Queues, stationarity, and stabilisation of last passage percolation

Joint with
Ofer Busani and Timo Seppäläinen

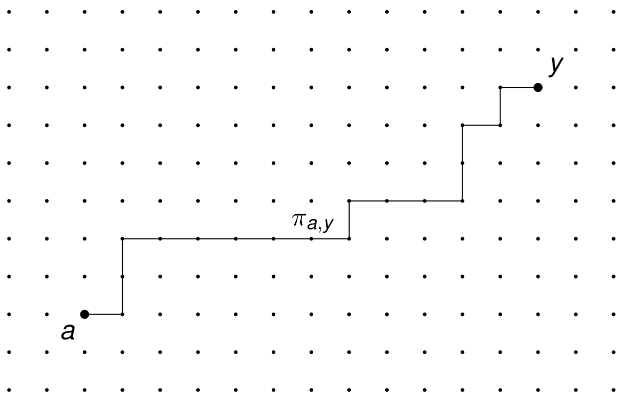
Márton Balázs

University of Bristol

Large Scale Stochastic Dynamics
Oberwolfach, 16 September, 2022.

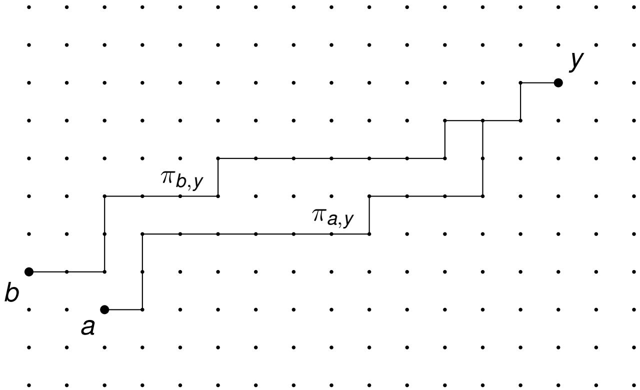
Last passage percolation

- Place ω_z i.i.d. Exp(1) for $z \in \mathbb{Z}^2$.
- The *geodesic* $\pi_{a,y}$ from a to y is the a.s. unique heaviest up-right path from a to y. Its weight is $G_{a,y}$.

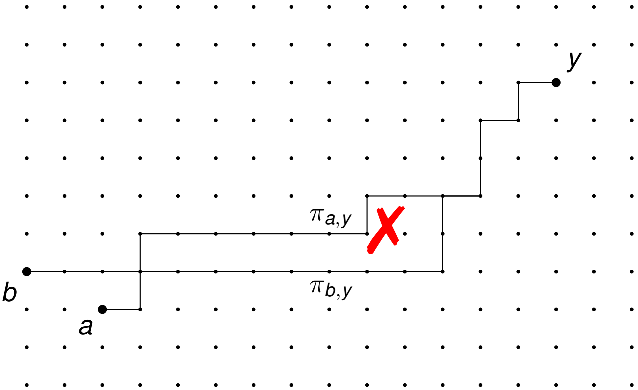


$G_{0,y}$ is the time TASEP hole y_1 swaps with particle y_2 if started from 1-0 initial condition.

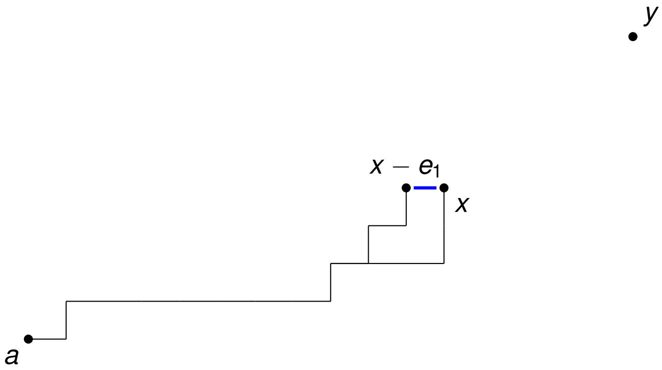
Coalescing: OK



But loops: not OK

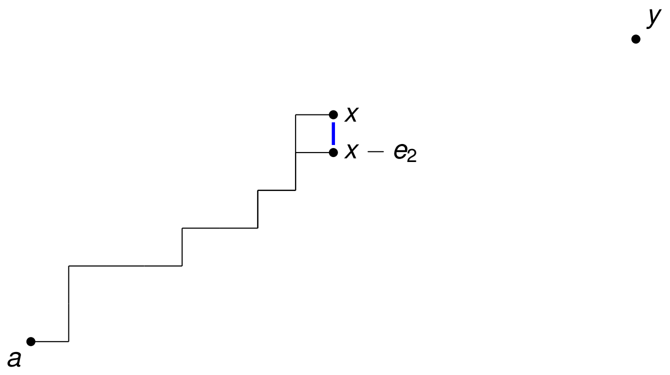


Increments as new boundary



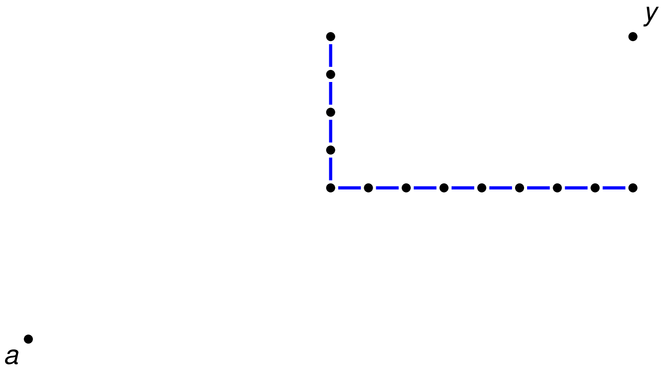
$$I_x = G_{a,x} - G_{a,x-e_1}$$

Increments as new boundary



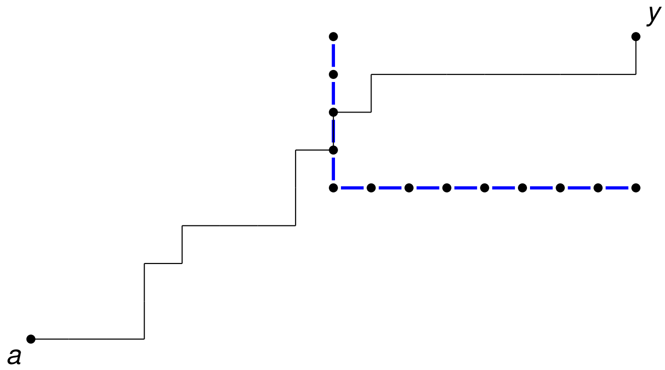
$$I_x = G_{a,x} - G_{a,x-e_1} \qquad J_x = G_{a,x} - G_{a,x-e_2}$$

Increments as new boundary



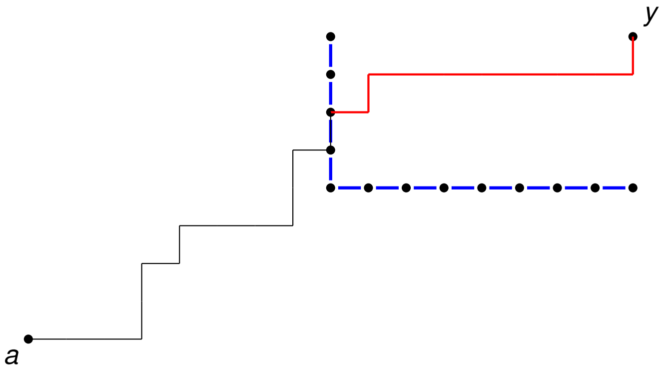
$$I_x = G_{a,x} - G_{a,x-e_1} \qquad J_x = G_{a,x} - G_{a,x-e_2}$$

Increments as new boundary



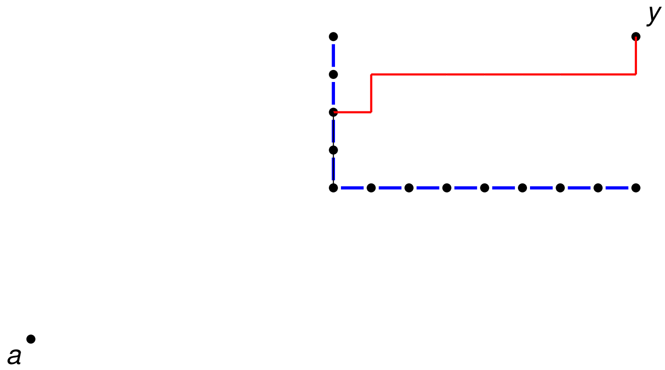
$$I_x = G_{a,x} - G_{a,x-e_1} \qquad J_x = G_{a,x} - G_{a,x-e_2}$$

Increments as new boundary



$$I_x = G_{a,x} - G_{a,x-e_1} \qquad J_x = G_{a,x} - G_{a,x-e_2}$$

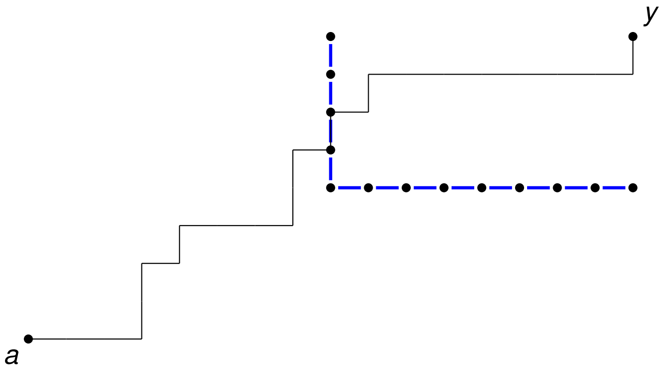
Increments as new boundary



$$I_x = G_{a,x} - G_{a,x-e_1} \qquad J_x = G_{a,x} - G_{a,x-e_2}$$

$\rightsquigarrow$ Act as boundary weights for a smaller, embedded model.

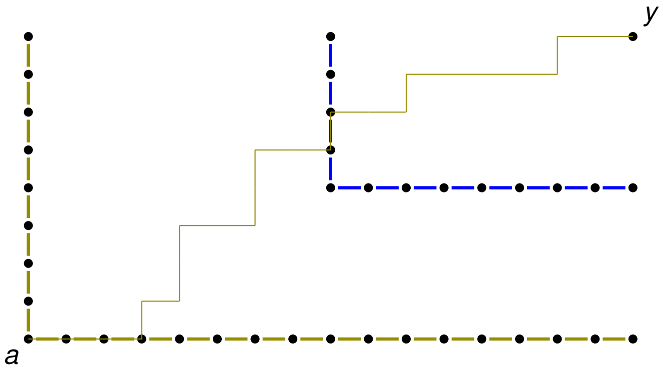
Stationary LPP



$$I_x = G_{a,x} - G_{a,x-e_1} \qquad J_x = G_{a,x} - G_{a,x-e_2}$$

Stationary LPP

Replace the boundary to | $\sim$ Exp(ϱ), — $\sim$ Exp($1-\varrho$) independent.

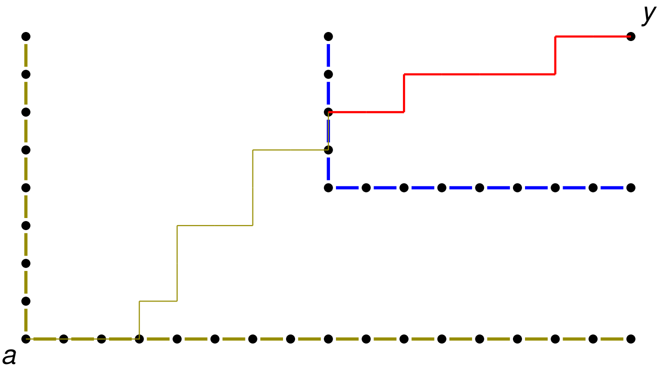


$$I_x = G_{a,x} - G_{a,x-e_1} \qquad J_x = G_{a,x} - G_{a,x-e_2}$$

Then $J_x \sim \text{Exp}(\varrho)$, $I_x \sim \text{Exp}(1-\varrho)$, independent.

Stationary LPP

Replace the boundary to | $\sim \text{Exp}(\varrho)$, — $\sim \text{Exp}(1-\varrho)$ independent.



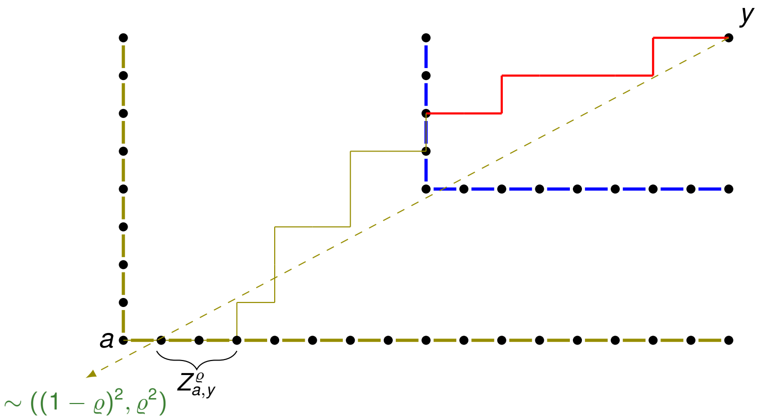
$$I_x = G_{a,x} - G_{a,x-e_1} \qquad J_x = G_{a,x} - G_{a,x-e_2}$$

Then $J_x \sim \text{Exp}(\varrho)$, $I_x \sim \text{Exp}(1-\varrho)$, independent.

The embedded model has the same structure.

Stationary LPP

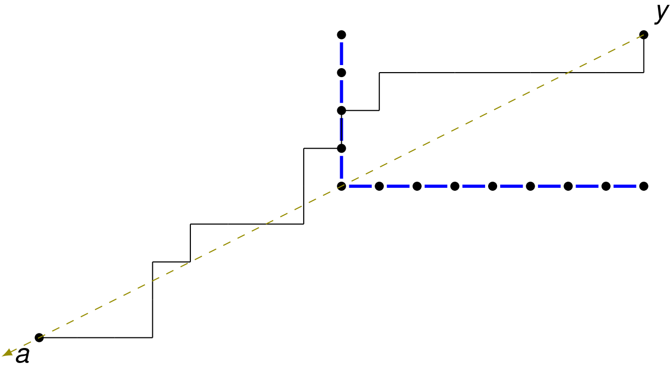
Replace the boundary to | $\sim \text{Exp}(\varrho)$, — $\sim \text{Exp}(1-\varrho)$ independent.



B., Cator, Seppäläinen '06: $\mathbb{P}\{|Z^{\varrho}_{a,y}| \geq \ell\} \leq \text{box}^2/\ell^3$, *good directional control*.

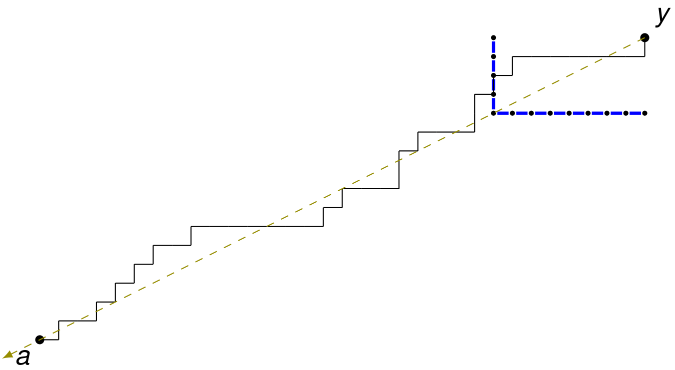
Infinite geodesics

Even without the boundary:
$J \underset{a \to -\infty}{\longrightarrow}$ i.i.d. Exp(ϱ), $I \underset{a \to -\infty}{\longrightarrow}$ i.i.d. Exp($1 - \varrho$), independent.



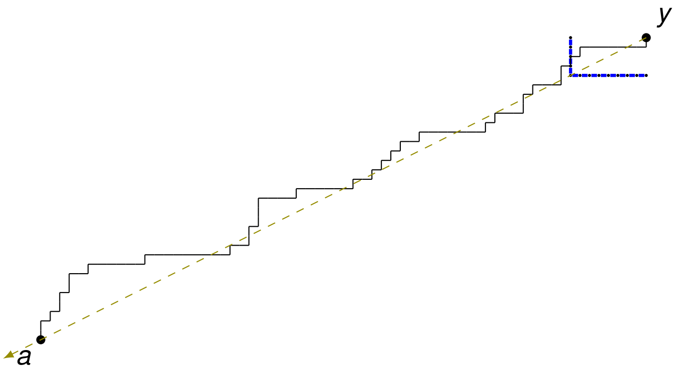
Infinite geodesics

Even without the boundary:
$J \underset{a\to-\infty}{\longrightarrow}$ i.i.d. Exp(ϱ), $I \underset{a\to-\infty}{\longrightarrow}$ i.i.d. Exp($1-\varrho$), independent.



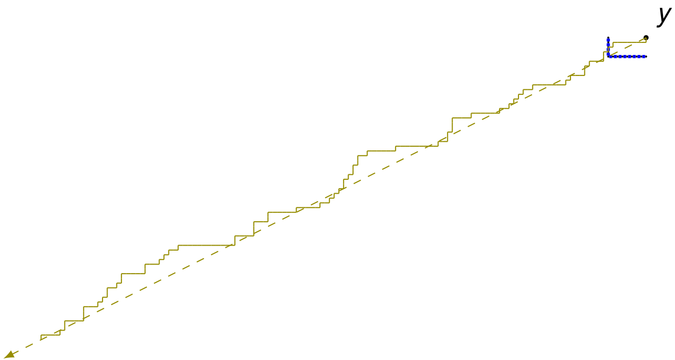
Infinite geodesics

Even without the boundary:
$J \underset{a\to-\infty}{\longrightarrow}$ i.i.d. Exp(ϱ), $I \underset{a\to-\infty}{\longrightarrow}$ i.i.d. Exp($1-\varrho$), independent.

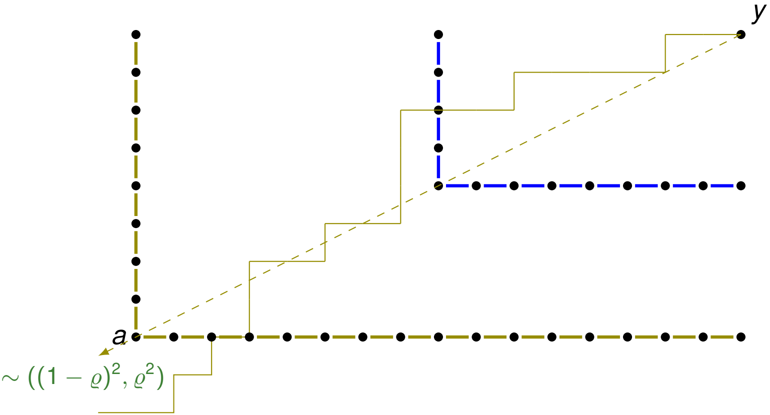


Infinite geodesics

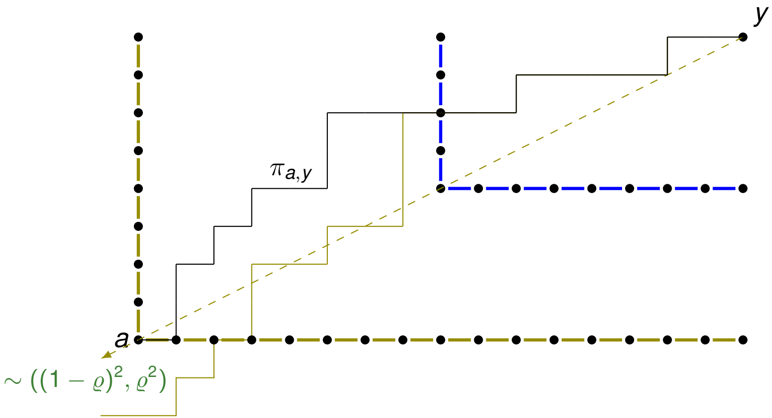
Even without the boundary:
$J \underset{a\to-\infty}{\longrightarrow}$ i.i.d. Exp(ϱ), $I \underset{a\to-\infty}{\longrightarrow}$ i.i.d. Exp($1-\varrho$), independent.



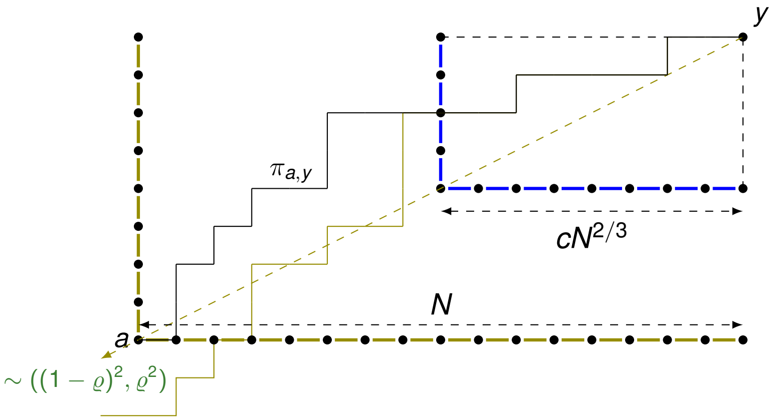
Result 1)



Result 1)

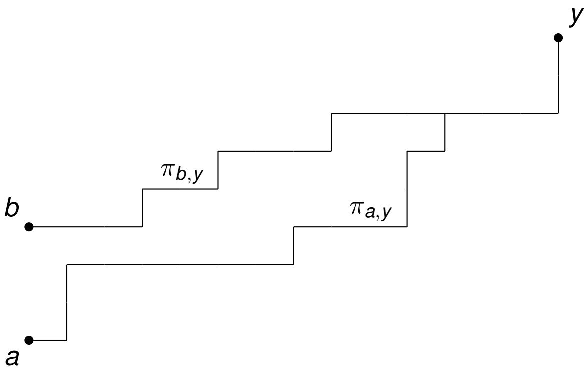


Result 1)

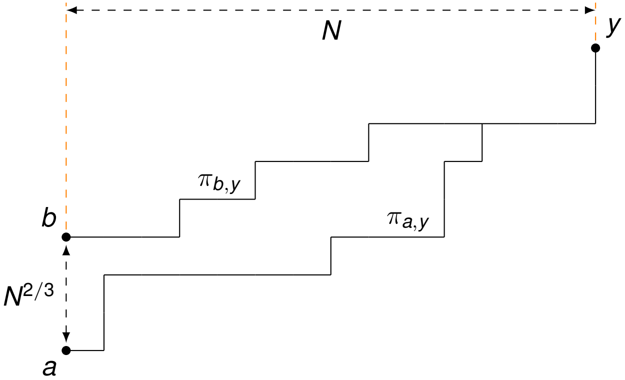


With probability at least $1 - Cc^{\frac{3}{8}}$, stationary and point-to-point paths already coalesce in the small box. (Busani, Ferrari '20)

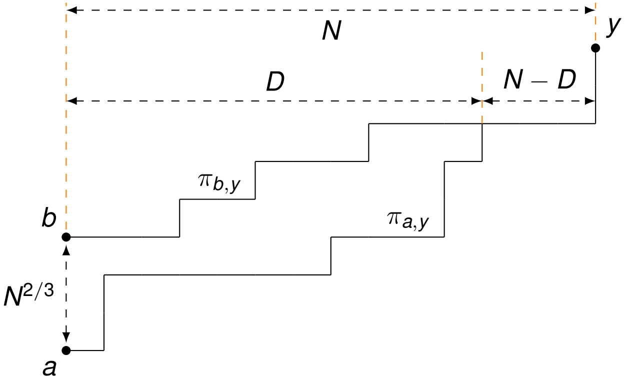
Result 2)



Result 2)



Result 2)



$$\begin{cases} \mathbf{P}\{D \le \alpha N\} \le C\alpha^2, \\ \mathbf{P}\{N - D \le \alpha N\} \le C\alpha^{\frac{2}{9}}. \end{cases}$$

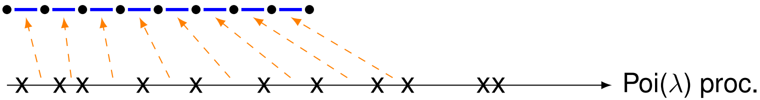
(Basu, Sarkar, Sly '19; Zhang '20)

Result 3)

The Airy_2 process minus a parabola is locally well approximated in total variation by Brownian motion.

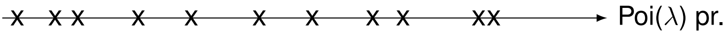
Queues

What is an i.i.d. Exp(λ) boundary?



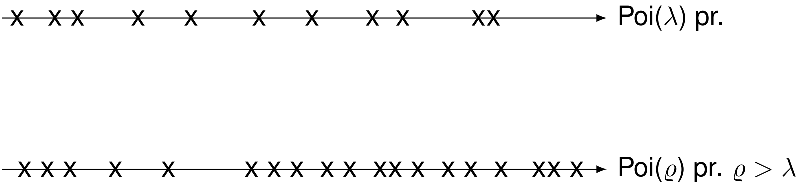
Queues

What is also an i.i.d. Exp(λ) boundary?



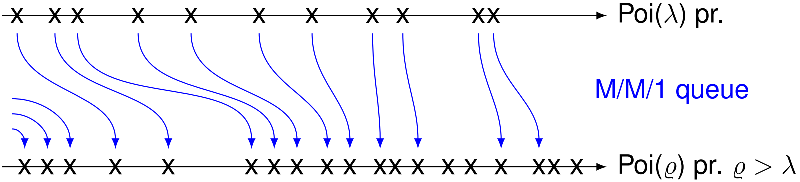
Queues

What is also an i.i.d. Exp(λ) boundary?



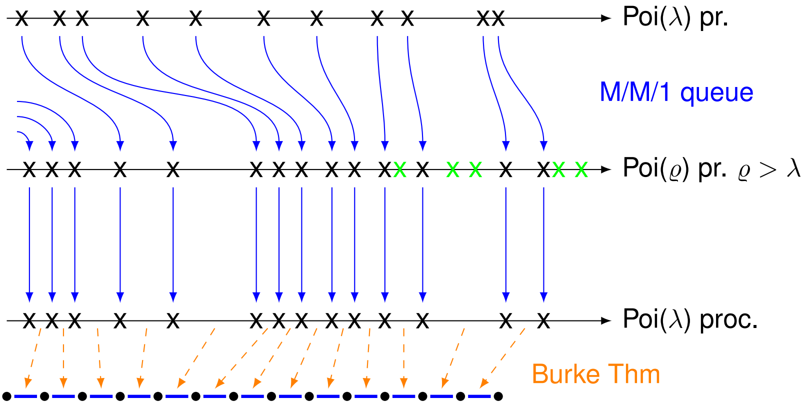
Queues

What is also an i.i.d. Exp(λ) boundary?



Queues

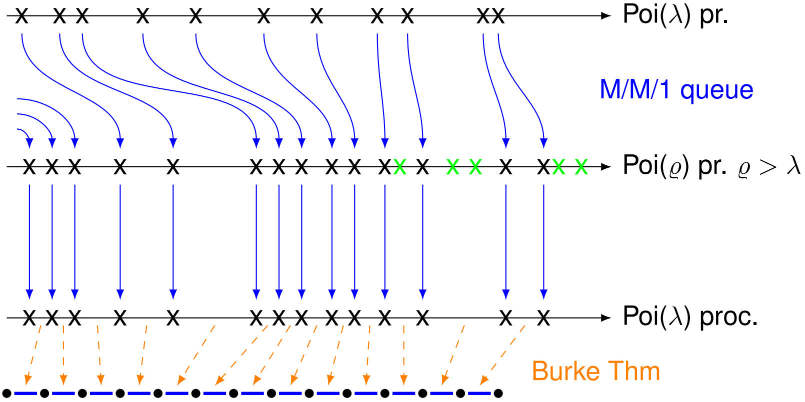
What is also an i.i.d. Exp(λ) boundary?



Queues

What is also an i.i.d. Exp(λ) boundary?

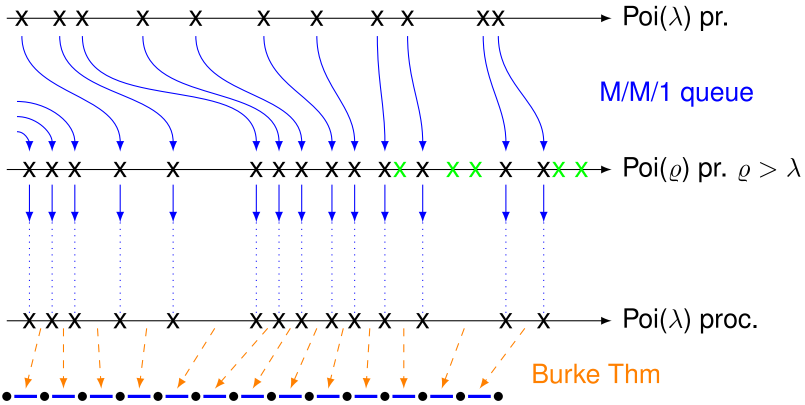
What is an i.i.d. Exp(ϱ) boundary?



Queues

What is also an i.i.d. Exp(λ) boundary?

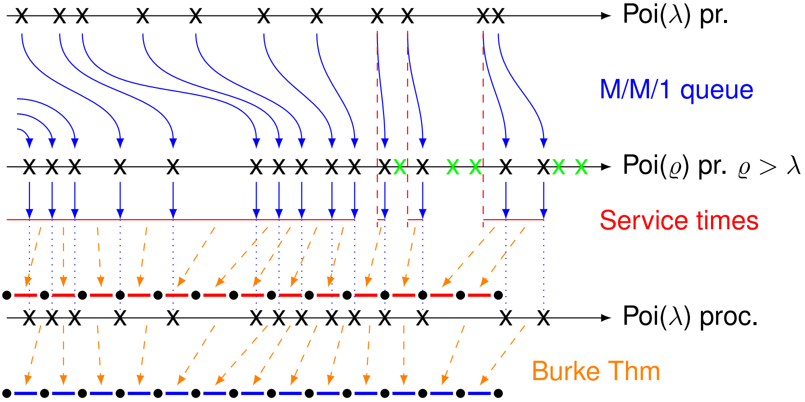
What is an i.i.d. Exp(ϱ) boundary?



Queues

What is also an i.i.d. Exp(λ) boundary?

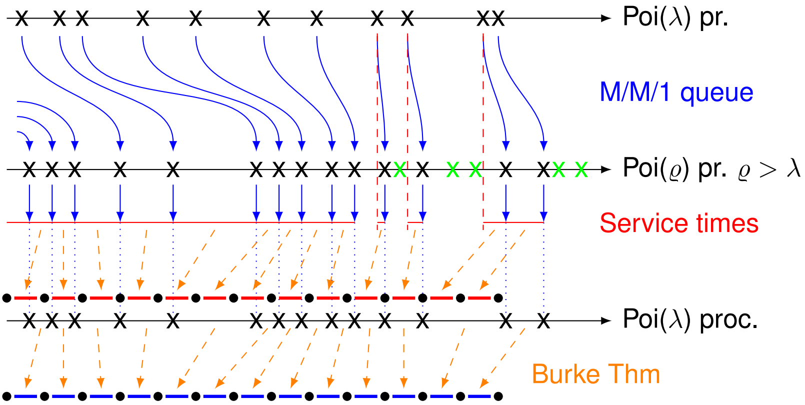
What is an i.i.d. Exp(ϱ) boundary?



Queues

What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?



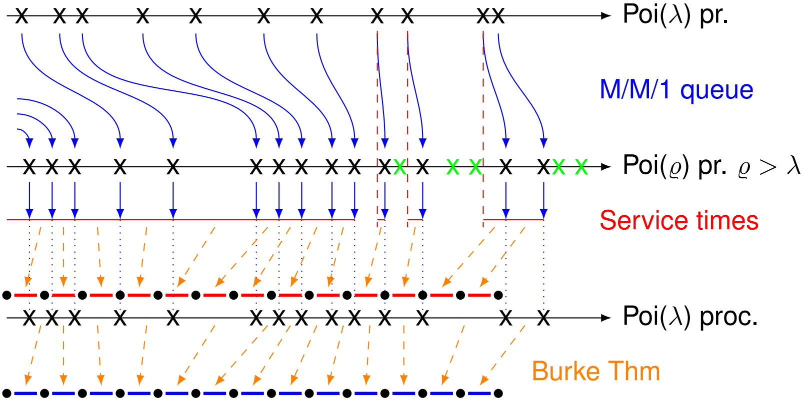
These two boundaries are **jointly** stationary;

(Ferrari, Martin '06; Fan, Seppäläinen '20)

Queues

What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?

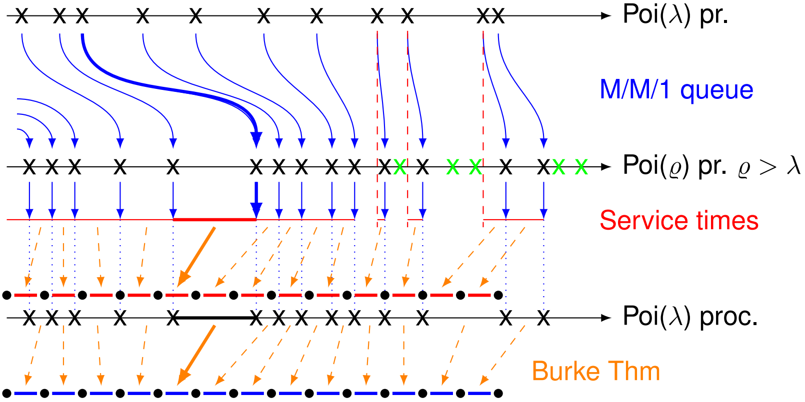


These two boundaries are **jointly** stationary; only differ when the queue empties. (Ferrari, Martin '06; Fan, Seppäläinen '20)

Queues

What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?

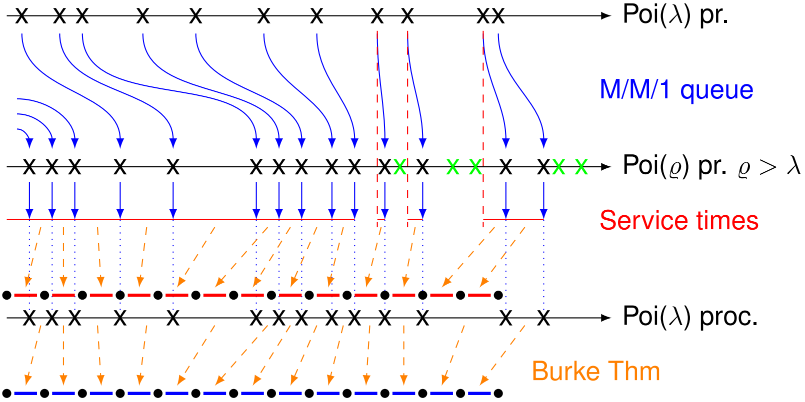


These two boundaries are **jointly** stationary; only differ when the queue empties. (Ferrari, Martin ’06; Fan, Seppäläinen ’20)

Queues

What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?

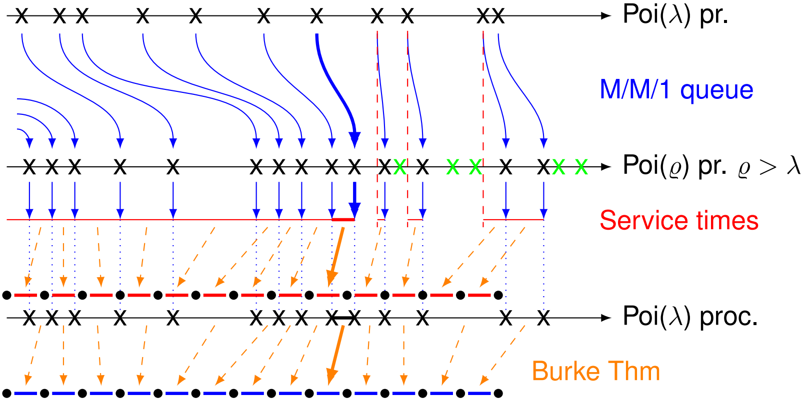


These two boundaries are **jointly** stationary; only differ when the queue empties. (Ferrari, Martin '06; Fan, Seppäläinen '20)

Queues

What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?

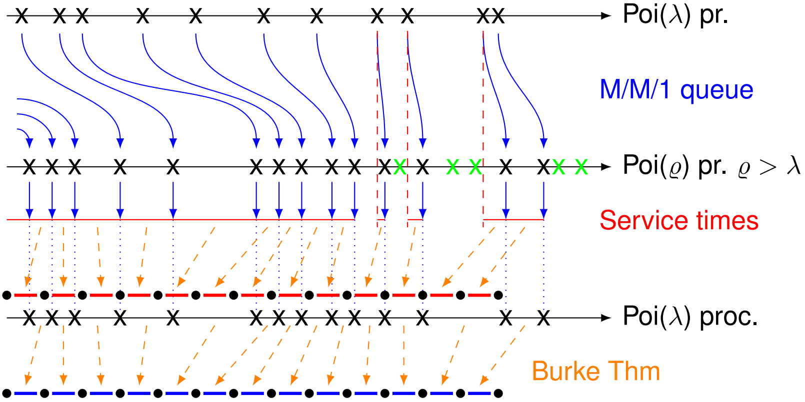


These two boundaries are **jointly** stationary; only differ when the queue empties. (Ferrari, Martin '06; Fan, Seppäläinen '20)

Queues

What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?

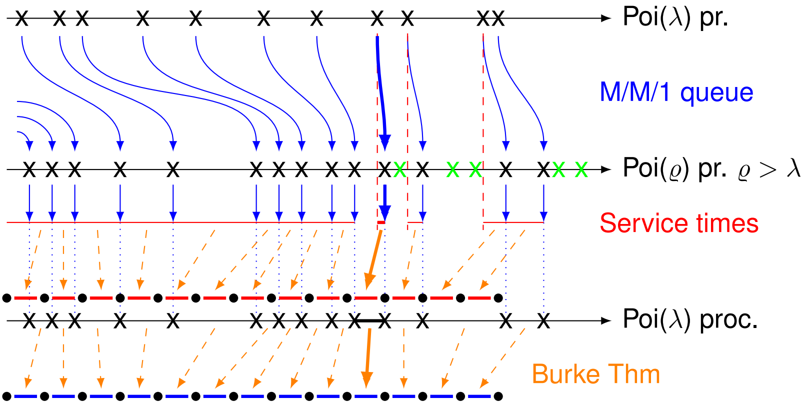


These two boundaries are **jointly** stationary; only differ when the queue empties. (Ferrari, Martin '06; Fan, Seppäläinen '20)

Queues

What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?

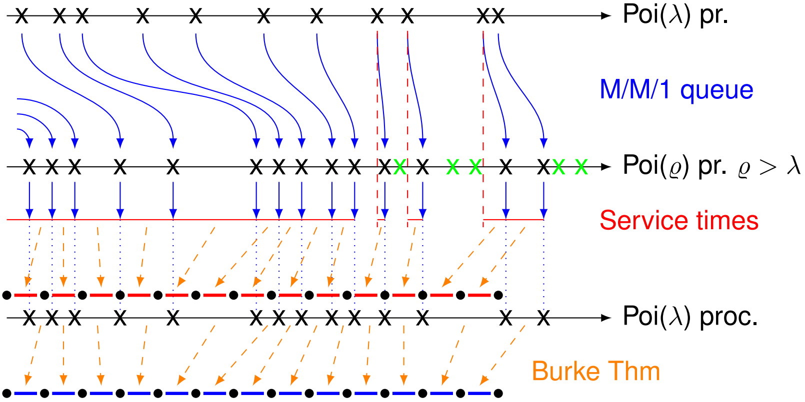


These two boundaries are **jointly** stationary; only differ when the queue empties. (Ferrari, Martin ’06; Fan, Seppäläinen ’20)

Queues

What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?

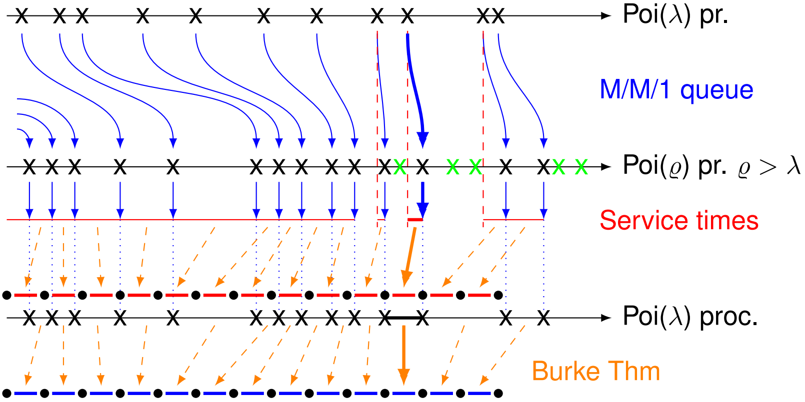


These two boundaries are **jointly** stationary; only differ when the queue empties. (Ferrari, Martin '06; Fan, Seppäläinen '20)

Queues

What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?

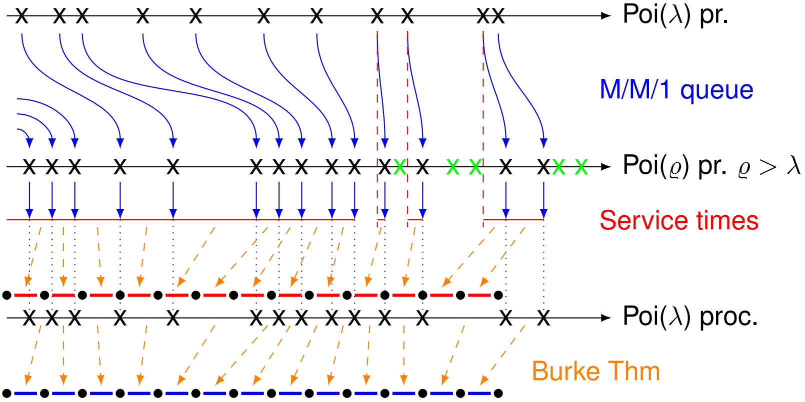


These two boundaries are **jointly** stationary; only differ when the queue empties. (Ferrari, Martin '06; Fan, Seppäläinen '20)

Queues

What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?

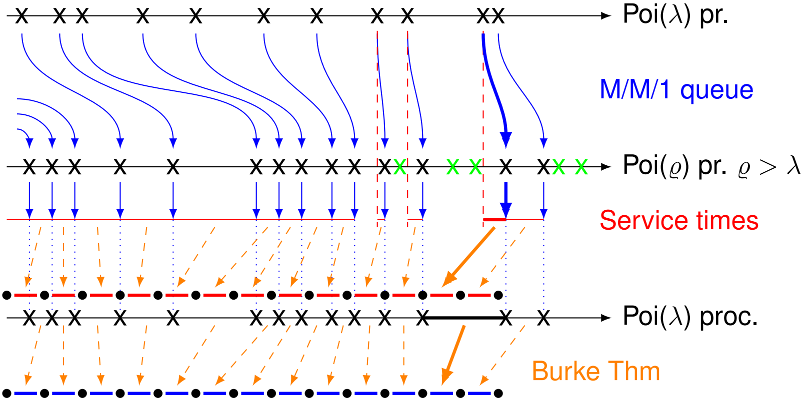


These two boundaries are **jointly** stationary; only differ when the queue empties. (Ferrari, Martin '06; Fan, Seppäläinen '20)

Queues

What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?

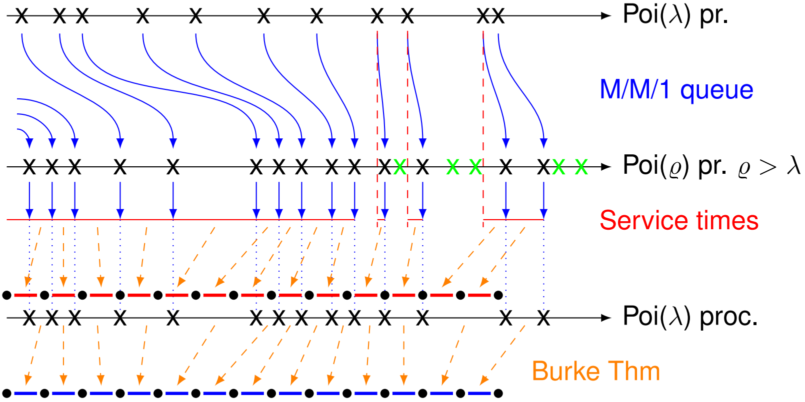


These two boundaries are **jointly** stationary; only differ when the queue empties. (Ferrari, Martin '06; Fan, Seppäläinen '20)

Queues

What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?

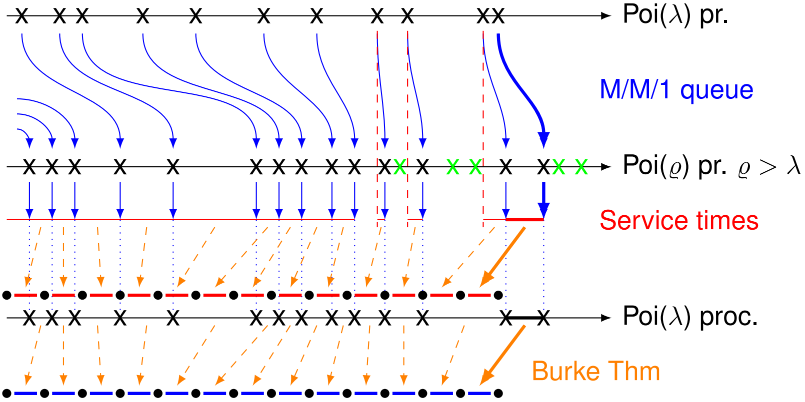


These two boundaries are **jointly** stationary; only differ when the queue empties. (Ferrari, Martin '06; Fan, Seppäläinen '20)

Queues

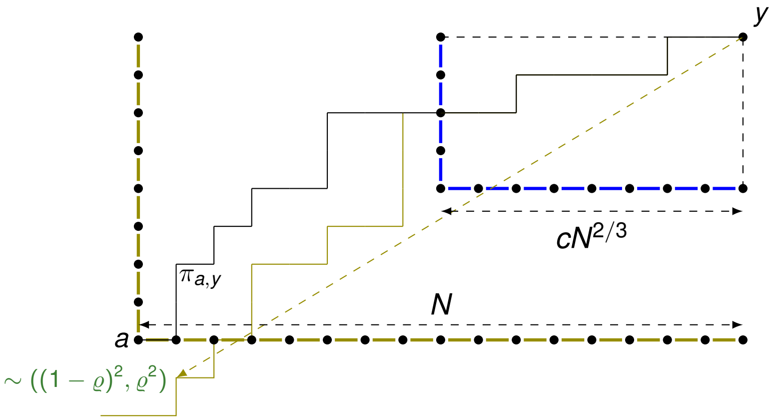
What is also an i.i.d. Exp(λ) boundary?

What is an i.i.d. Exp(ϱ) boundary?

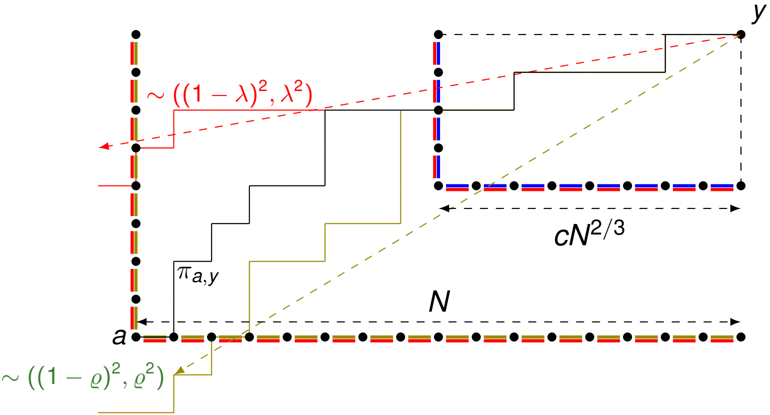


These two boundaries are **jointly** stationary; only differ when the queue empties. (Ferrari, Martin ’06; Fan, Seppäläinen ’20)

Result 1): P-2-P is like stati path

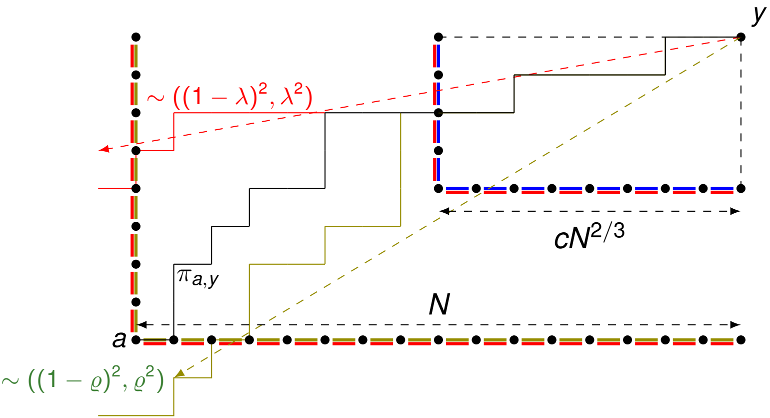


Result 1): P-2-P is like stati path



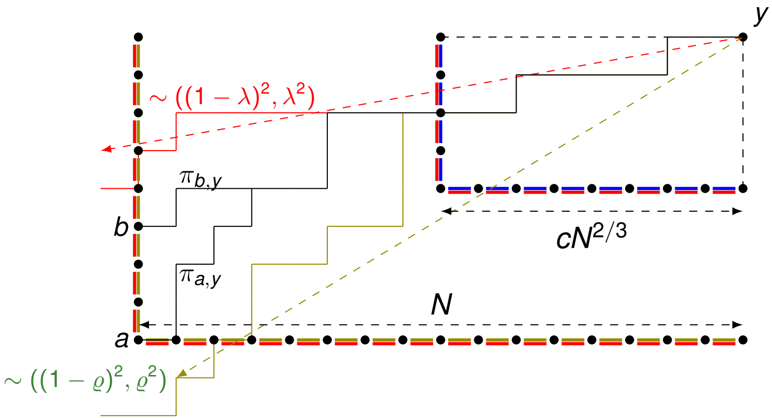
$$\lambda = \varrho - N^{-\frac{1}{3}}$$

Result 2): P-2-P paths coalesce soon



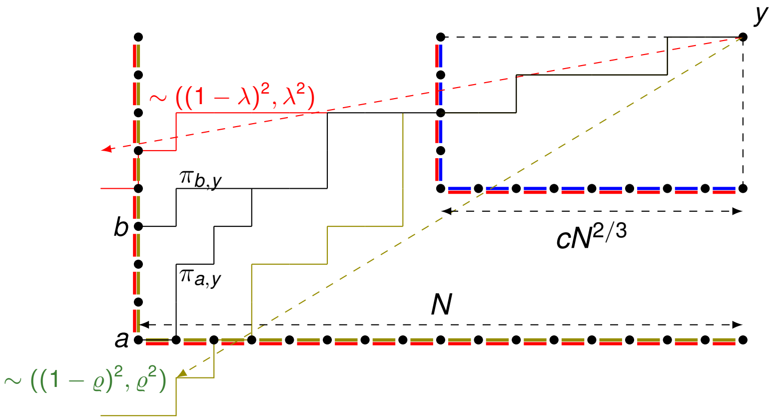
$$\lambda = \varrho - N^{-\frac{1}{3}}$$

Result 2): P-2-P paths coalesce soon



$$\lambda = \varrho - N^{-\frac{1}{3}}$$

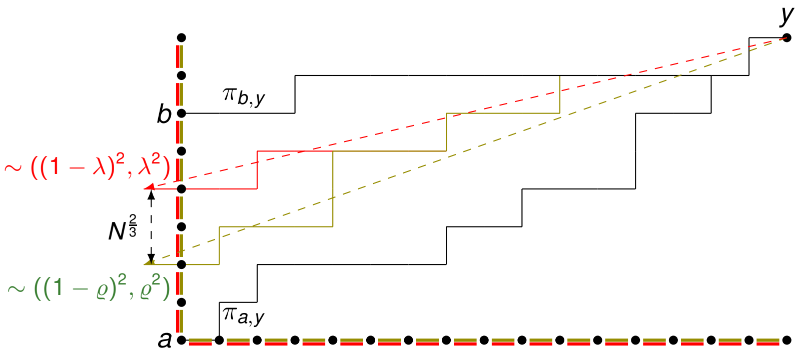
Result 2): P-2-P paths coalesce soon



$$\lambda = \varrho - N^{-\frac{1}{3}}$$

This can be boosted by pulling the small box left by αN.

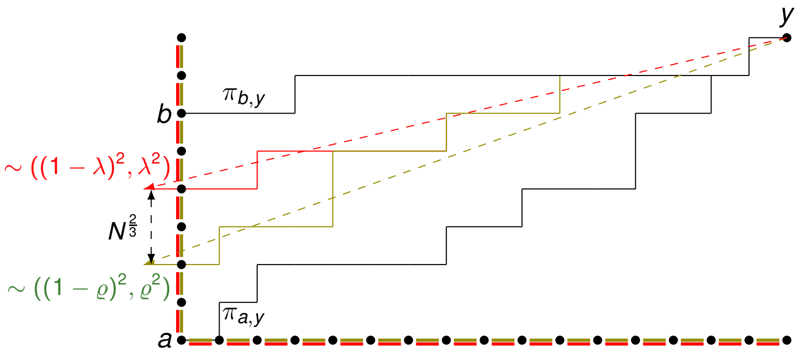
Result 2): P-2-P paths don't coalesce soon



$$\lambda = \varrho - N^{-\frac{1}{3}}$$

Coalescing too soon would mean stationary paths getting squeezed to each other too soon so they bend.

Result 2): P-2-P paths don't coalesce soon



$$\lambda = \varrho - N^{-\frac{1}{3}}$$

Coalescing too soon would mean stationary paths getting squeezed to each other too soon so they bend.

Thank you.