Blocking measures, hills, and hydrodynamics

Joint with
Jacob Calvert, Patrícia Gonçalves and Katerina Michaelides

Márton Balázs

University of Bristol

Dynamics, random media and universality of complex physical systems
Münster, 29 August, 2019.

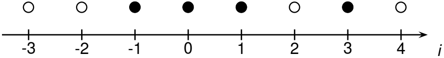
Models
- Asymmetric simple exclusion
- Zero range

Classical knowledge
- Asymmetric hydrodynamics
- Symmetric hydrodynamics

Blocking measures
- ASEP
- ZRP
- Further models

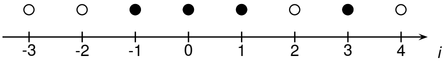
Hills
- Microscopic model
- Hydrodynamics

Asymmetric simple exclusion



$$\eta_i \in \{0, 1\}.$$

Asymmetric simple exclusion



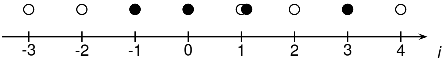
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



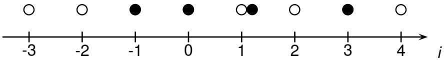
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



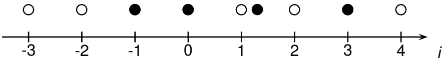
$\eta_i \in \{0,\,1\}$.

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



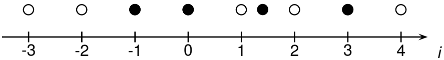
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



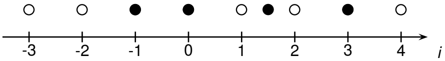
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



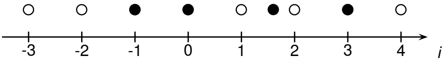
$$\eta_i \in \{0,\, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



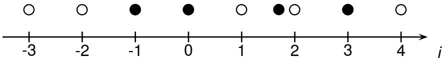
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



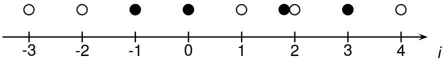
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



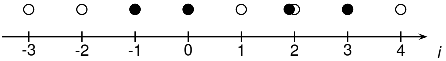
$$\eta_i \in \{0,\, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



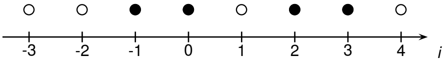
$\eta_i \in \{0, 1\}$.

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



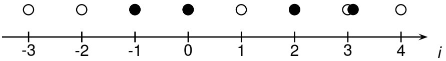
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



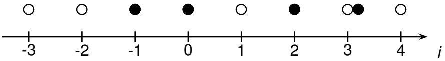
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



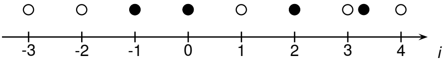
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



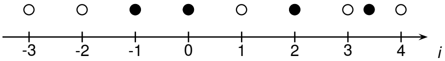
$$\eta_i \in \{0,\ 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



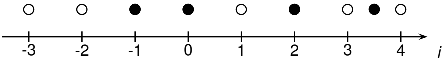
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



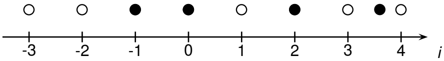
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



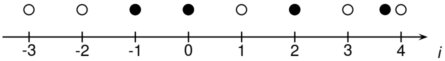
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



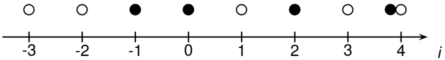
$$\eta_i \in \{0,\, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



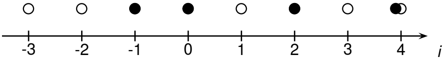
$$\eta_i \in \{0,\, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



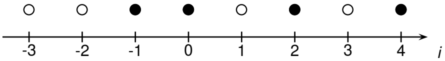
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



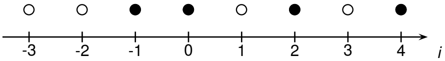
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



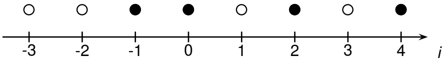
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



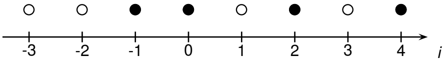
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



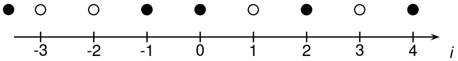
$\eta_i \in \{0, 1\}$.

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



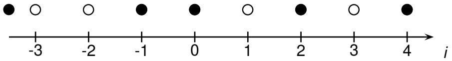
$$\eta_i \in \{0,\, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



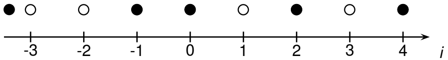
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



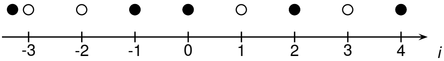
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



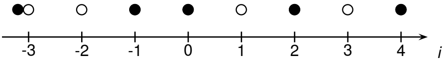
$$\eta_i \in \{0,\ 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



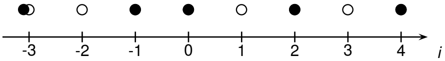
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



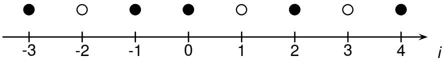
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



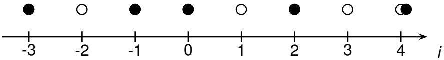
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



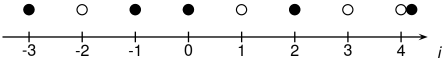
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



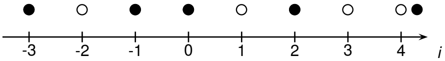
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



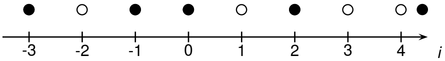
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



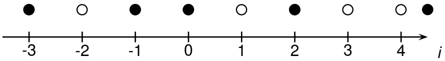
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



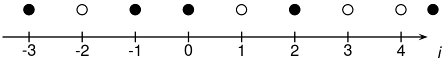
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



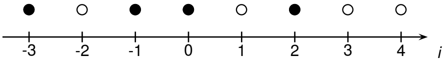
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



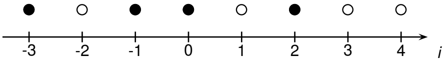
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



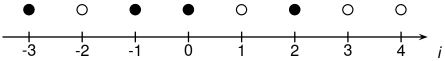
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



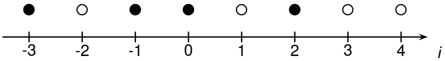
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



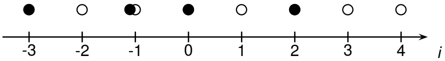
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



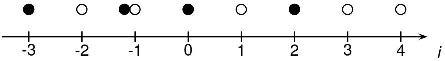
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



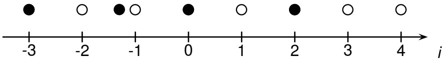
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



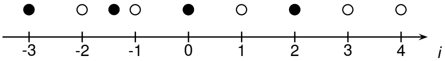
$\eta_i \in \{0, 1\}$.

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



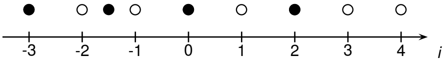
$$\eta_i \in \{0,\ 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



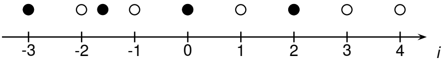
$$\eta_i \in \{0,\ 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



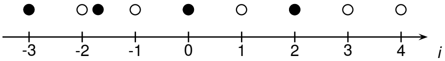
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



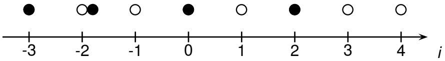
$$\eta_i \in \{0,\, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



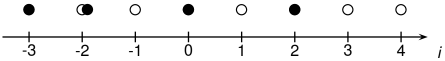
$$\eta_i \in \{0,\,1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



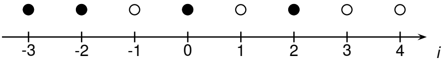
$$\eta_i \in \{0, 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

The jump is suppressed if the destination site is occupied by another particle.

Asymmetric simple exclusion



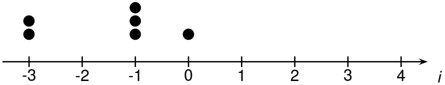
$$\eta_i \in \{0,\ 1\}.$$

Particles try to jump

to the right with rate p,
to the left with rate $q = 1 - p < p$.

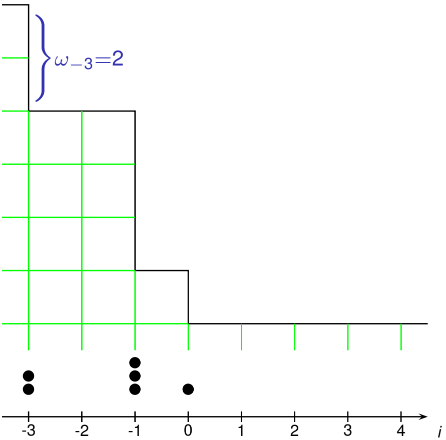
The jump is suppressed if the destination site is occupied by another particle.

The asymmetric zero range process



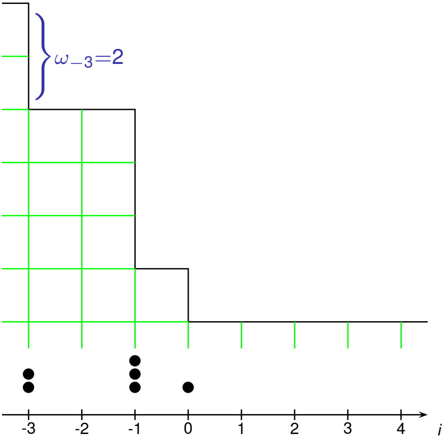
$$\omega_i \in \mathbb{Z}^+.$$

The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

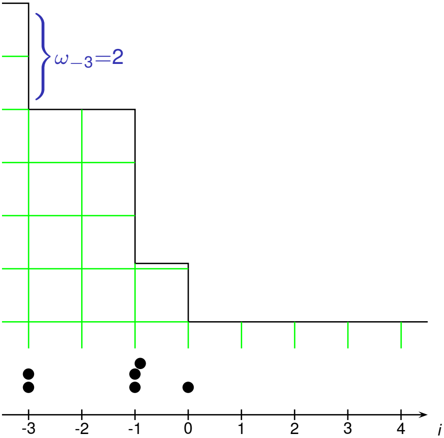
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

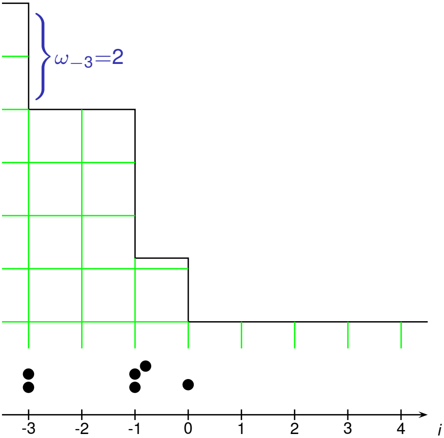
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

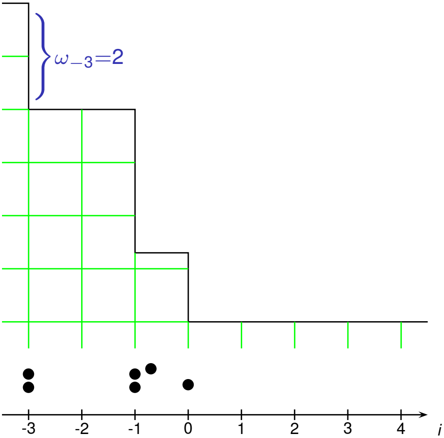
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

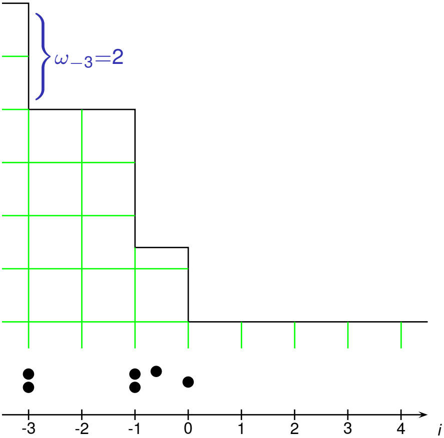
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

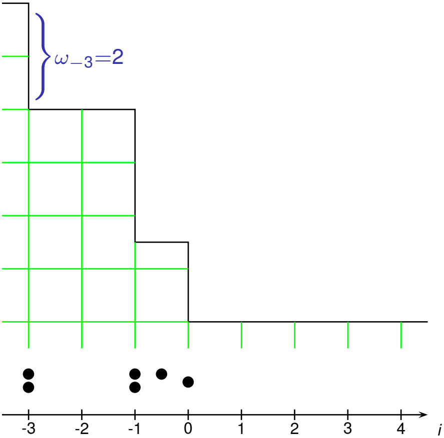
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

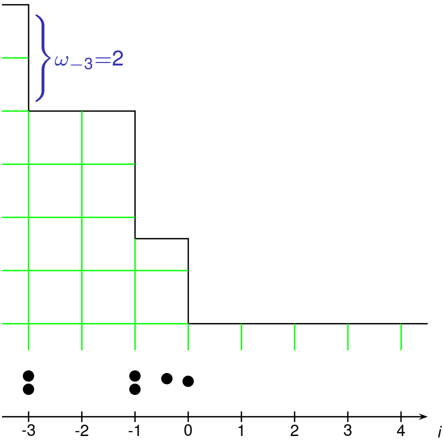
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

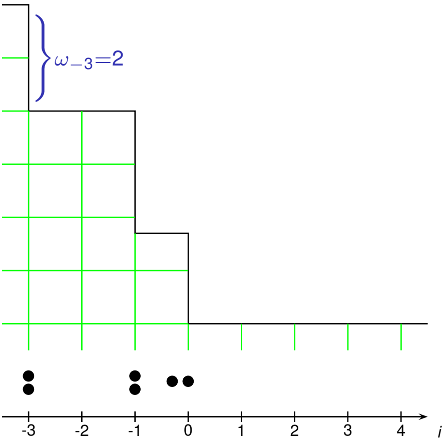
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

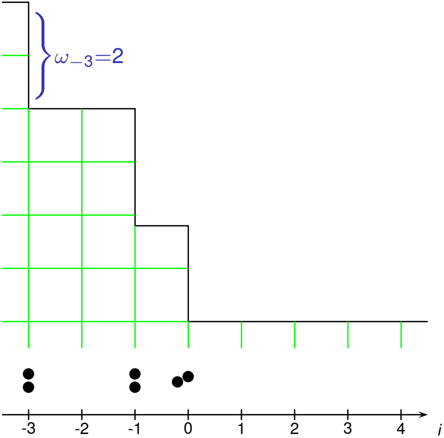
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

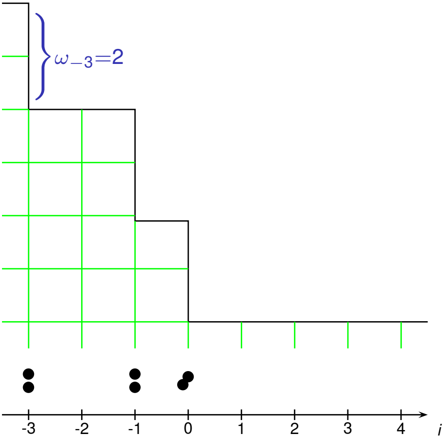
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

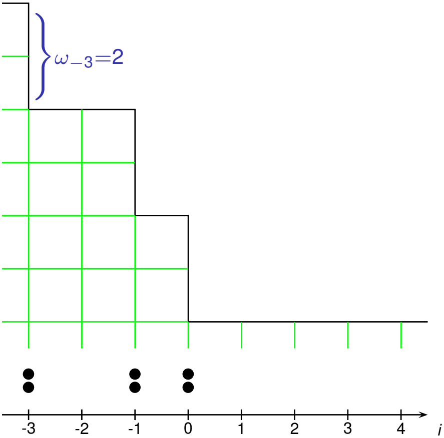
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

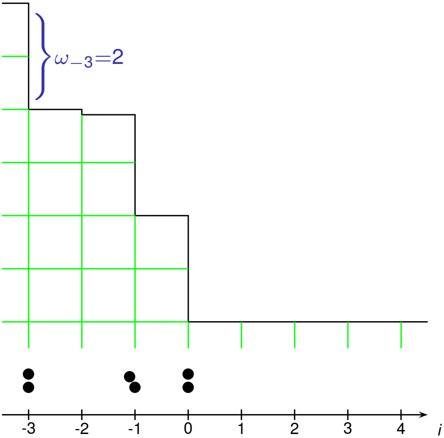
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

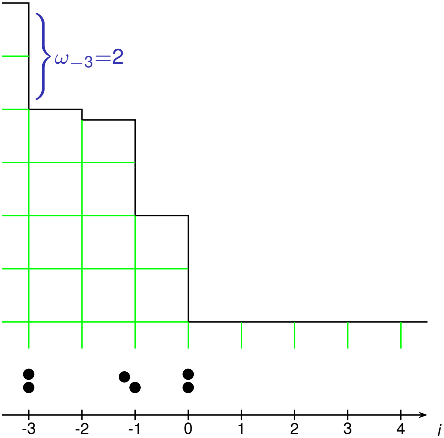
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

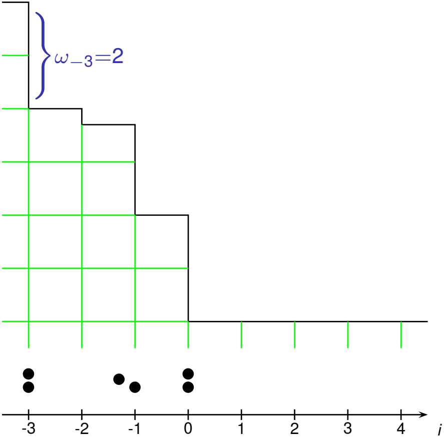
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

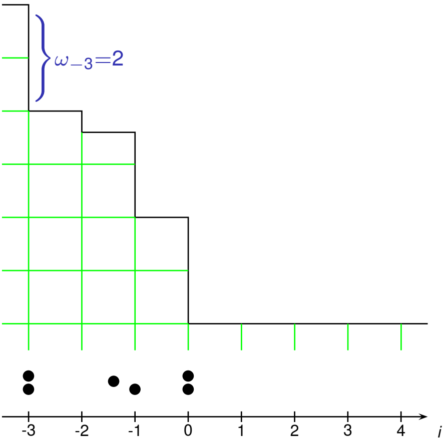
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

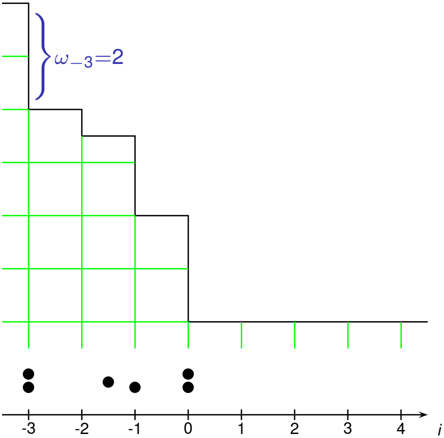
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

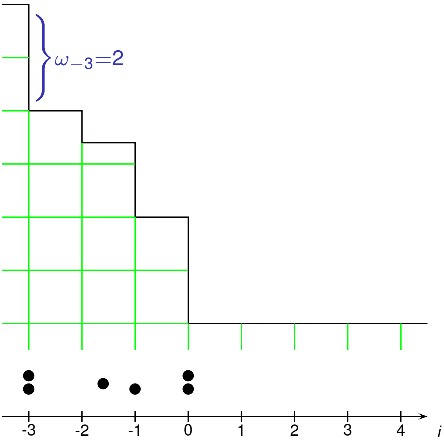
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

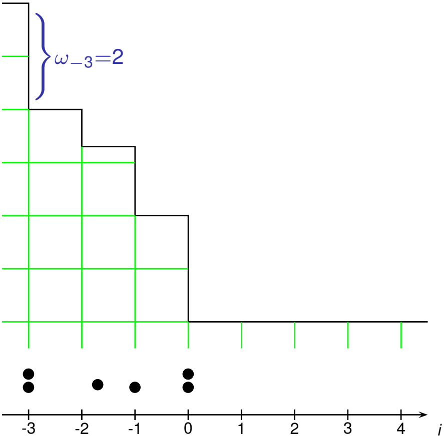
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

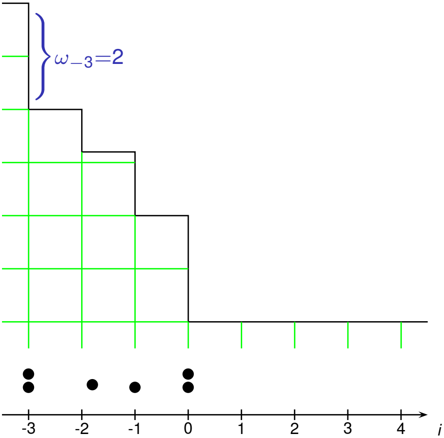
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

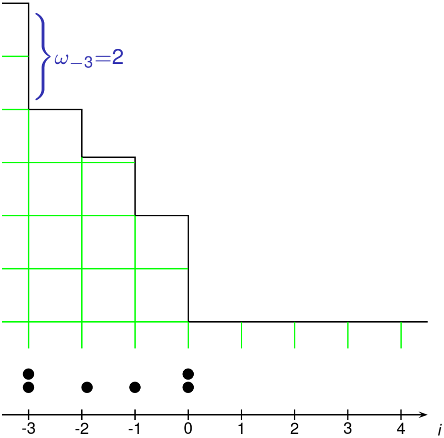
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

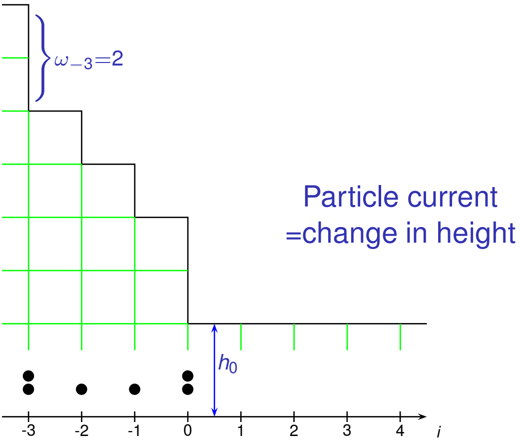
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

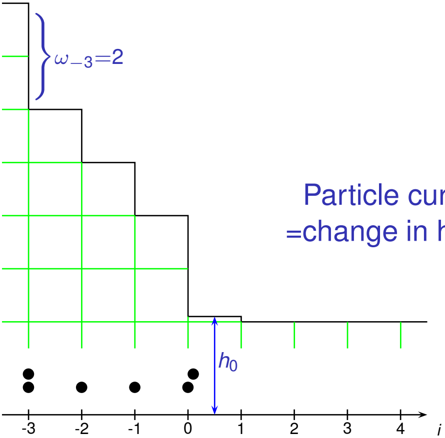
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

The asymmetric zero range process

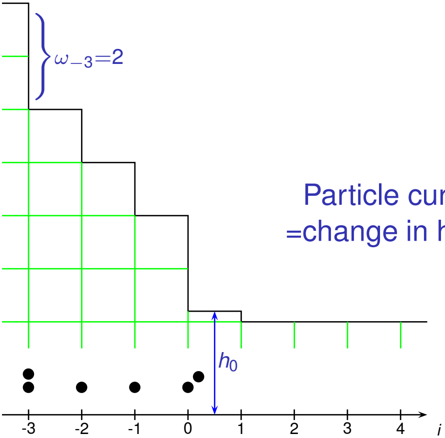


Particle current
=change in height

$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

The asymmetric zero range process

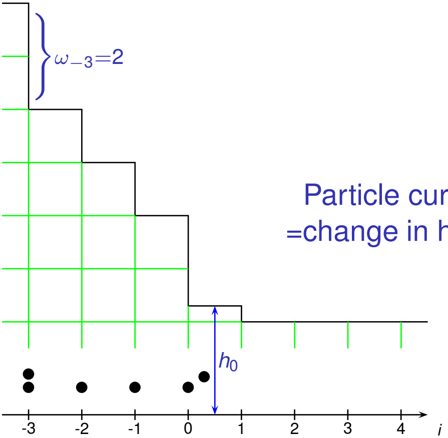


Particle current
=change in height

$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

The asymmetric zero range process

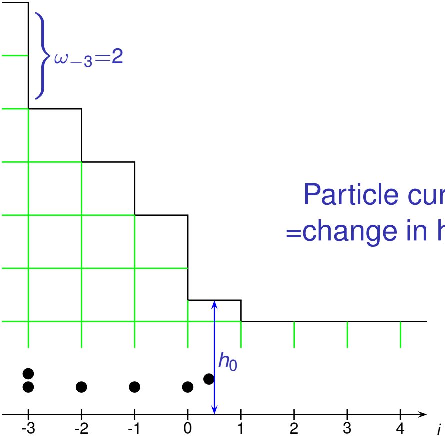


Particle current
=change in height

$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

The asymmetric zero range process

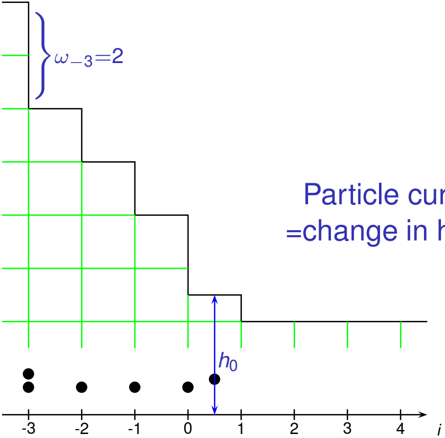


Particle current
=change in height

$\omega_i \in \mathbb{Z}^+$.

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

The asymmetric zero range process

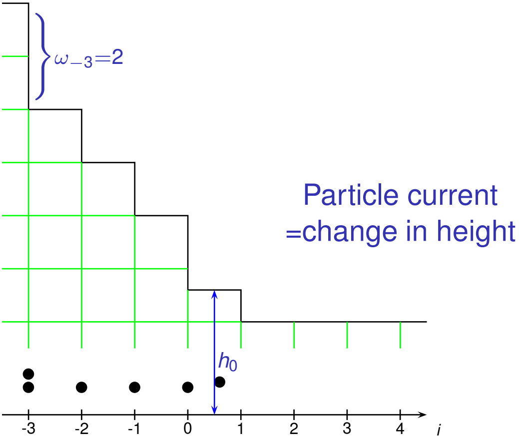


Particle current
=change in height

$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

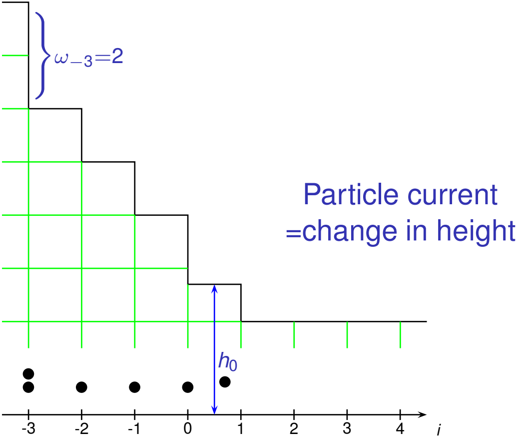
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

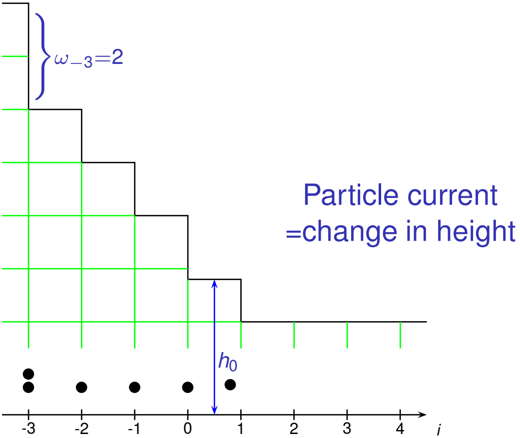
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

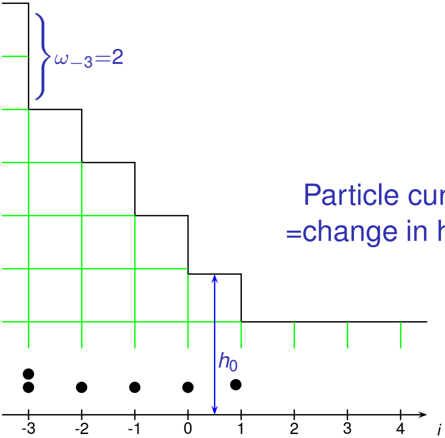
The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

The asymmetric zero range process

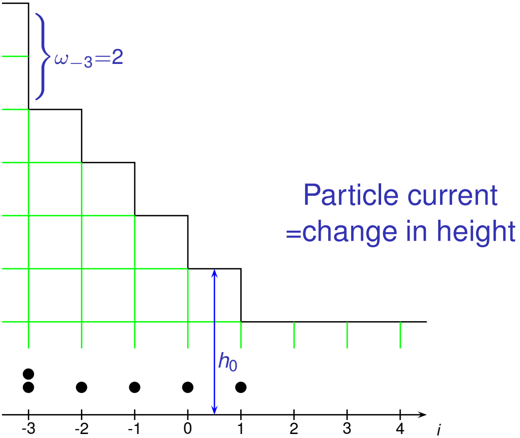


Particle current
=change in height

$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

The asymmetric zero range process



$$\omega_i \in \mathbb{Z}^+.$$

Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

The asymmetric zero range process

We need r non-decreasing.

Examples:

- 'Classical' ZRP: $r(\omega_i) = \mathbf{1}\{\omega_i > 0\}$.
- Independent walkers: $r(\omega_i) = \omega_i$.

Translation invariant measures

No surprise that constant density is stationary:

ASEP: $\eta_i \sim$ iid. Bernoulli(ϱ).

Classical ZRP: $\omega_i \sim$ iid. Geometric($\frac{1}{1+\varrho}$).

Independent walkers: $\omega_i \sim$ iid. Poisson(ϱ).

These are the only extremal translation-invariant distributions.

Hydrodynamics: asymmetric ZRP

Take $p > q = 1 - p$, and AZRP with right rate $p \cdot r(\omega_i)$, left rate $q \cdot r(\omega_i)$.

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i = p\mathbf{E}r(\omega_{i-1}) + q\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)$$

Hydrodynamics: asymmetric ZRP

Take $p > q = 1 - p$, and AZRP with right rate $p \cdot r(\omega_i)$, left rate $q \cdot r(\omega_i)$.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i &= p\mathbf{E}r(\omega_{i-1}) + q\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)\\ &= q\big(\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)\big) - p\big(\mathbf{E}r(\omega_i) - \mathbf{E}r(\omega_{i-1})\big).\end{aligned}$$

Hydrodynamics: asymmetric ZRP

Take $p > q = 1 - p$, and AZRP with right rate $p \cdot r(\omega_i)$, left rate $q \cdot r(\omega_i)$.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i &= p\mathbf{E}r(\omega_{i-1}) + q\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)\\ &= q\big(\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)\big) - p\big(\mathbf{E}r(\omega_i) - \mathbf{E}r(\omega_{i-1})\big).\end{aligned}$$

Ballistic scaling (zoom out and speed up by factor L):

- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(Lt)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

$$\frac{\mathrm{d}}{\mathrm{d}(\tau/L)}\mathbf{E}\omega_i = qL\big(\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)\big) - pL\big(\mathbf{E}r(\omega_i) - \mathbf{E}r(\omega_{i-1})\big)$$

Hydrodynamics: asymmetric ZRP

Take $p > q = 1 - p$, and AZRP with right rate $p \cdot r(\omega_i)$, left rate $q \cdot r(\omega_i)$.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i &= p\mathbf{E}r(\omega_{i-1}) + q\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)\\ &= q\big(\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)\big) - p\big(\mathbf{E}r(\omega_i) - \mathbf{E}r(\omega_{i-1})\big).\end{aligned}$$

Ballistic scaling (zoom out and speed up by factor L):

- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(Lt)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

$$\frac{\mathrm{d}}{\mathrm{d}(\tau/L)}\mathbf{E}\omega_i = qL\big(\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)\big) - pL\big(\mathbf{E}r(\omega_i) - \mathbf{E}r(\omega_{i-1})\big)$$

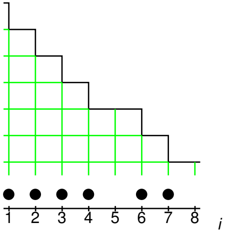
$$\frac{\partial}{\partial t}\varrho(t,\, x) = (q-p)\frac{\partial}{\partial x}G(\varrho(t,\, x)).$$

Hydrodynamics: asymmetric ZRP

$$\frac{\partial}{\partial t}\varrho(t,\,x)+(p-q)\frac{\partial}{\partial x}G(\varrho(t,\,x))=0$$

Classical ZRP: $G(\varrho)=\mathbf{E}^{\varrho}r(\omega)=\mathbf{E}^{\varrho}\mathbf{1}\{\omega>0\}=\frac{\varrho}{1+\varrho}$
concave, Burgers-type equation.

Independent walkers: $G(\varrho)=\mathbf{E}^{\varrho}r(\omega)=\mathbf{E}^{\varrho}\omega=\varrho$
linear, transport equation.



The stationary solution is constant density, linear slope.

Hydrodynamics: symmetric ZRP

Take $p = q = \frac{1}{2}$, and SZRP with right rate $\frac{1}{2}r(\omega_i)$, left rate $\frac{1}{2}r(\omega_i)$.

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i = \frac{1}{2}\mathbf{E}r(\omega_{i-1}) + \frac{1}{2}\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)$$

Hydrodynamics: symmetric ZRP

Take $p = q = \frac{1}{2}$, and SZRP with right rate $\frac{1}{2}r(\omega_i)$, left rate $\frac{1}{2}r(\omega_i)$.

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i = \frac{1}{2}\mathbf{E}r(\omega_{i-1}) + \frac{1}{2}\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)$$

Diffusive scaling:

- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

$$\frac{\mathrm{d}}{\mathrm{d}(\tau/L^2)}\mathbf{E}\omega_i = \frac{L^2}{2}\big(\mathbf{E}r(\omega_{i+1}) - 2\mathbf{E}r(\omega_i) + \mathbf{E}r(\omega_{i-1})\big)$$

Hydrodynamics: symmetric ZRP

Take $p = q = \frac{1}{2}$, and SZRP with right rate $\frac{1}{2}r(\omega_i)$, left rate $\frac{1}{2}r(\omega_i)$.

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i = \frac{1}{2}\mathbf{E}r(\omega_{i-1}) + \frac{1}{2}\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_i)$$

Diffusive scaling:

- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

$$\frac{\mathrm{d}}{\mathrm{d}(\tau/L^2)}\mathbf{E}\omega_i = \frac{L^2}{2}\big(\mathbf{E}r(\omega_{i+1}) - 2\mathbf{E}r(\omega_i) + \mathbf{E}r(\omega_{i-1})\big)$$

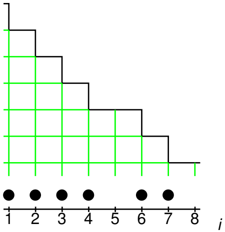
$$\frac{\partial}{\partial t}\varrho(t,\, x) = \frac{1}{2}\frac{\partial^2}{\partial x^2}G(\varrho(t,\, x)).$$

Hydrodynamics: symmetric ZRP

$$\frac{\partial}{\partial t}\varrho(t,\,x) = \frac{1}{2}\frac{\partial^2}{\partial x^2}G(\varrho(t,\,x))$$

Classical ZRP: $G(\varrho) = \mathbf{E}^{\varrho} r(\omega) = \mathbf{E}^{\varrho}\mathbf{1}\{\omega > 0\} = \frac{\varrho}{1+\varrho}$
concave, nonlinear heat equation.

Independent walkers: $G(\varrho) = \mathbf{E}^{\varrho} r(\omega) = \mathbf{E}^{\varrho}\omega = \varrho$
linear, heat equation.



The stationary solution is constant density, linear slope, or linearly changing G with current.

Hills

Can we model sedimentation and erosion processes with these surfaces?

Issues:

Hills

Can we model sedimentation and erosion processes with these surfaces?

Issues:

- Hills are not always straight $\leftrightarrow$ translation invariance.

Hills

Can we model sedimentation and erosion processes with these surfaces?

Issues:

- Hills are not always straight $\leftrightarrow$ translation invariance.
- Most hillslopes are rather stationary $\leftrightarrow$ particle current.

Convex hills

Wikipedia

Concave hills

Stockphotos4free

Product blocking measures

Solution: block particles (no current) and make their rates asymmetric (non-constant density).

Can we have a reversible stationary distribution in product form:

$$\underline{\mu}(\underline{\omega}) = \bigotimes_i \mu_i(\omega_i);$$

$$\underline{\mu}(\underline{\omega}) \cdot \mathrm{rate}\big(\underline{\omega} \to \underline{\omega}^{i\curvearrowright i+1}\big) = \underline{\mu}\big(\underline{\omega}^{i\curvearrowright i+1}\big) \cdot \mathrm{rate}\big(\underline{\omega}^{i\curvearrowright i+1} \to \underline{\omega}\big) \quad ?$$

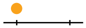
Here

$$\underline{\omega}^{i\curvearrowright i+1} = \underline{\omega} - \underline{\delta}_i + \underline{\delta}_{i+1}.$$

Asymmetric simple exclusion

$$\underline{\mu}(\underline{\eta}) \cdot \text{rate}\big(\underline{\eta} \to \underline{\eta}^{i\frown i+1}\big) = \underline{\mu}(\underline{\eta}^{i\frown i+1}) \cdot \text{rate}\big(\underline{\eta}^{i\frown i+1} \to \underline{\eta}\big)$$

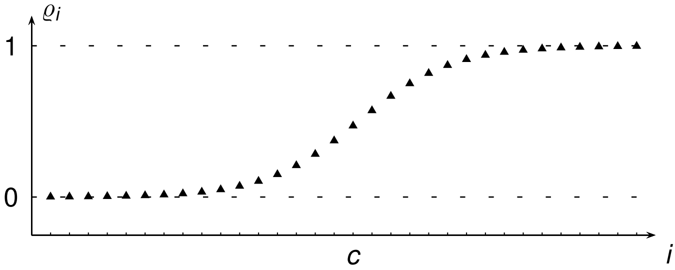
<u>ASEP:</u> $\mu_i \sim \text{Bernoulli}(\varrho_i)$;



$\underline{\eta}$

$$\varrho_i(1 - \varrho_{i+1}) \cdot p = (1 - \varrho_i)\varrho_{i+1} \cdot q$$

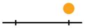
Solution: $$\varrho_i = \frac{(\frac{p}{q})^{i-c}}{1 + (\frac{p}{q})^{i-c}} = \frac{1}{(\frac{q}{p})^{i-c} + 1}$$



Asymmetric simple exclusion

$$\underline{\mu}(\underline{\eta}) \cdot \text{rate}\big(\underline{\eta} \to \underline{\eta}^{i\curvearrowright i+1}\big) = \underline{\mu}(\underline{\eta}^{i\curvearrowright i+1}) \cdot \text{rate}\big(\underline{\eta}^{i\curvearrowright i+1} \to \underline{\eta}\big)$$

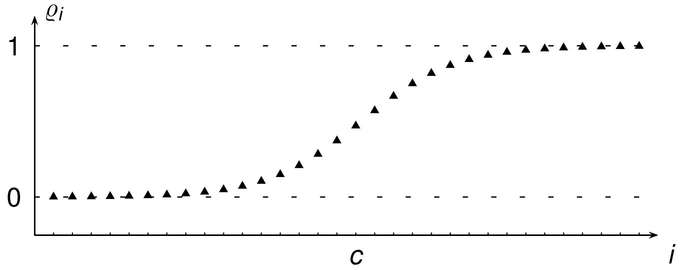
ASEP: $\mu_i \sim$ Bernoulli(ϱ_i);



$\underline{\eta}^{i\curvearrowright i+1}$

$$\varrho_i(1-\varrho_{i+1}) \cdot p = (1-\varrho_i)\varrho_{i+1} \cdot q$$

Solution: $$\varrho_i = \frac{(\frac{p}{q})^{i-c}}{1+(\frac{p}{q})^{i-c}} = \frac{1}{(\frac{q}{p})^{i-c}+1}$$



Asymmetric zero range process

$$\underline{\mu}(\underline{\omega}) \cdot \text{rate}\big(\underline{\omega} \to \underline{\omega}^{i\curvearrowright i+1}\big) = \underline{\mu}(\underline{\omega}^{i\curvearrowright i+1}) \cdot \text{rate}\big(\underline{\omega}^{i\curvearrowright i+1} \to \underline{\omega}\big) \quad ?$$

AZRP, classical:

$$\mu_i(\omega_i)\mu_{i+1}(\omega_{i+1}) \cdot p\mathbf{1}\{\omega_i > 0\} = \mu_i(\omega_i - 1)\mu_{i+1}(\omega_{i+1} + 1) \cdot q$$

Solution: $\mu_i \sim \text{Geometric}\Big(1 - \Big(\frac{p}{q}\Big)^{i-\text{const}}\Big).$

Asymmetric zero range process

$$\underline{\mu}(\underline{\omega}) \cdot \text{rate}\big(\underline{\omega} \to \underline{\omega}^{i\curvearrowright i+1}\big) = \underline{\mu}(\underline{\omega}^{i\curvearrowright i+1}) \cdot \text{rate}\big(\underline{\omega}^{i\curvearrowright i+1} \to \underline{\omega}\big) \quad ?$$

AZRP, independent walkers:

$$\mu_i(\omega_i)\mu_{i+1}(\omega_{i+1}) \cdot p\omega_i = \mu_i(\omega_i - 1)\mu_{i+1}(\omega_{i+1} + 1) \cdot q(\omega_{i+1} + 1)$$

Solution: $\mu_i \sim \text{Poisson}\Big(\Big(\dfrac{p}{q}\Big)^{i-\text{const}}\Big).$

Further models

In fact product blocking measures are very general.

Further models

In fact product blocking measures are very general.

- ASEP

Further models

In fact product blocking measures are very general.

- ASEP
- K-exclusion (!)

Further models

In fact product blocking measures are very general.

- ASEP
- K-exclusion (!)
- All zero range processes (“classical”, independent walkers, q-zero range)

Further models

In fact product blocking measures are very general.

- ASEP
- K-exclusion (!)
- All zero range processes (“classical”, independent walkers, q-zero range)
- Misanthrope / bricklayers processes

Further models

In fact product blocking measures are very general.

- ASEP
- K-exclusion (!)
- All zero range processes ("classical", independent walkers, q-zero range)
- Misanthrope / bricklayers processes

Other models can be *stood up*:

Further models

In fact product blocking measures are very general.

- ASEP
- K-exclusion (!)
- All zero range processes ("classical", independent walkers, q-zero range)
- Misanthrope / bricklayers processes

Other models can be *stood up*:

- ASEP

Further models

In fact product blocking measures are very general.

- ASEP
- K-exclusion (!)
- All zero range processes ("classical", independent walkers, q-zero range)
- Misanthrope / bricklayers processes

Other models can be *stood up*:

- ASEP
- q-exclusion

Further models

In fact product blocking measures are very general.

- ASEP
- K-exclusion (!)
- All zero range processes ("classical", independent walkers, q-zero range)
- Misanthrope / bricklayers processes

Other models can be *stood up*:

- ASEP
- q-exclusion
- Katz-Lebowitz-Spohn model

Product blocking measures

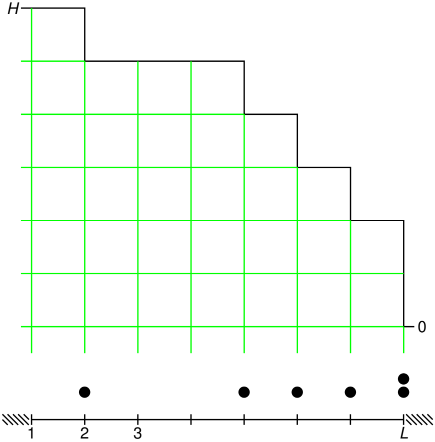
They are also very handy, due to reversibility.

Take a stationary, reversible Markov chain. Cut any of its edges. It stays reversible stationary w.r.t. the same distribution.

In our case: freeze the boundaries to obtain a stationary hill slope.

Microscopic model

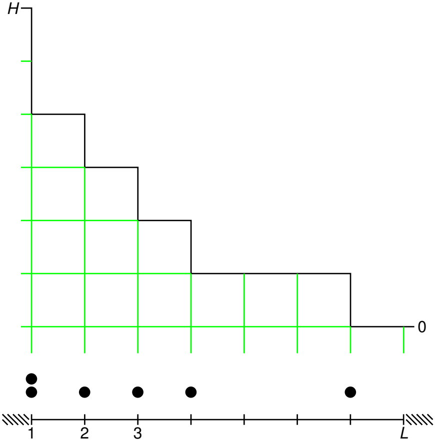
Our choice: AZRP with frozen boundaries. $p > q$: convex



Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

Microscopic model

Our choice: AZRP with frozen boundaries. $p < q$: concave



Particles jump to the right with rate $p \cdot r(\omega_i)$
to the left with rate $q \cdot r(\omega_i)$.

Microscopic model

Notice:

- The height of the hill H is conserved, the product measure is *not* ergodic.
- One-site marginals, given H, are in general not explicit.
- Except for independent walkers, where ω_i are Binomial.

Microscopic model

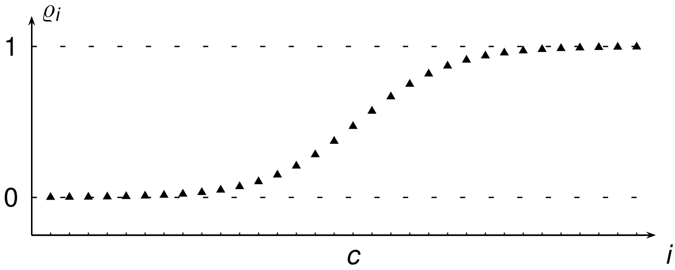
Notice:

- The height of the hill H is conserved, the product measure is *not* ergodic.
- One-site marginals, given H, are in general not explicit.
- Except for independent walkers, where ω_i are Binomial.

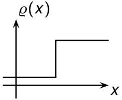
We won't be bothered by this.

Hydrodynamics

Work in progress. . .

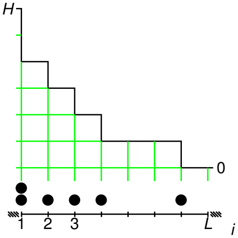


A blocking measure is a microscopic object. Here is its scaling

limit: 

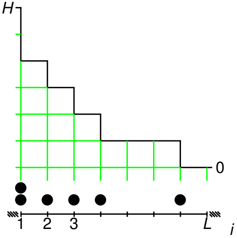
, not very interesting.

Hydrodynamics



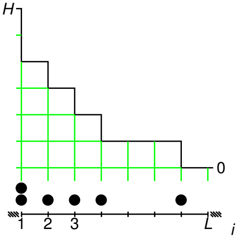
- Scaling parameter: L

Hydrodynamics



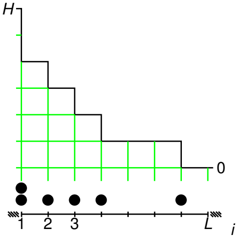
- Scaling parameter: L
- Blocking measure marginals depend on $\left(\frac{p}{q}\right)^{i} = \left(\frac{p}{q}\right)^{Lx}$.

Hydrodynamics



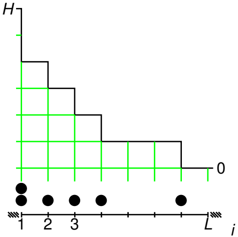
- Scaling parameter: L
- Blocking measure marginals depend on $\left(\frac{p}{q}\right)^i = \left(\frac{p}{q}\right)^{Lx}$.
- Scale $p = \frac{1}{2} + \frac{\gamma}{L}\,, \;\; q = \frac{1}{2} - \frac{\gamma}{L}\,.$

Hydrodynamics



- Scaling parameter: L
- Blocking measure marginals depend on $\left(\frac{p}{q}\right)^i = \left(\frac{p}{q}\right)^{Lx}$.
- Scale $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$.
- Then check $\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i(\tau)$.

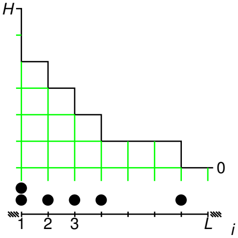
Hydrodynamics



- Scaling parameter: L
- Blocking measure marginals depend on $\left(\frac{p}{q}\right)^i = \left(\frac{p}{q}\right)^{Lx}$.
- Scale $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$.
- Then check $\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i(\tau)$.

Notice: this is *not* KPZ scaling ($p = \frac{1}{2} + \frac{\gamma}{\sqrt{L}}$, $q = \frac{1}{2} - \frac{\gamma}{\sqrt{L}}$).

Hydrodynamics



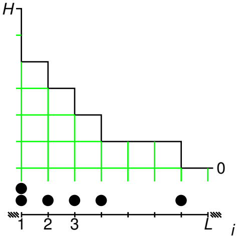
- Scaling parameter: L
- Blocking measure marginals depend on $\left(\frac{p}{q}\right)^i = \left(\frac{p}{q}\right)^{Lx}$.
- Scale $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$.
- Then check $\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i(\tau)$.

Notice: this is *not* KPZ scaling ($p = \frac{1}{2} + \frac{\gamma}{\sqrt{L}}$, $q = \frac{1}{2} - \frac{\gamma}{\sqrt{L}}$).

For AZRP (rates $p \cdot r(\omega_i)$ right and $q \cdot r(\omega_i)$ left):

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i = \frac{1}{2}\big(\mathbf{E}r(\omega_{i-1}) - 2\mathbf{E}r(\omega_i) + \mathbf{E}r(\omega_{i+1})\big) - \frac{\gamma}{L}\big(\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_{i-1})\big).$$

Hydrodynamics



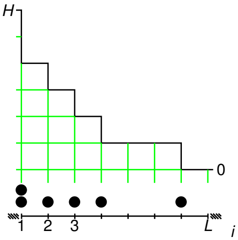
- Scaling parameter: L
- Blocking measure marginals depend on $\left(\frac{p}{q}\right)^{i} = \left(\frac{p}{q}\right)^{Lx}$.
- Scale $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$.
- Then check $\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i(\tau)$.

Notice: this is *not* KPZ scaling ($p = \frac{1}{2} + \frac{\gamma}{\sqrt{L}}$, $q = \frac{1}{2} - \frac{\gamma}{\sqrt{L}}$).

For AZRP (rates $p \cdot r(\omega_i)$ right and $q \cdot r(\omega_i)$ left):

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i = \frac{1}{2}\big(\mathbf{E}r(\omega_{i-1}) - 2\mathbf{E}r(\omega_i) + \mathbf{E}r(\omega_{i+1})\big)$$
$$- \frac{\gamma}{L}\big(\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_{i-1})\big).$$

Hydrodynamics



- Scaling parameter: L
- Blocking measure marginals depend on $\left(\frac{p}{q}\right)^i = \left(\frac{p}{q}\right)^{Lx}$.
- Scale $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$.
- Then check $\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i(\tau)$.

Notice: this is *not* KPZ scaling ($p = \frac{1}{2} + \frac{\gamma}{\sqrt{L}}$, $q = \frac{1}{2} - \frac{\gamma}{\sqrt{L}}$).

For AZRP (rates $p \cdot r(\omega_i)$ right and $q \cdot r(\omega_i)$ left):

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i = \frac{1}{2}\big(\mathbf{E}r(\omega_{i-1}) - 2\mathbf{E}r(\omega_i) + \mathbf{E}r(\omega_{i+1})\big) - \frac{\gamma}{L}\big(\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_{i-1})\big).$$

Hydrodynamics

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_i = \frac{1}{2}\big(\mathbf{E}r(\omega_{i-1}) - 2\mathbf{E}r(\omega_i) + \mathbf{E}r(\omega_{i+1})\big)$$
$$- \frac{\gamma}{L}\big(\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_{i-1})\big).$$

which dictates diffusive scaling:

- $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$;
- $\varrho(t,\,x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

$$\frac{\mathrm{d}}{\mathrm{d}(\tau/L^2)}\mathbf{E}\omega_i = \frac{L^2}{2}\big(\mathbf{E}r(\omega_{i-1}) - 2\mathbf{E}r(\omega_i) + \mathbf{E}r(\omega_{i+1})\big)$$
$$- \gamma L\big(\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_{i-1})\big),$$
$$\frac{\partial}{\partial t}\varrho(t,\,x) = \frac{1}{2}\frac{\partial^2}{\partial x^2}G(\varrho(t,\,x)) - 2\gamma\frac{\partial}{\partial x}G(\varrho(t,\,x)), \quad (0 < x < 1).$$

Hydrodynamics

- $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$;
- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

How about the boundaries?

Hydrodynamics

- $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$;
- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

How about the boundaries?

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_1 = \frac{1}{2}\big(\mathbf{E}r(\omega_2) - \mathbf{E}r(\omega_1)\big) - \frac{\gamma}{L}\big(\mathbf{E}r(\omega_2) + \mathbf{E}r(\omega_1)\big),$$

Hydrodynamics

- $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$;
- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

How about the boundaries?

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_1 = \frac{1}{2}\big(\mathbf{E}r(\omega_2) - \mathbf{E}r(\omega_1)\big) - \frac{\gamma}{L}\big(\mathbf{E}r(\omega_2) + \mathbf{E}r(\omega_1)\big),$$

Hydrodynamics

- $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$;
- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

How about the boundaries?

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_1 = \frac{1}{2}\big(\mathbf{E}r(\omega_2) - \mathbf{E}r(\omega_1)\big) - \frac{\gamma}{L}\big(\mathbf{E}r(\omega_2) + \mathbf{E}r(\omega_1)\big),$$

Hydrodynamics

- $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$;
- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

How about the boundaries?

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_1 = \frac{1}{2}\big(\mathbf{E}r(\omega_2) - \mathbf{E}r(\omega_1)\big) - \frac{\gamma}{L}\big(\mathbf{E}r(\omega_2) + \mathbf{E}r(\omega_1)\big),$$

$$\frac{1}{L}\frac{\mathrm{d}}{\mathrm{d}(\tau/L^2)}\mathbf{E}\omega_1 = \frac{L}{2}\big(\mathbf{E}r(\omega_2) - \mathbf{E}r(\omega_1)\big) - \gamma\big(\mathbf{E}r(\omega_2) + \mathbf{E}r(\omega_1)\big),$$

Hydrodynamics

- $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$;
- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

How about the boundaries?

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_1 = \frac{1}{2}\big(\mathbf{E}r(\omega_2) - \mathbf{E}r(\omega_1)\big) - \frac{\gamma}{L}\big(\mathbf{E}r(\omega_2) + \mathbf{E}r(\omega_1)\big),$$

$$\frac{1}{L}\frac{\mathrm{d}}{\mathrm{d}(\tau/L^2)}\mathbf{E}\omega_1 = \frac{L}{2}\big(\mathbf{E}r(\omega_2) - \mathbf{E}r(\omega_1)\big) - \gamma\big(\mathbf{E}r(\omega_2) + \mathbf{E}r(\omega_1)\big),$$

$$0 = \frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,\, 0)) - 2\gamma G(\varrho(t,\, 0)).$$

Hydrodynamics

- $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$;
- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

How about the boundaries?

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\mathbf{E}\omega_1 = \frac{1}{2}\big(\mathbf{E}r(\omega_2) - \mathbf{E}r(\omega_1)\big) - \frac{\gamma}{L}\big(\mathbf{E}r(\omega_2) + \mathbf{E}r(\omega_1)\big),$$

$$\frac{1}{L}\frac{\mathrm{d}}{\mathrm{d}(\tau/L^2)}\mathbf{E}\omega_1 = \frac{L}{2}\big(\mathbf{E}r(\omega_2) - \mathbf{E}r(\omega_1)\big) - \gamma\big(\mathbf{E}r(\omega_2) + \mathbf{E}r(\omega_1)\big),$$

$$0 = \frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,\, 0)) - 2\gamma G(\varrho(t,\, 0)).$$

Also: $$0 = \frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,\, 1)) - 2\gamma G(\varrho(t,\, 1)).$$

Hydrodynamics

$$\frac{\partial}{\partial t}\varrho(t,\,x) = \frac{1}{2}\frac{\partial^2}{\partial x^2}G(\varrho(t,\,x)) - 2\gamma\frac{\partial}{\partial x}G(\varrho(t,\,x)) \quad (0 < x < 1)$$

$$0 = \frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,\,0)) - 2\gamma G(\varrho(t,\,0))$$
$$0 = \frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,\,1)) - 2\gamma G(\varrho(t,\,1))$$

Convection-diffusion type equation with Robin boundary.

Hydrodynamics

$$\begin{aligned}
\frac{\partial}{\partial t}\varrho(t,\,x) &= \frac{1}{2}\frac{\partial^2}{\partial x^2}G(\varrho(t,\,x)) - 2\gamma\frac{\partial}{\partial x}G(\varrho(t,\,x)) \quad (0<x<1)\\
&= \frac{\partial}{\partial x}\Big(\frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,\,x)) - 2\gamma G(\varrho(t,\,x))\Big)\\
0 &= \frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,\,0)) - 2\gamma G(\varrho(t,\,0))\\
0 &= \frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,\,1)) - 2\gamma G(\varrho(t,\,1))
\end{aligned}$$

Convection-diffusion type equation with Robin boundary.

Hydrodynamics

$$
\begin{aligned}
\frac{\partial}{\partial t}\varrho(t,\,x) &= \frac{1}{2}\frac{\partial^2}{\partial x^2}G(\varrho(t,\,x)) - 2\gamma\frac{\partial}{\partial x}G(\varrho(t,\,x)) \quad (0 < x < 1)\\
&= \frac{\partial}{\partial x}\Big(\frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,\,x)) - 2\gamma G(\varrho(t,\,x))\Big)\\
0 &= \frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,\,0)) - 2\gamma G(\varrho(t,\,0))\\
0 &= \frac{1}{2}\frac{\partial}{\partial x}G(\varrho(t,\,1)) - 2\gamma G(\varrho(t,\,1))
\end{aligned}
$$

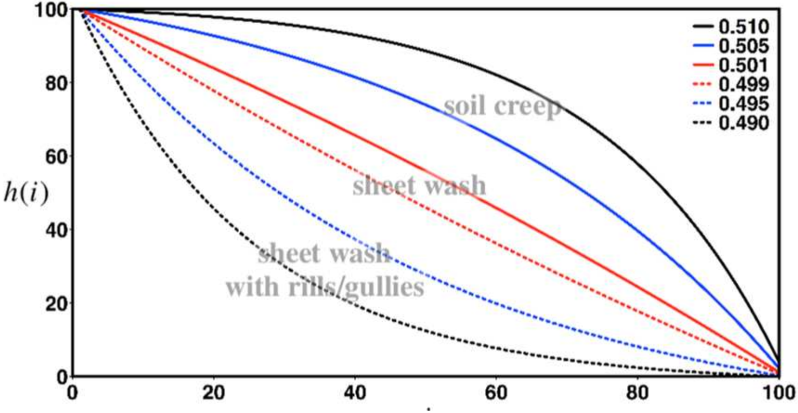
Convection-diffusion type equation with Robin boundary.

Doing the proper derivation is work in progress.

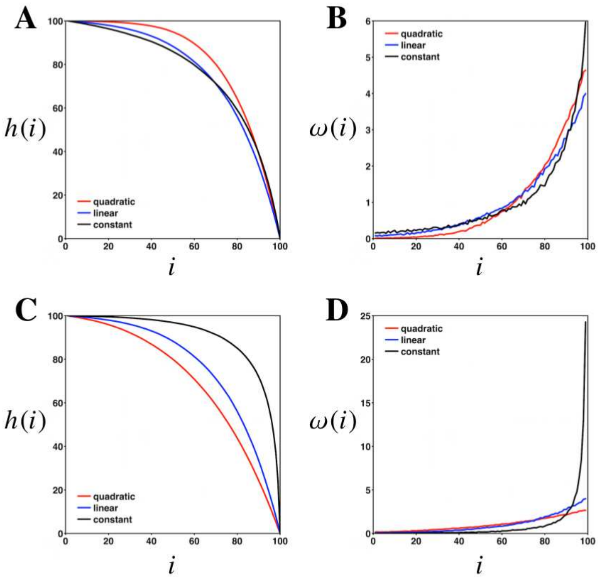
The time-stationary solution $G(\varrho(x)) = C\mathrm{e}^{4\gamma x}$ is consistent with the stationary blocking measure.

The stationary slope

$$G(\varrho(x)) = Ce^{4\gamma x}$$



The stationary slope



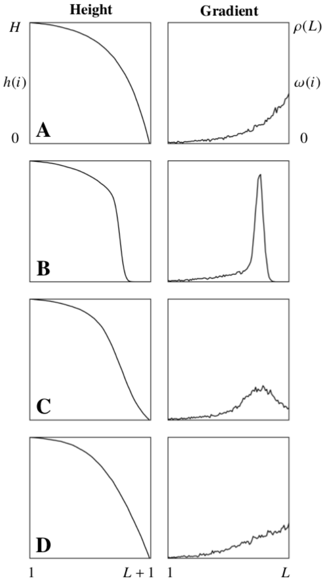
Dynamics

Space scale: $x \in [0, 1] \Leftrightarrow$ we $\in$ hill.

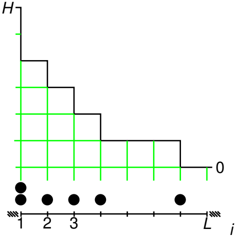
Problem 1: The stationary hillslope will not tell us the time scale.

$\rightsquigarrow$ Observe relaxation to stationarity in Nature and in the PDE.

Dynamics



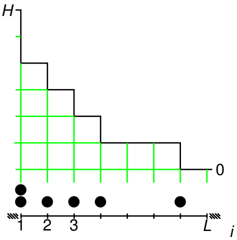
Dynamics



Problem 2: Geologists want a prediction for the *hill particle flux*, and the *distance travelled by hill particles*.

Notice: Hill particles $\neq$ our particles.

Dynamics

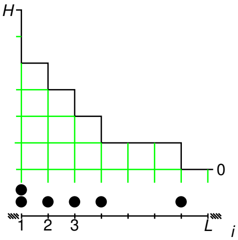


Problem 2: Geologists want a prediction for the *hill particle flux*, and the *distance travelled by hill particles*.

Notice: Hill particles $\neq$ our particles.

This is not part of the core argument, instead, is done by heuristics:

Dynamics



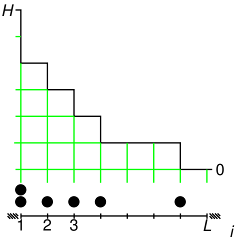
Problem 2: Geologists want a prediction for the *hill particle flux*, and the *distance travelled by hill particles*.

Notice: Hill particles $\neq$ our particles.

This is not part of the core argument, instead, is done by heuristics:

- Erosion flow speed $\sim$ average deposition rate $p\mathbf{E}r$.

Dynamics



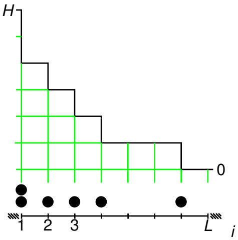
Problem 2: Geologists want a prediction for the *hill particle flux*, and the *distance travelled by hill particles*.

Notice: Hill particles $\neq$ our particles.

This is not part of the core argument, instead, is done by heuristics:

- Erosion flow speed $\sim$ average deposition rate $p\mathbf{E}r$.
- Time of hill particle spent in the flow to be picked as a constant or function of the slope ϱ.

Dynamics



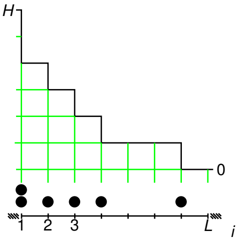
Problem 2: Geologists want a prediction for the *hill particle flux*, and the *distance travelled by hill particles*.

Notice: Hill particles $\neq$ our particles.

This is not part of the core argument, instead, is done by heuristics:

- Erosion flow speed $\sim$ average deposition rate $p\mathbf{E}r$.
- Time of hill particle spent in the flow to be picked as a constant or function of the slope ϱ.
- Average hill particle flux is the same across the hill (reversibility), *but this is not provided by the model*.

Dynamics



Problem 2: Geologists want a prediction for the *hill particle flux*, and the *distance travelled by hill particles*.

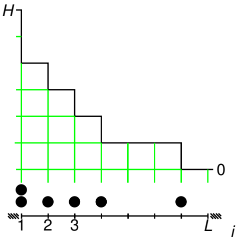
Notice: Hill particles $\neq$ our particles.

This is not part of the core argument, instead, is done by heuristics:

- Erosion flow speed $\sim$ average deposition rate $p\mathbf{E}r$.
- Time of hill particle spent in the flow to be picked as a constant or function of the slope ϱ.
- Average hill particle flux is the same across the hill (reversibility), *but this is not provided by the model*.

One can then give an expected distance travelled by a hill particle.

Dynamics



Problem 2: Geologists want a prediction for the *hill particle flux*, and the *distance travelled by hill particles*.

Notice: Hill particles $\neq$ our particles.

This is not part of the core argument, instead, is done by heuristics:

- Erosion flow speed $\sim$ average deposition rate $p\mathbf{E}r$.
- Time of hill particle spent in the flow to be picked as a constant or function of the slope ϱ.
- Average hill particle flux is the same across the hill (reversibility), *but this is not provided by the model*.

One can then give an expected distance travelled by a hill particle.

Thank you.