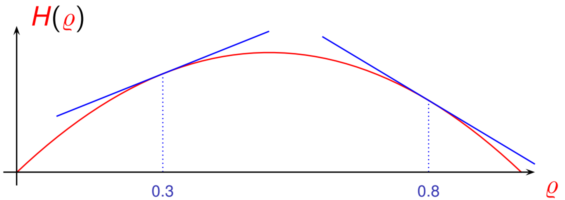

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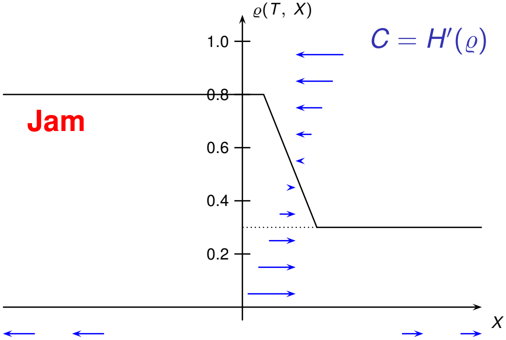

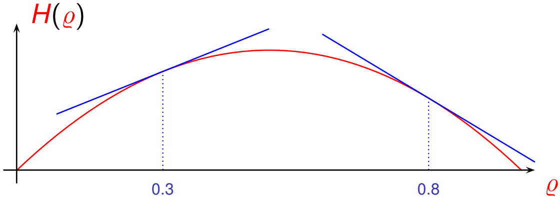

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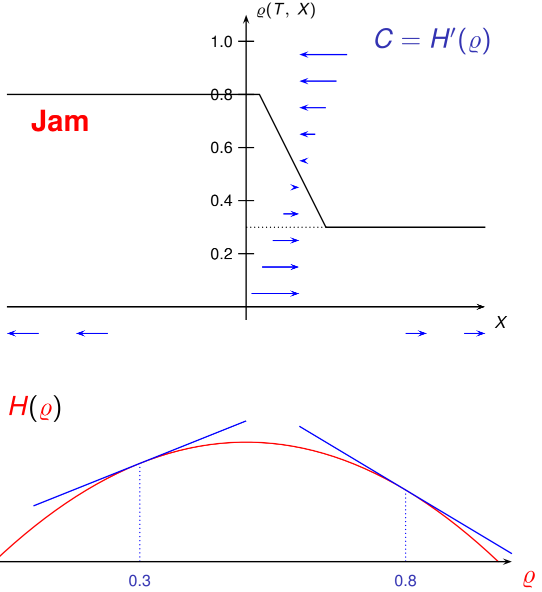

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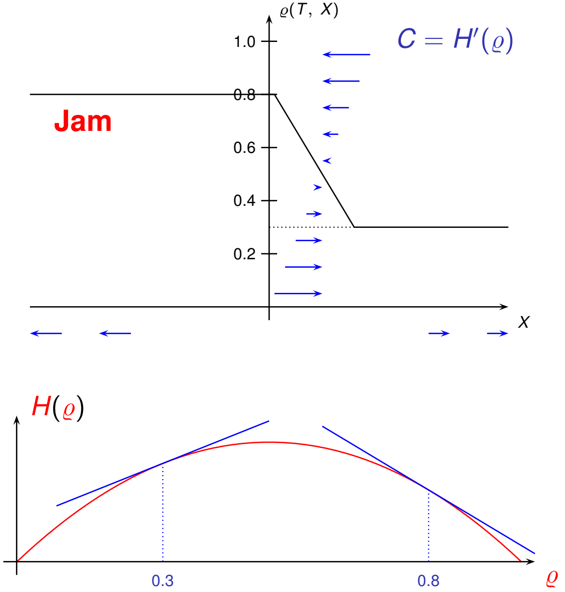

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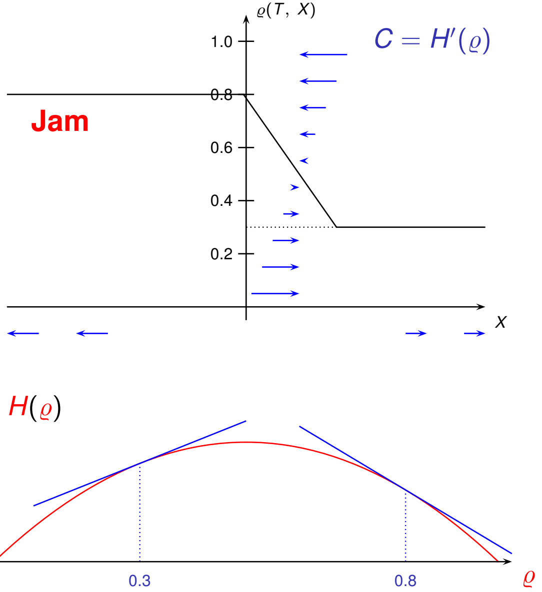

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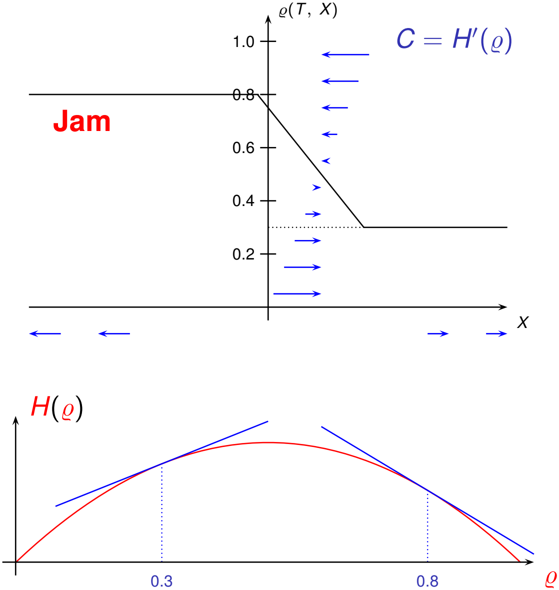

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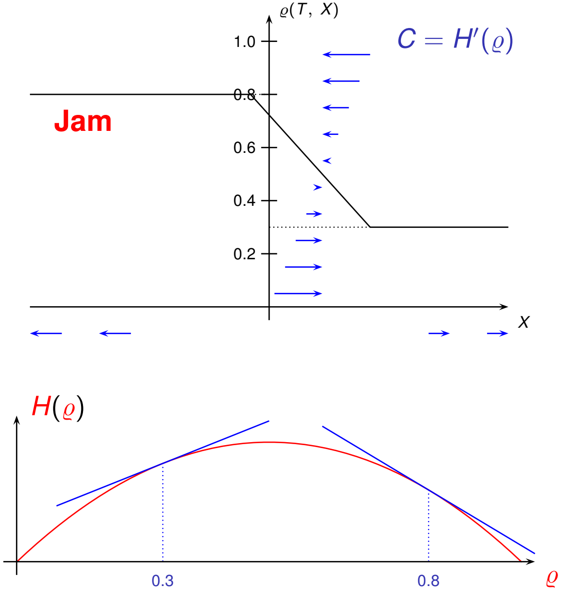

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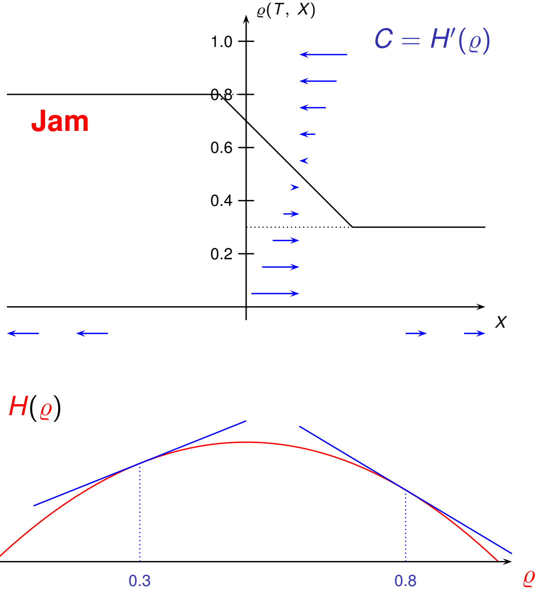

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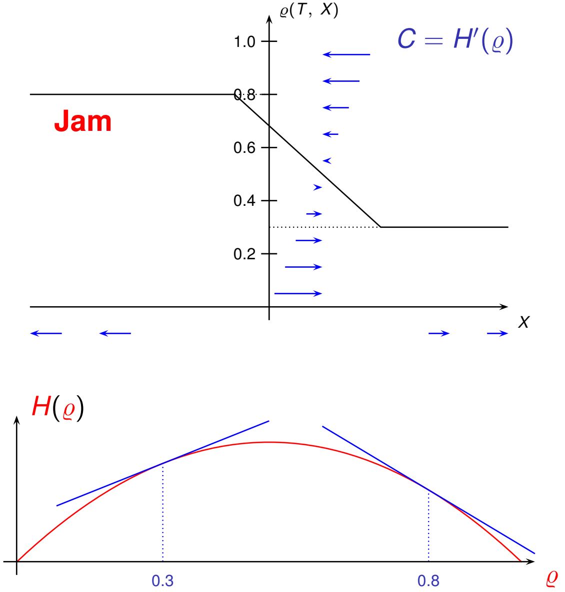

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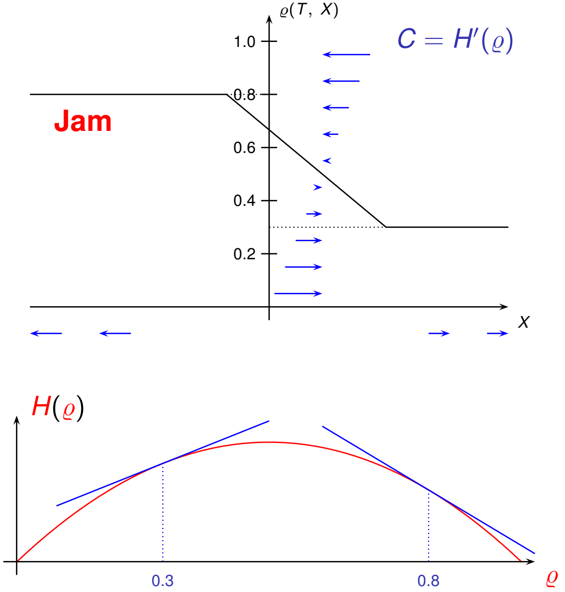

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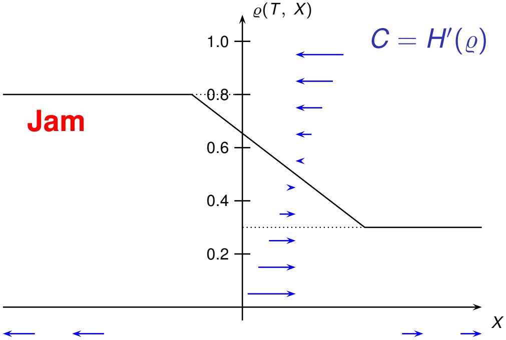

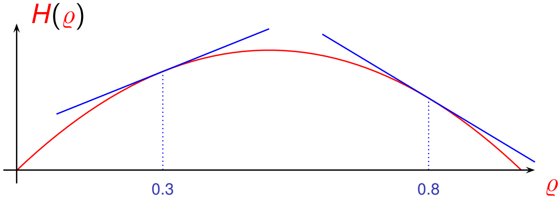

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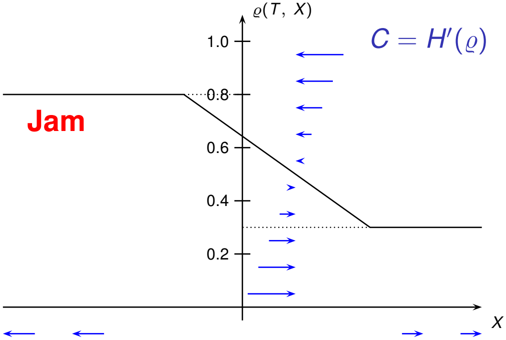

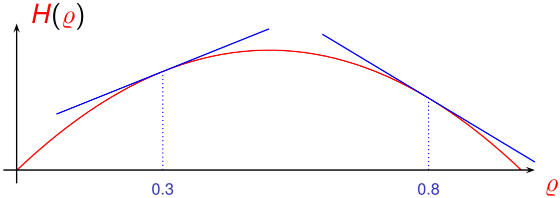

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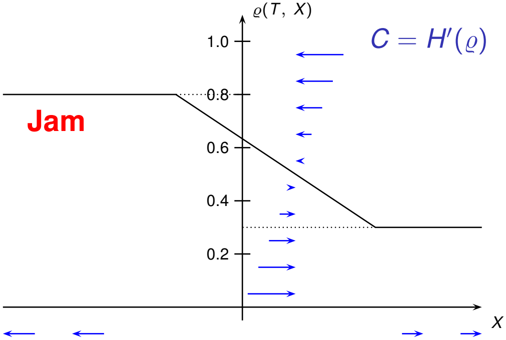

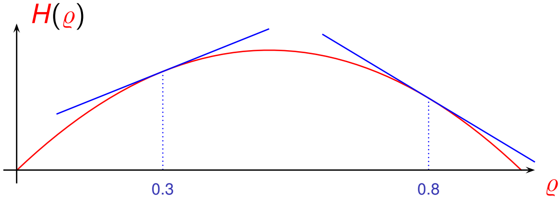

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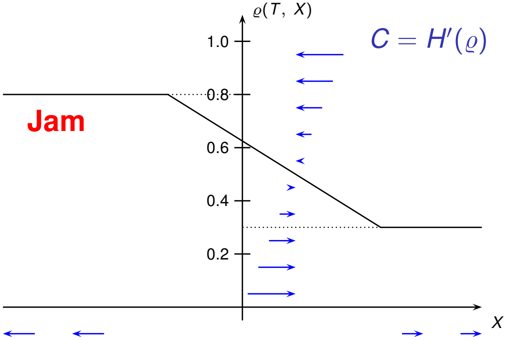

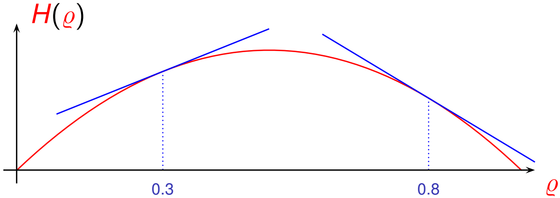

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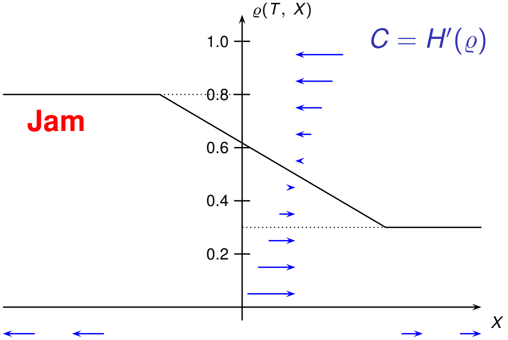

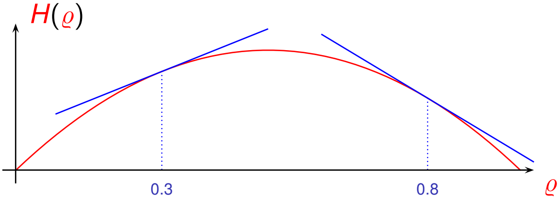

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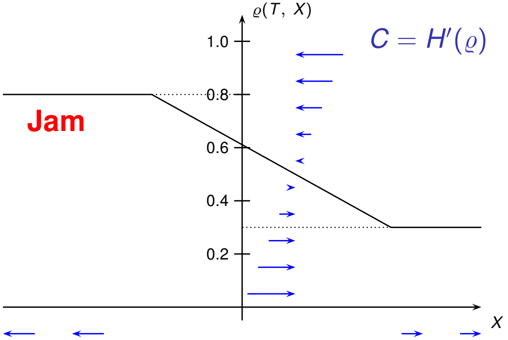

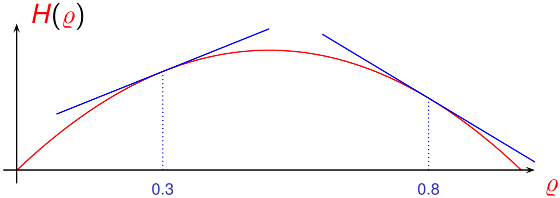

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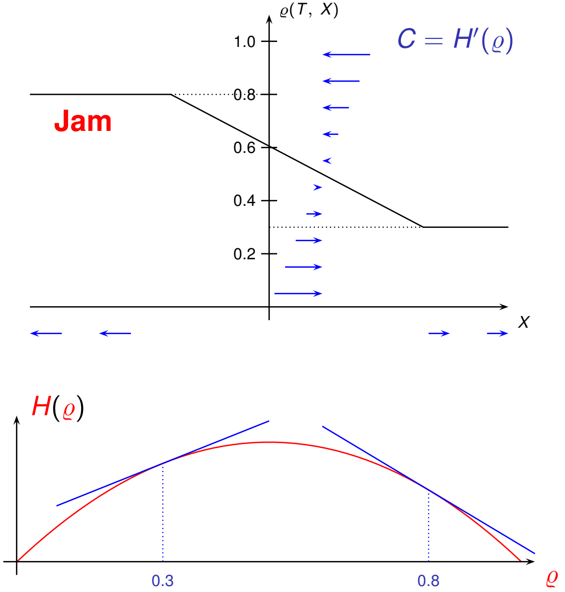

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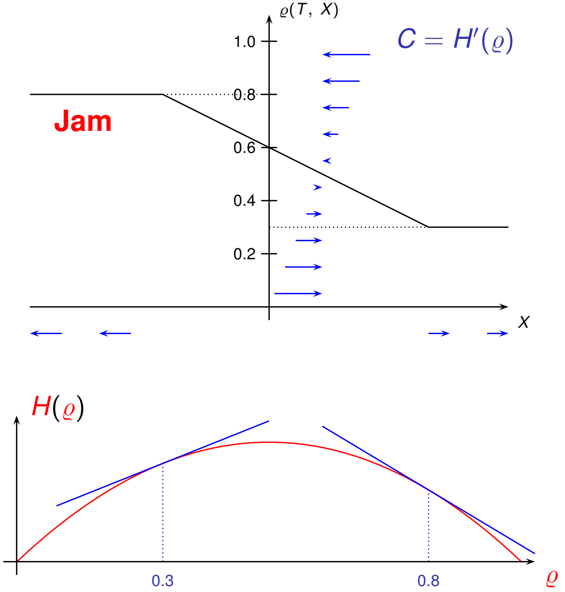

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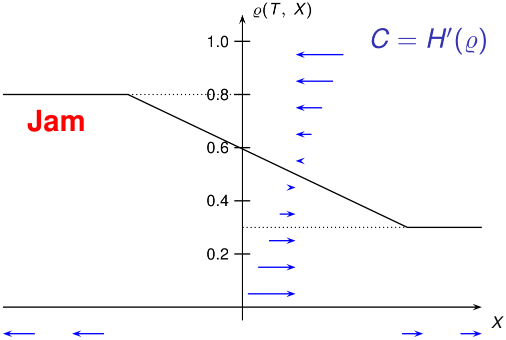

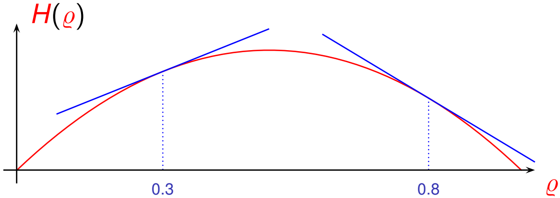

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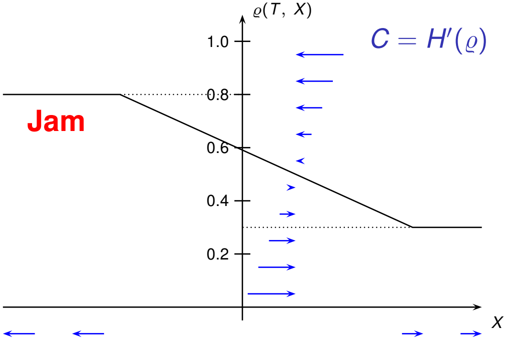

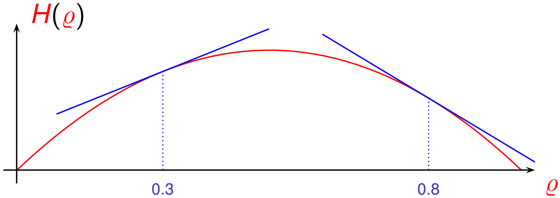

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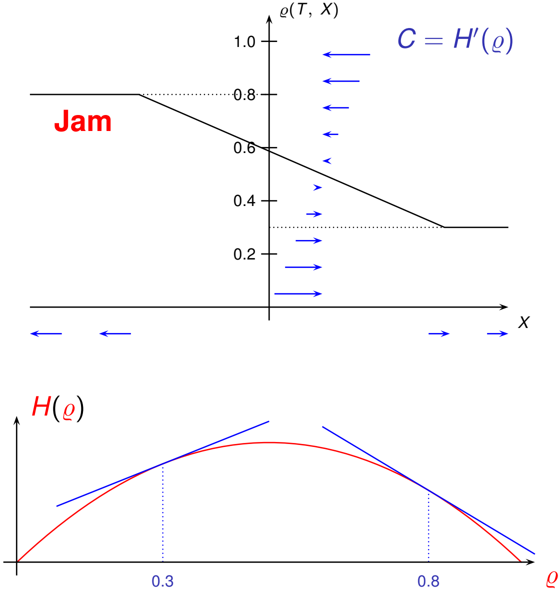

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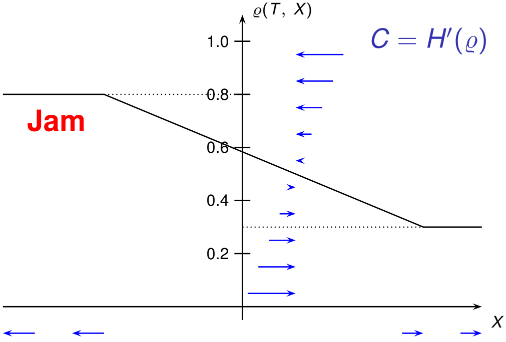

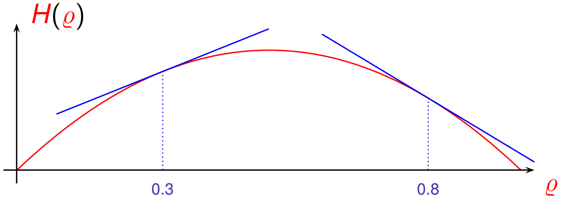

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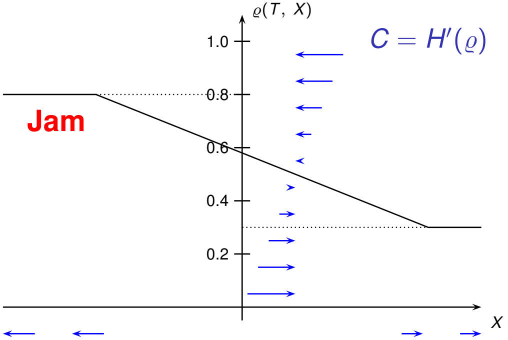

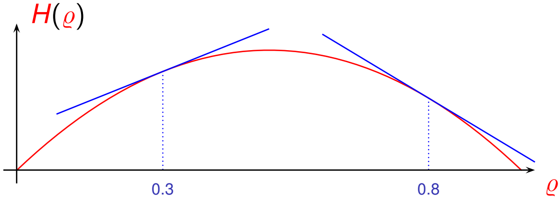

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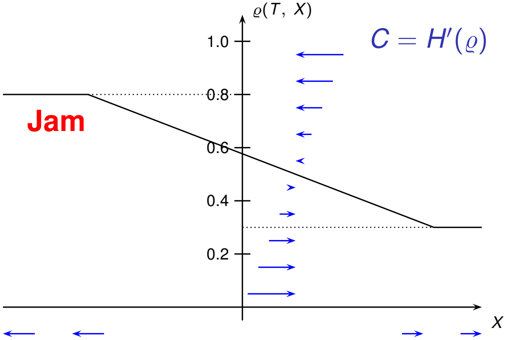

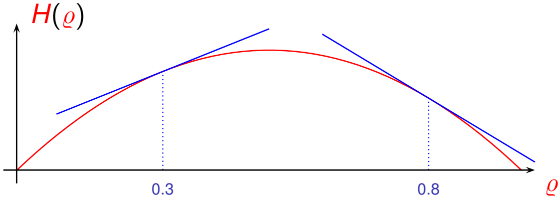

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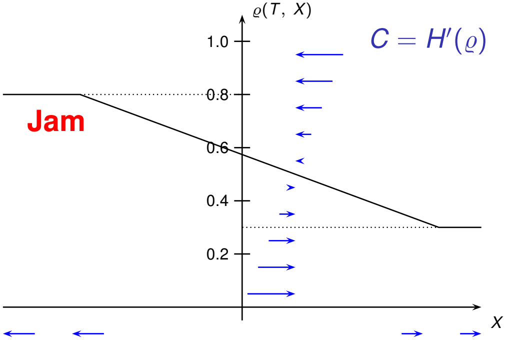

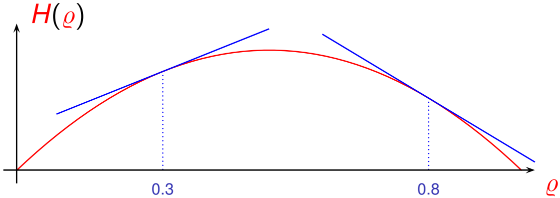

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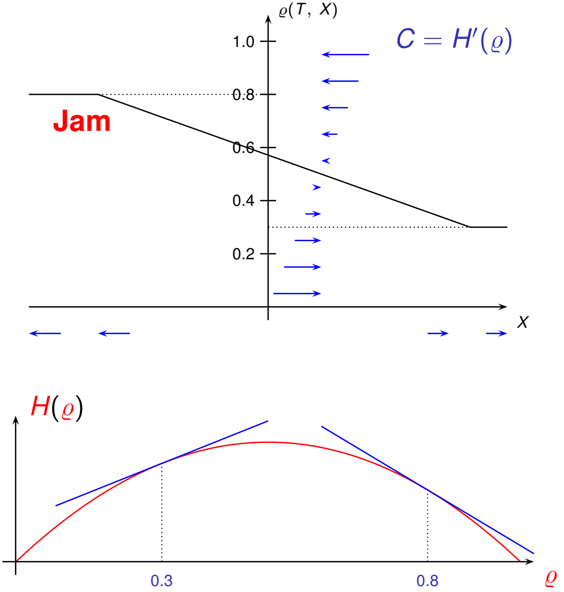

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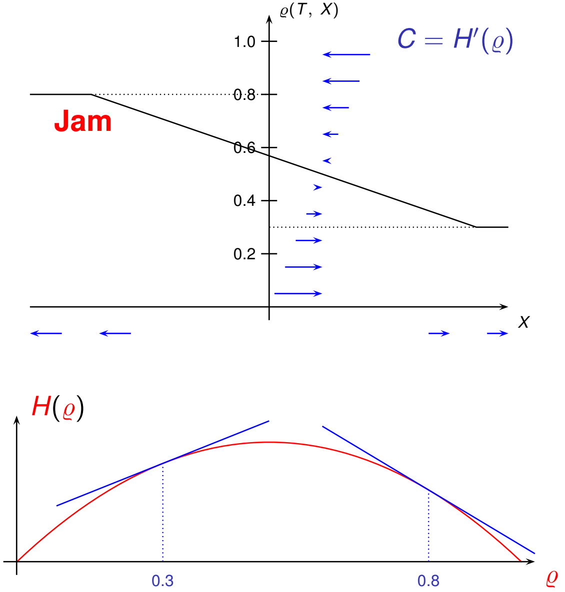

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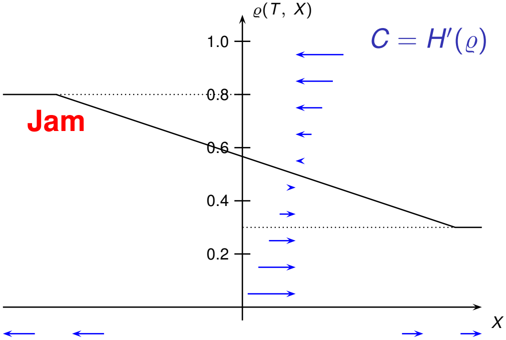

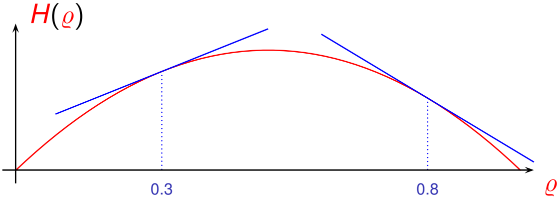

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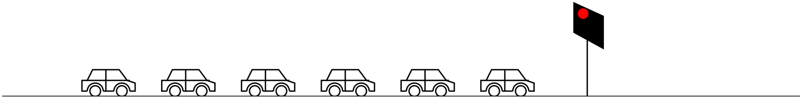

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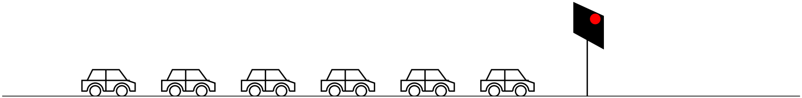

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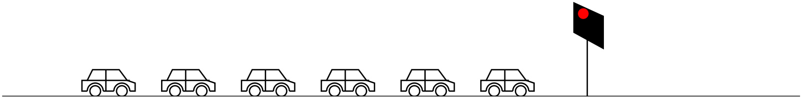

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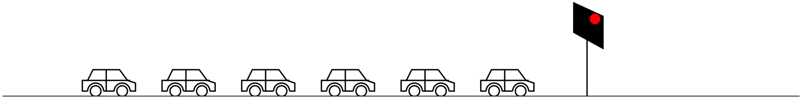

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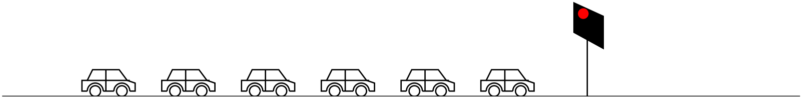

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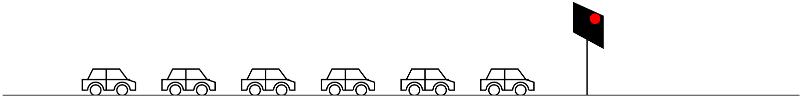

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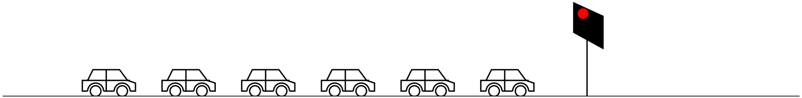

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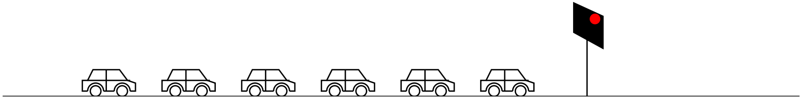

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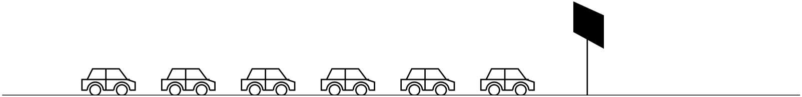

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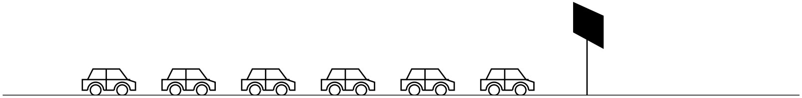

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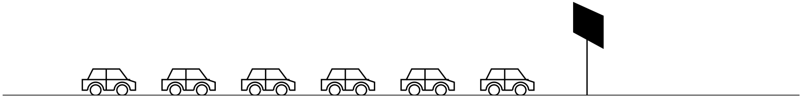

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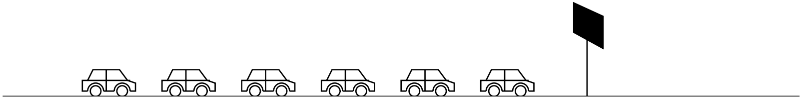

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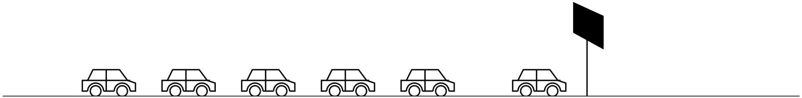

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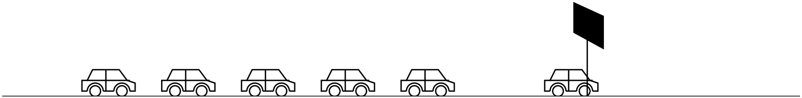

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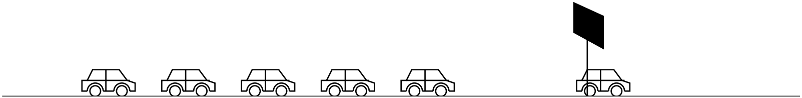

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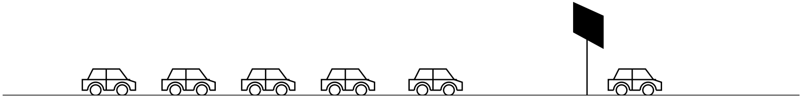

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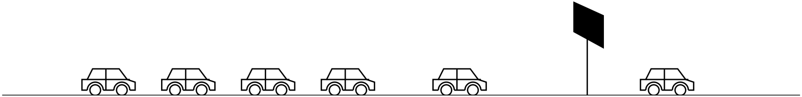

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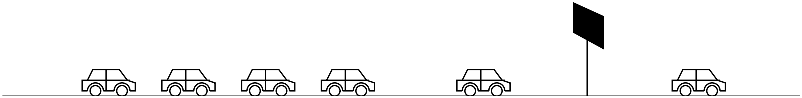

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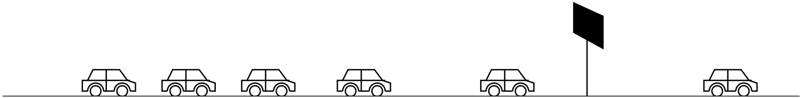

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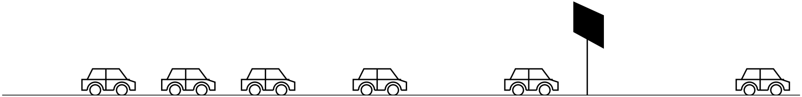

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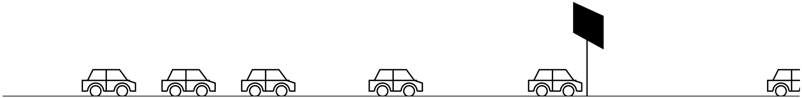

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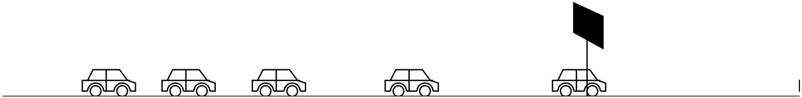

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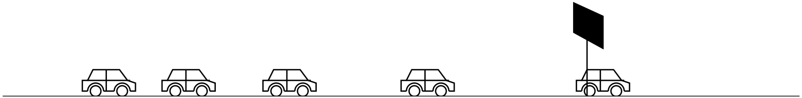

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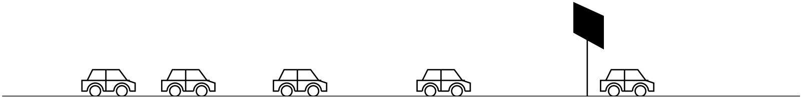

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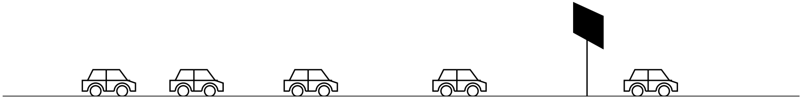

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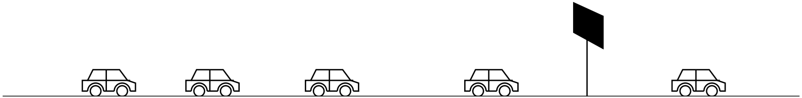

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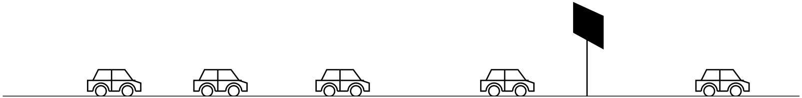

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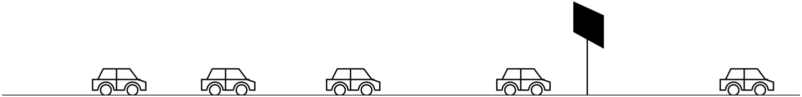

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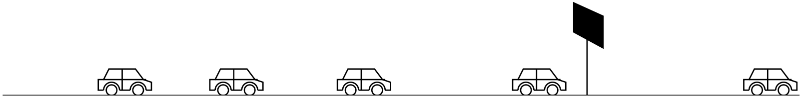

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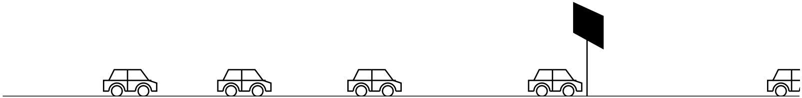

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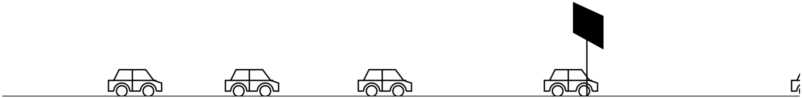

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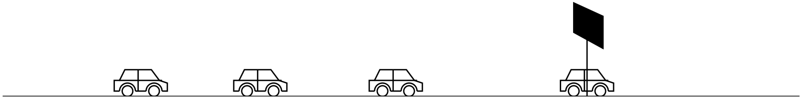

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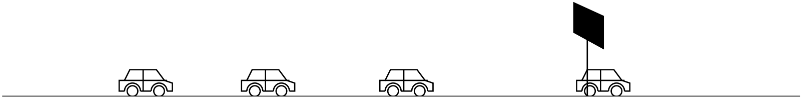

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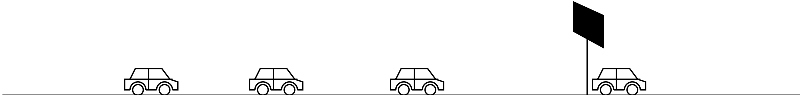

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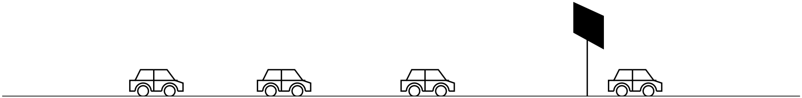

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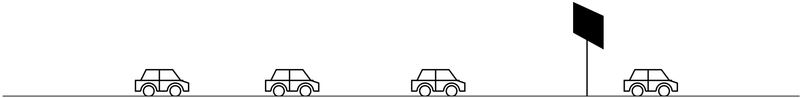

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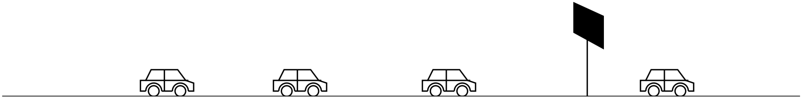

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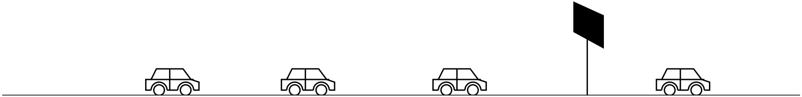

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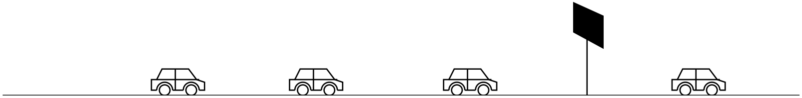

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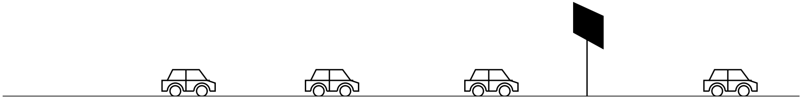

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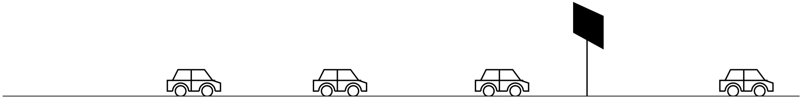

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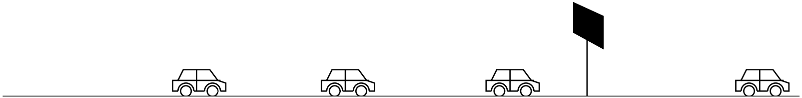

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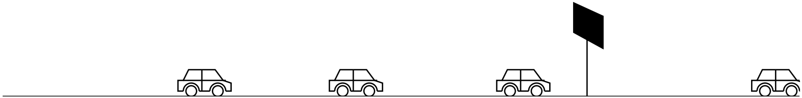

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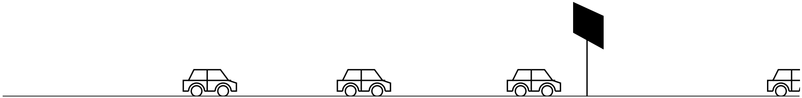

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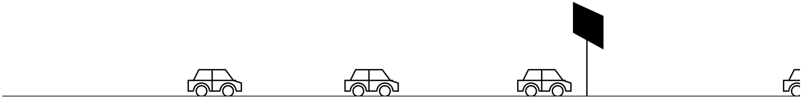

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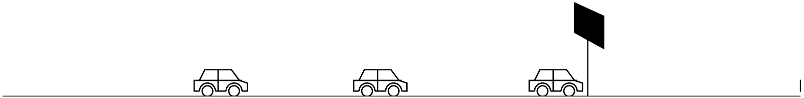

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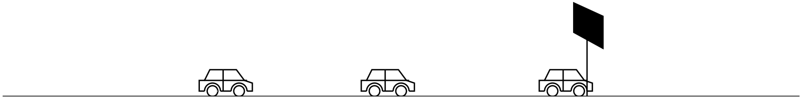

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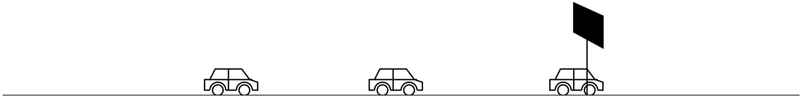

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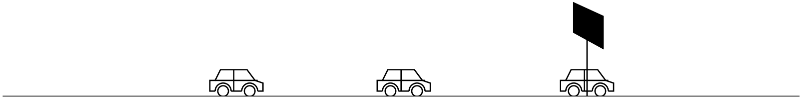

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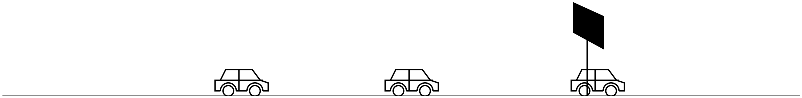

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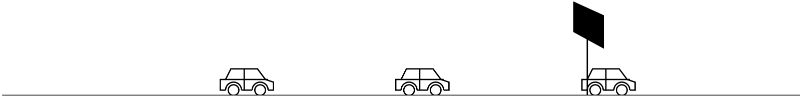

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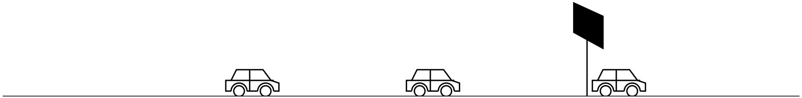

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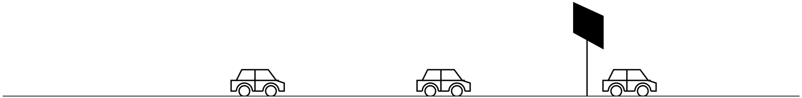

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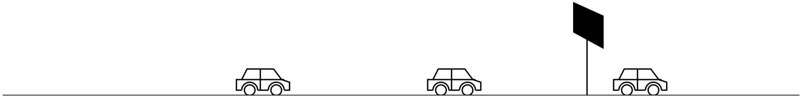

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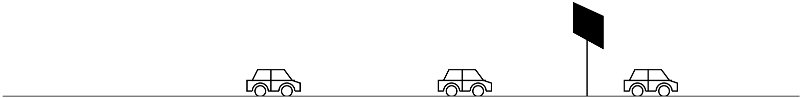

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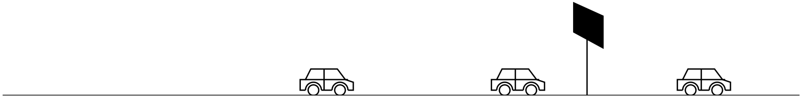

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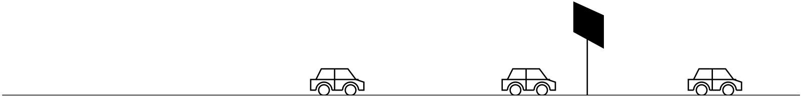

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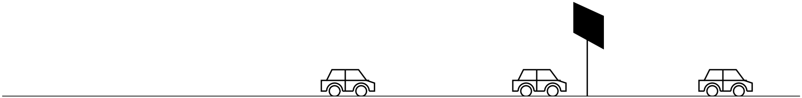

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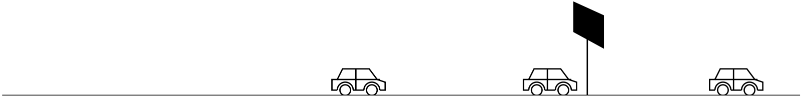

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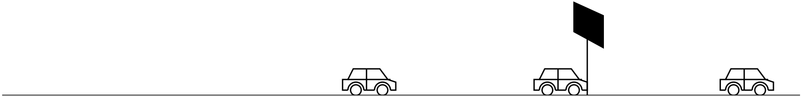

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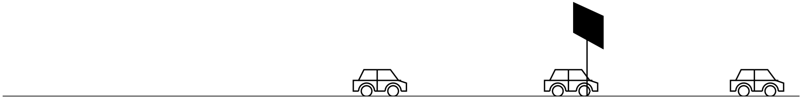

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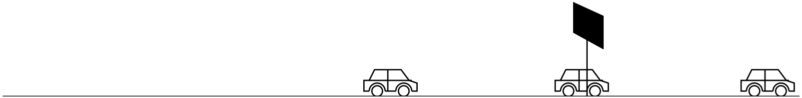

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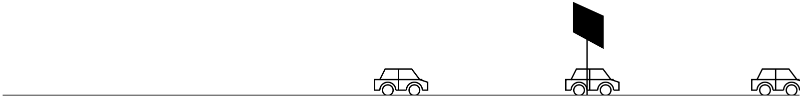

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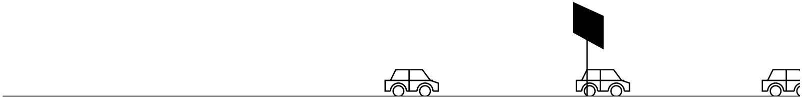

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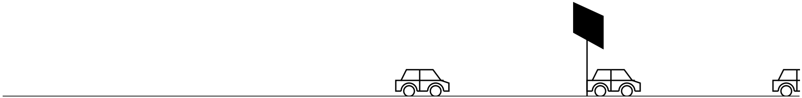

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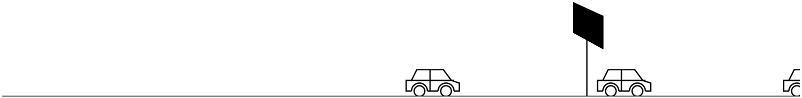

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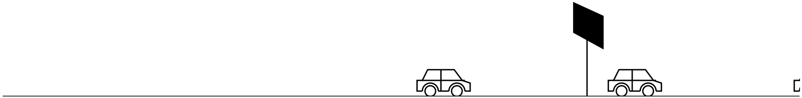

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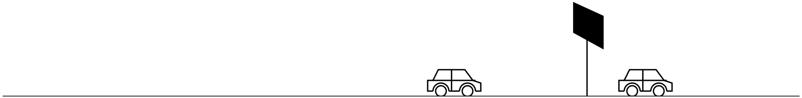

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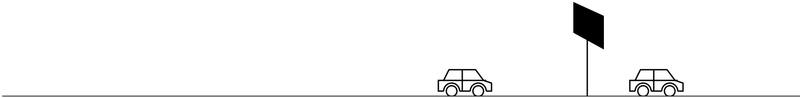

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Leaving a traffic jam

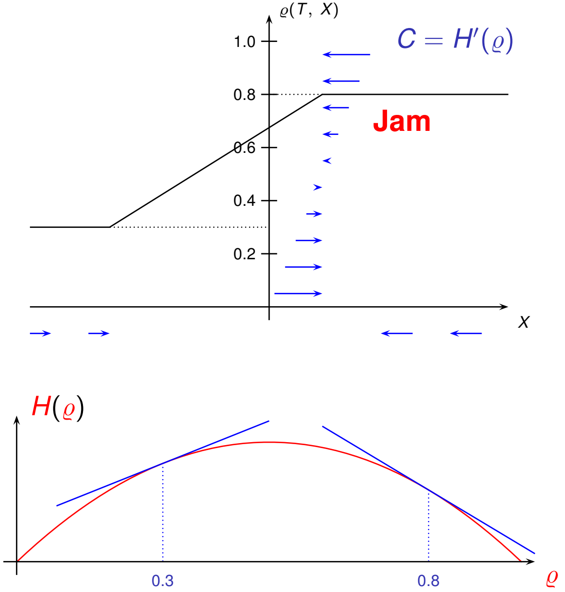
Continuous, long acceleration for those starting from the rear

Leaving a traffic jam

Continuous, long acceleration for those starting from the rear

Leaving a traffic jam is always soft, “blurry”.

Rescaled version: shock



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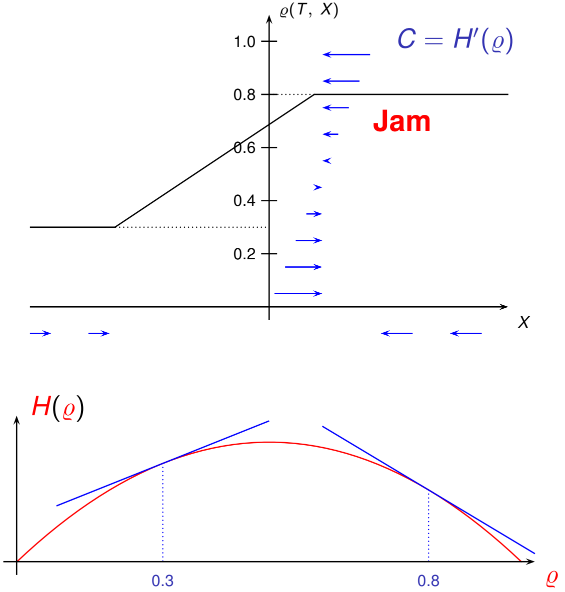

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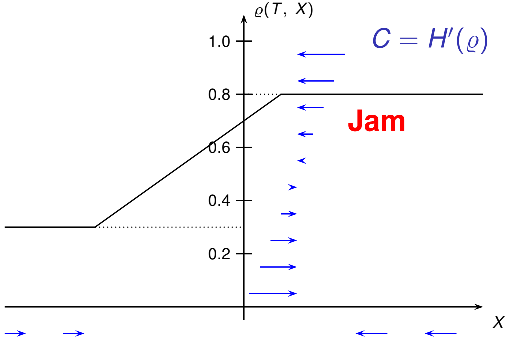

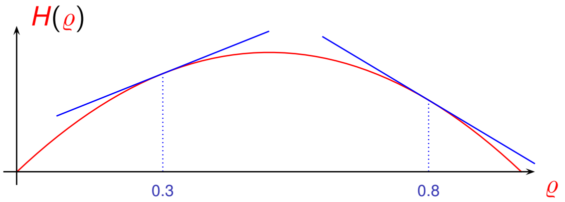

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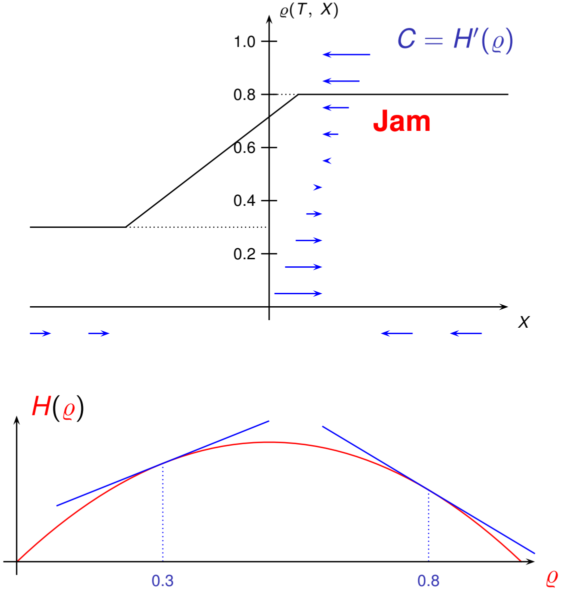

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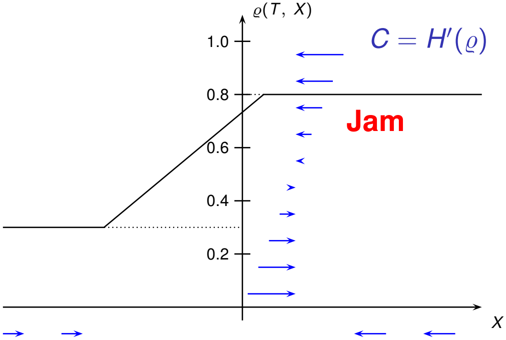

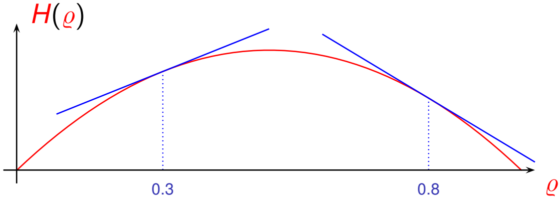

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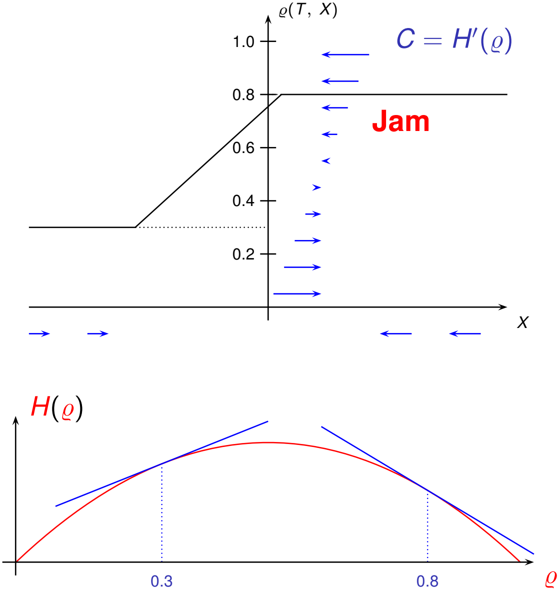

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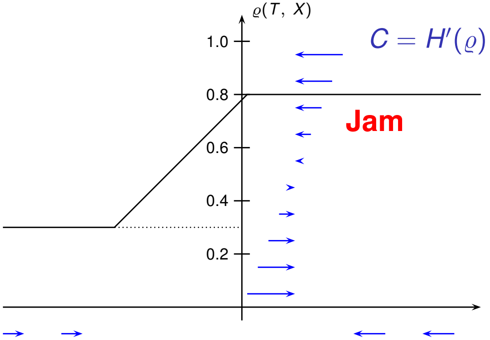

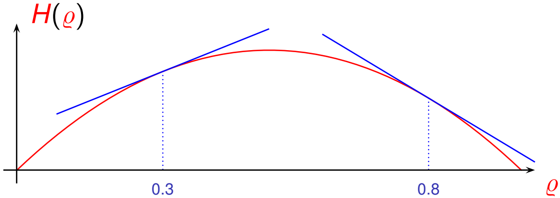

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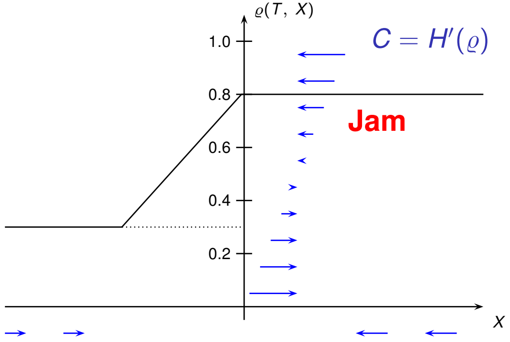

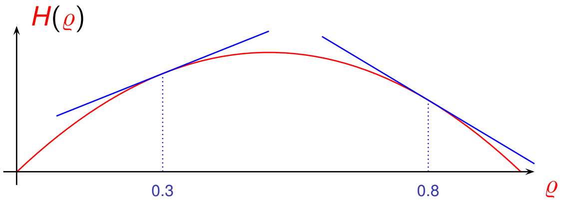

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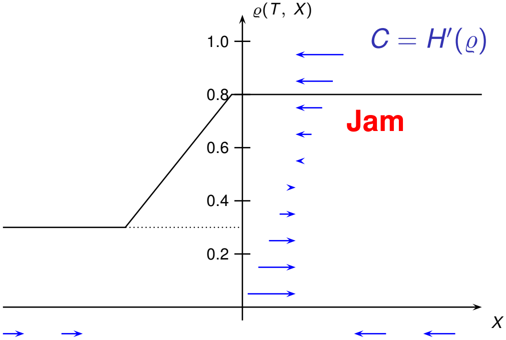

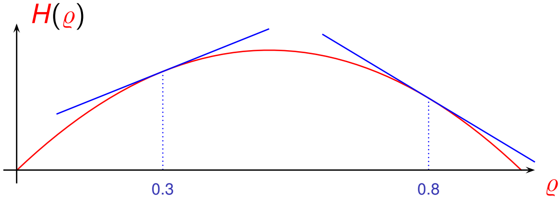

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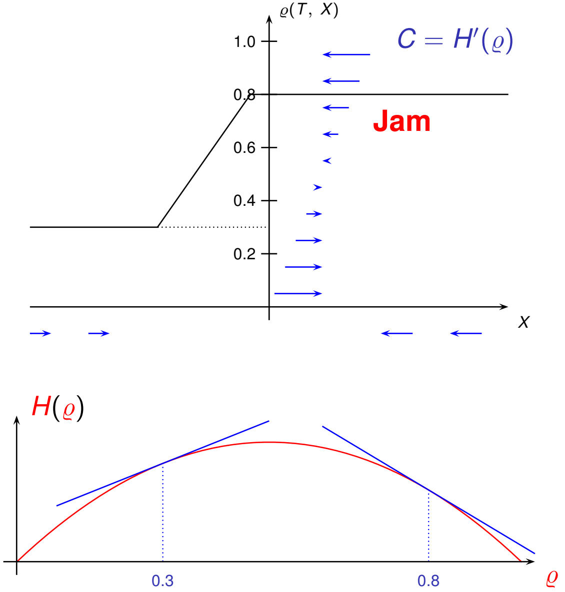

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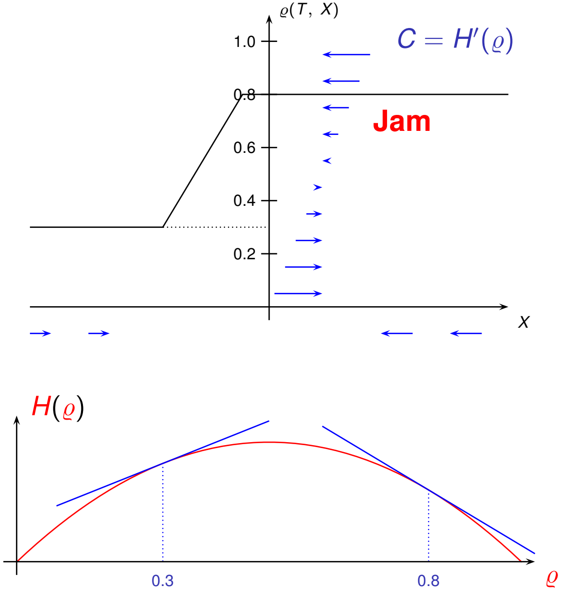

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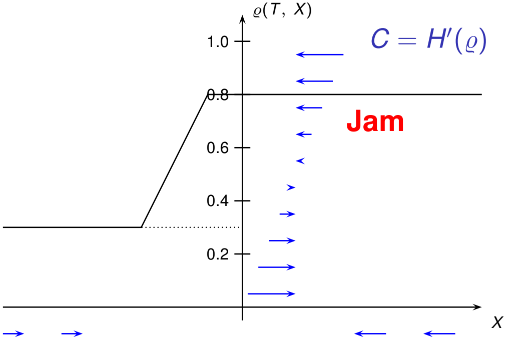

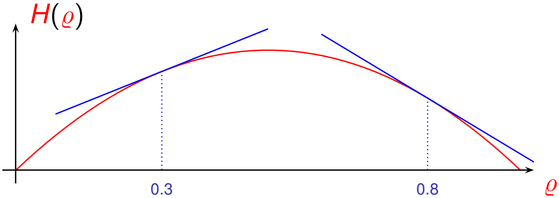

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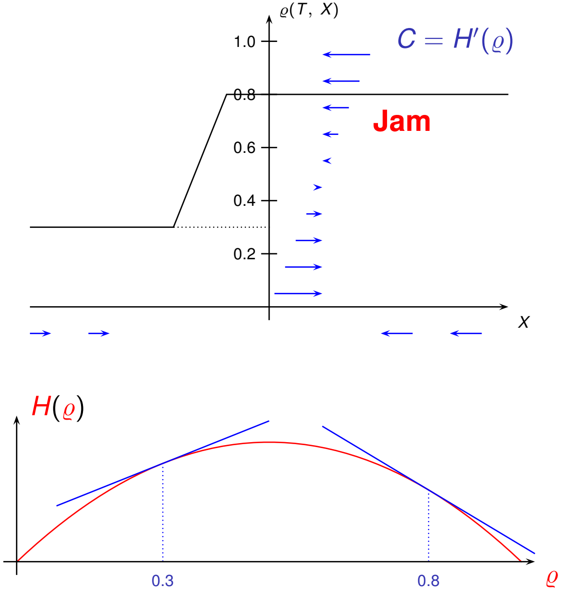

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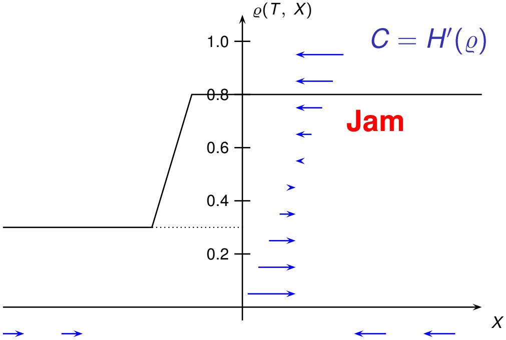

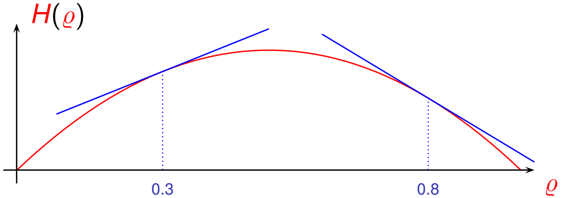

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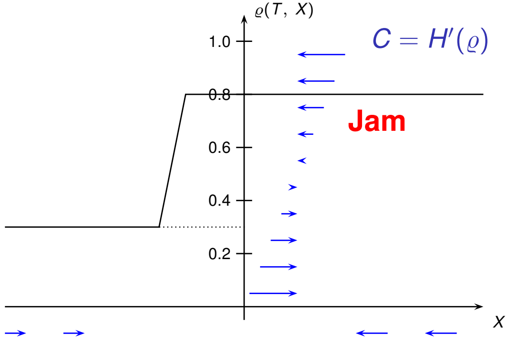

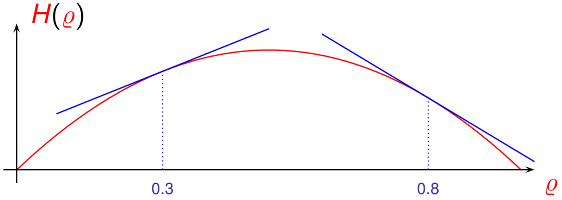

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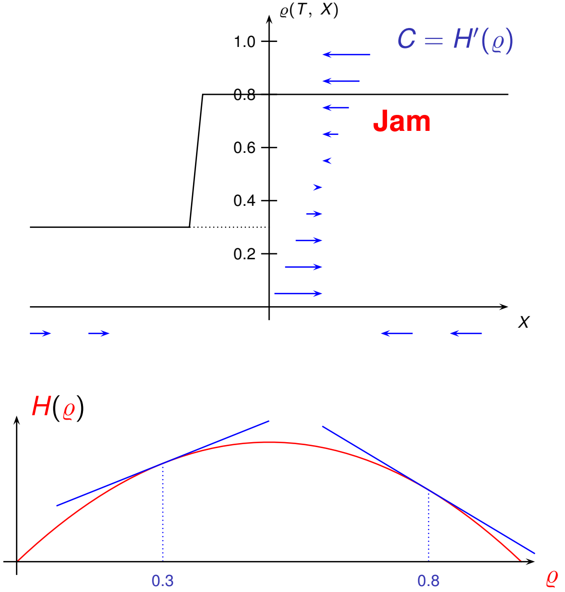

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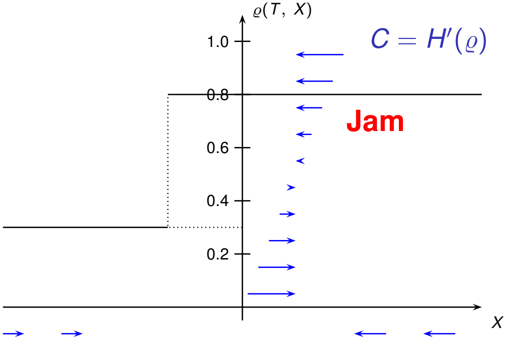

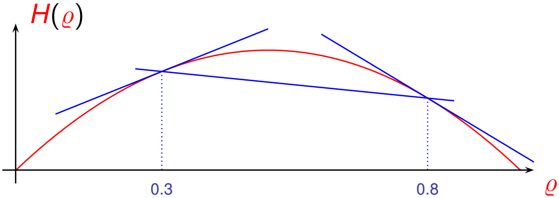

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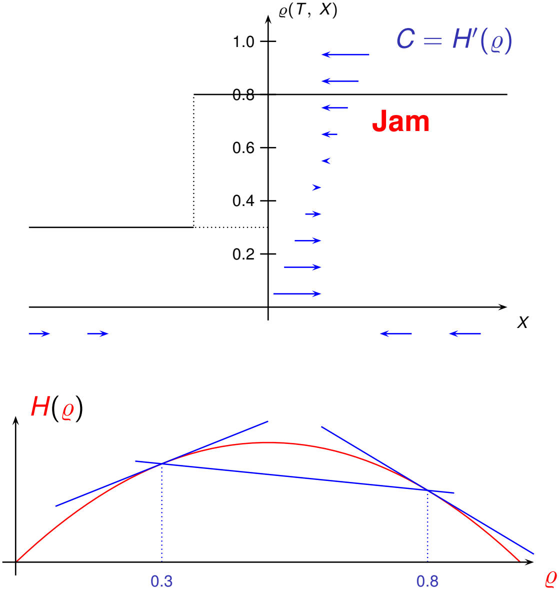

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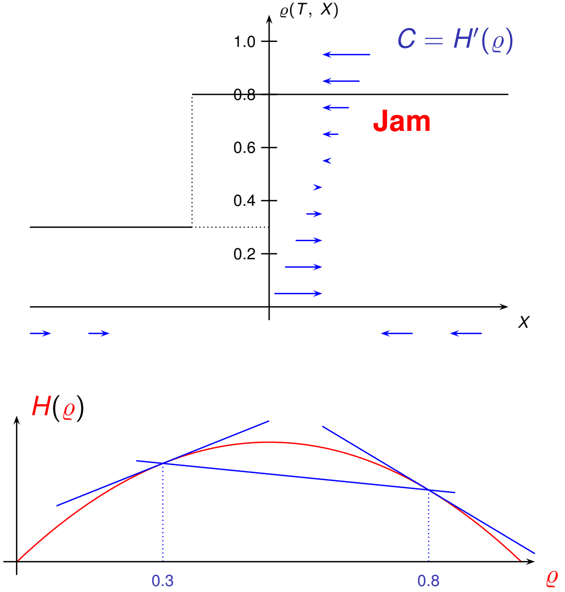

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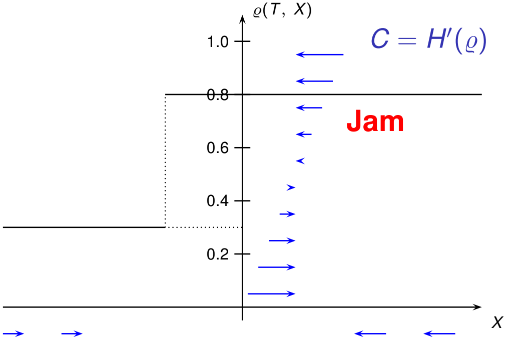

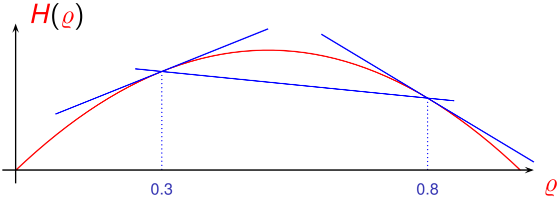

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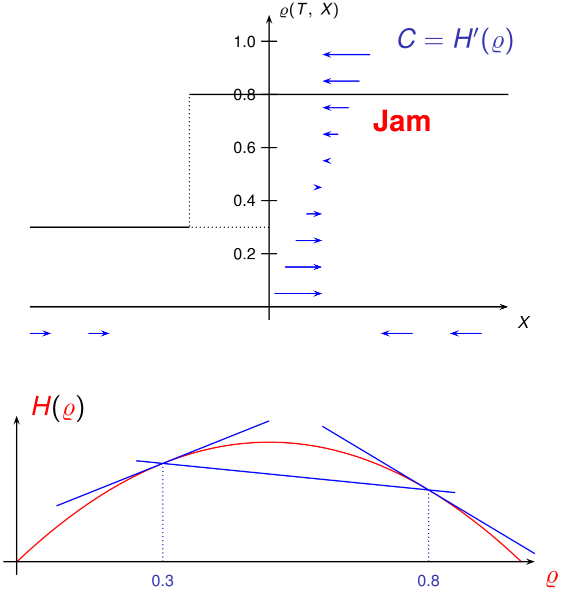

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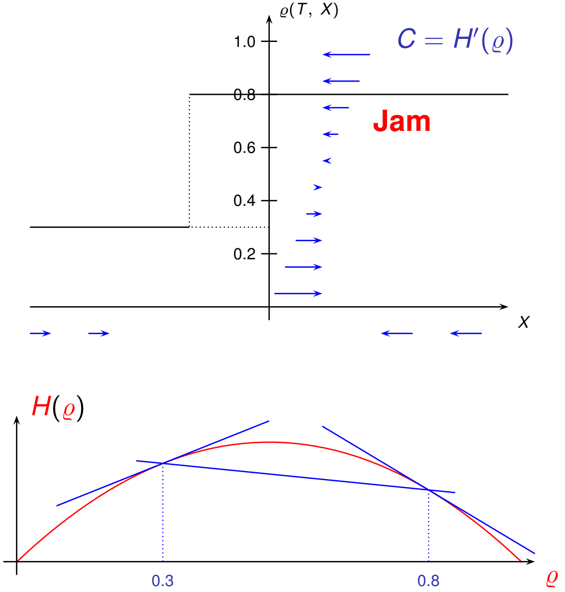

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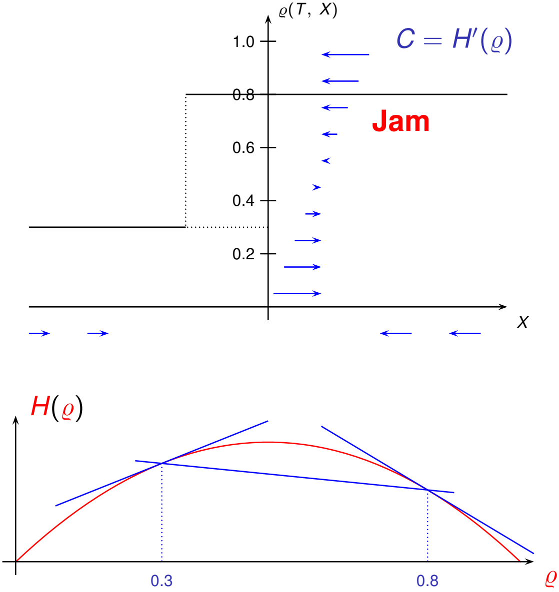

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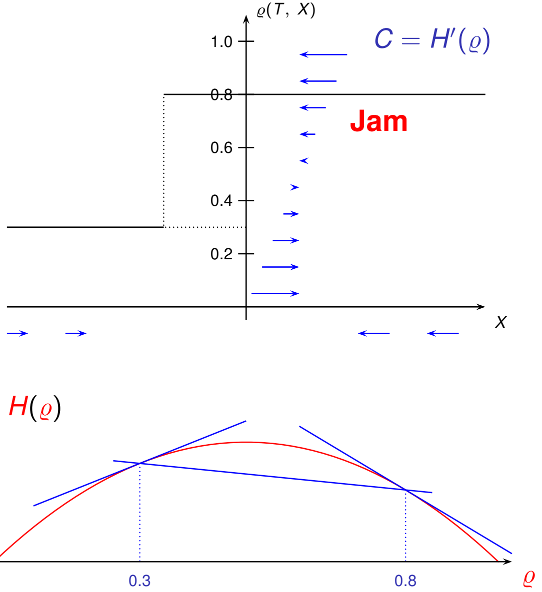

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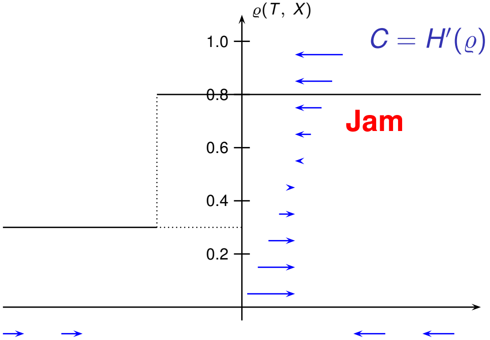

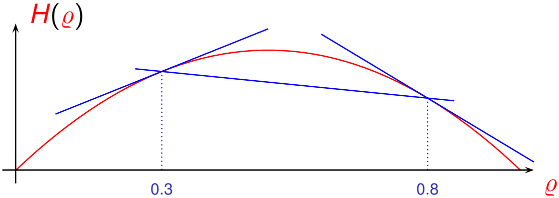

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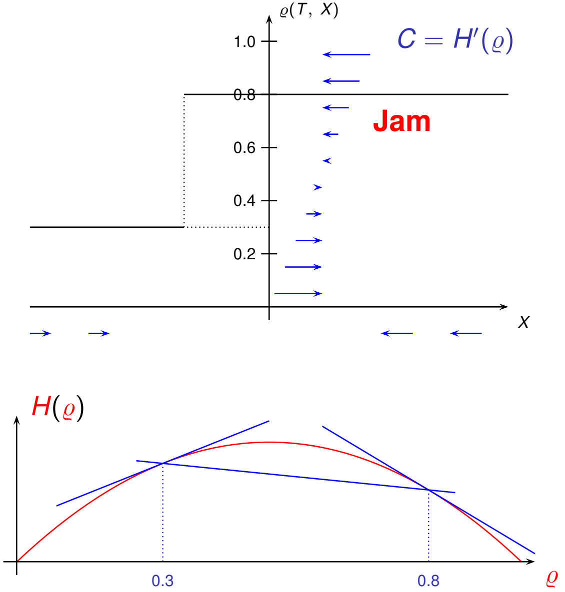

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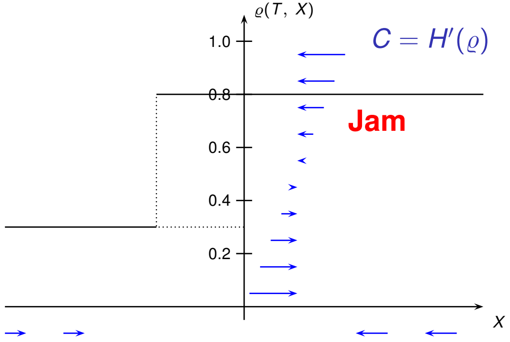

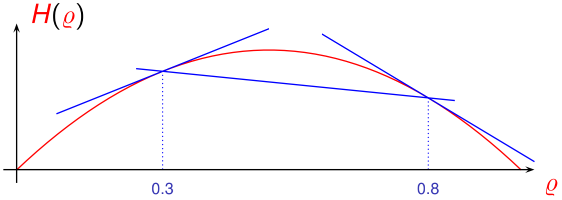

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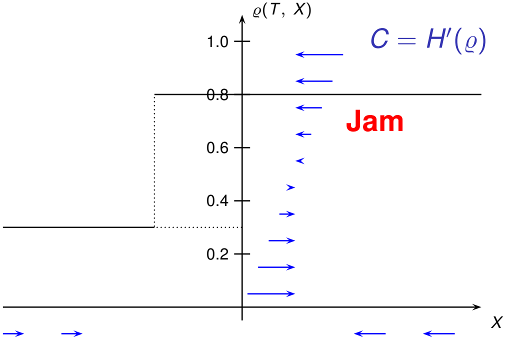

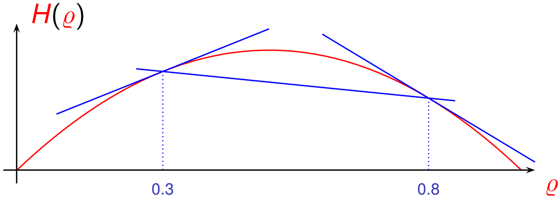

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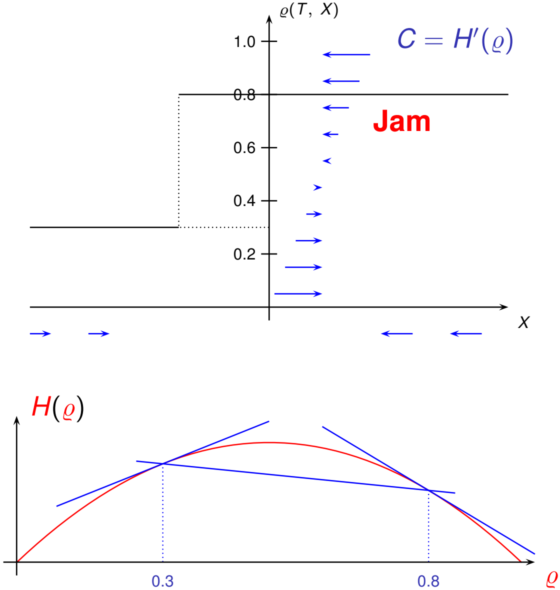

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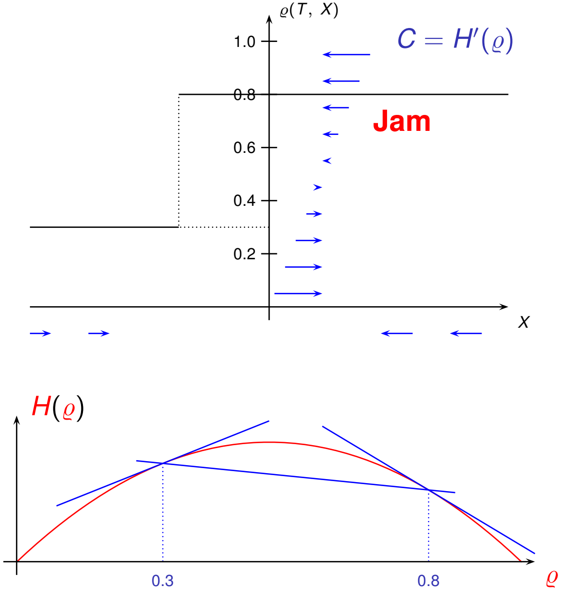

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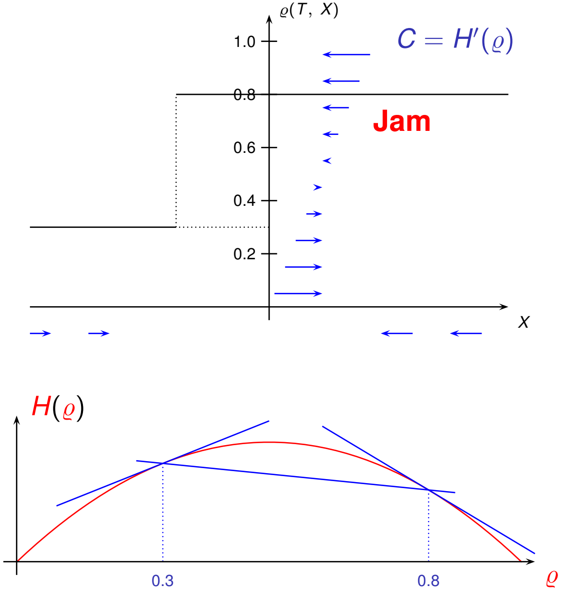

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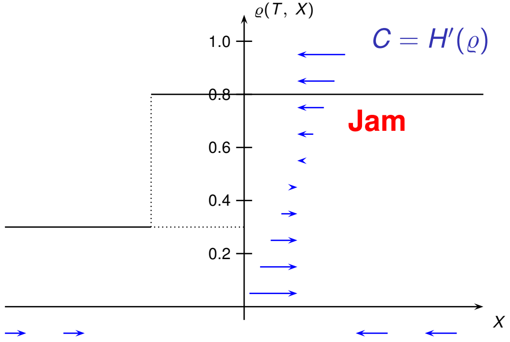

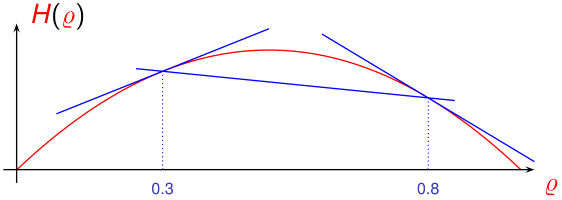

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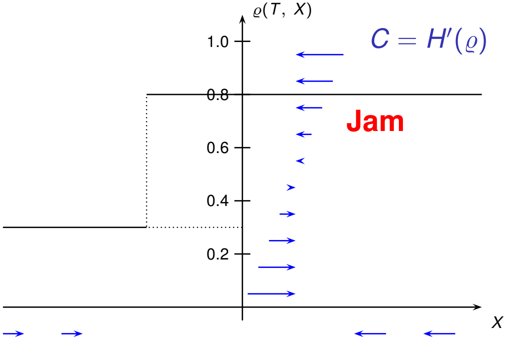

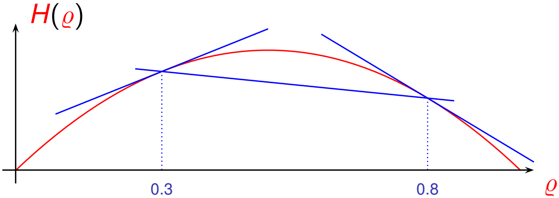

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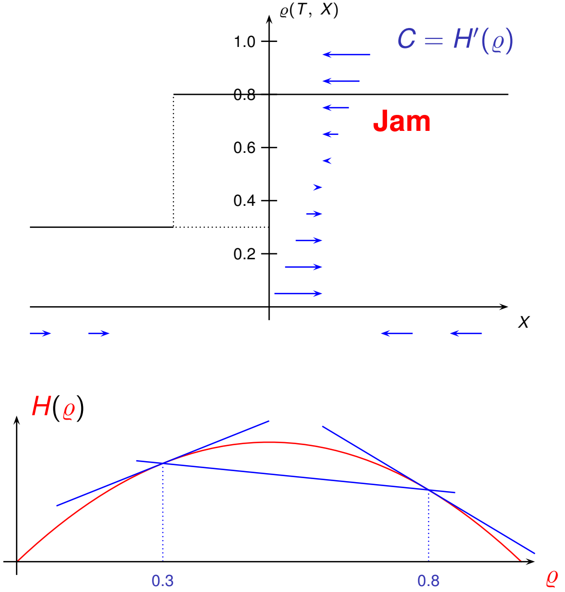

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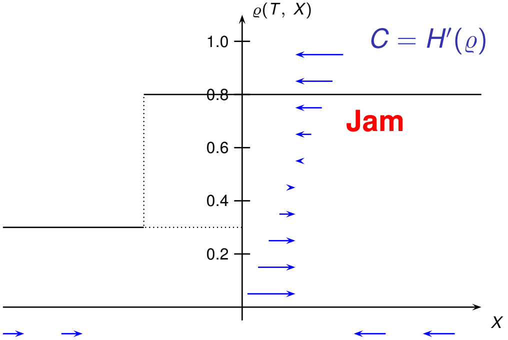

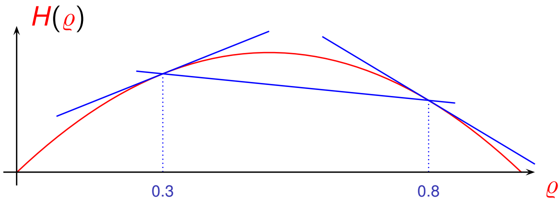

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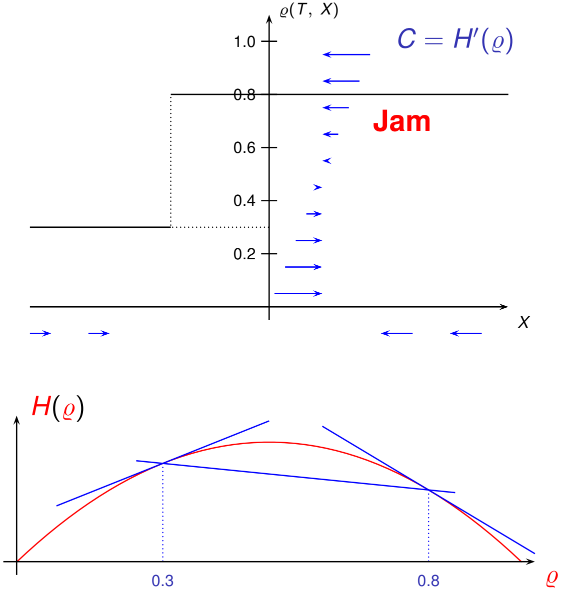

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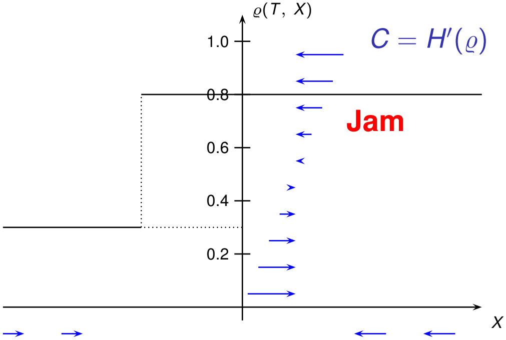

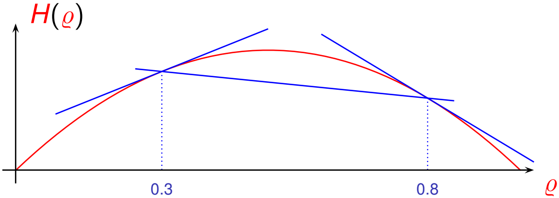

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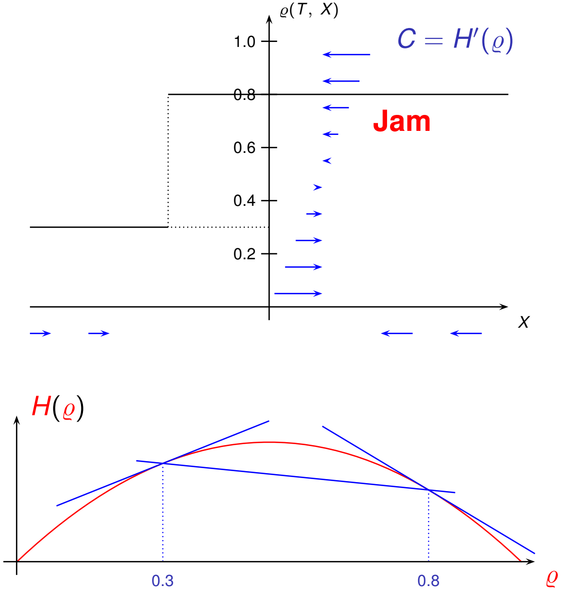

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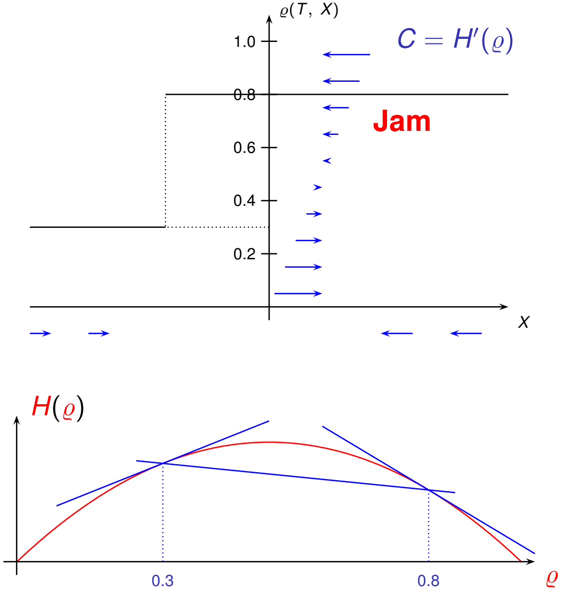

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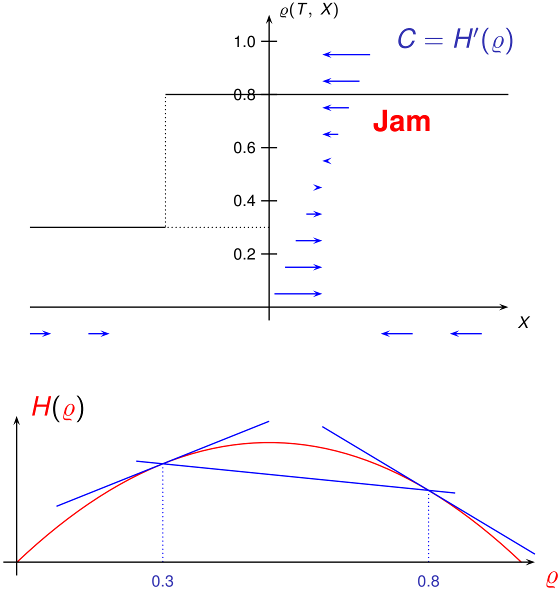

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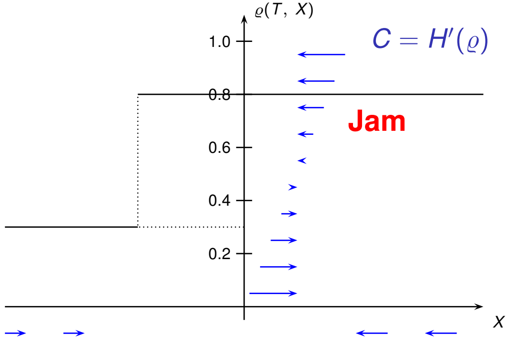

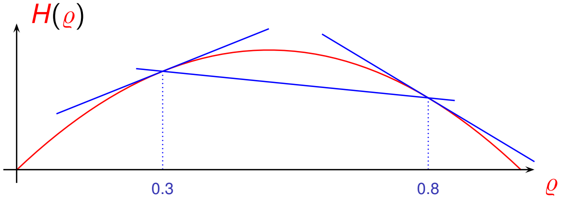

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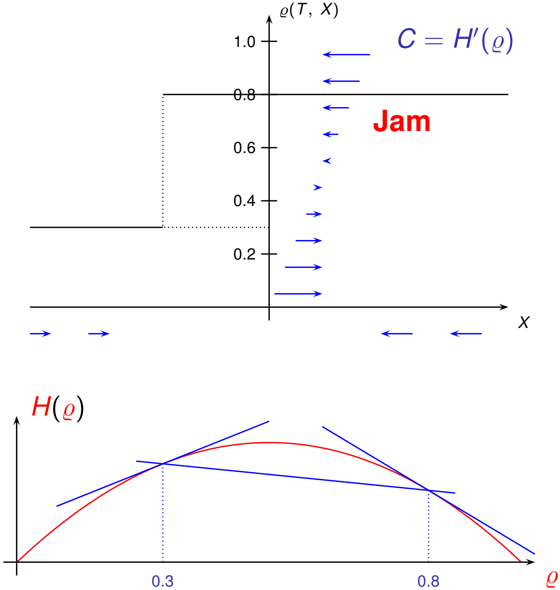

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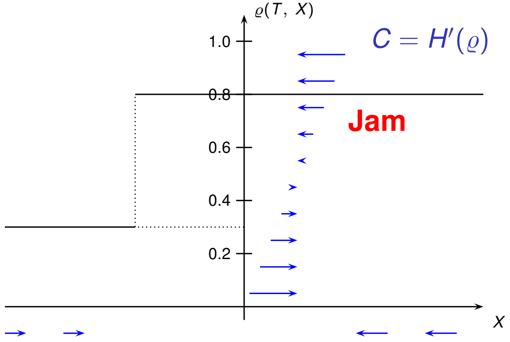

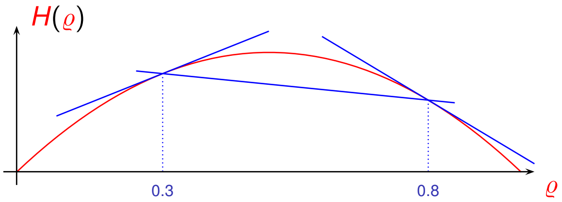

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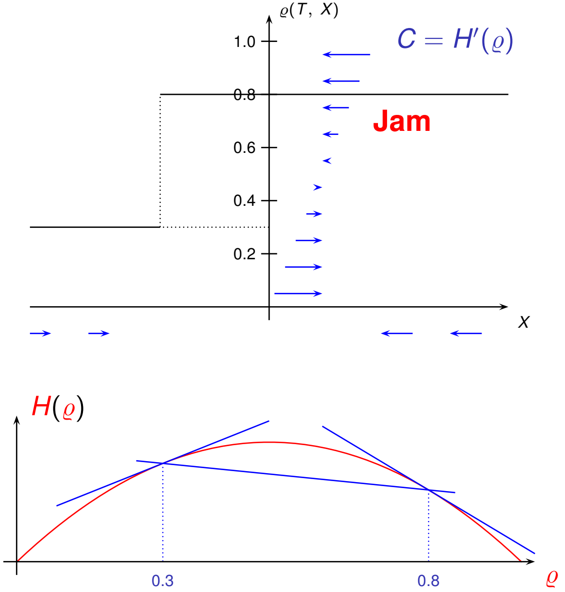

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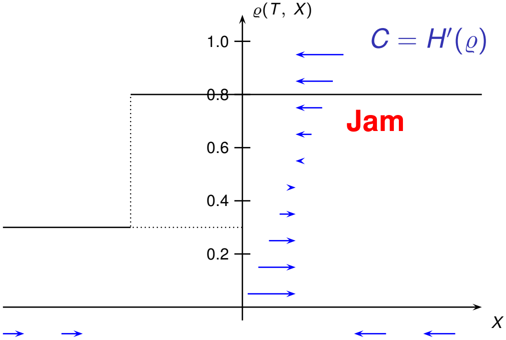

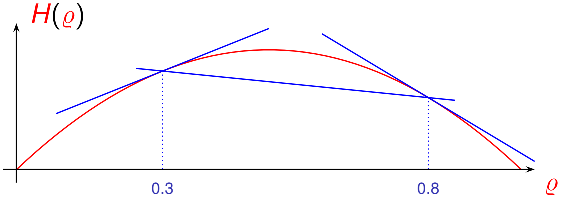

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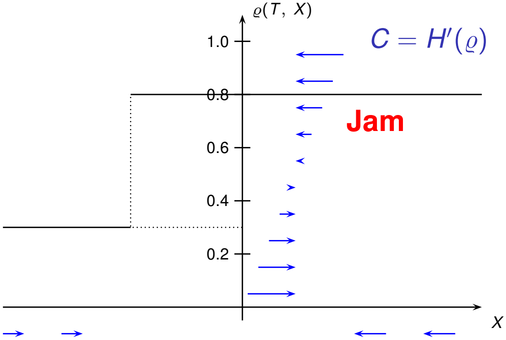

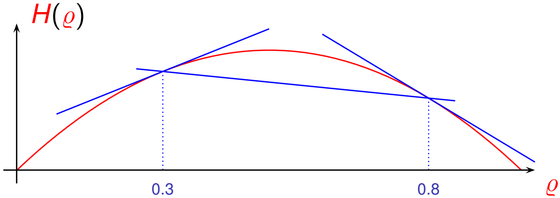

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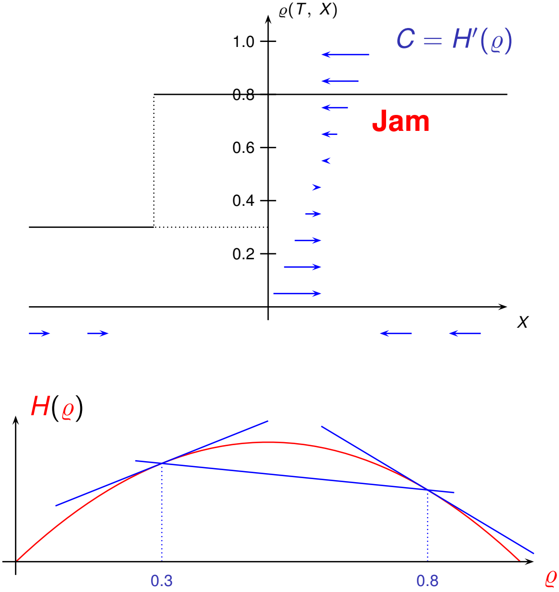

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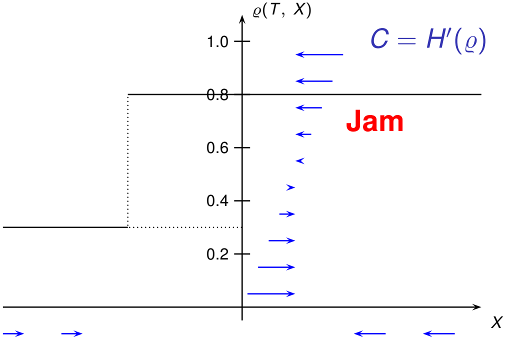

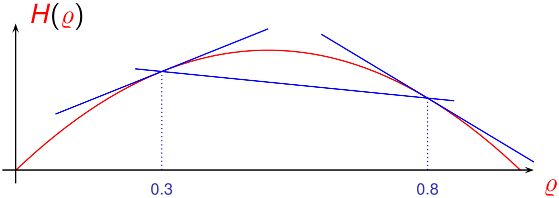

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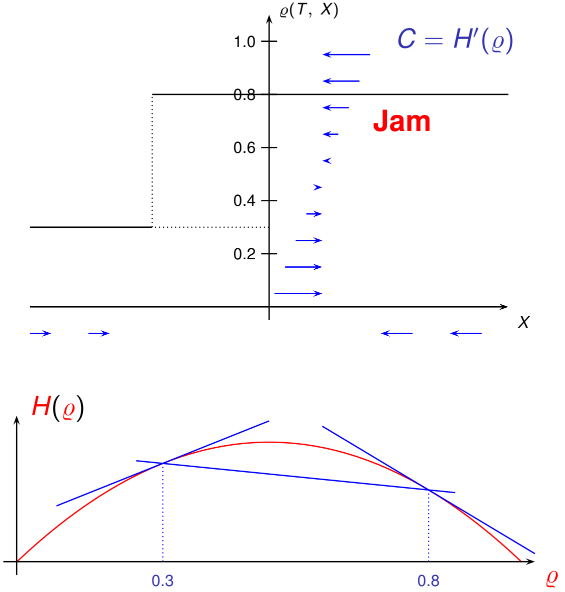

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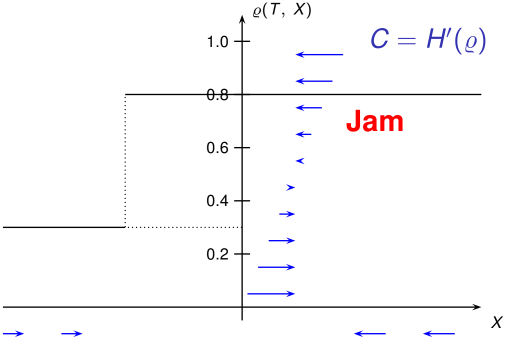

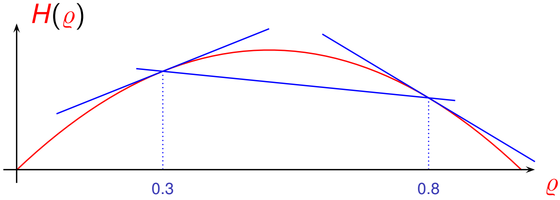

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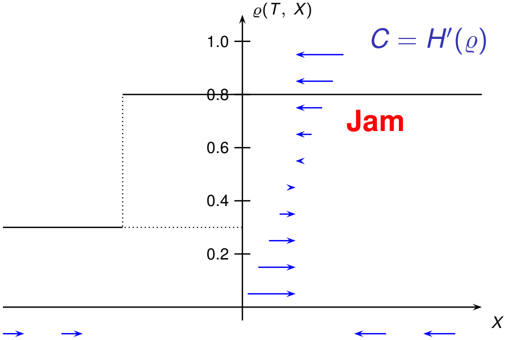

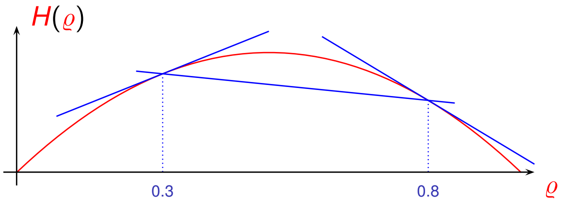

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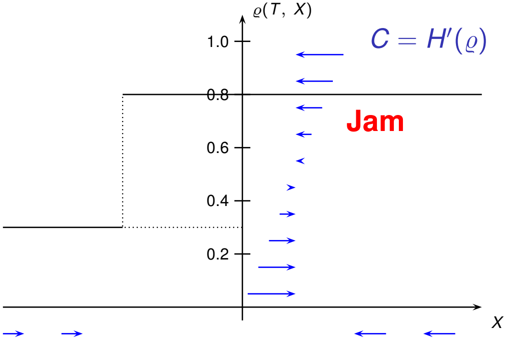

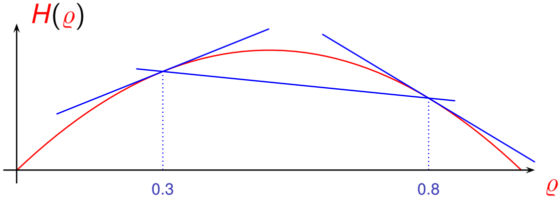

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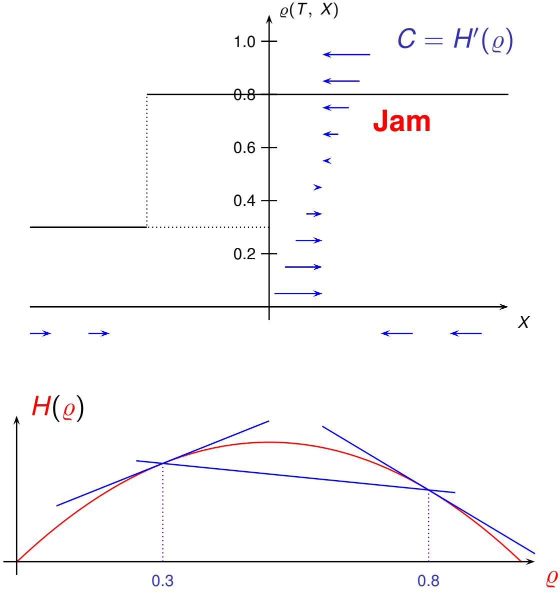

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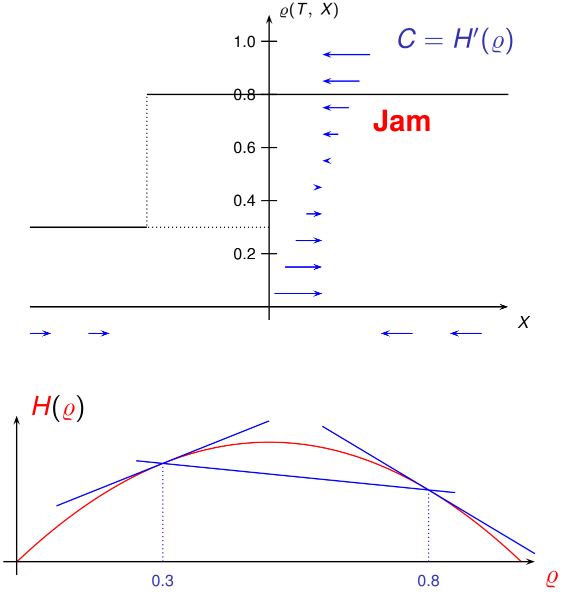

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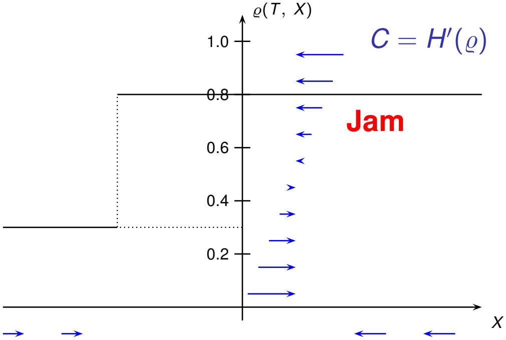

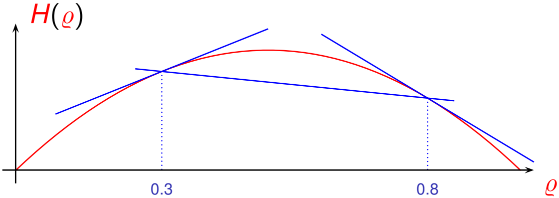

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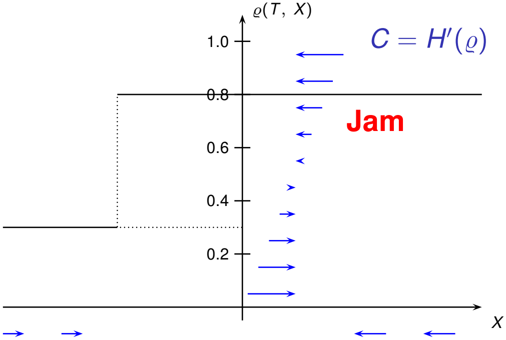

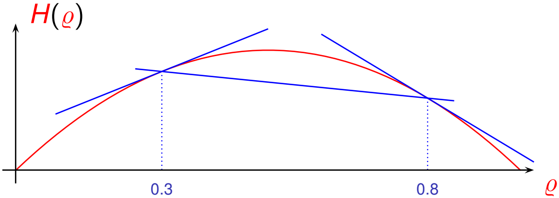

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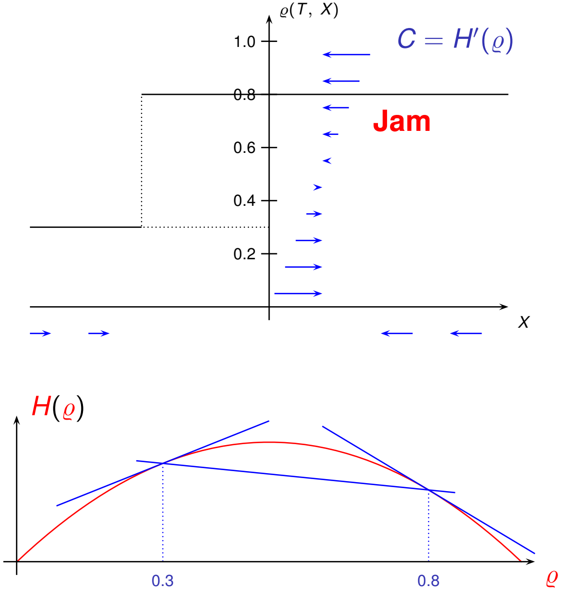

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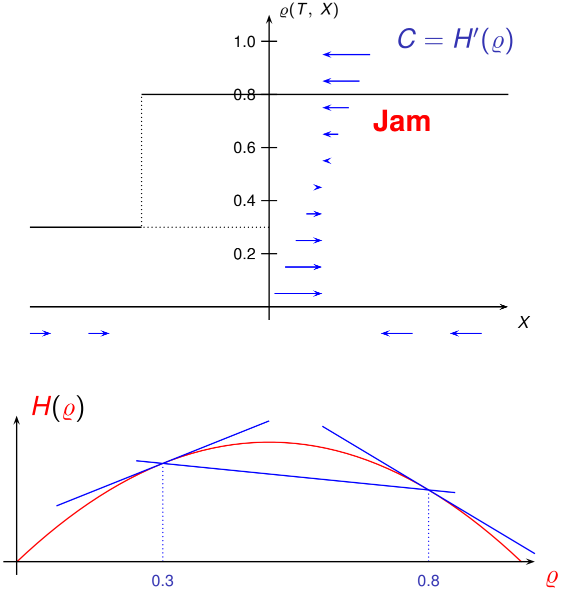

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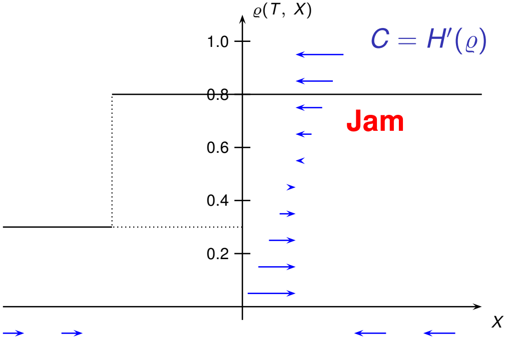

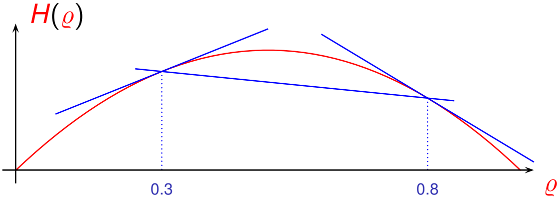

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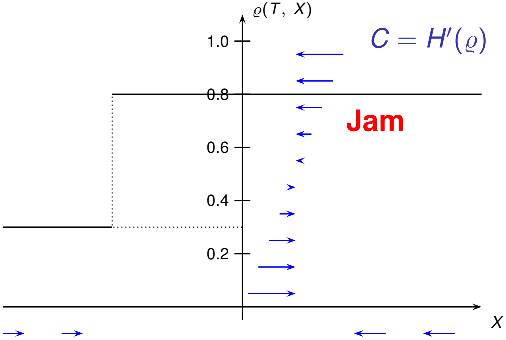

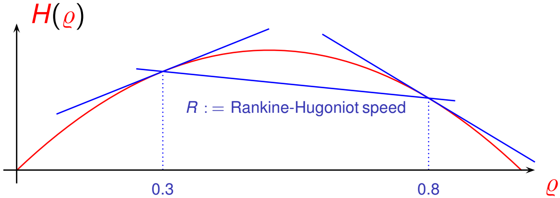

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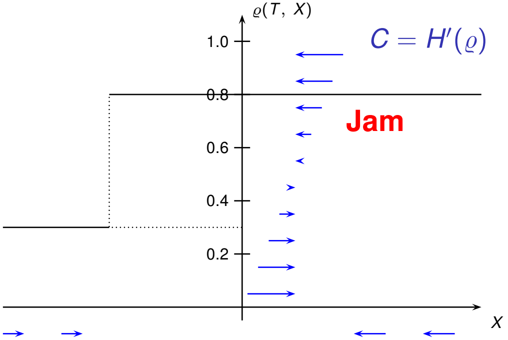

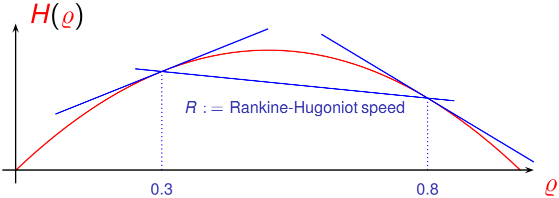

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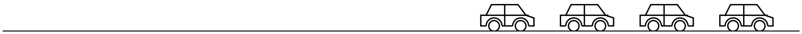

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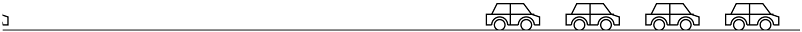

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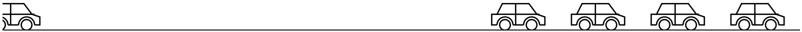

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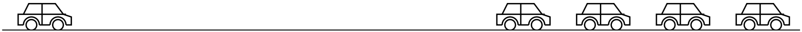

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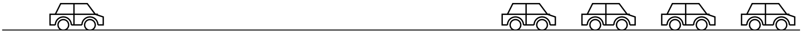

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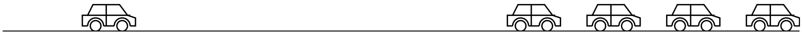

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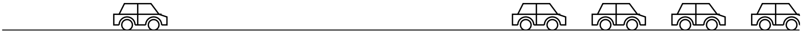

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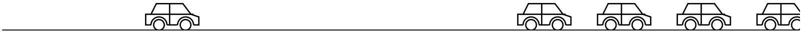

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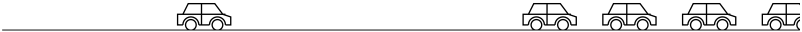

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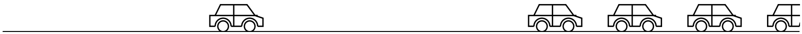

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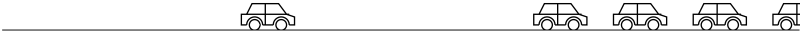
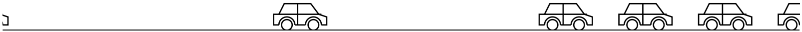


Arriving to a traffic jam



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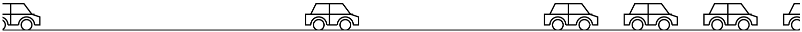

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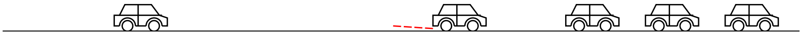

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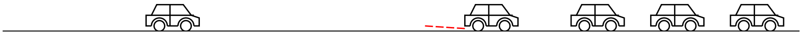

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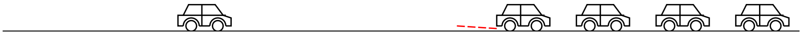

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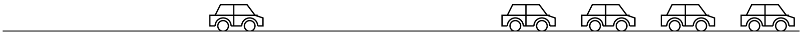

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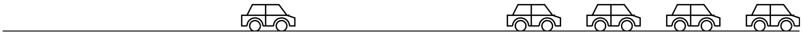

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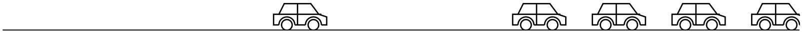

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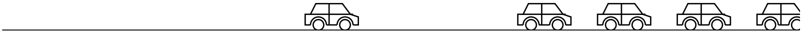

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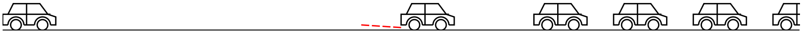

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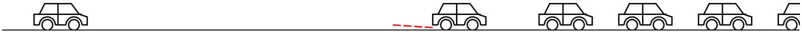

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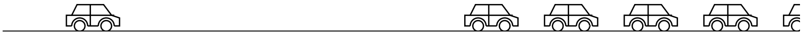

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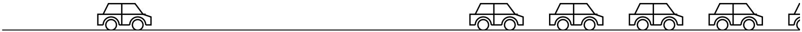

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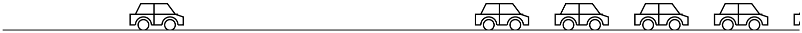

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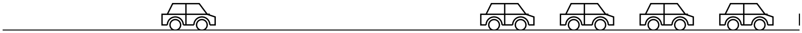

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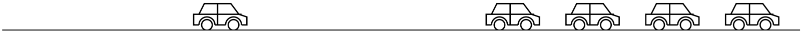

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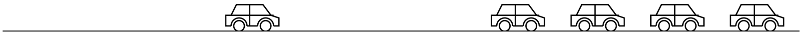

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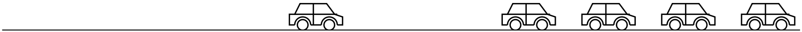

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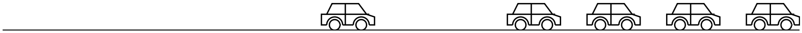

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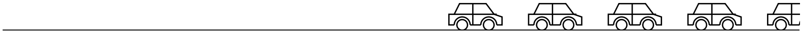

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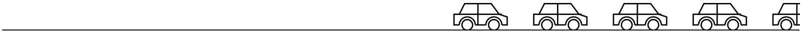

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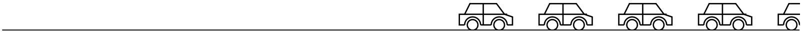

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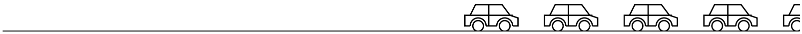

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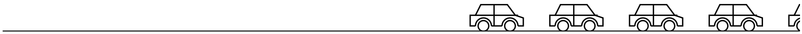

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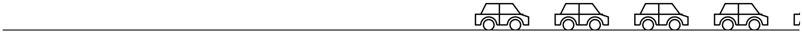

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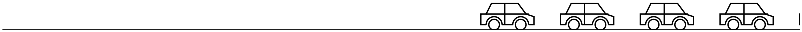

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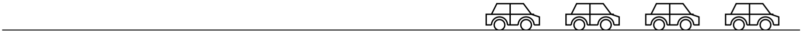

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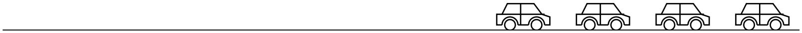

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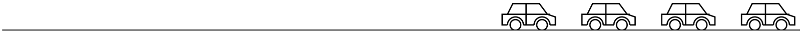

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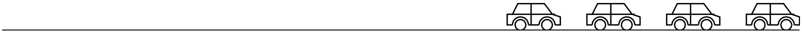

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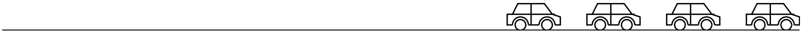
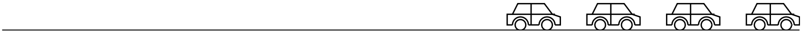


We notice the slow cars ⇝ strong braking immediately.

Arriving to a traffic jam is always sharp.

Arriving to a traffic jam



We notice the slow cars ⇝ strong braking immediately.

Arriving to a traffic jam is always sharp.

This is one aspect that makes motorways dangerous places.

Remarks.

- Of course there are much more sophisticated models for traffic modelling.

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- `http://youtu.be/Suugn-p5C1M`

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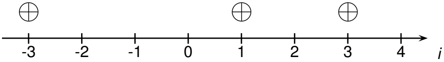
- `http://youtu.be/Suugn-p5C1M`
- TASEP is already very interesting from the mathematics point of view, with many nice theorems and interesting open questions.
- But we'll now go crazy with shocks and rarefaction fans.

A $\oplus$ $\ominus$ 0 model

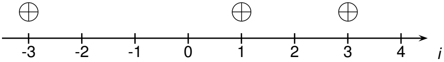
(Joint with A.L. Nagy, B. Tóth, I. Tóth)



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

⊕ to the right: rate 1



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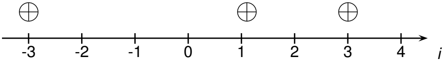

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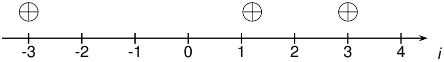

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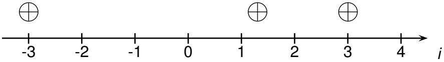

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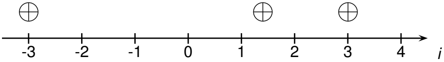

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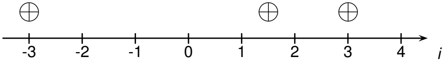

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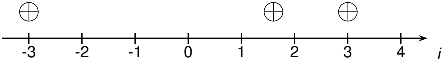

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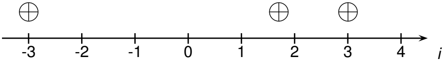

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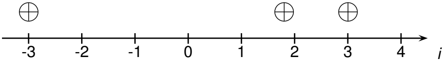

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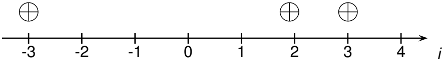

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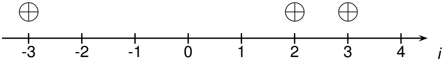

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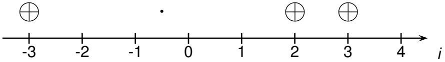
pair creation from vacuum: rate c



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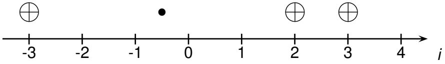

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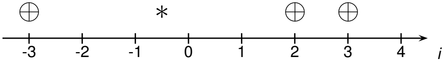

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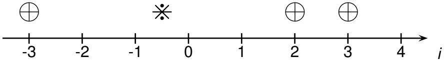

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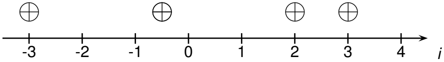

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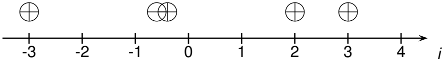

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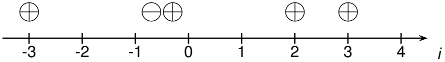

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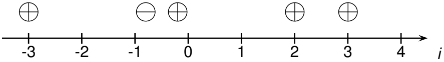

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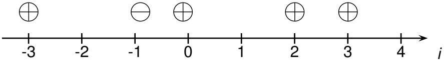

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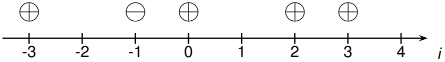

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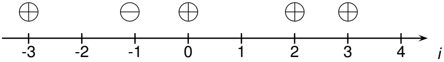
(Joint with A.L. Nagy, B. Tóth, I. Tóth)

⊖ to the left: rate 1



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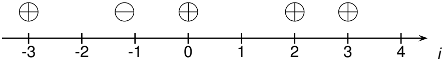

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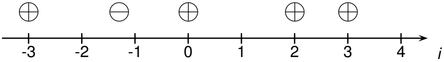

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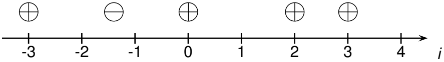

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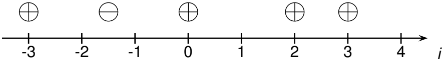

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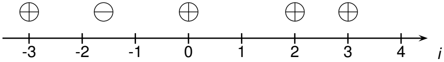
(Joint with A.L. Nagy, B. Tóth, I. Tóth)

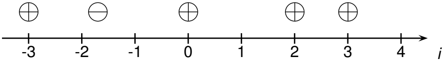
$\ominus$ to the left: rate 1



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

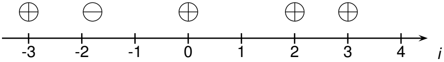
⊖ to the left: rate 1



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

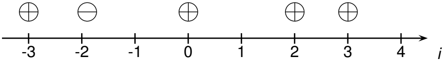
⊖ to the left: rate 1



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

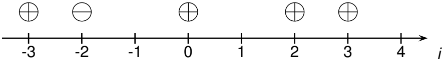
⊖ to the left: rate 1



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

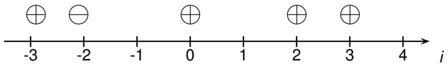
⊖ to the left: rate 1



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

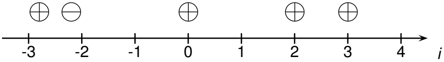
annihilation: rate 2



A $\oplus$ $\ominus$ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

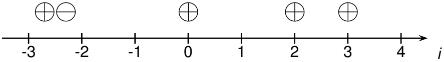
annihilation: rate 2



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

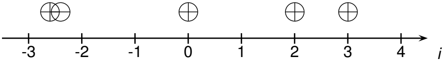
annihilation: rate 2



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

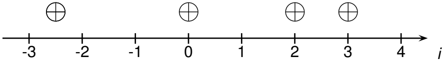
annihilation: rate 2



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

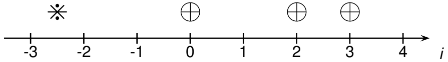
annihilation: rate 2



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

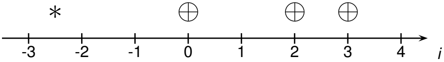
annihilation: rate 2



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

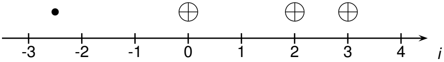
annihilation: rate 2



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

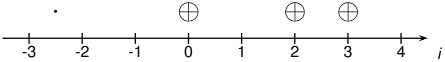
annihilation: rate 2



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)

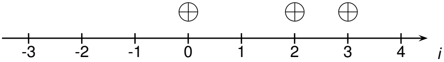
annihilation: rate 2



A ⊕ ⊖ 0 model

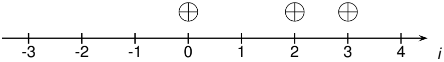
(Joint with A.L. Nagy, B. Tóth, I. Tóth)

annihilation: rate 2



A ⊕ ⊖ 0 model

(Joint with A.L. Nagy, B. Tóth, I. Tóth)



The important stationary distributions are again i.i.d. on the set $\{\ominus, 0, \oplus\}$.

Calling $\ominus = -1,\ 0 = 0,\ \oplus = 1$, the mean ϱ makes sense as a signed density of particles.

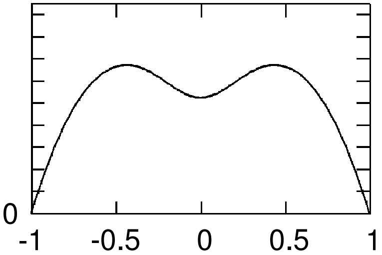
And $H(\varrho)$ makes sense as a signed particle current.

A ⊕ ⊖ 0 model

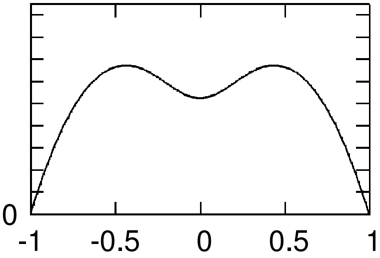
We still have

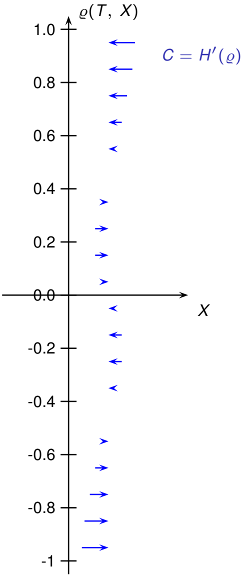
$$\partial_T \varrho(T, X) + \partial_X H(\varrho(T, X)) = 0.$$

The hydrodynamic flux $H(\varrho)$, for certain c:

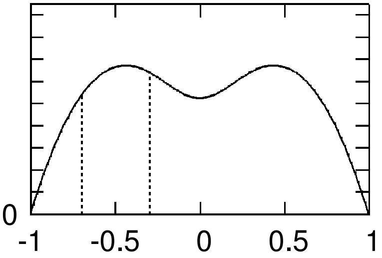


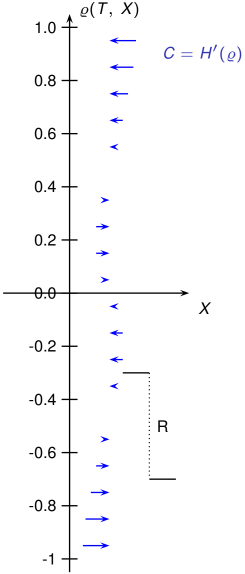
A ⊕ ⊖ 0 model



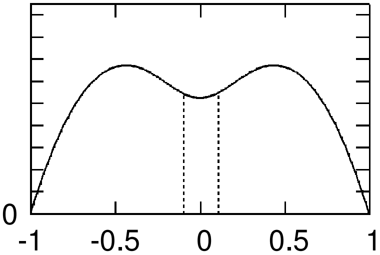


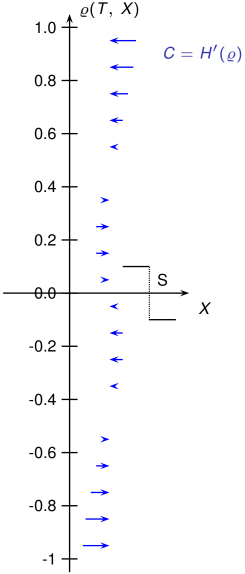
A ⊕ ⊖ 0 model



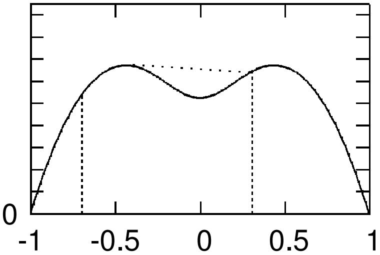


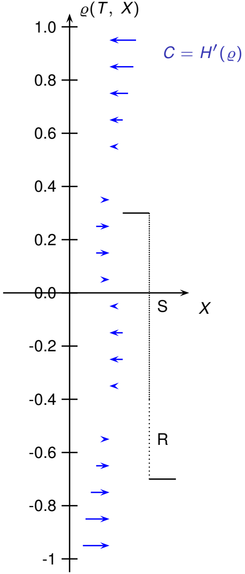
A ⊕ ⊖ 0 model



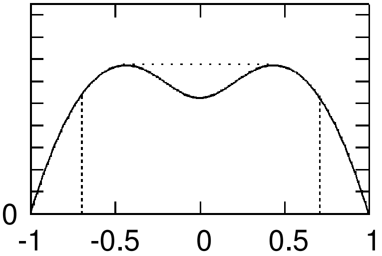


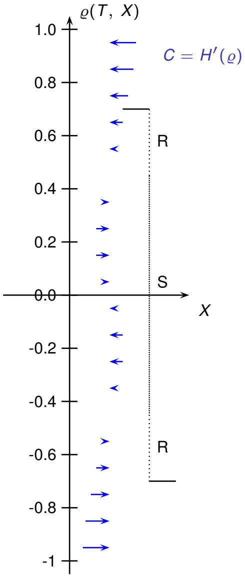
A ⊕ ⊖ 0 model



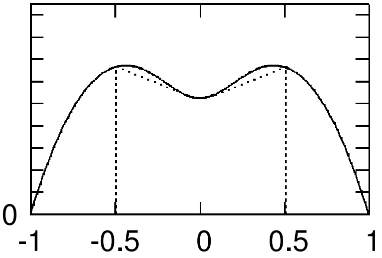


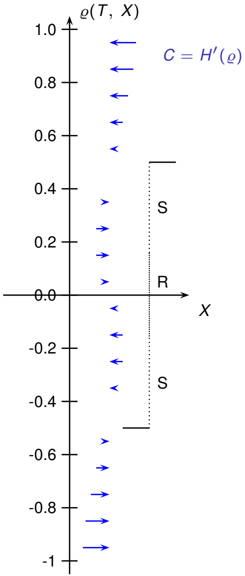
A ⊕ ⊖ 0 model





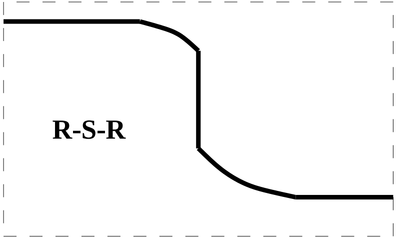
A ⊕ ⊖ 0 model

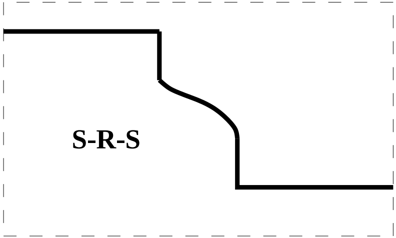




A ⊕ ⊖ 0 model

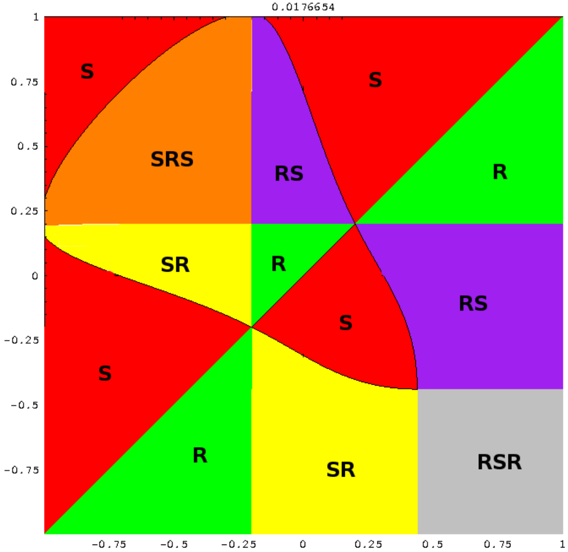
Examples for $\varrho(T, X)$:





A ⊕ ⊖ 0 model

Here is the full picture (**R**: rarefaction wave, **S**: Shock):



Thank you.