Blocking measures, hills, and hydrodynamics

Joint with
Jacob Calvert, Patrícia Gonçalves and Katerina Michaelides

Márton Balázs

University of Bristol

Particle Systems and PDE's - VI
Nice, 27 November, 2017.

Models

- Asymmetric simple exclusion
- Zero range

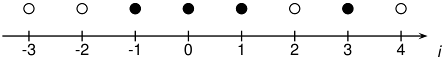
Blocking measures

- ASEP
- ZRP
- Further models

Hills

- Microscopic model
- Hydrodynamics

Asymmetric simple exclusion



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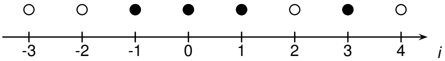

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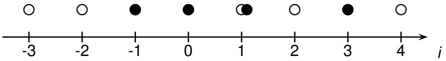

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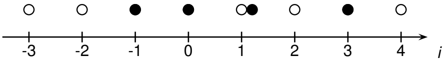

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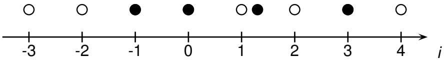

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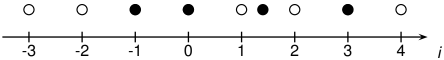

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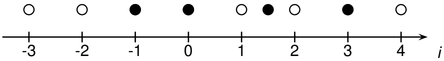

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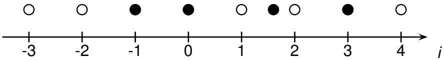

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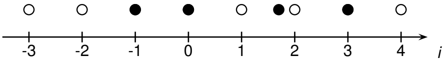

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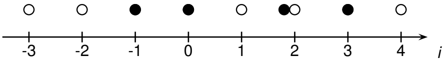

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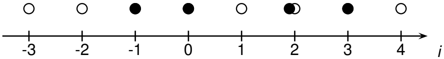

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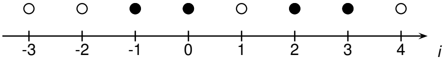

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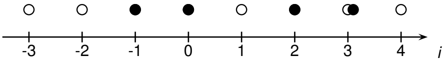

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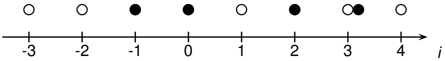

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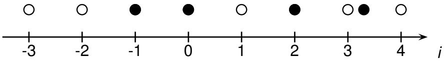

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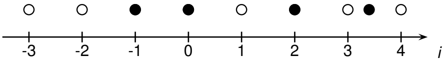

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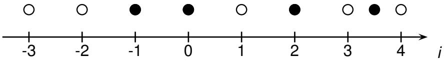

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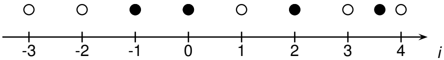

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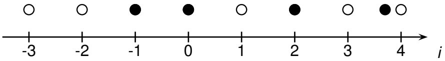

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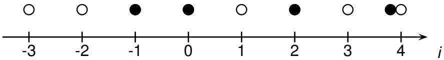

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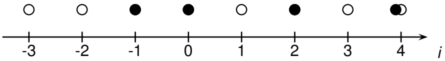

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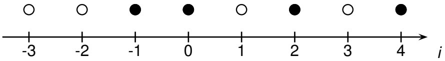

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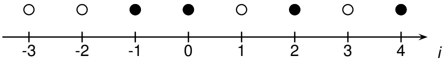

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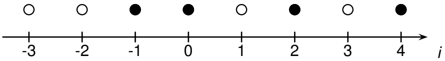

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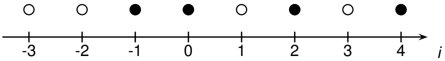

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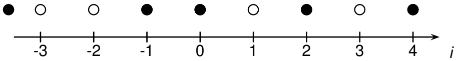

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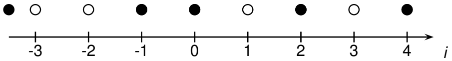

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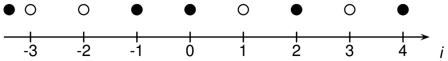

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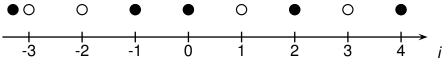

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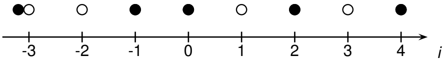

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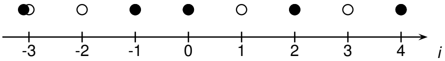

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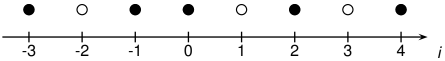

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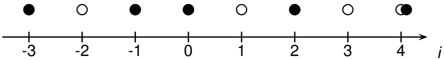

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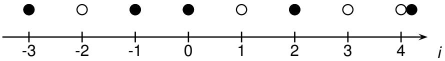

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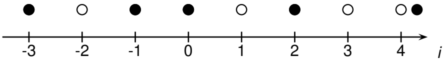

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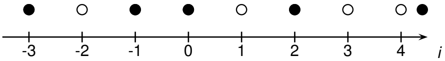

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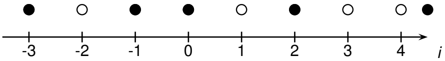

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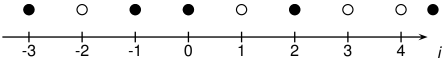

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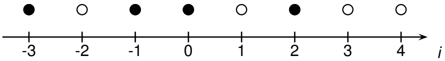

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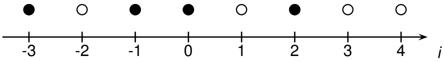

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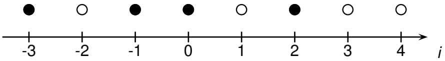

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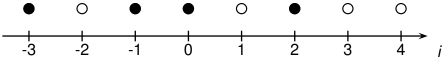

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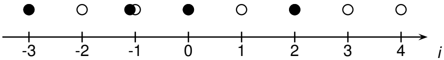

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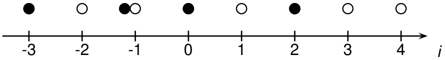

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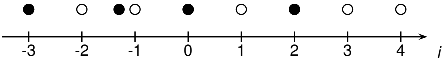

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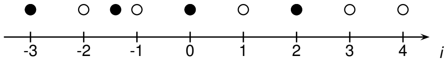

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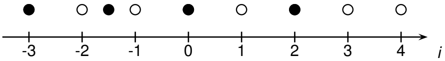

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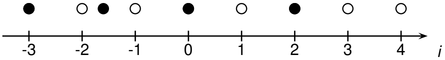

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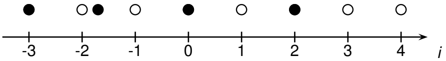

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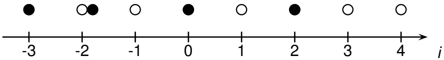

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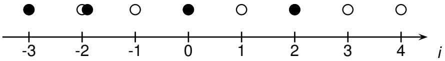

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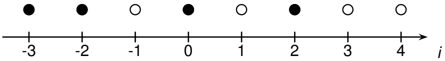

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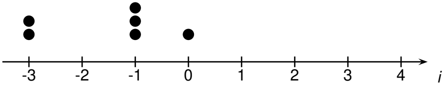

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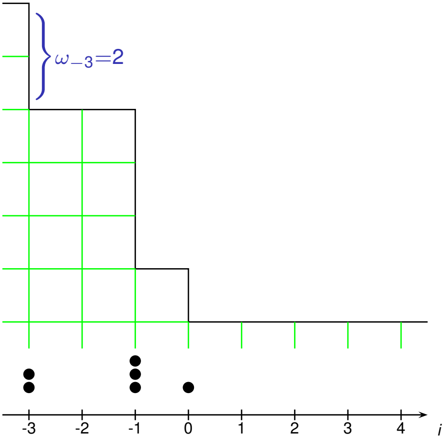

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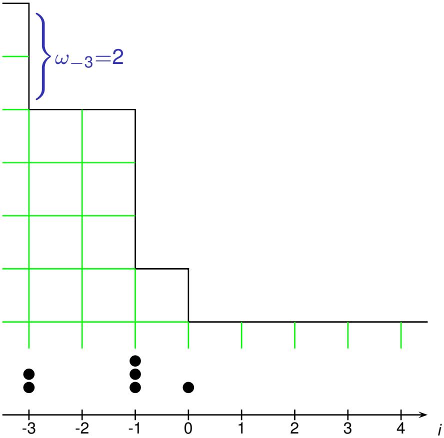

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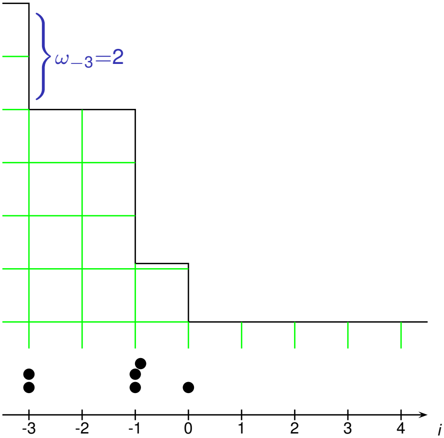

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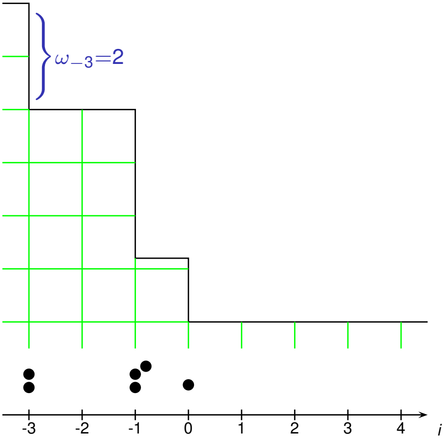

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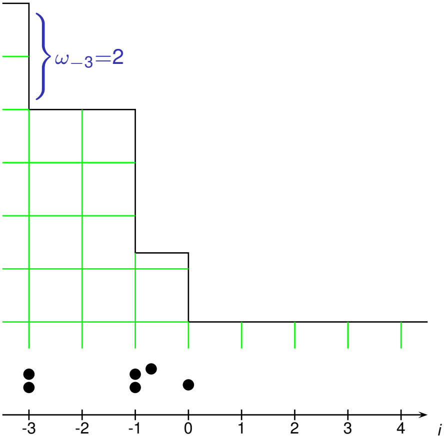

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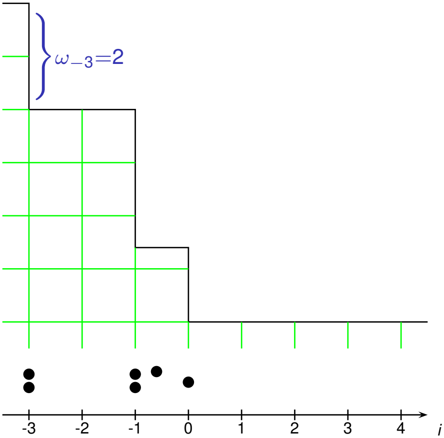

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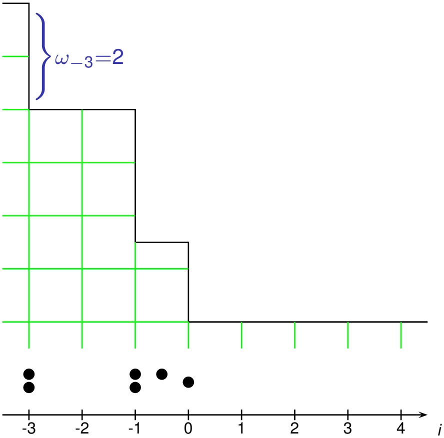

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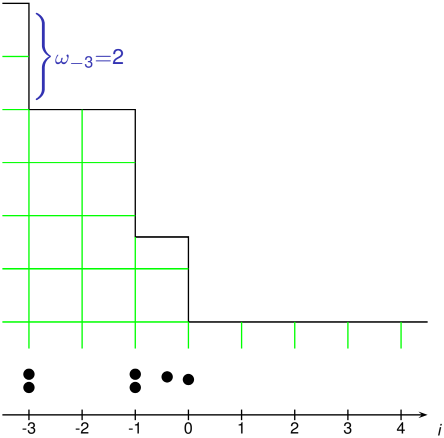

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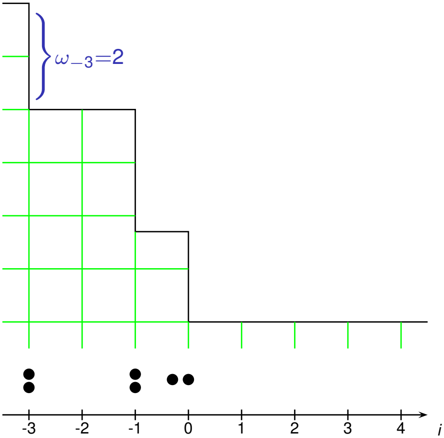

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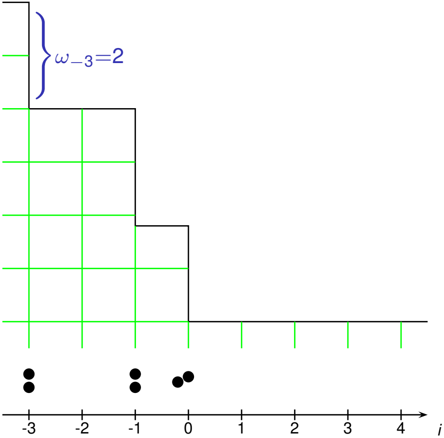

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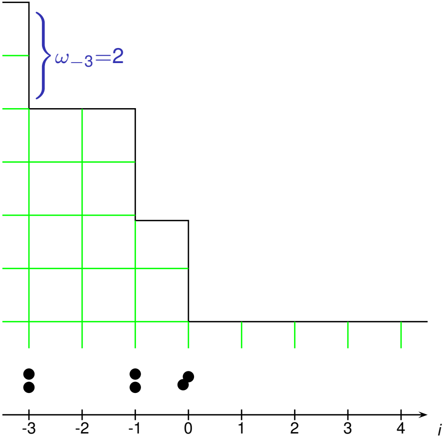

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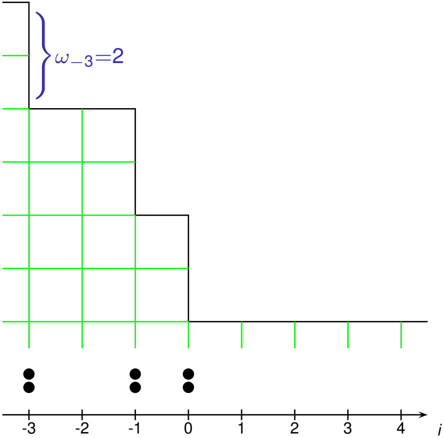

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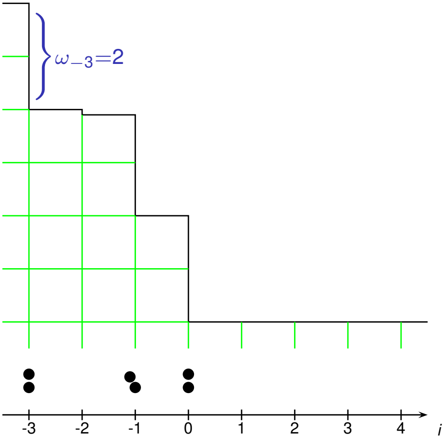

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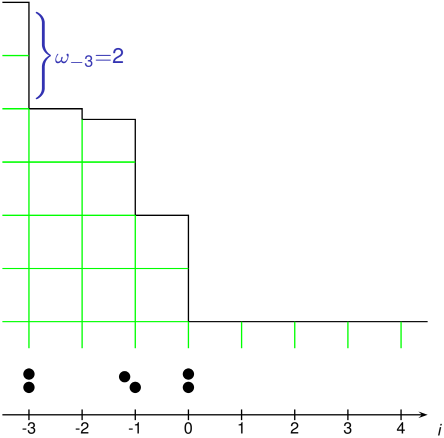

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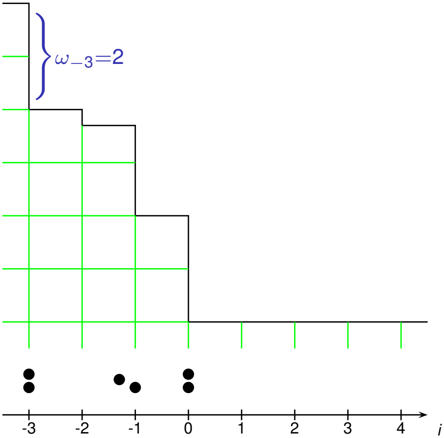

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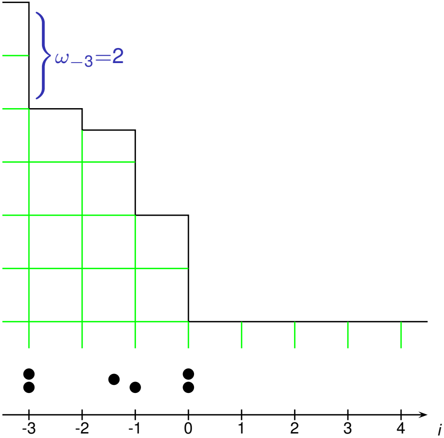

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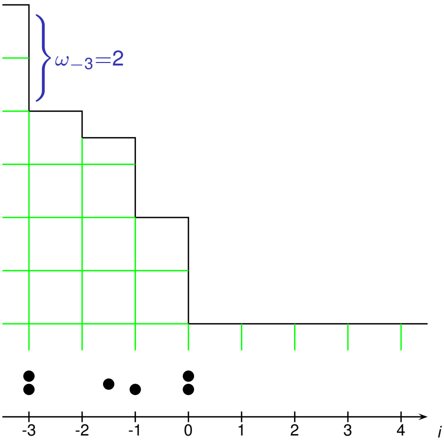

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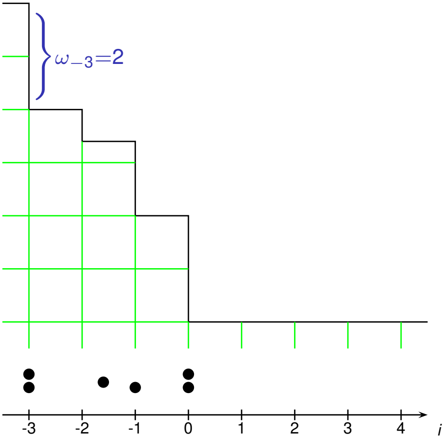

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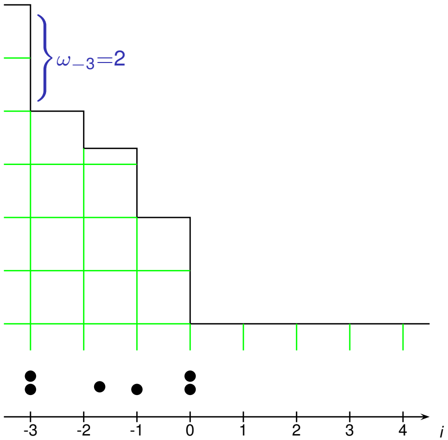

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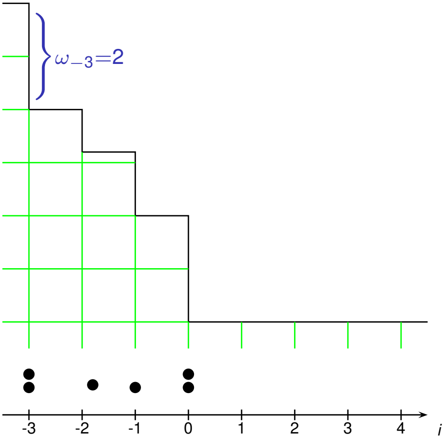

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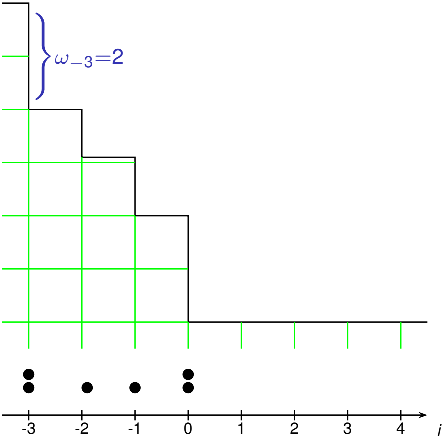

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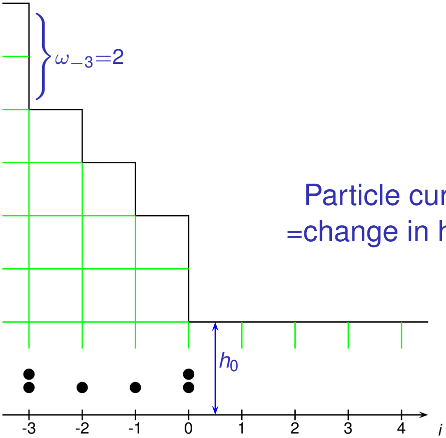

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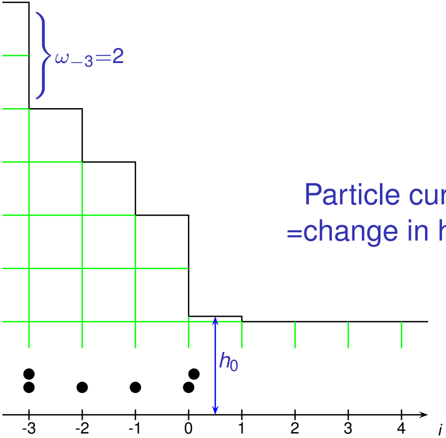

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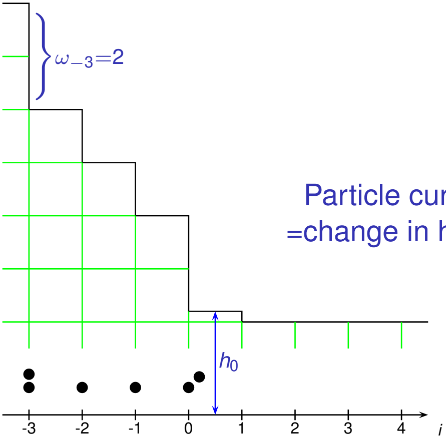

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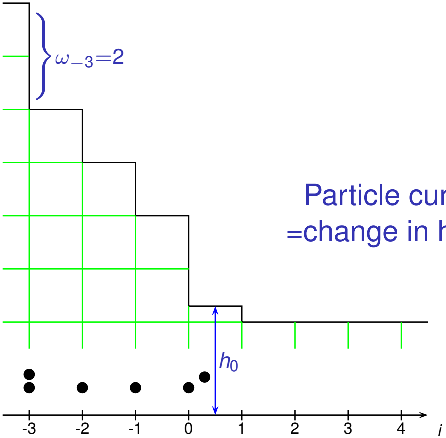

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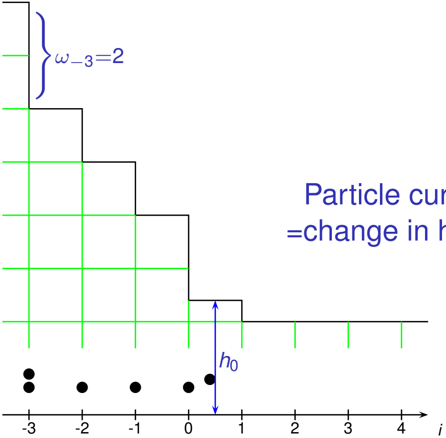

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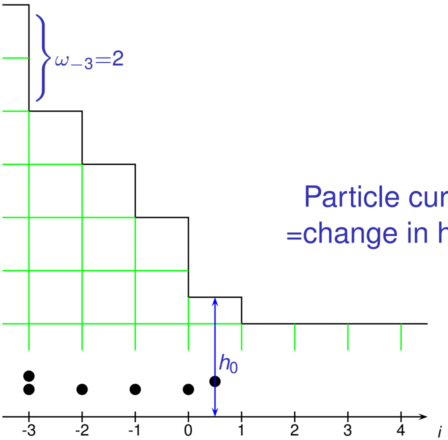

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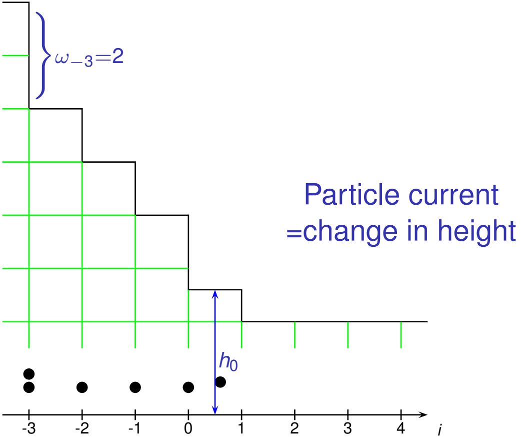

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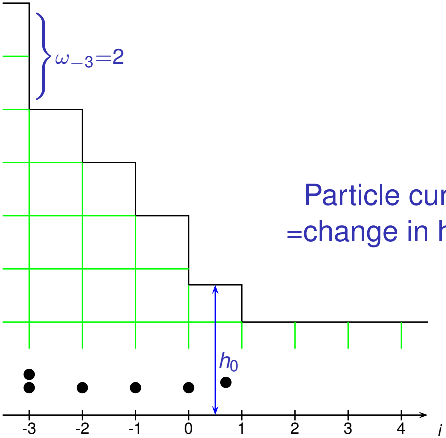

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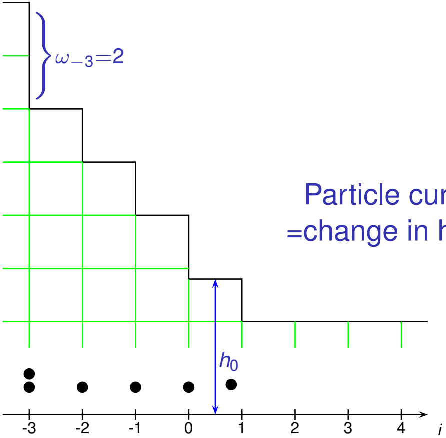

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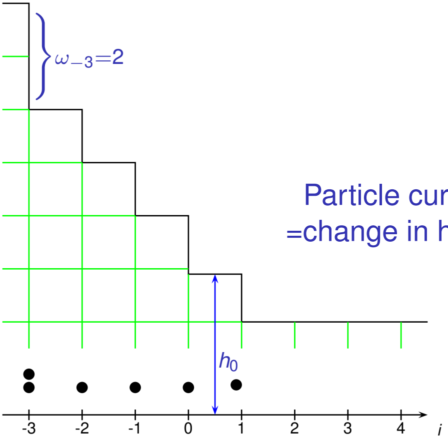

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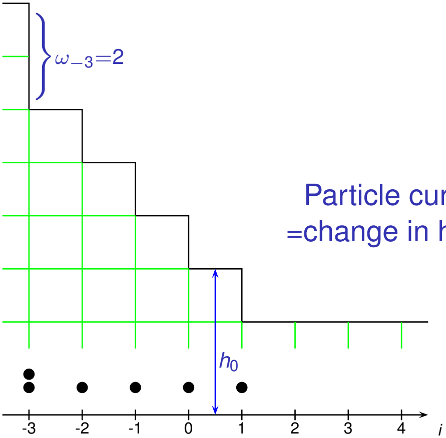

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We need r non-decreasing.

Examples:

- 'Classical' ZRP: $r(\omega_i) = \mathbf{1}\{\omega_i > 0\}$.
- Independent walkers: $r(\omega_i) = \omega_i$.

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Can we model sedimentation and erosion processes with these surfaces?

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Issues:

- Hills are not always straight $\leftrightarrow$ translation invariance.
- Most hillslopes are rather stationary $\leftrightarrow$ particle current.

Convex hills

Wikipedia

Concave hills

Stockphotos4free

Product blocking measures

Solution: block particles (no current) and make their rates asymmetric (non-constant density).

Can we have a reversible stationary distribution in product form:

$$\underline{\mu}(\underline{\omega}) = \bigotimes_i \mu_i(\omega_i);$$

$$\underline{\mu}(\underline{\omega}) \cdot \text{rate}\big(\underline{\omega} \to \underline{\omega}^{i \curvearrowright i+1}\big) = \underline{\mu}\big(\underline{\omega}^{i \curvearrowright i+1}\big) \cdot \text{rate}\big(\underline{\omega}^{i \curvearrowright i+1} \to \underline{\omega}\big) \quad ?$$

Here

$$\underline{\omega}^{i \curvearrowright i+1} = \underline{\omega} - \underline{\delta}_i + \underline{\delta}_{i+1}.$$

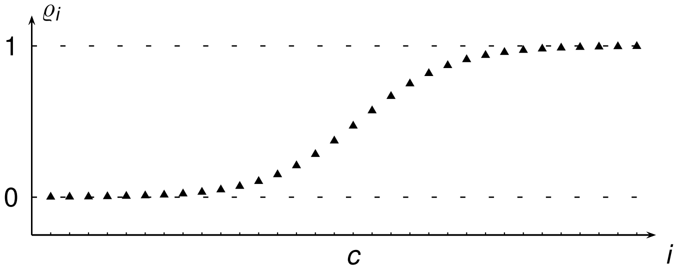
Asymmetric simple exclusion

$$\underline{\mu}(\underline{\eta}) \cdot \text{rate}\big(\underline{\eta} \to \underline{\eta}^{i \curvearrowright i+1}\big) = \underline{\mu}(\underline{\eta}^{i \curvearrowright i+1}) \cdot \text{rate}\big(\underline{\eta}^{i \curvearrowright i+1} \to \underline{\eta}\big)$$

ASEP: $\mu_i \sim \text{Bernoulli}(\varrho_i)$; $\underline{\eta}$

$$\varrho_i(1 - \varrho_{i+1}) \cdot p = (1 - \varrho_i)\varrho_{i+1} \cdot q$$

Solution: $$\varrho_i = \frac{\left(\frac{p}{q}\right)^{i-c}}{1 + \left(\frac{p}{q}\right)^{i-c}} = \frac{1}{\left(\frac{q}{p}\right)^{i-c} + 1}$$



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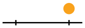

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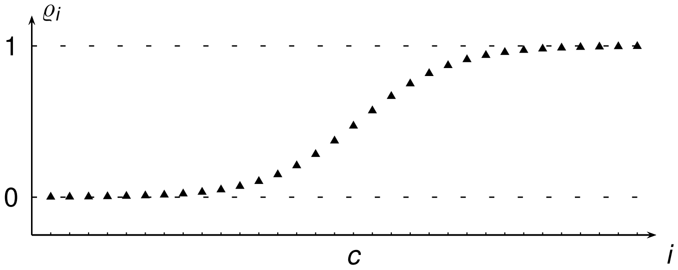

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AZRP, classical:

$$\mu_i(\omega_i)\mu_{i+1}(\omega_{i+1}) \cdot p\mathbf{1}\{\omega_i > 0\} = \mu_i(\omega_i - 1)\mu_{i+1}(\omega_{i+1} + 1) \cdot q$$

Solution: $\mu_i \sim \text{Geometric}\left(1 - \left(\frac{p}{q}\right)^{i-\text{const}}\right).$

Asymmetric zero range process

$$\underline{\mu}(\underline{\omega}) \cdot \text{rate}\big(\underline{\omega} \to \underline{\omega}^{i \curvearrowright i+1}\big) = \underline{\mu}(\underline{\omega}^{i \curvearrowright i+1}) \cdot \text{rate}\big(\underline{\omega}^{i \curvearrowright i+1} \to \underline{\omega}\big) \quad ?$$

AZRP, independent walkers:

$$\mu_i(\omega_i)\mu_{i+1}(\omega_{i+1}) \cdot p\omega_i = \mu_i(\omega_i - 1)\mu_{i+1}(\omega_{i+1} + 1) \cdot q(\omega_{i+1} + 1)$$

Solution: $\mu_i \sim \text{Poisson}\Big(\Big(\frac{p}{q}\Big)^{i - \text{const}}\Big).$

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In fact product blocking measures are very general.

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- K-exclusion (!)
- All zero range processes ("classical", independent walkers, q-zero range)
- Misanthrope / bricklayers processes

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Other models can be *stood up*:

- ASEP
- q-exclusion
- Katz-Lebowitz-Spohn model

Product blocking measures

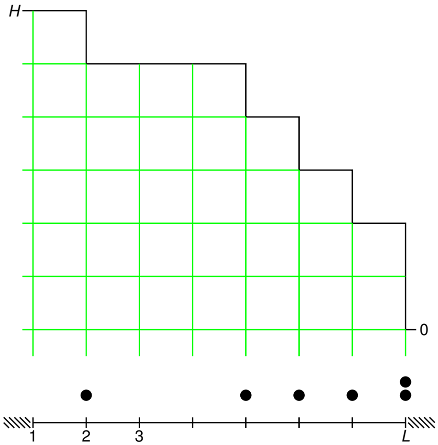
They are also very handy, due to reversibility.

Take a stationary, reversible Markov chain. Cut any of its edges. It stays reversible stationary w.r.t. the same distribution.

<u>In our case:</u> freeze the boundaries to obtain a stationary hill slope.

Microscopic model

Our choice: AZRP with frozen boundaries. $p > q$: convex



Particles jump
to the right with rate $p \cdot r(\omega_i)$
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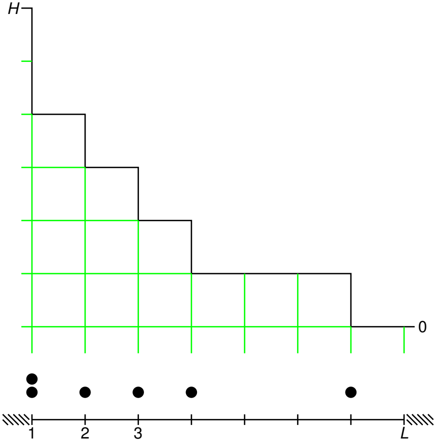

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Notice:

- The height of the hill H is conserved, the product measure is *not* ergodic.
- One-site marginals, given H, are in general not explicit.
- Except for independent walkers, where ω_i are Binomial.

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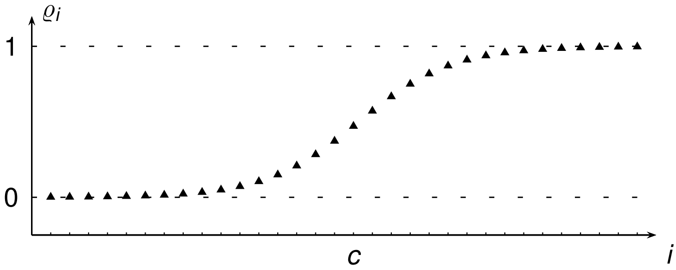
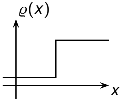
We won't be bothered by this.

Hydrodynamics

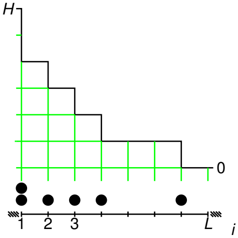
Work in progress. . .



A blocking measure is a microscopic object. Here is its scaling limit: 

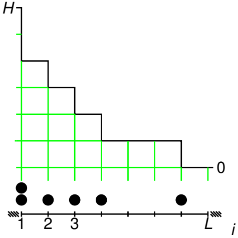
, not very interesting.

Hydrodynamics



- Scaling parameter: L

Hydrodynamics



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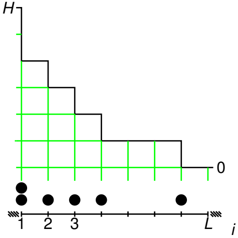
Hydrodynamics



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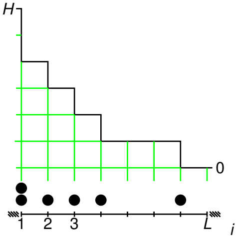

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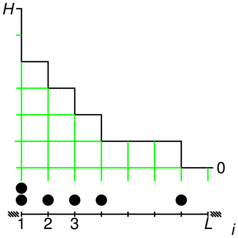

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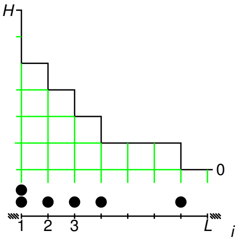

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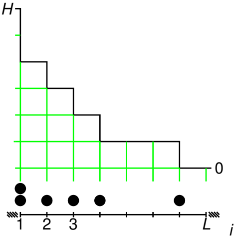

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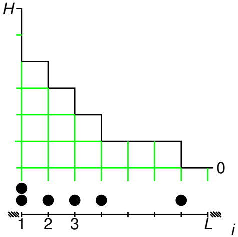

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which dictates diffusive scaling:

- $p = \frac{1}{2} + \frac{\gamma}{L}$, $q = \frac{1}{2} - \frac{\gamma}{L}$;
- $\varrho(t,\, x) = \mathbf{E}\omega_{Lx}(L^2 t)$;
- also define $G(\varrho) = \mathbf{E}^{\varrho} r(\omega)$:

$$\frac{\mathrm{d}}{\mathrm{d}(\tau/L^2)}\mathbf{E}\omega_i = \frac{L^2}{2}\big(\mathbf{E}r(\omega_{i-1}) - 2\mathbf{E}r(\omega_i) + \mathbf{E}r(\omega_{i+1})\big) - \gamma L\big(\mathbf{E}r(\omega_{i+1}) - \mathbf{E}r(\omega_{i-1})\big),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\varrho(t,\, x) = \frac{1}{2}\frac{\partial^2}{\partial x^2}G(\varrho(t,\, x)) - 2\gamma\frac{\partial}{\partial x}G(\varrho(t,\, x)), \qquad (0 < x < 1).$$

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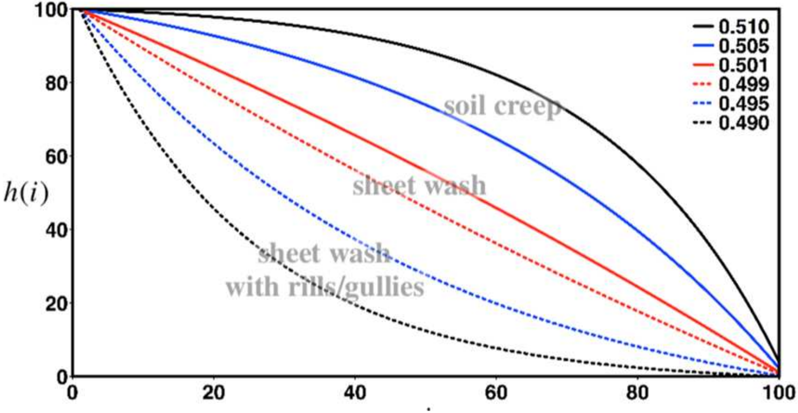
Convection-diffusion type equation with Robin boundary.

Doing the proper derivation is work in progress.

The time-stationary solution $G(\varrho(x)) = C\mathrm{e}^{4\gamma x}$ is consistent with the stationary blocking measure.

The stationary slope

$$G(\varrho(x)) = C\mathrm{e}^{4\gamma x}$$



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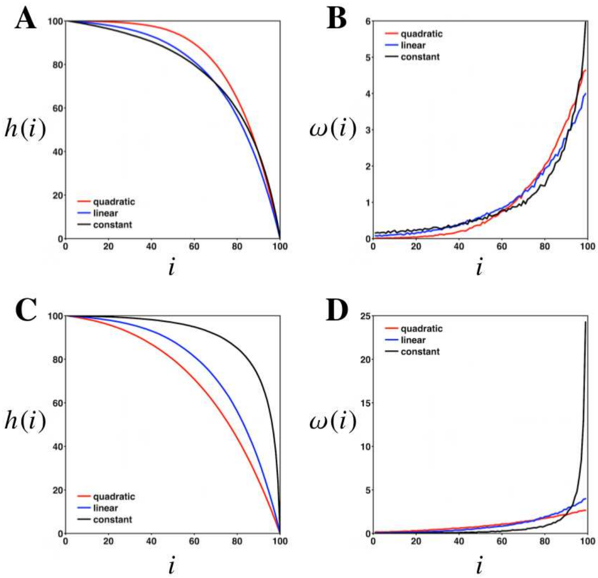


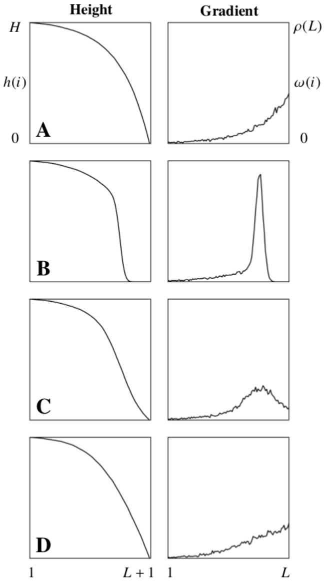
Dynamics

Space scale: $x \in [0, 1] \Leftrightarrow$ we $\in$ hill.

Problem 1: The stationary hillslope will not tell us the time scale.

$\leadsto$ Observe relaxation to stationarity in Nature and in the PDE.

Dynamics



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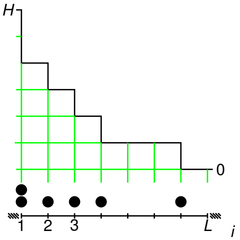

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Notice: Hill particles $\neq$ our particles.

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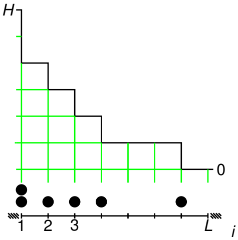

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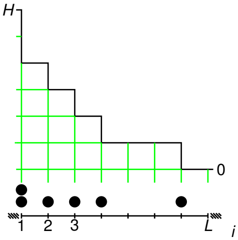

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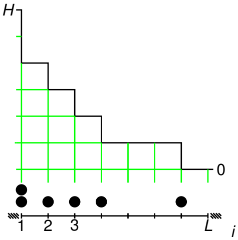

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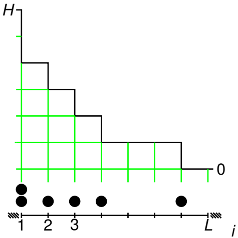

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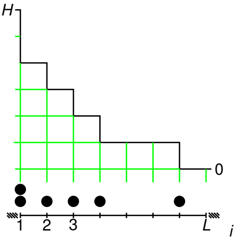

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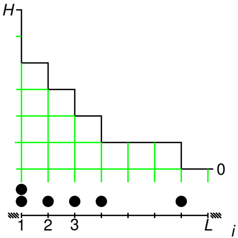

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