$t^{2/3}$-scaling of current variance in interacting particle systems

Joint with
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Márton Balázs

MTA-BME Stochastics Research Group
Budapest University of Technology and Economics

Stochastic Analysis Seminar, Oxford, June 9, 2008.

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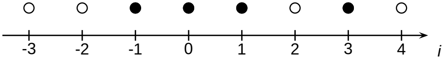

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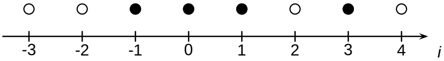

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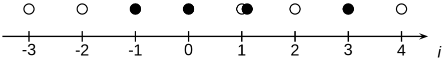

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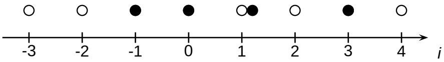

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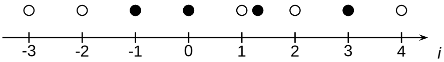

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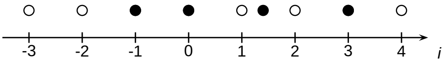

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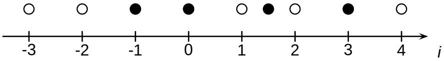

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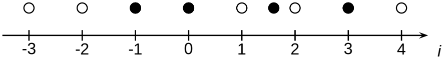

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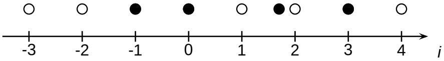

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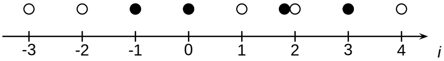

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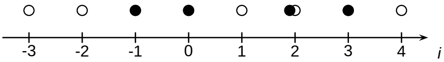

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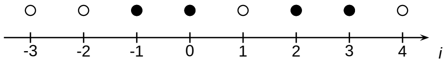

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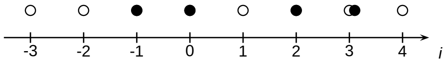

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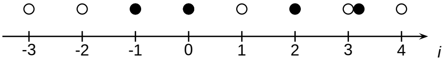

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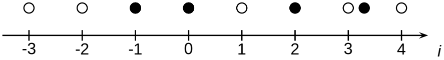

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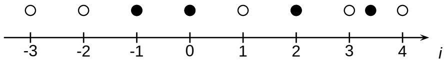

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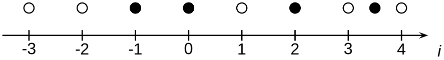

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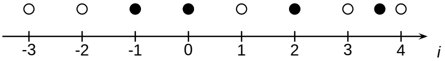

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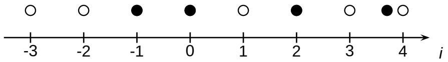

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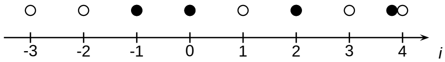

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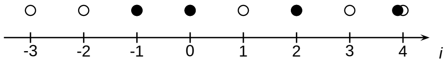

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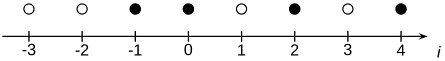

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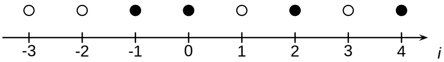

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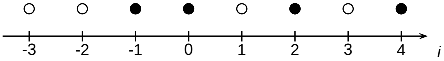

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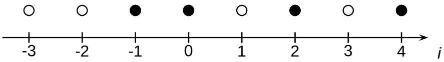

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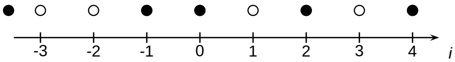

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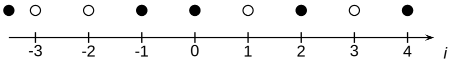

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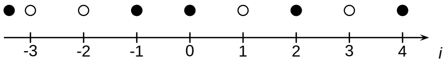

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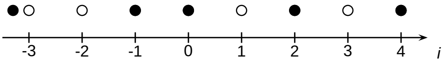

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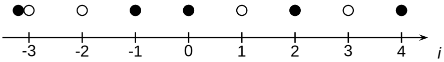

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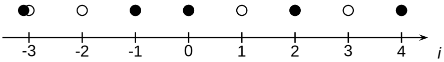

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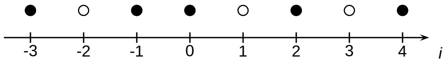

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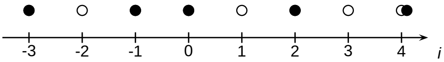

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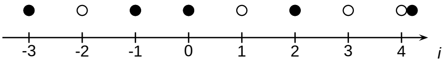

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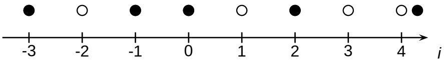

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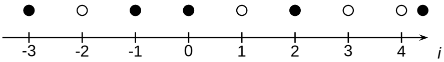

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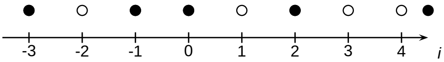

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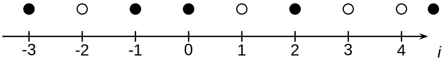

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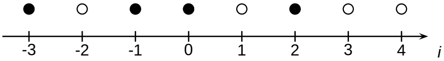

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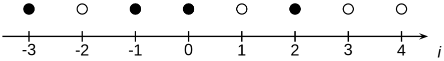

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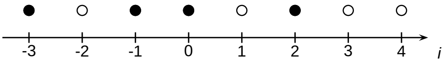

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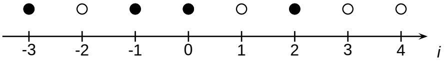

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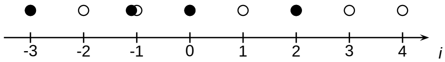

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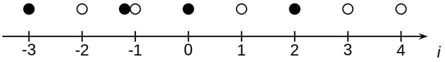

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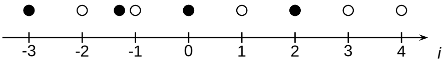

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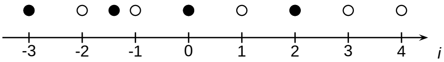

Bernoulli(ϱ) distribution; $\omega_i = 0$ or 1.

Particles try to jump

to the right with rate p,
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The jump is suppressed if the destination site is occupied by another particle.

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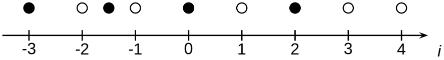

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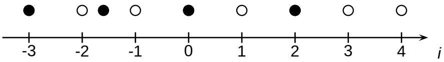

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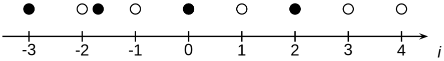

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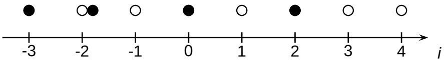

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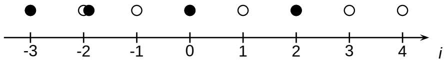

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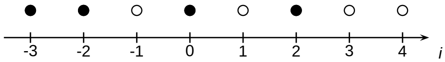

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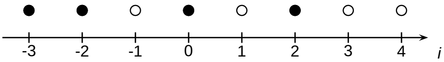

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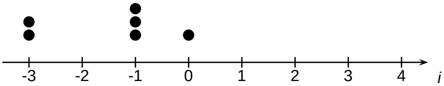
The Bernoulli(ϱ) distribution is time-stationary for any ($0 \le \varrho \le 1$). Any translation-invariant stationary distribution is a mixture of Bernoullis.

The asymmetric zero range process



Poisson-type distribution; $\omega_i \in \mathbb{Z}^+$.

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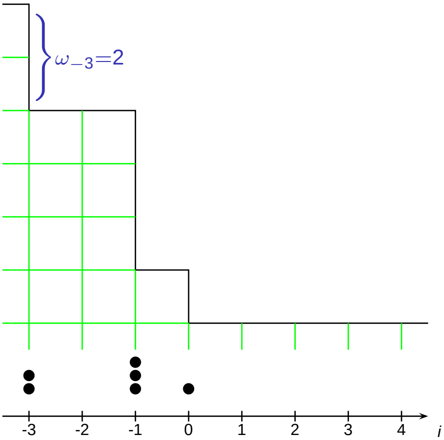

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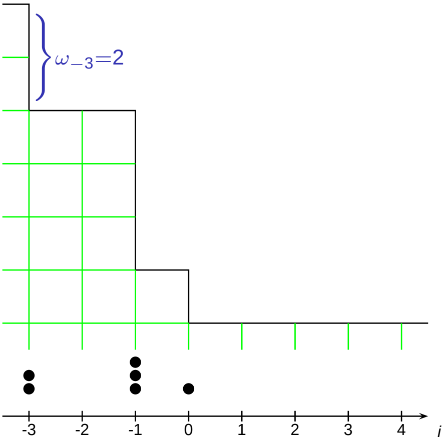

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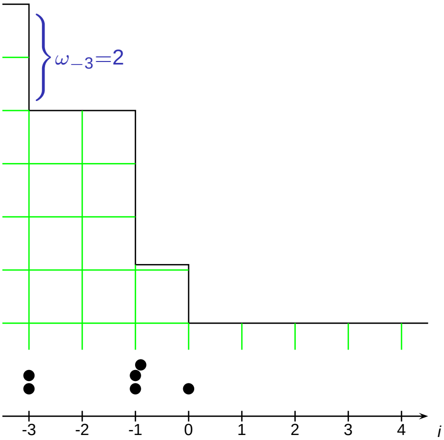

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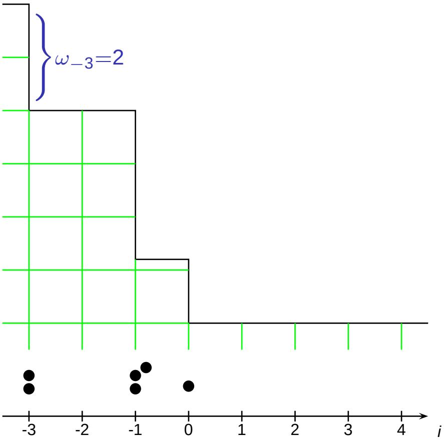

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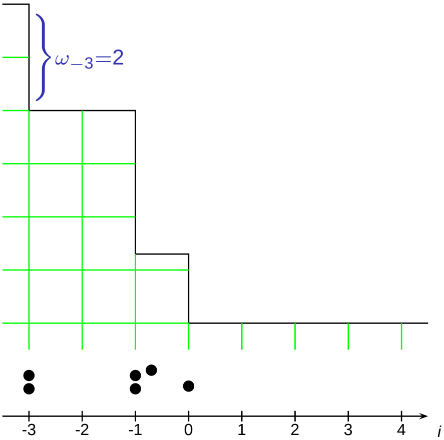

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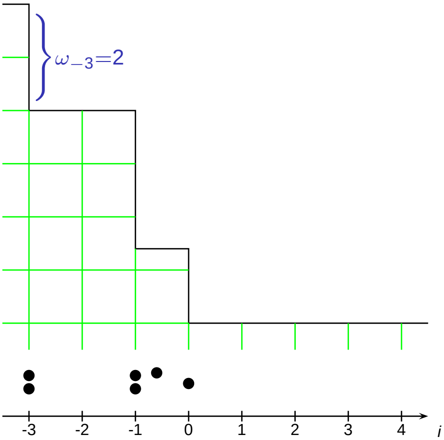

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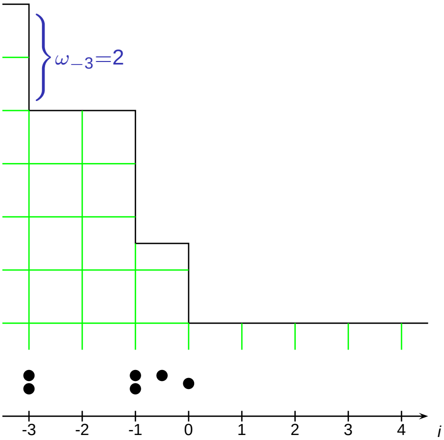

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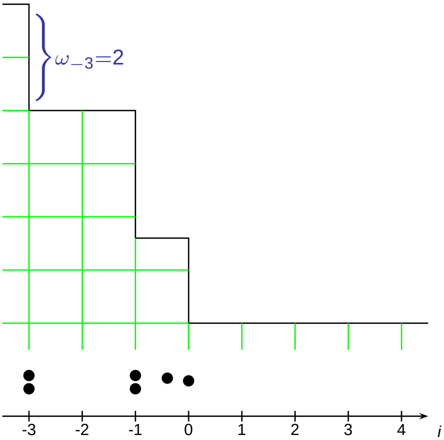

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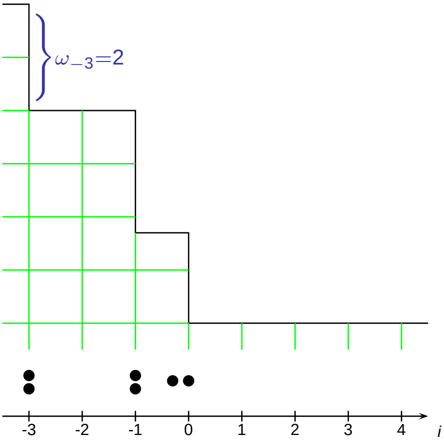

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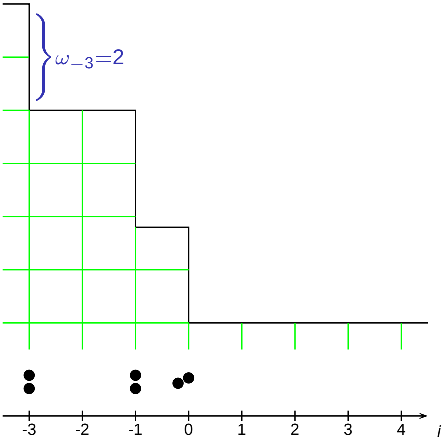

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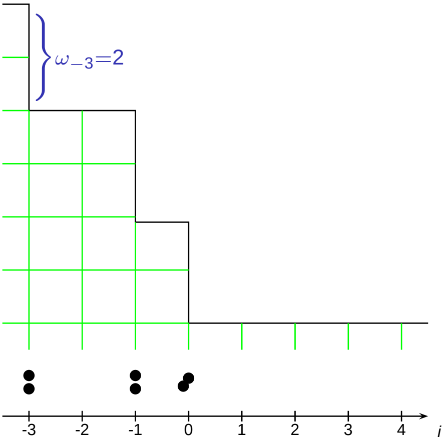

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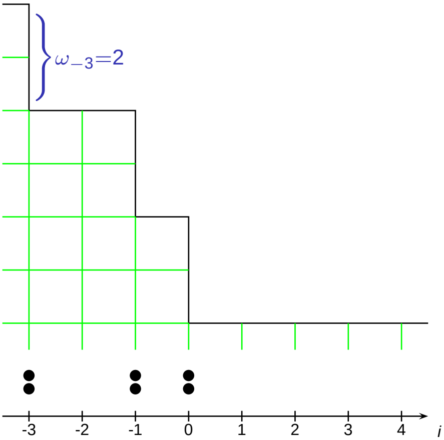

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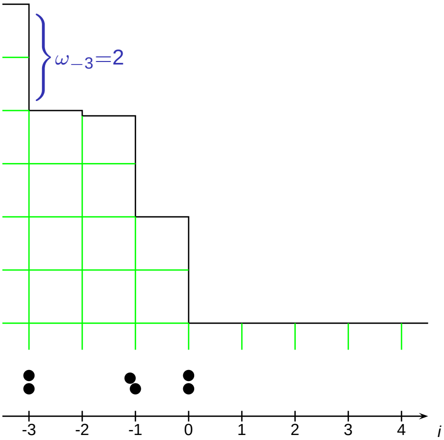

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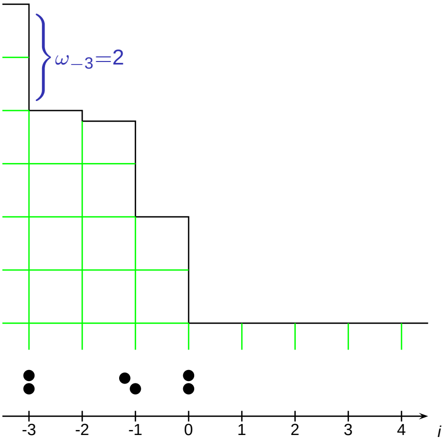

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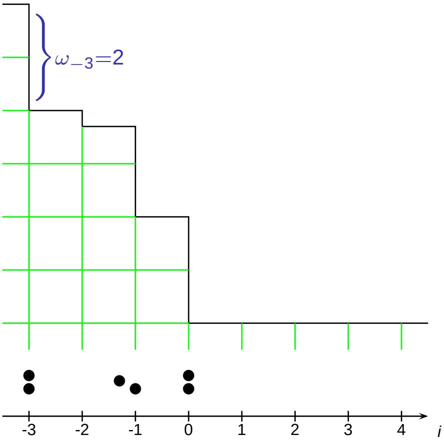

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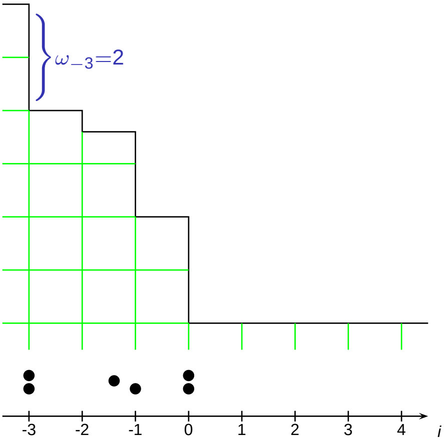

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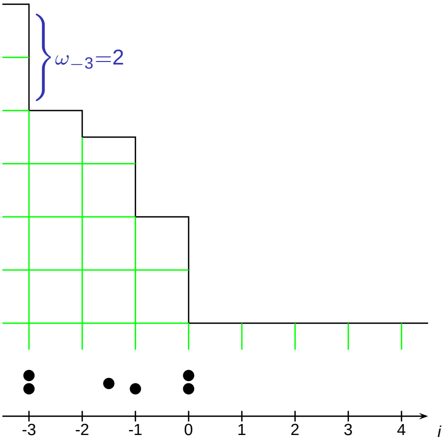

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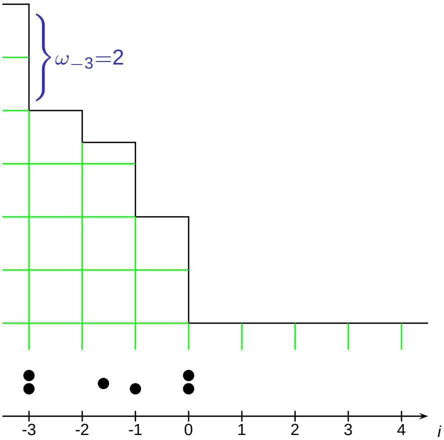

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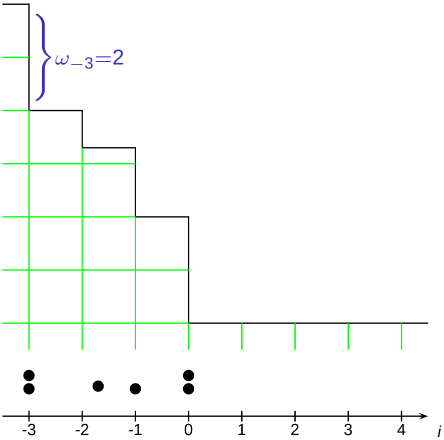

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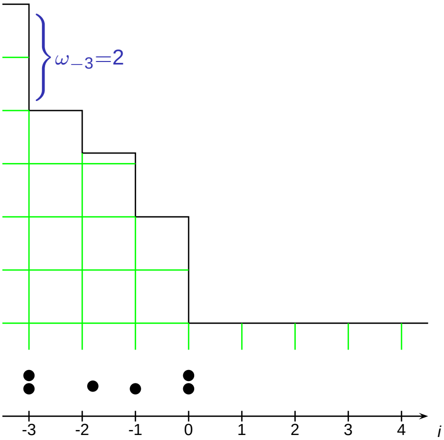

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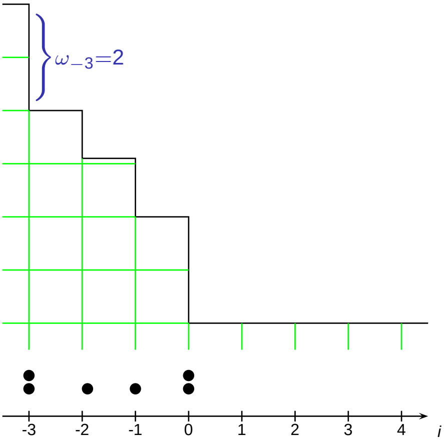

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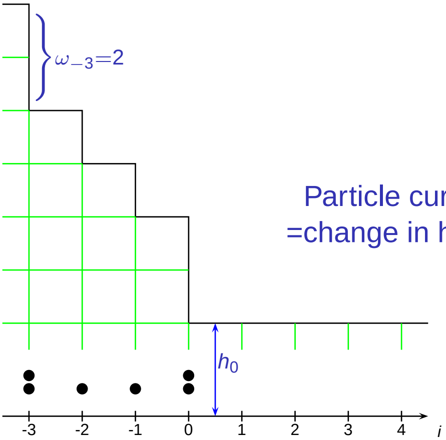

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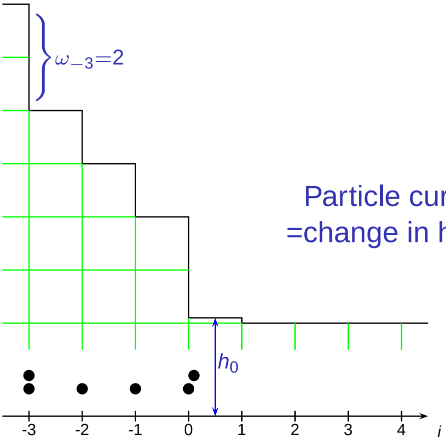

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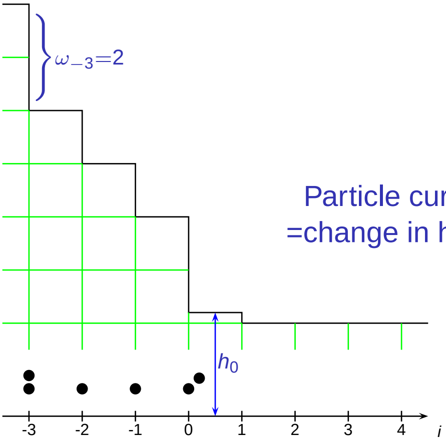

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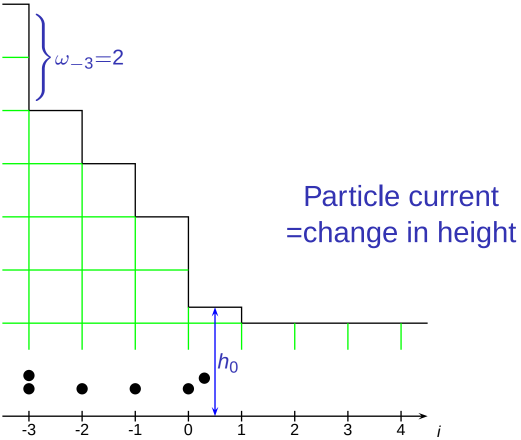

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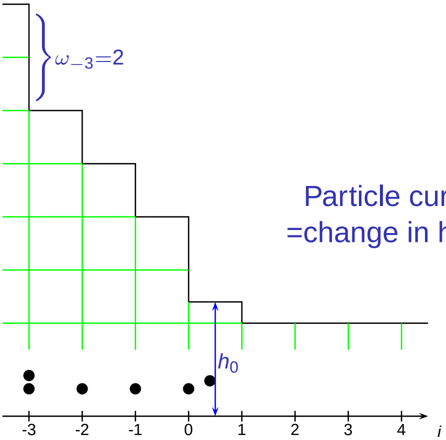

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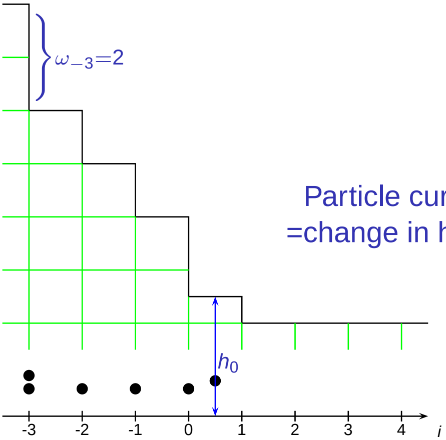

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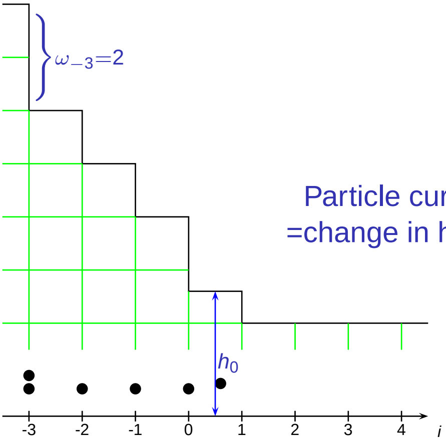

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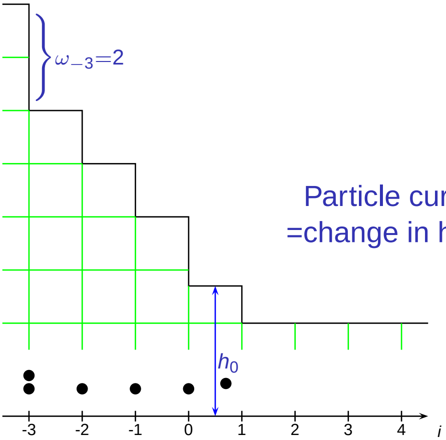

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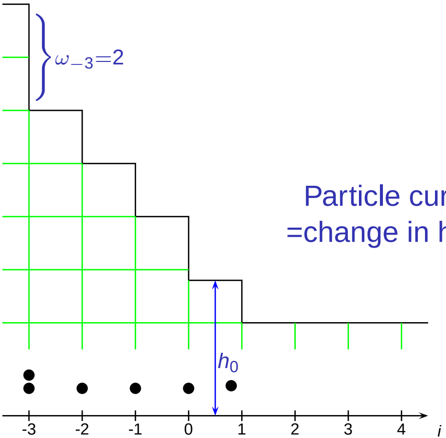

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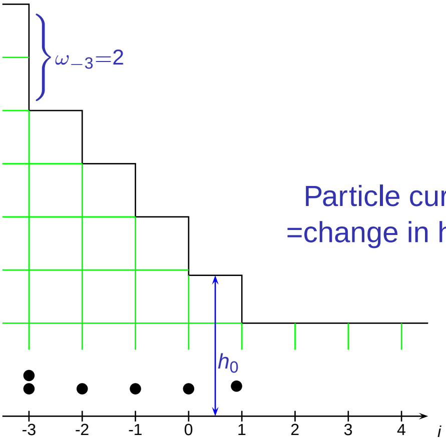

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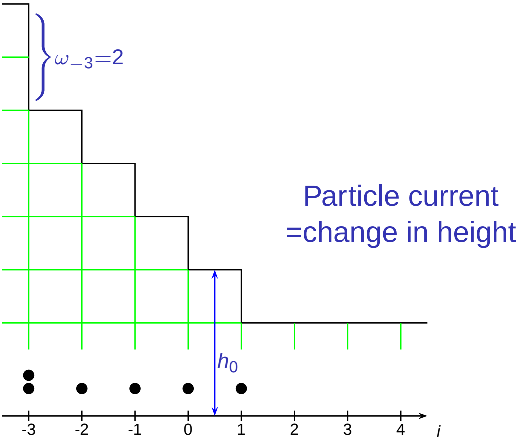

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$$\begin{pmatrix} \omega_i \\ \omega_{i+1} \end{pmatrix} \to \begin{pmatrix} \omega_i - 1 \\ \omega_{i+1} + 1 \end{pmatrix} \qquad \text{with rate } p(\omega_i, \omega_{i+1}),$$

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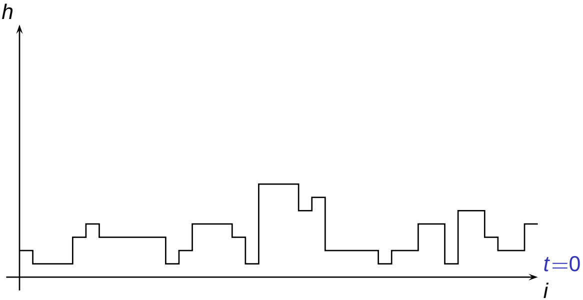
- p and q are such that they keep the state space (ASEP, ZRP),
- p is non-decreasing in the first, non-increasing in the second variable, and q vice-versa (attractivity),
- they satisfy some algebraic conditions to get a product stationary distribution for the process,
- they satisfy some regularity conditions to make sure the dynamics exists.

Integrated particle current



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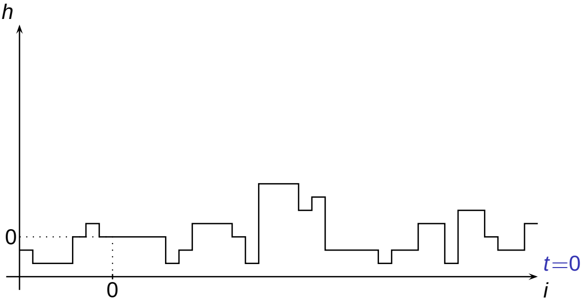

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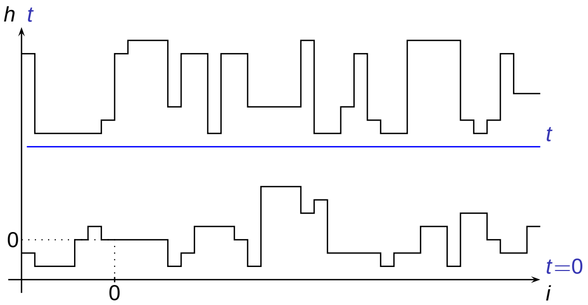

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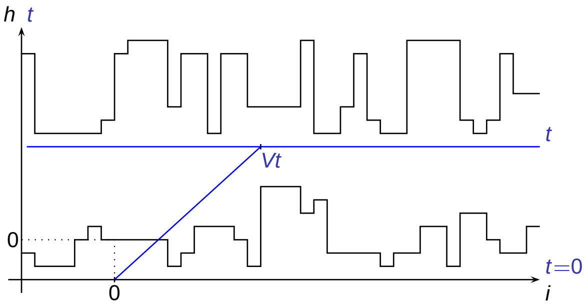

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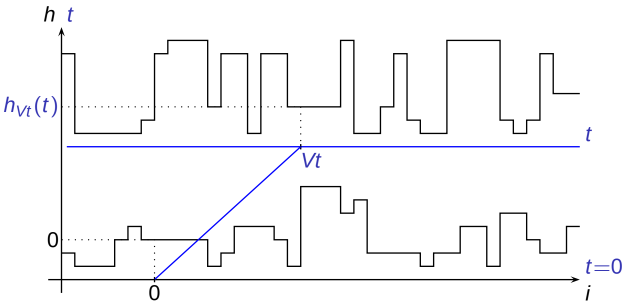

$h_{Vt}(t) =$ height as seen by a moving observer of velocity V.

$=$ net number of particles passing the window $s \mapsto Vs$.

(Remember: particle current=change in height.)

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- The *characteristics* is a path $X(T)$ where $\varrho(T, X(T))$ is constant.

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This would be the speed of a shock of densities ϱ and λ.

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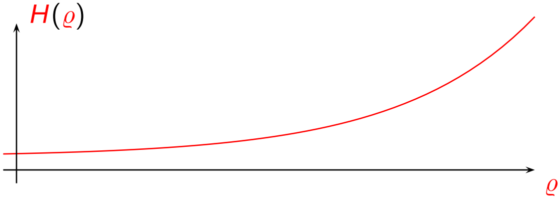

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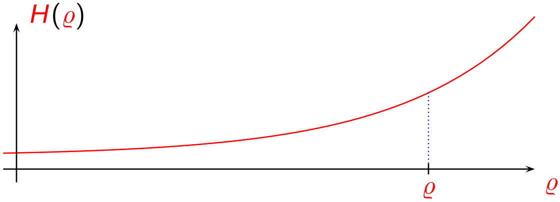

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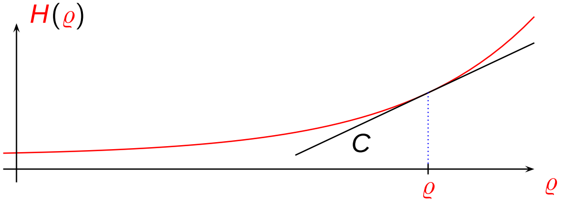

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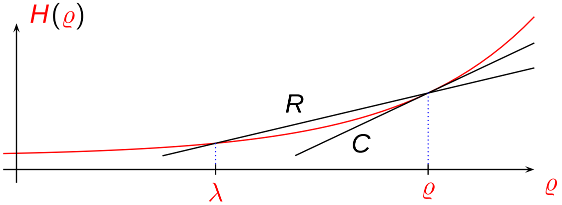

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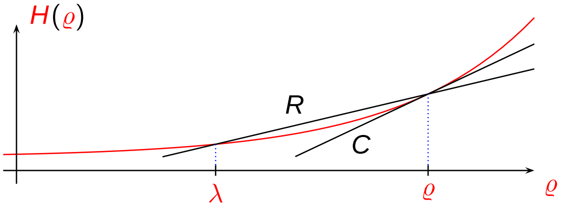

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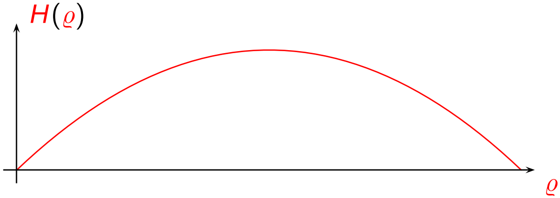

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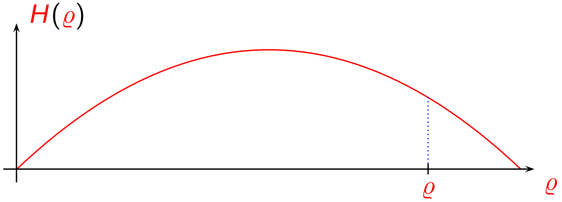

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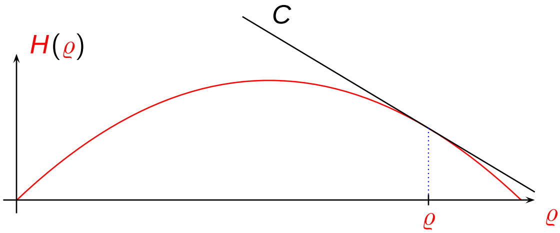

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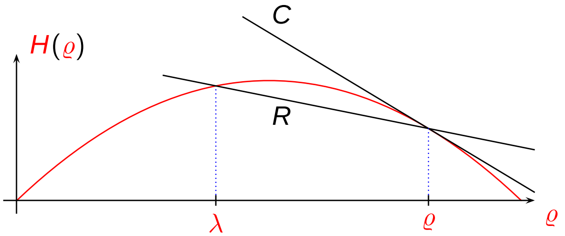

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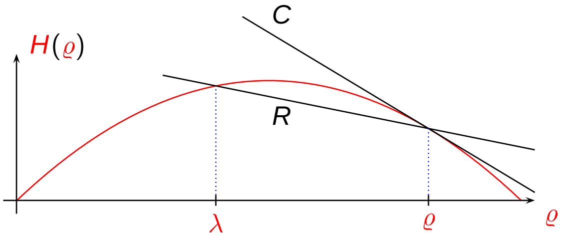

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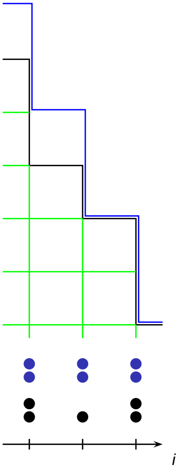
Tool: the second class particle

States ω and ω only differ at one site.



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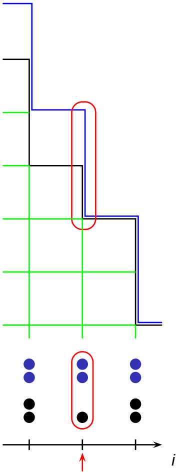

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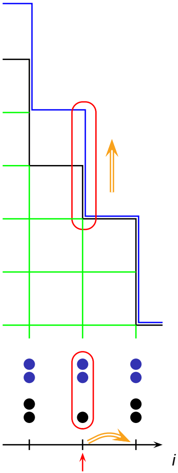

Growth on the right:

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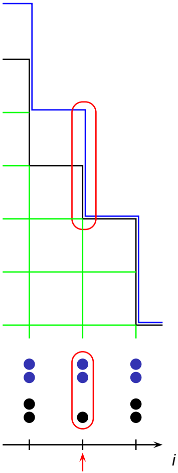

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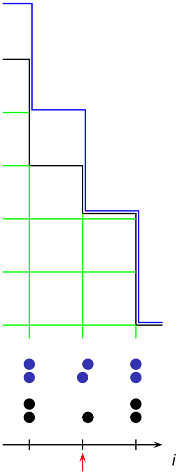

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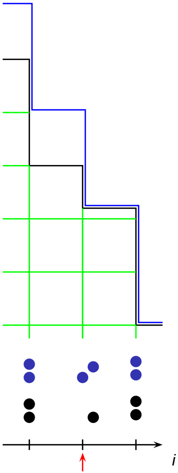

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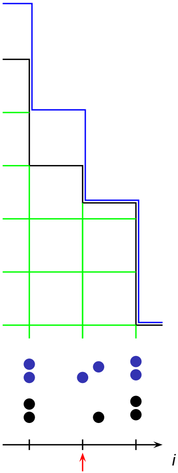

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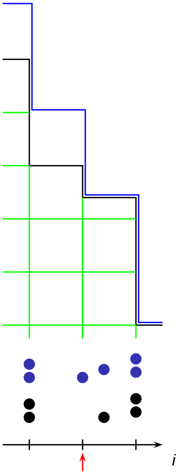

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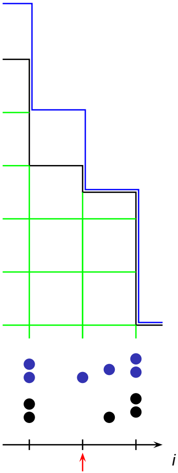

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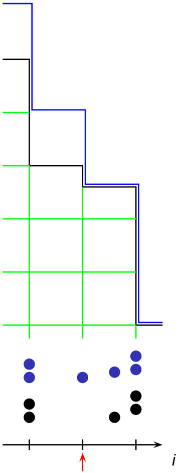

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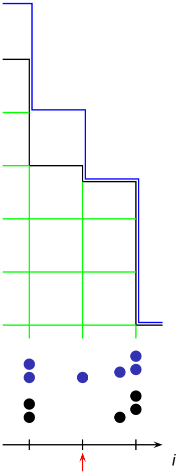

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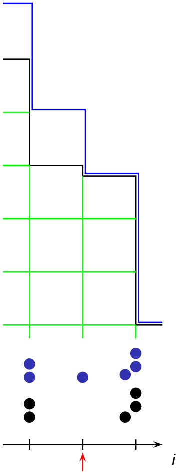

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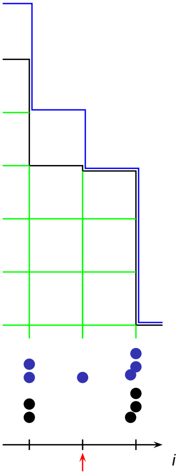

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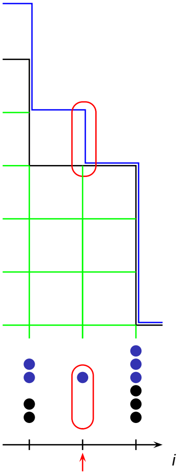

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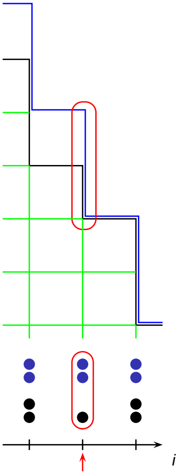

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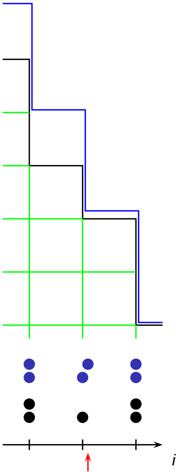

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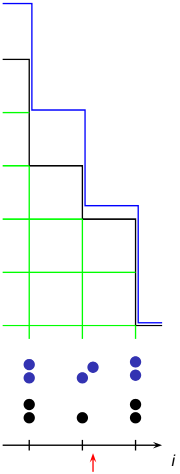

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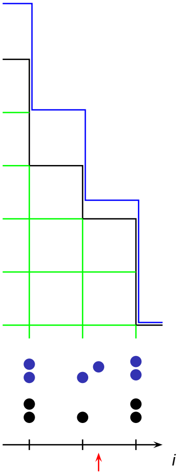

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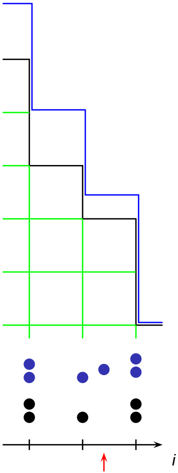

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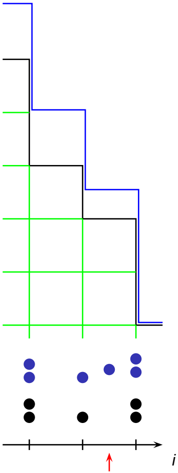

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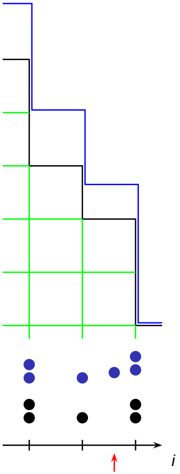

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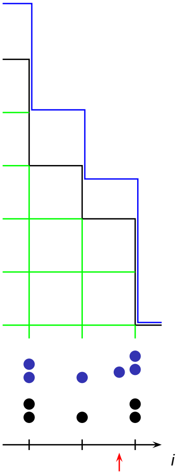

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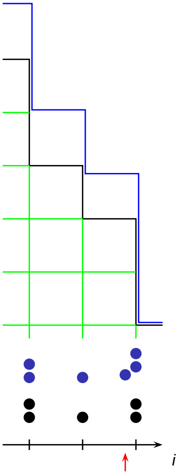

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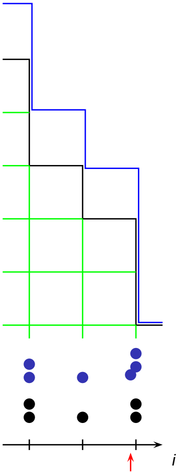

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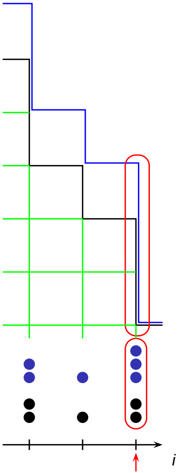

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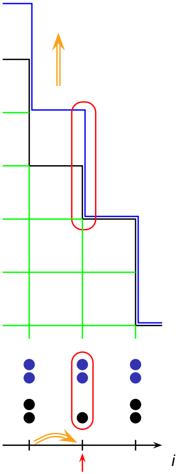

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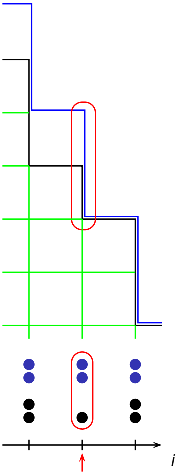

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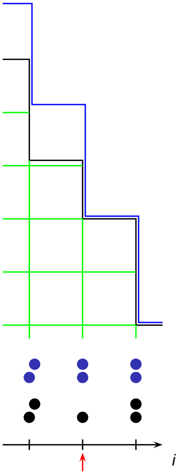

Tool: the second class particle

States ω and ω only differ at one site.

Growth on the left:
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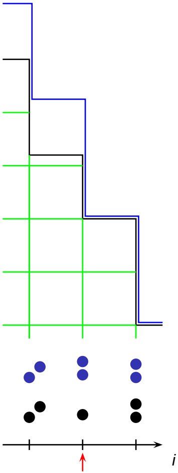

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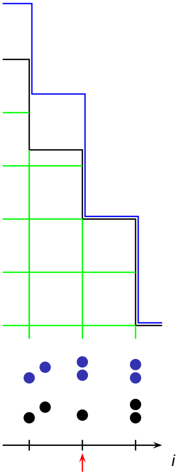

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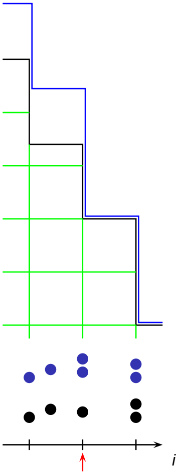

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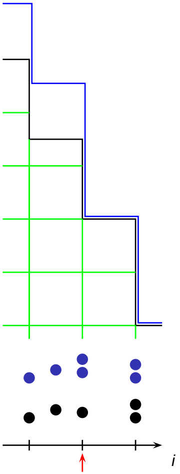

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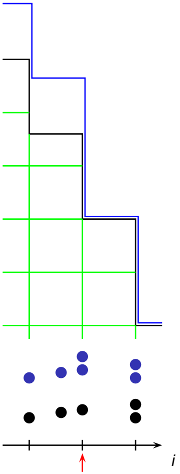

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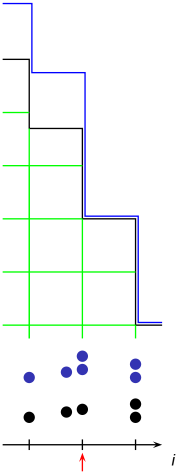

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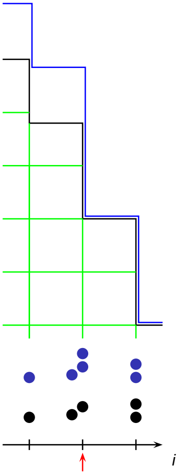

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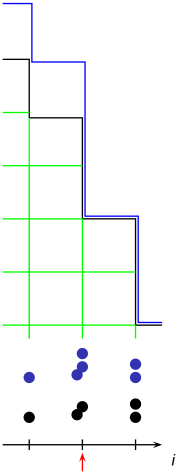

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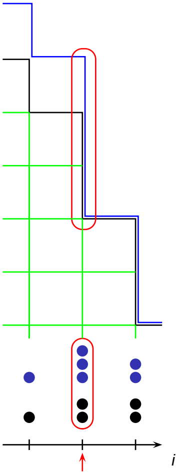

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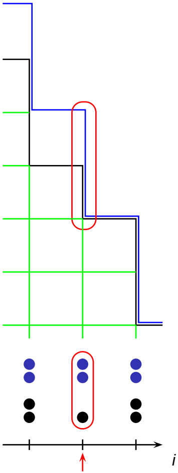

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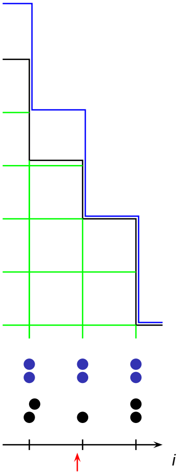

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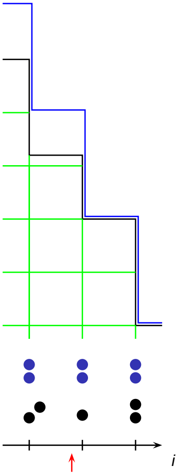

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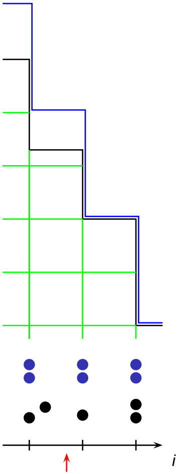

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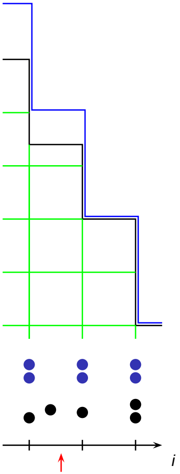

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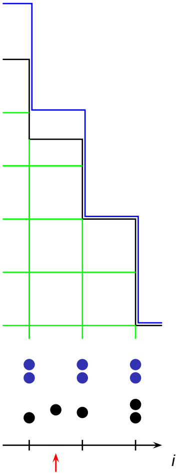

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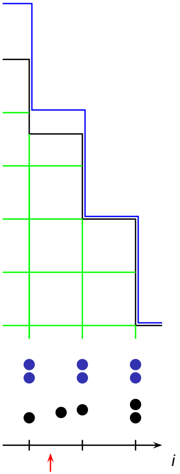

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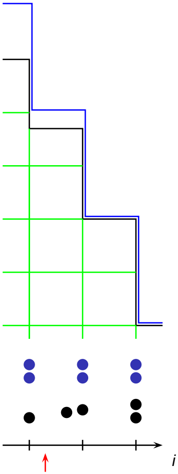

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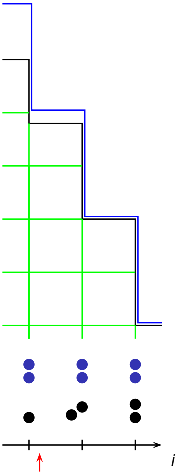

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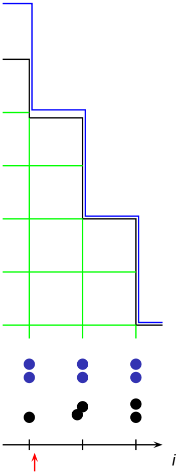

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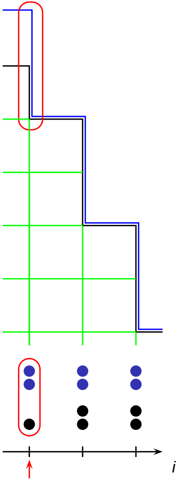

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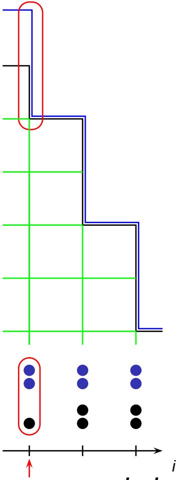

A single discrepancy↑, the *second class particle*, is conserved.
Its position at time t is $Q(t)$.

Tool: the second class particle

Theorem (B. - Seppäläinen; also ideas from B. Tóth, H. Spohn and M. Prähofer)

Started from (almost) equilibrium,

$$\mathbf{E}(Q(t)) = C \cdot t$$

in the whole family of processes.

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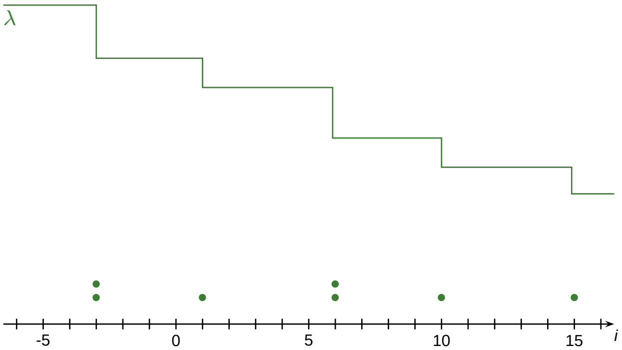
C is the characteristic speed.

The second class particle follows the characteristics, people have known this for a long time.

Many second class particles



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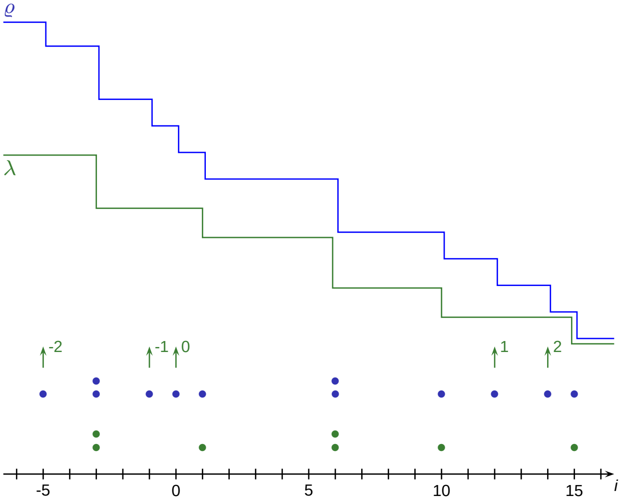

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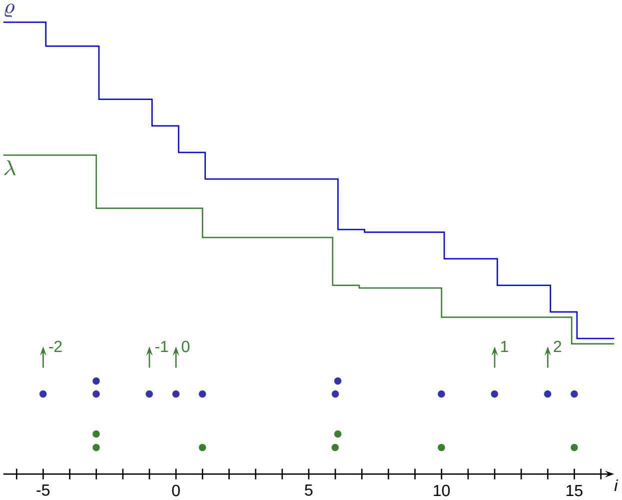

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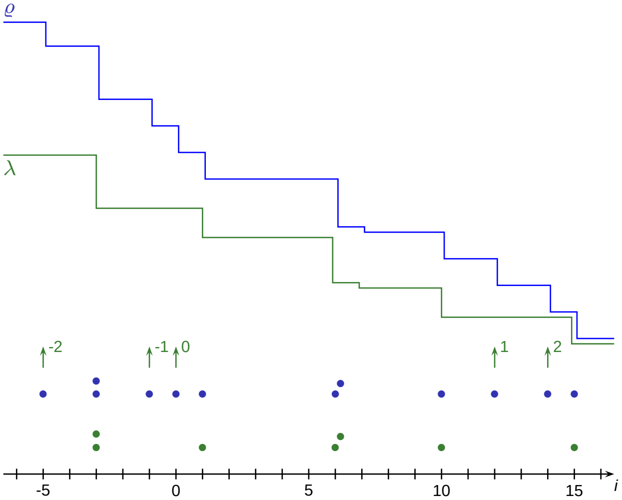

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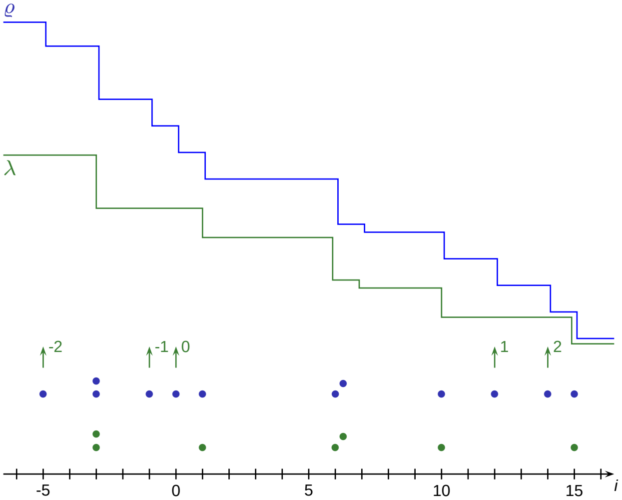

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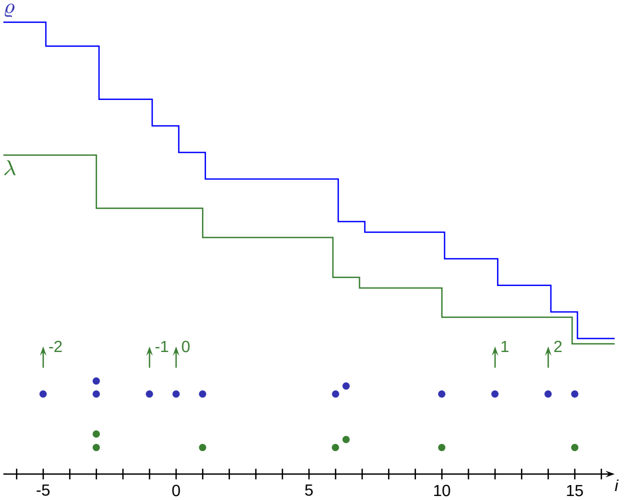

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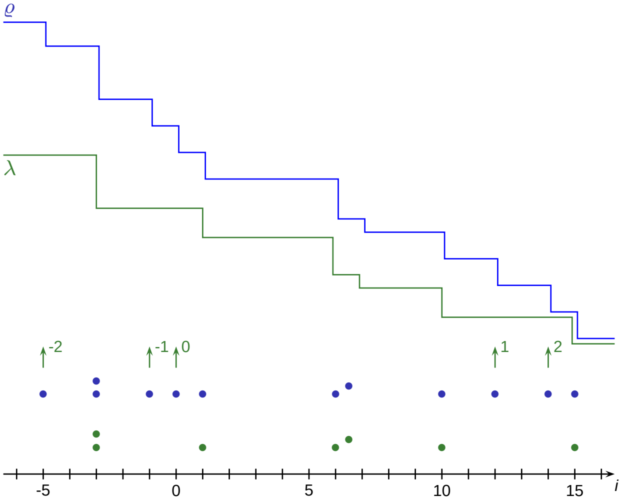

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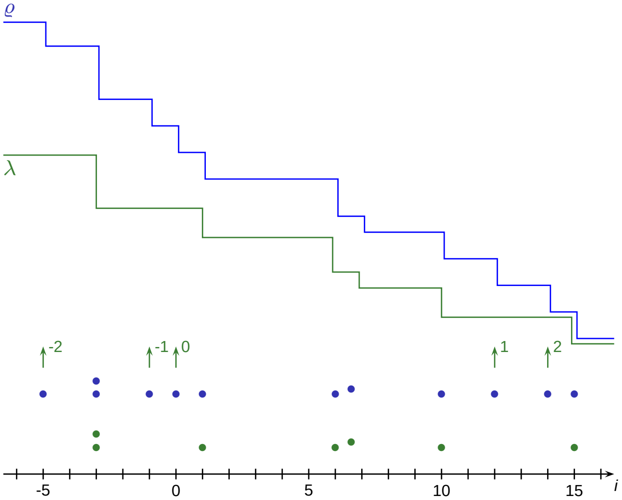

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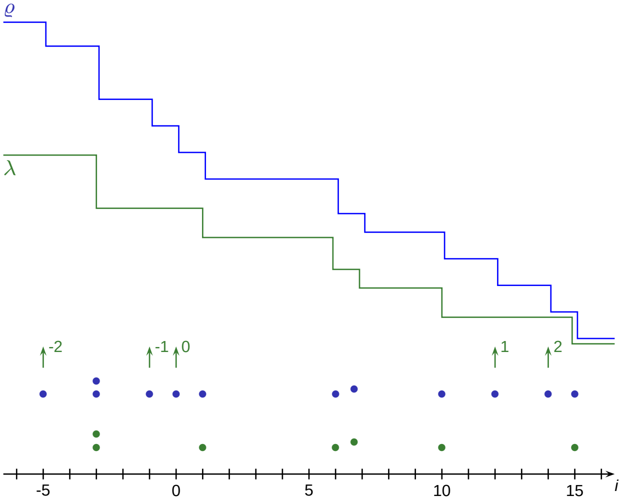

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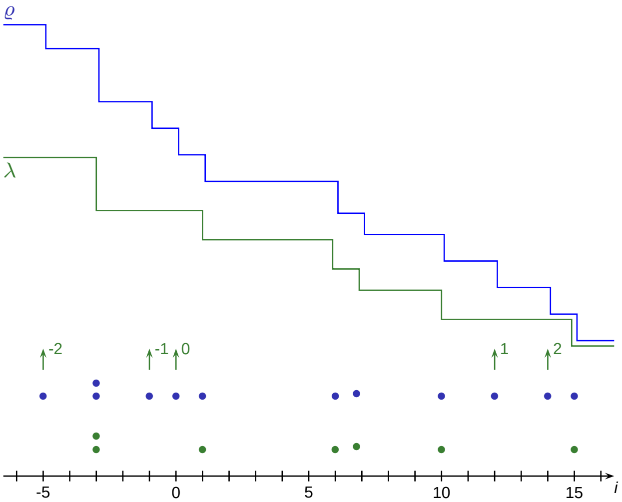

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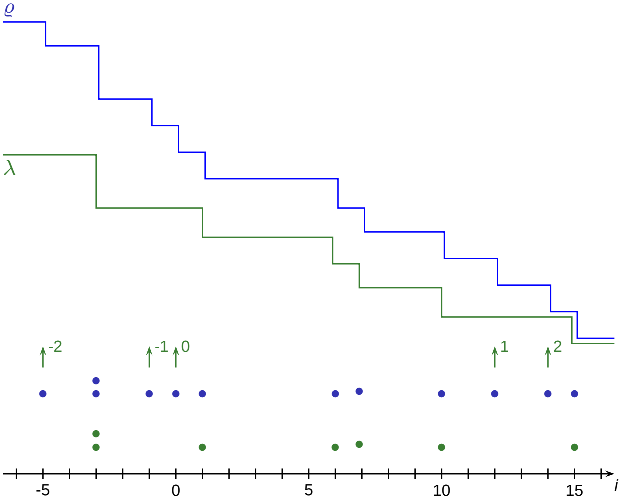

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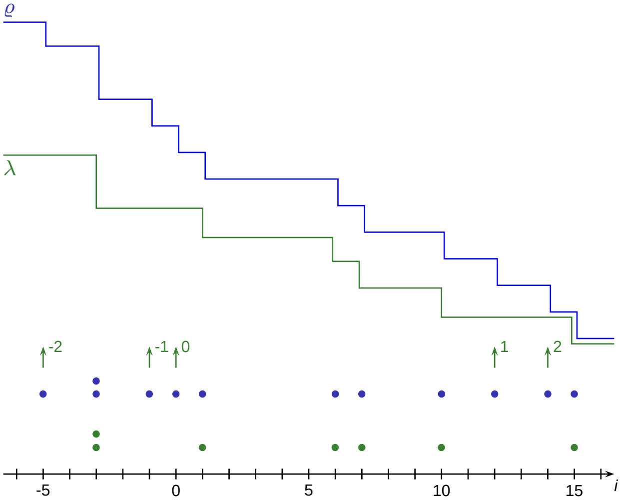

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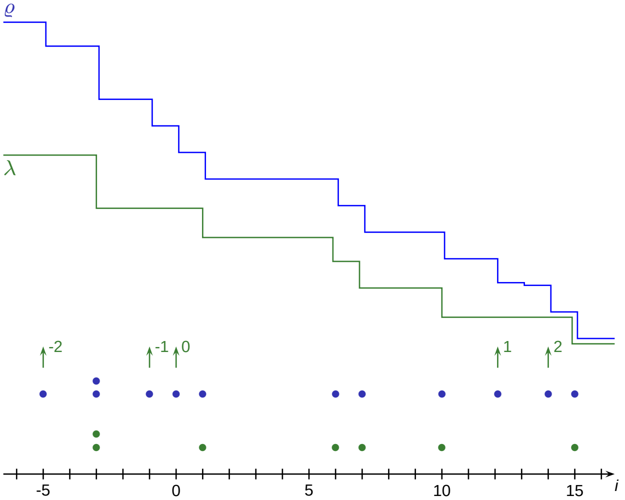

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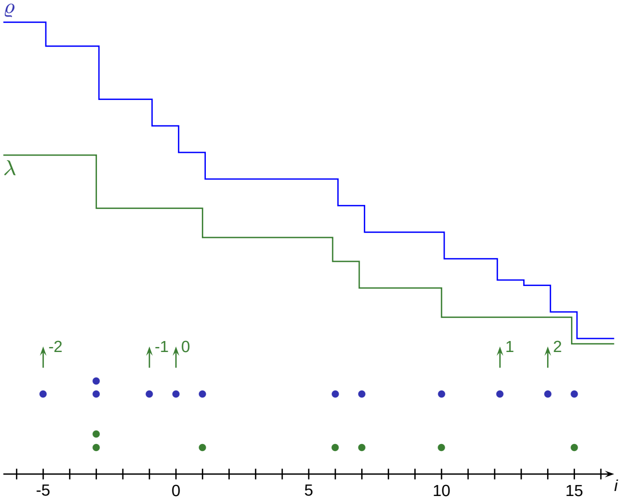

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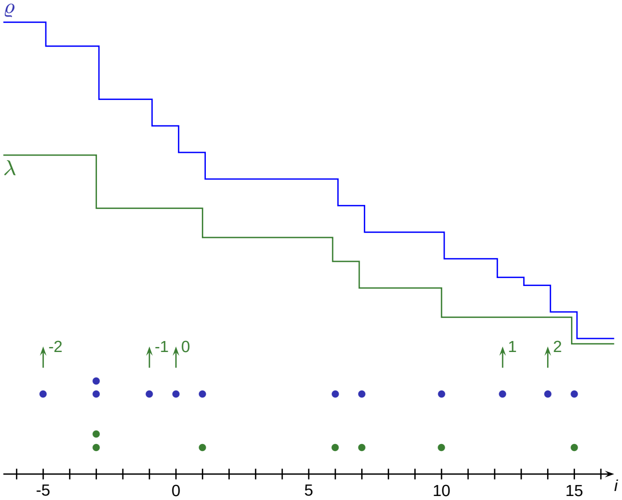

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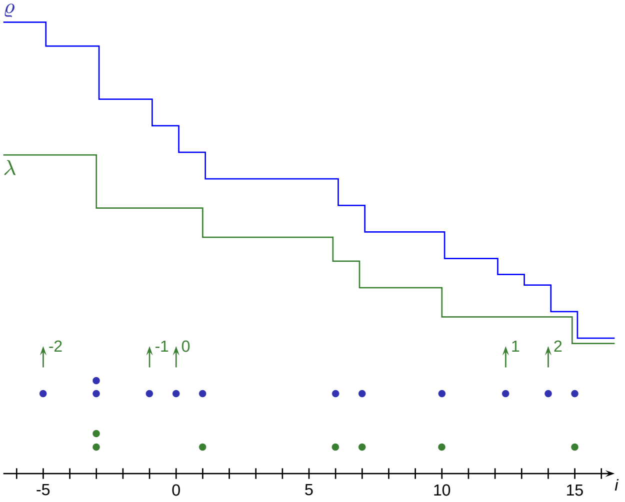

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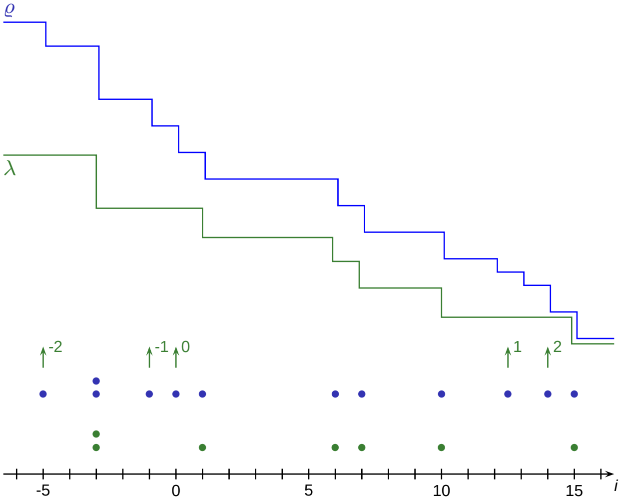

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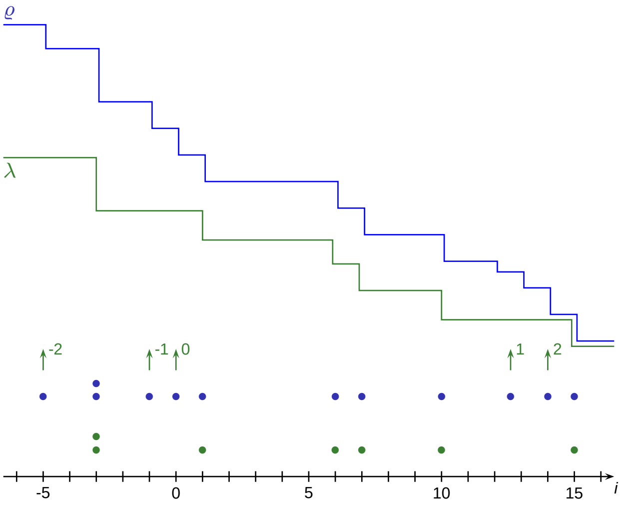

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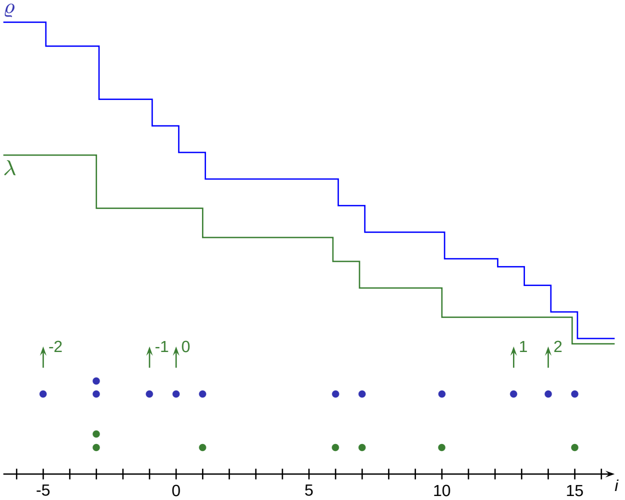

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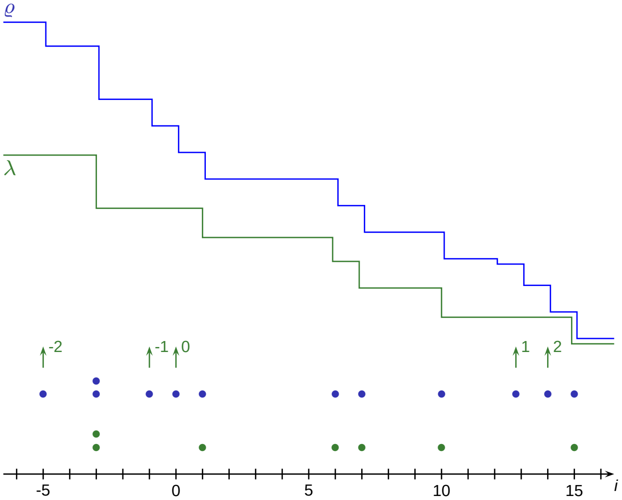

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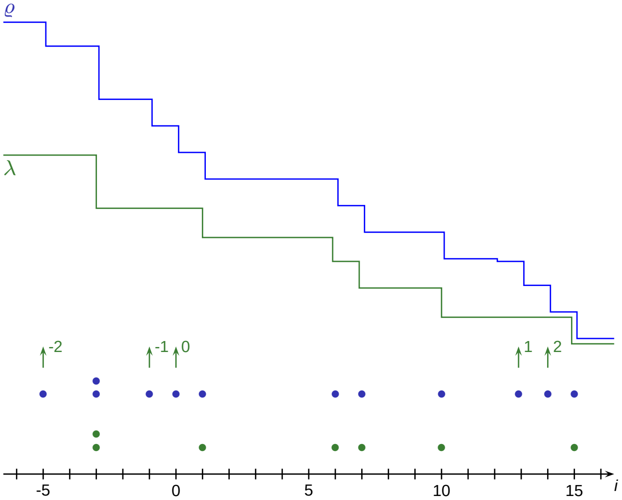

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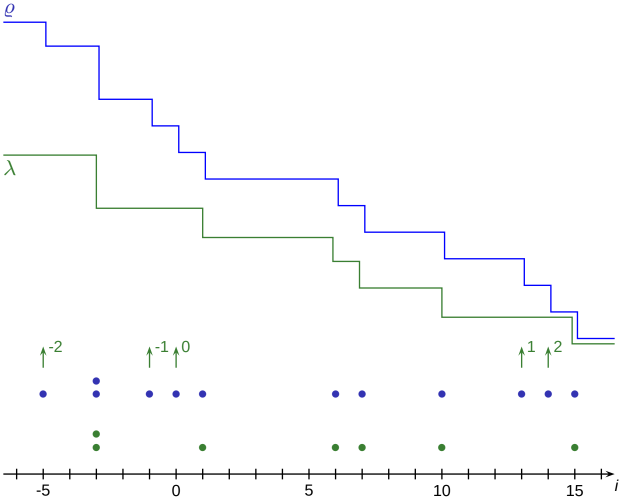

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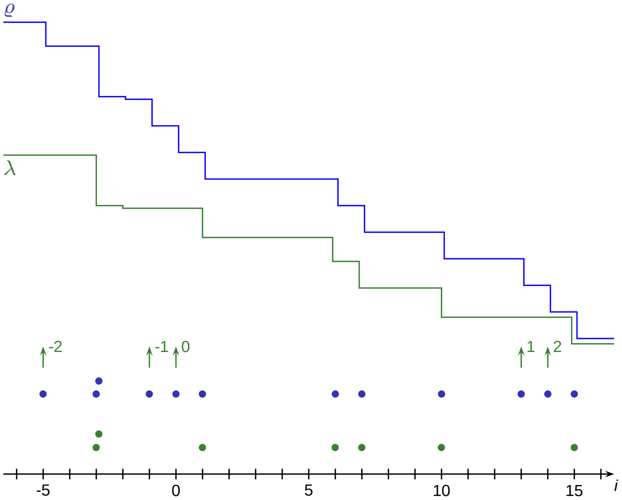

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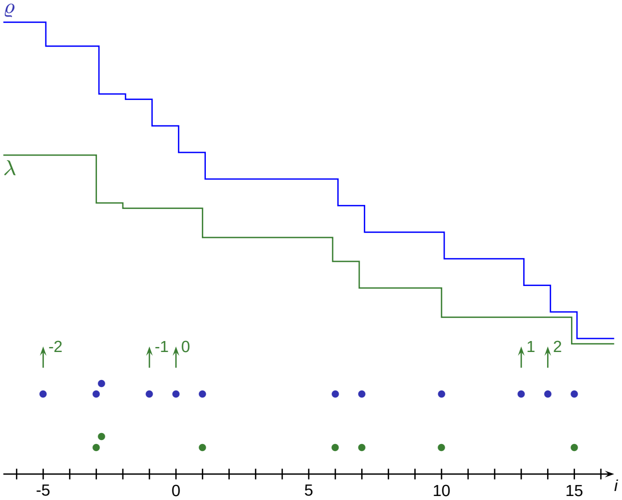

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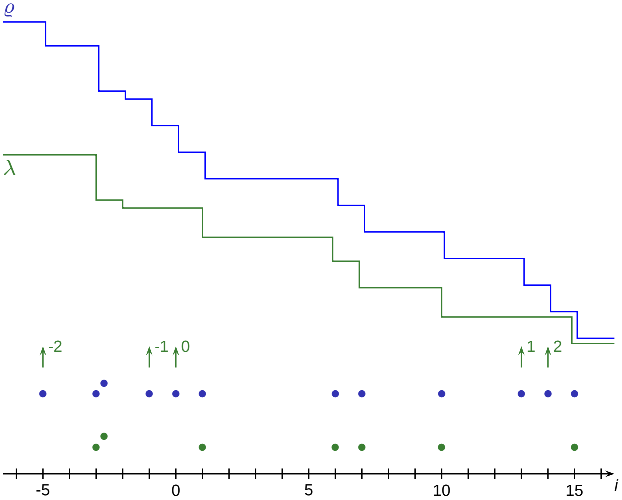

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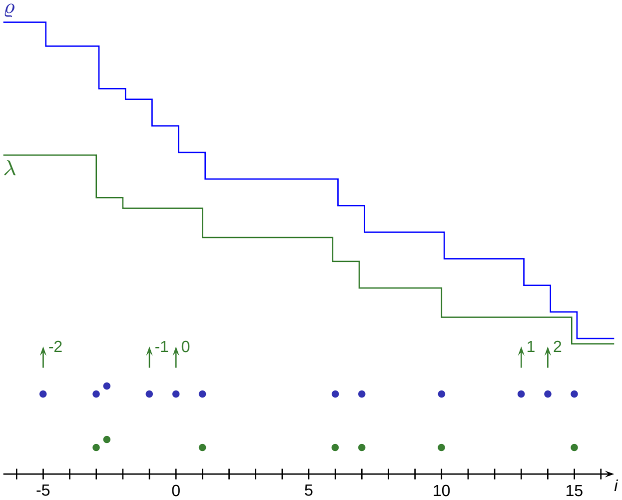

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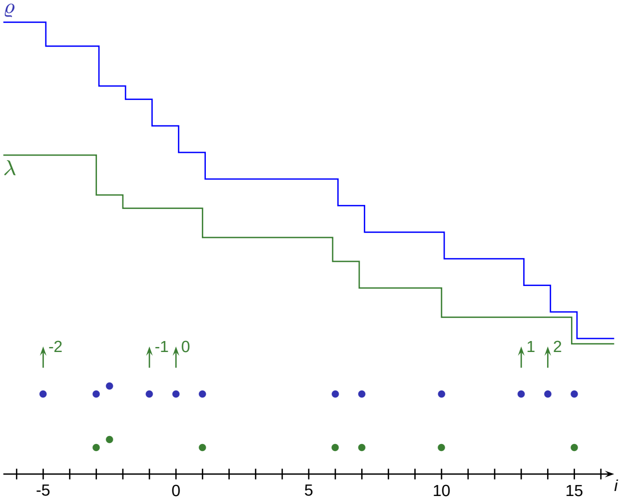

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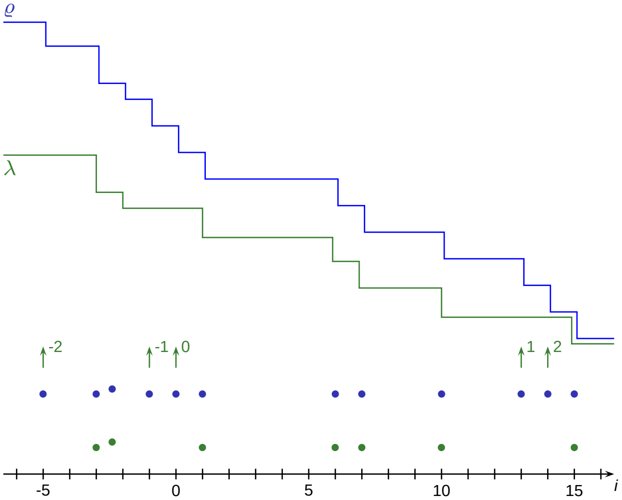

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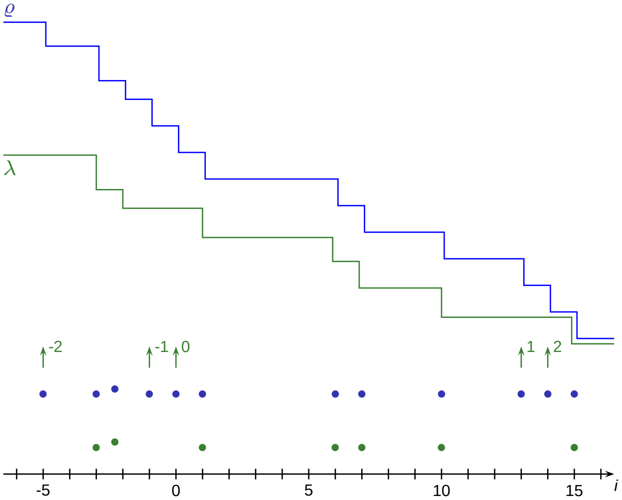

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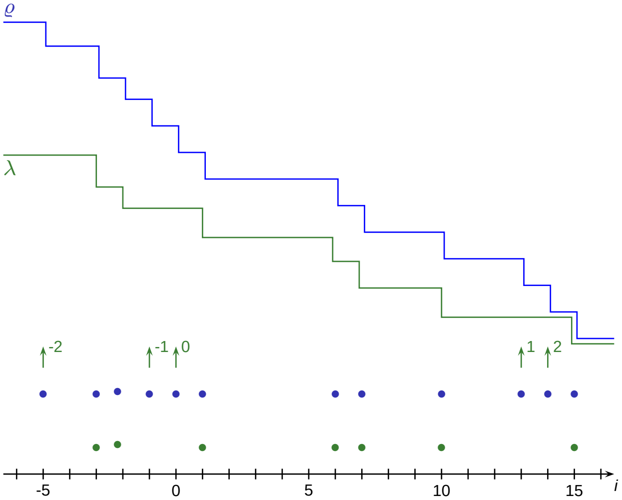

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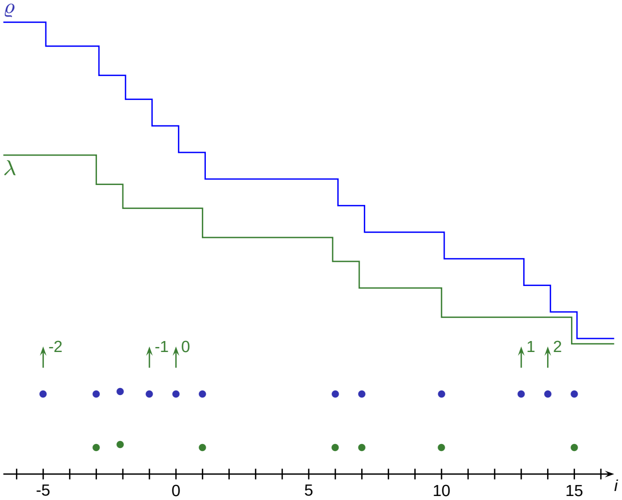

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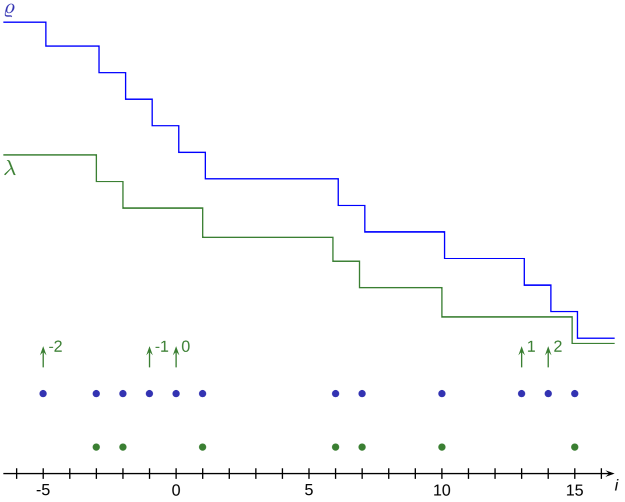

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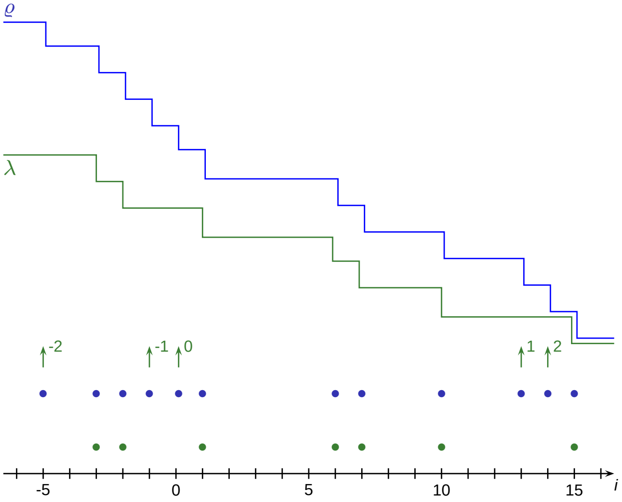

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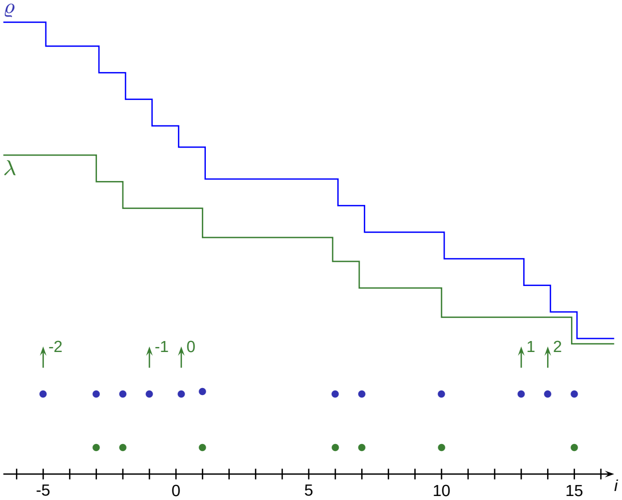

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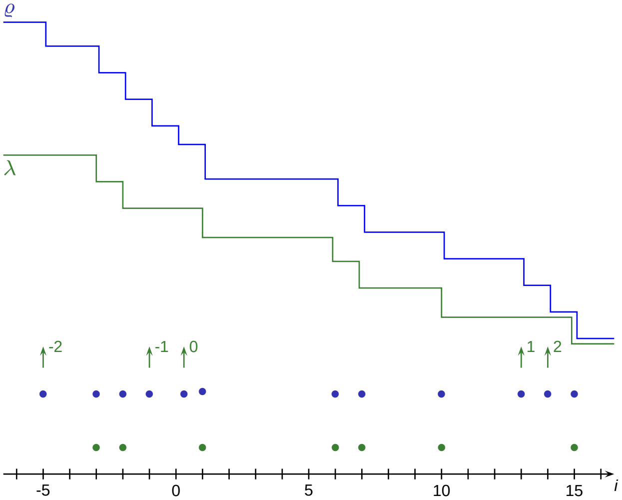

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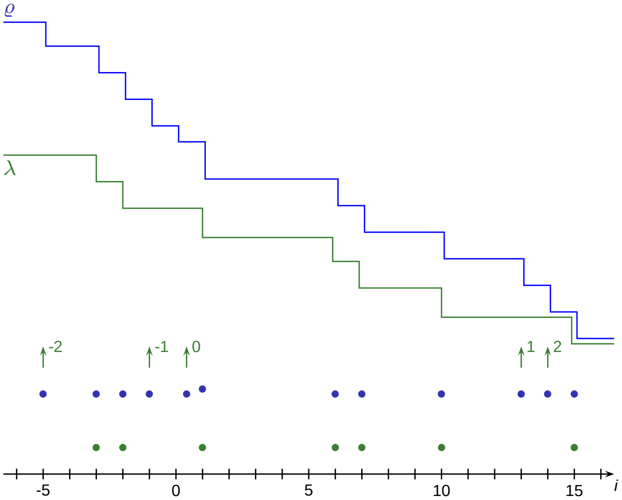

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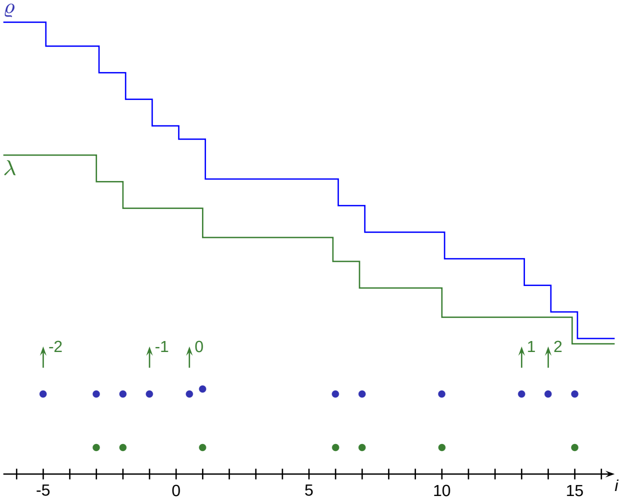

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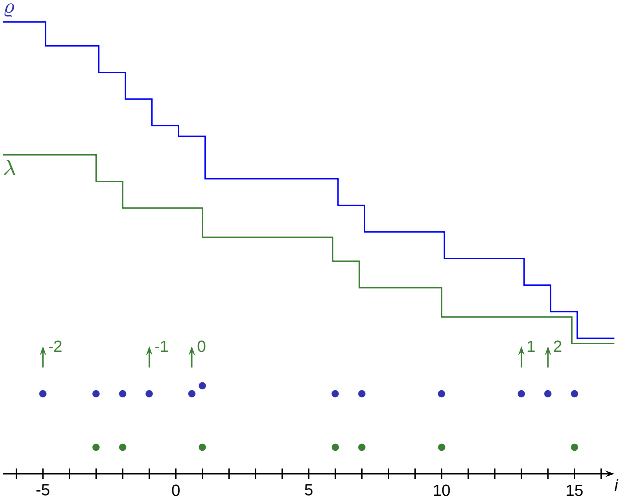

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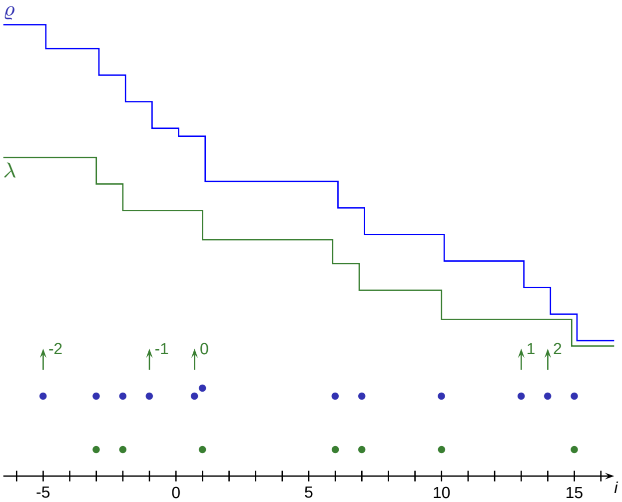

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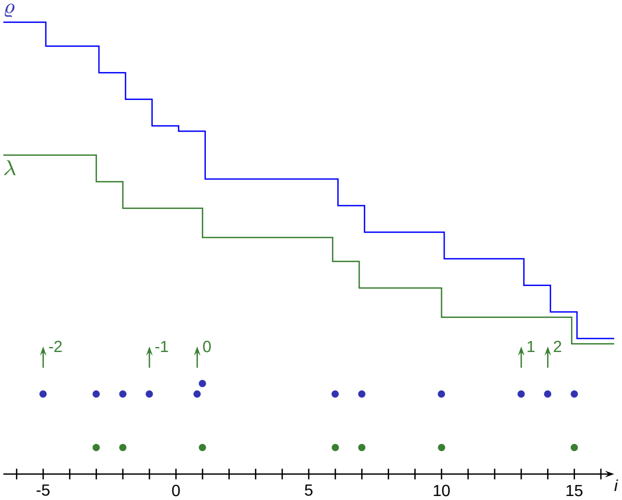

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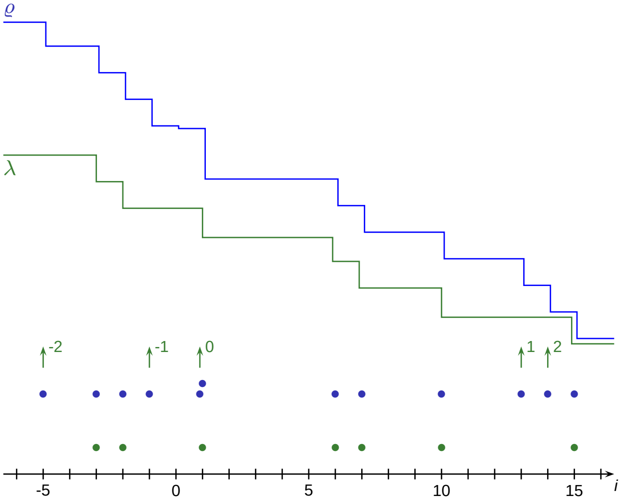

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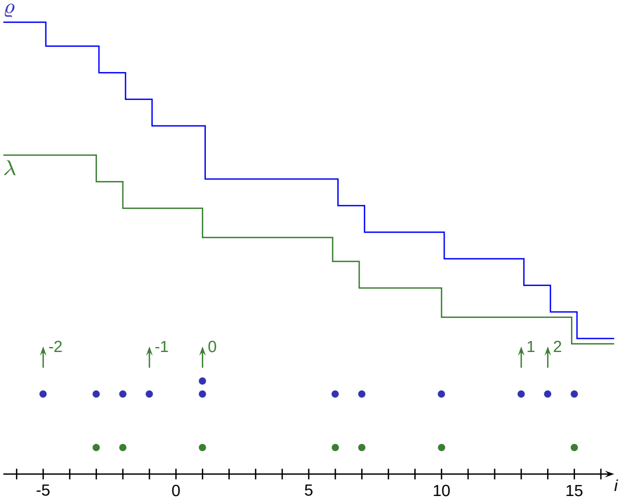

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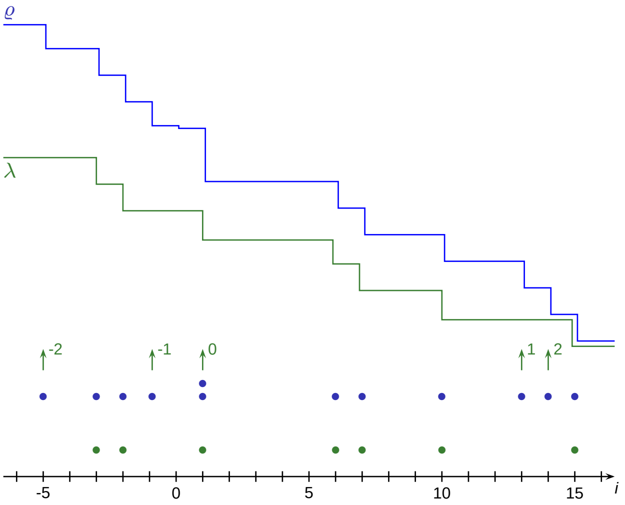

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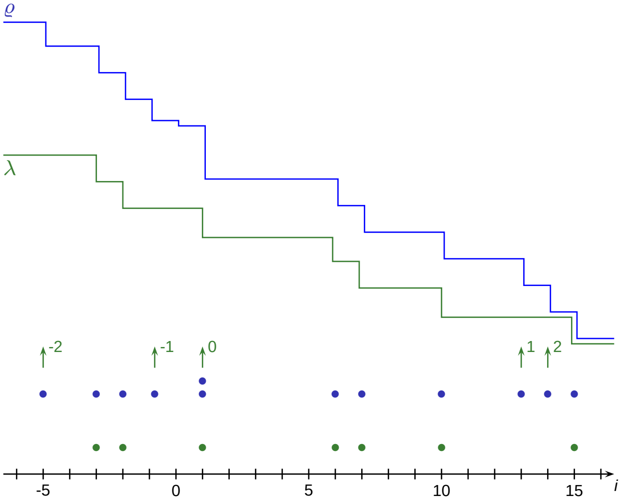

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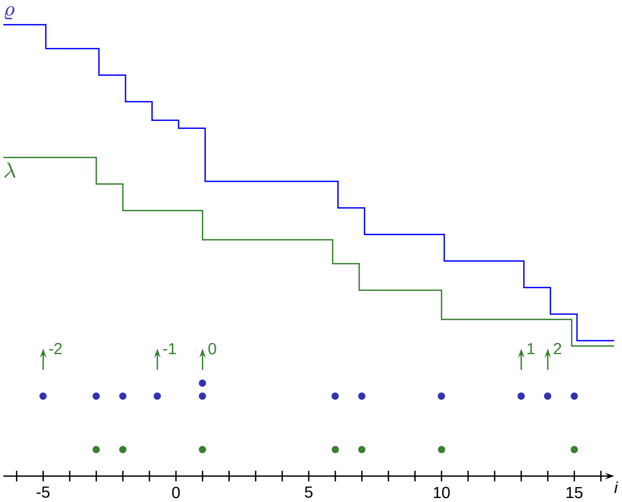

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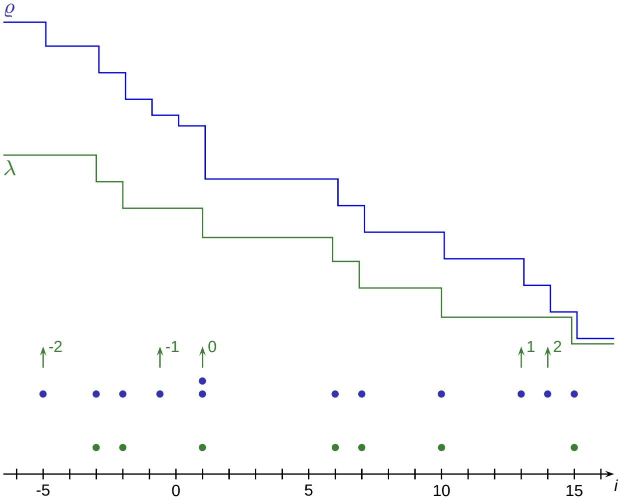

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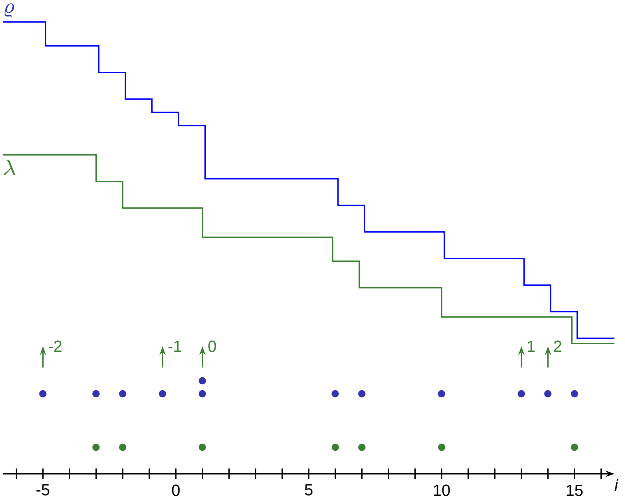

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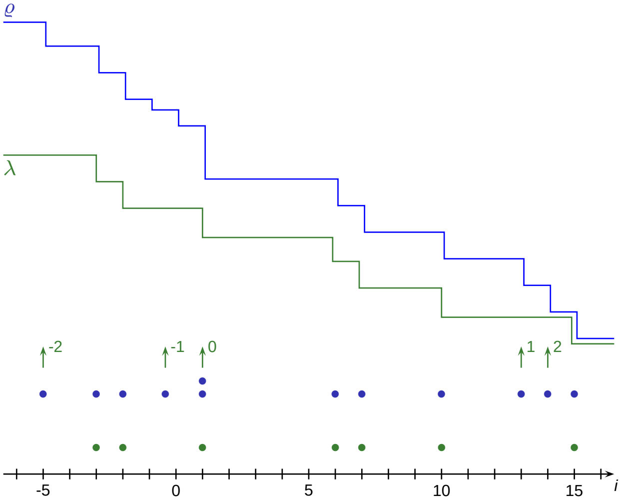

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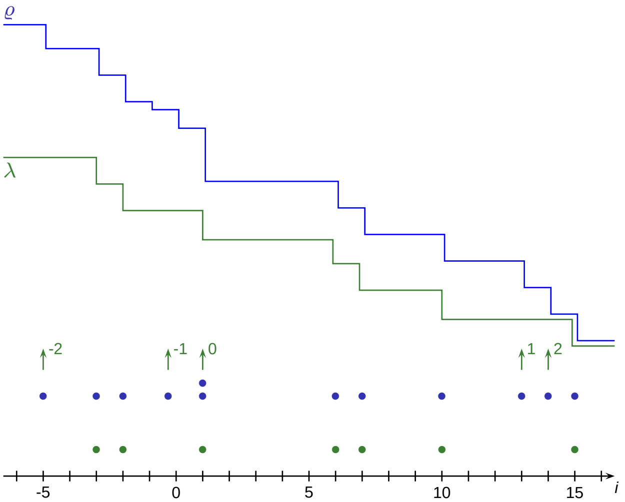

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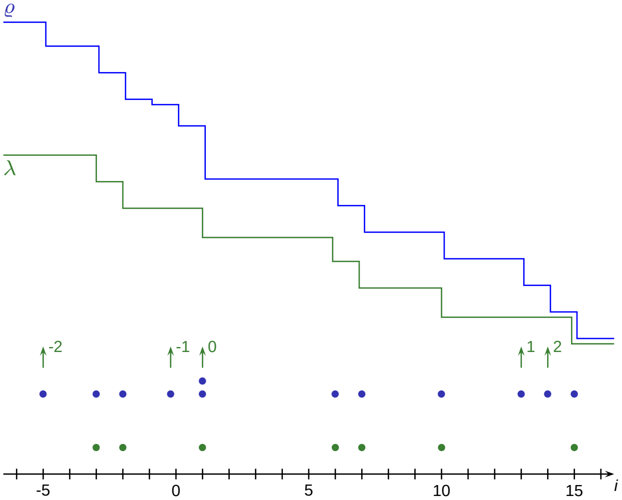

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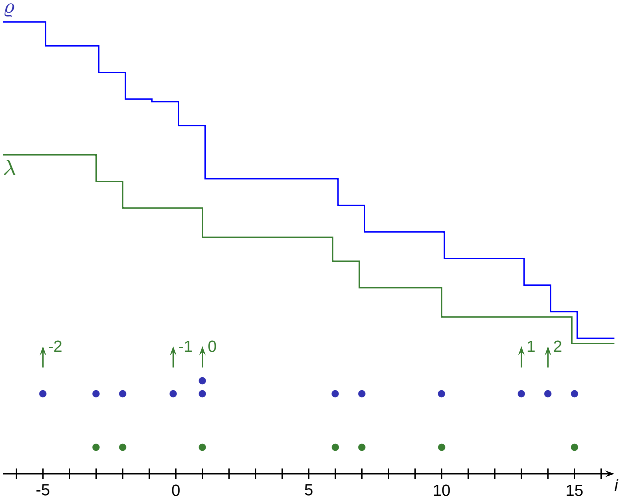

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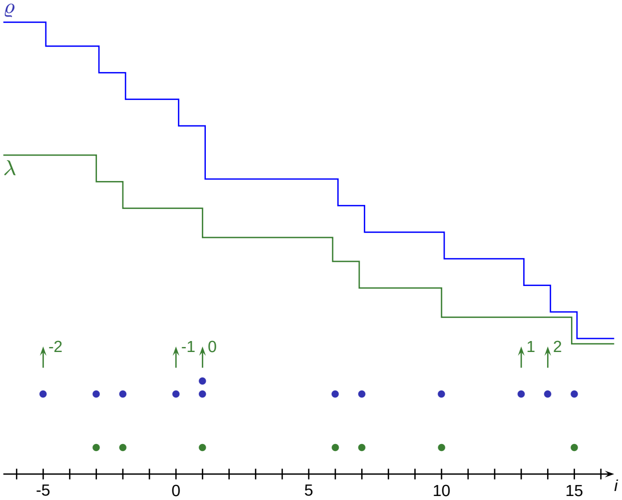

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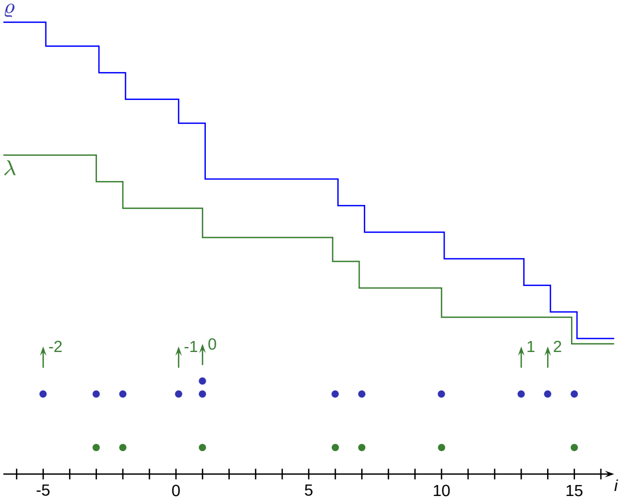

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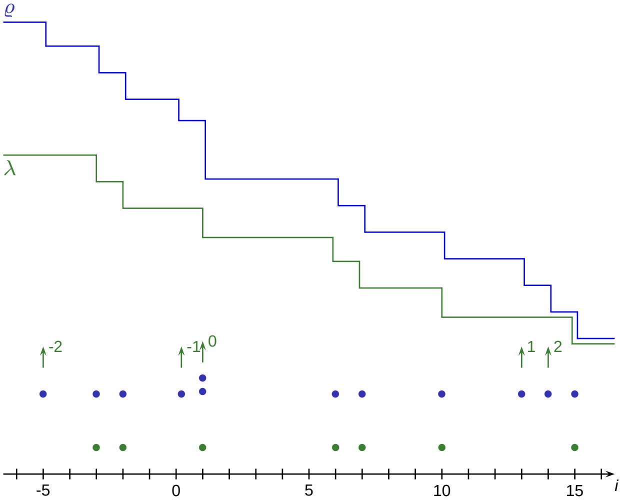

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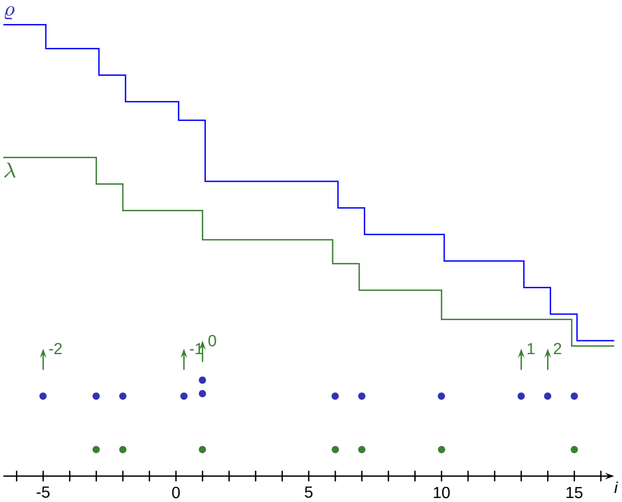

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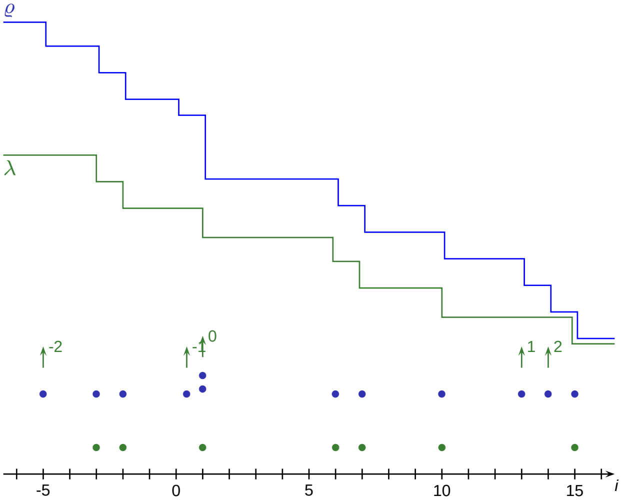

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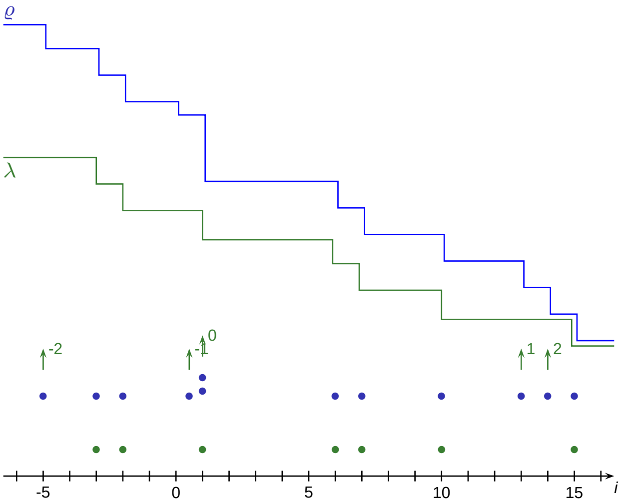

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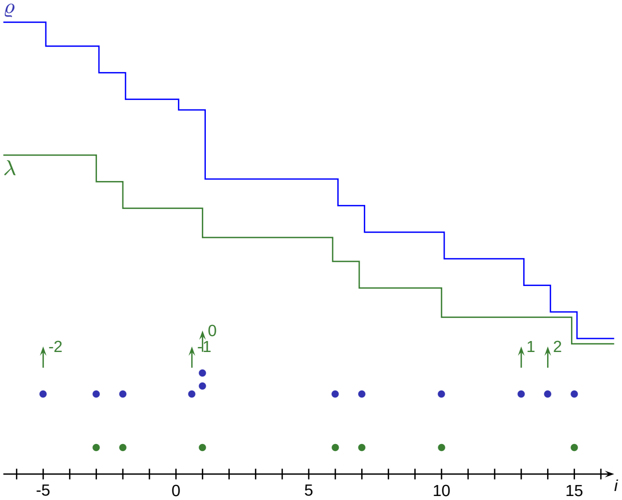

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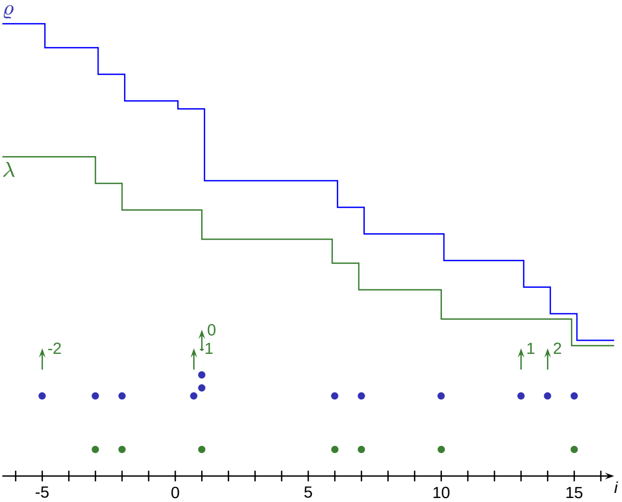

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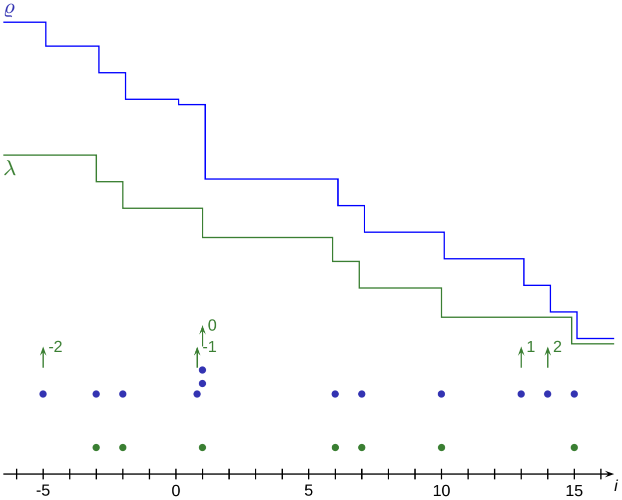

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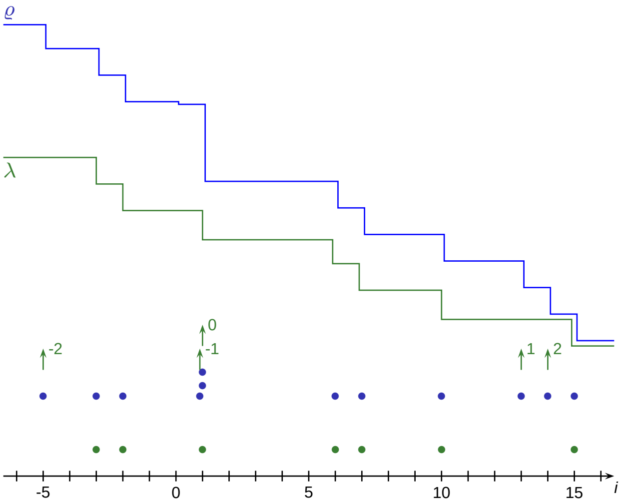

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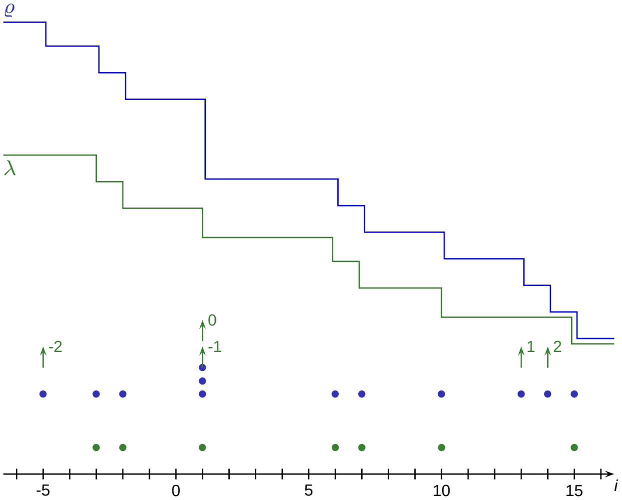

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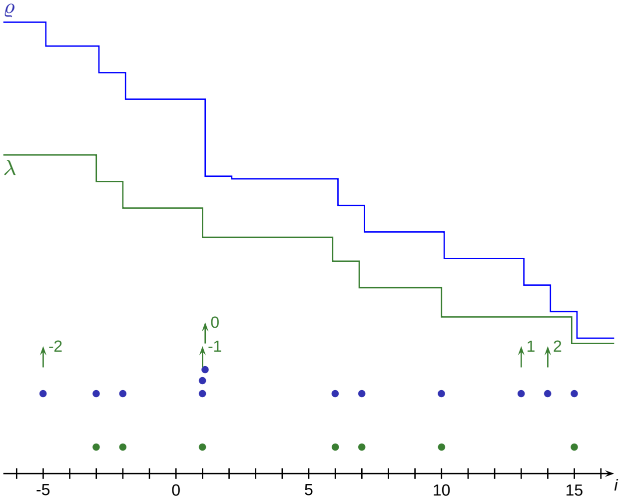

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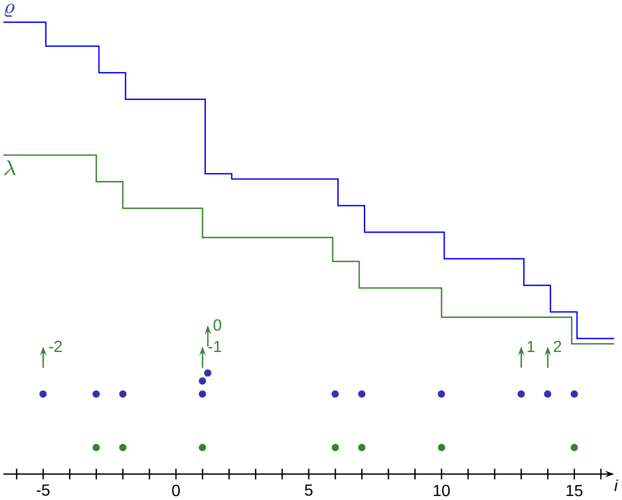

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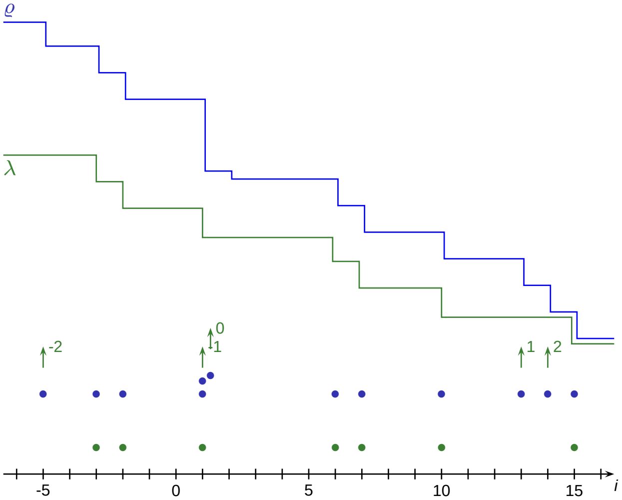

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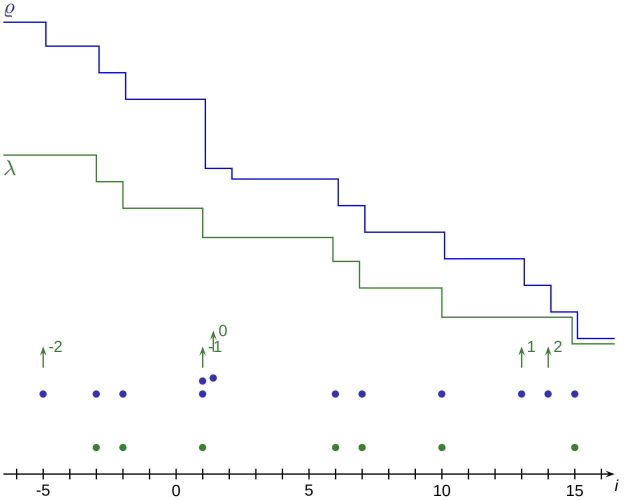

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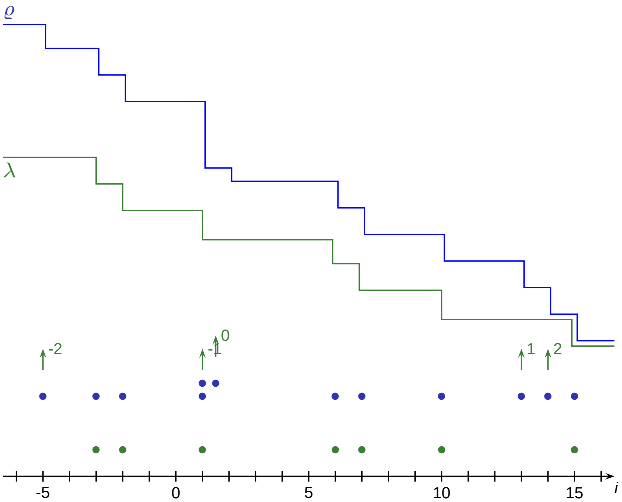

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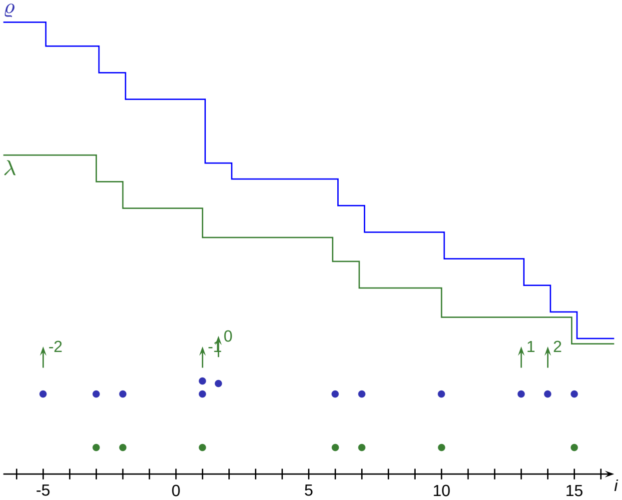

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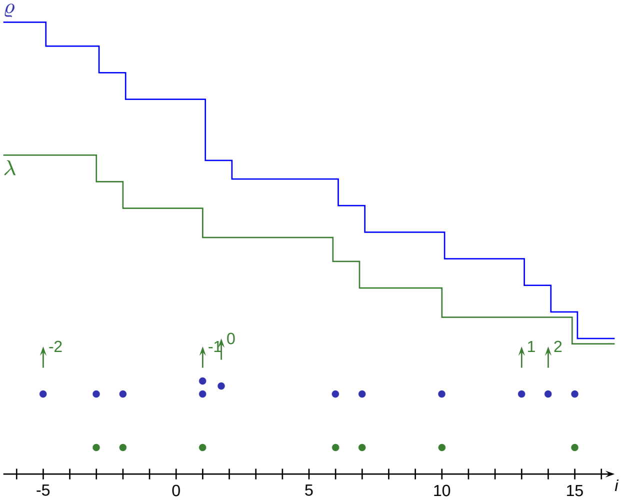

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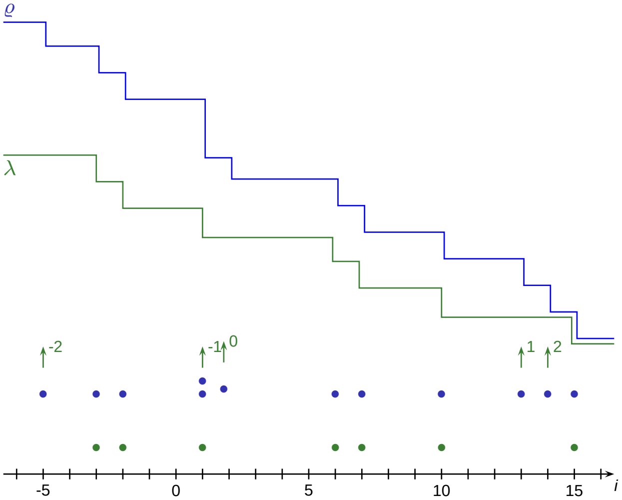

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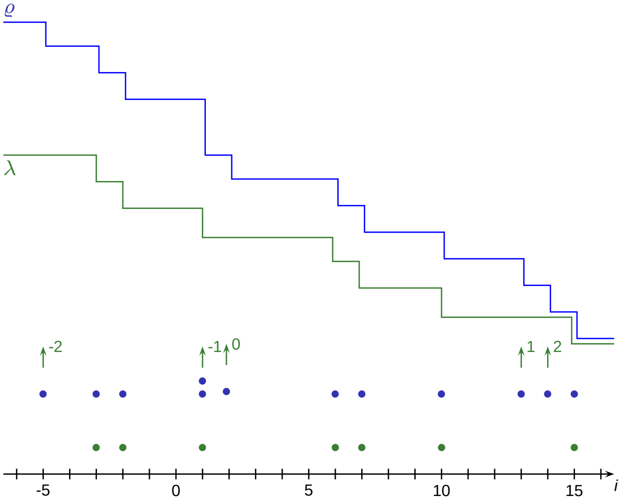

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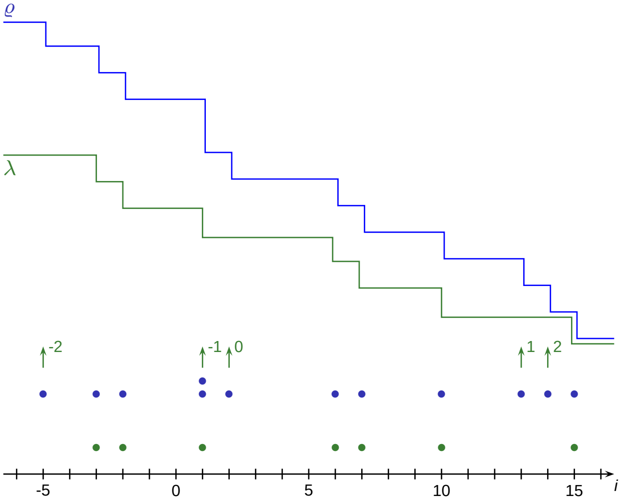

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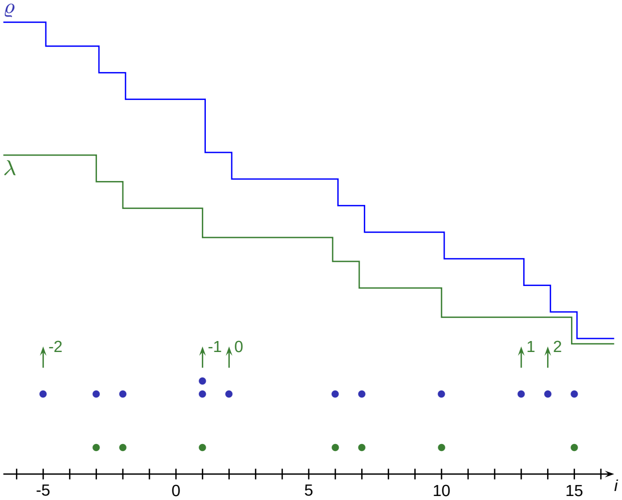
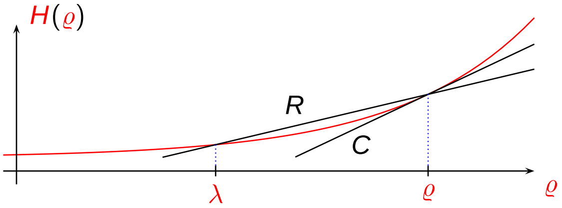


Picture:

The position $X(t)$ of $\uparrow^0$ follows the Rankine-Hugoniot speed R.

Characteristics (very briefly)

Convex flux (some cases of AZRP):



Recall $$C = H'(\varrho) > R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

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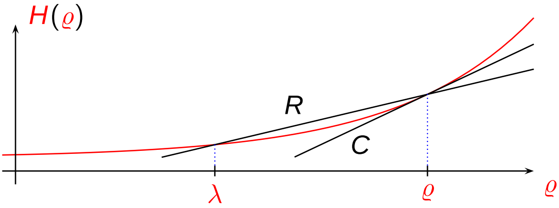

Recall $C = H'(\varrho) > R = \dfrac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$

Do we have $Q(t) \overset{?}{\geq} X(t)$

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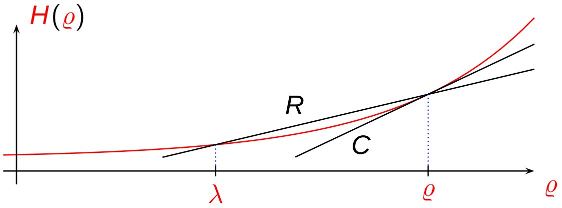
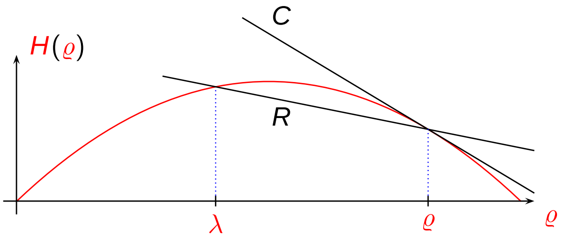


Recall $$C = H'(\varrho) > R = \frac{H(\varrho) - H(\lambda)}{\varrho - \lambda}$$

Do we have $$Q(t) \overset{?}{\geq} X(t) + \text{tight error}$$

Characteristics (very briefly)

Concave flux (ASEP, AZRP):



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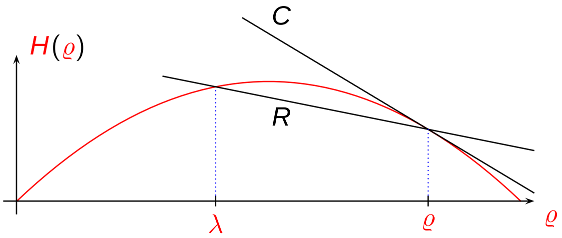

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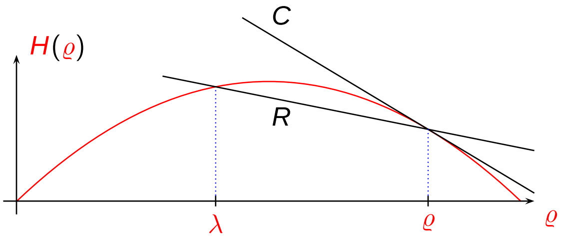

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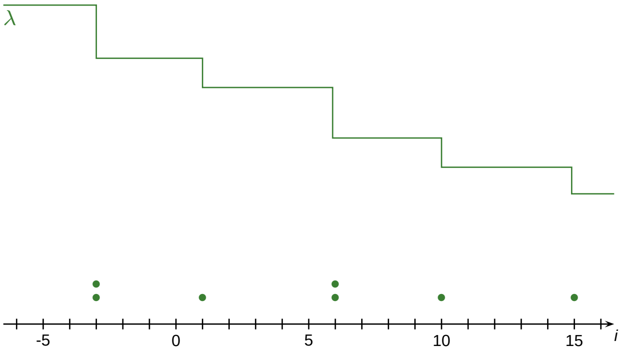
Many second class particles



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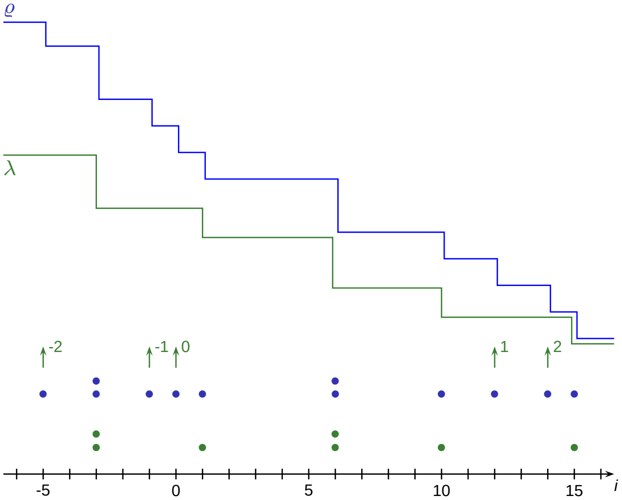
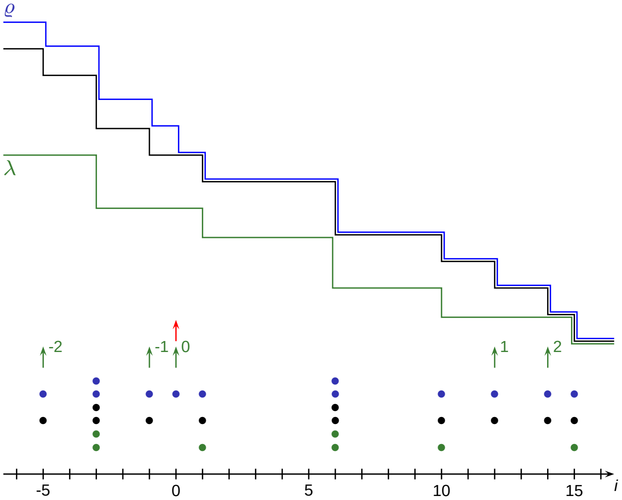
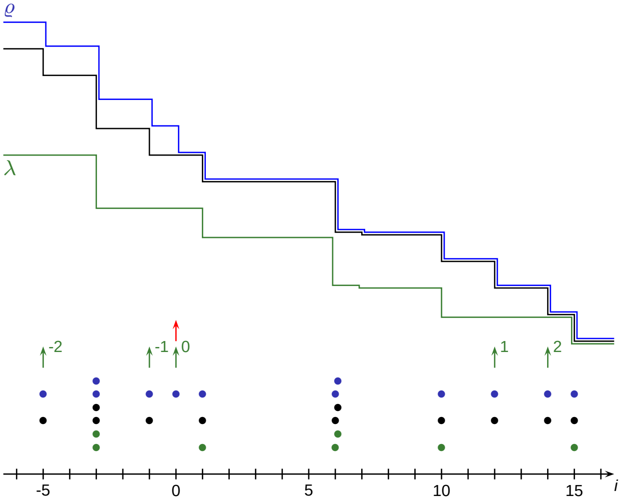


Many second class particles plus one



Couple three processes, and $X(t)$ to $Q(t)$.

Many second class particles plus one



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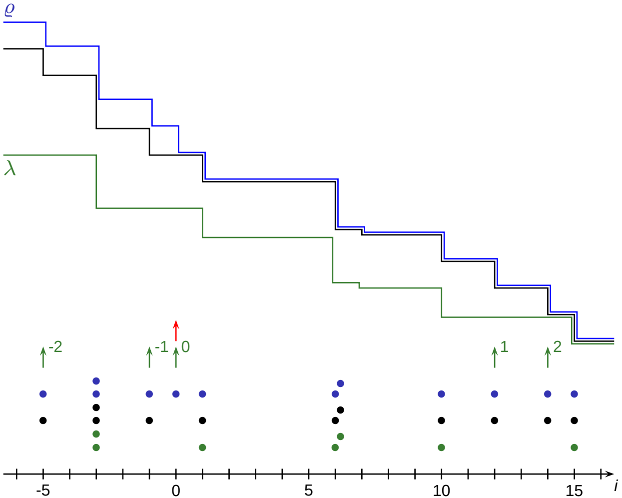

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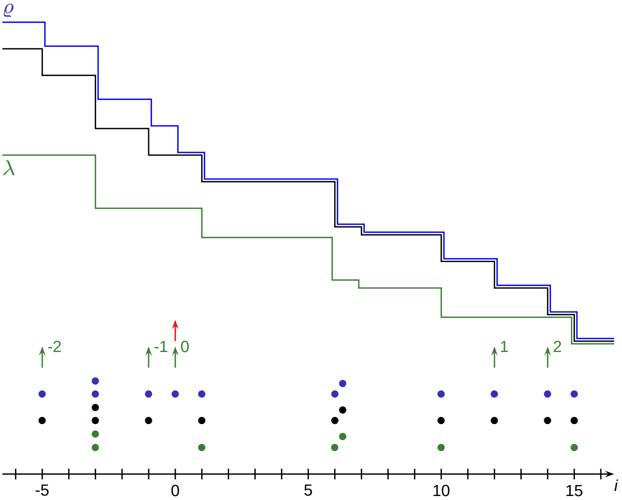

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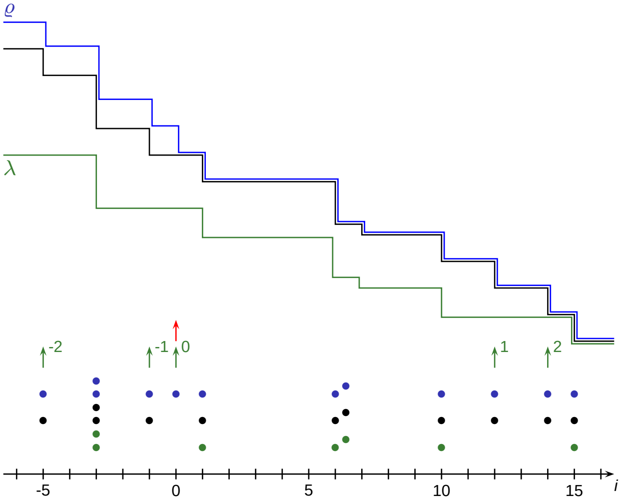

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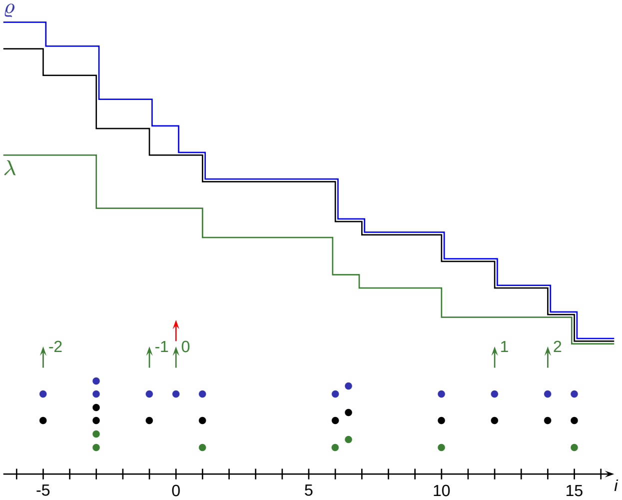

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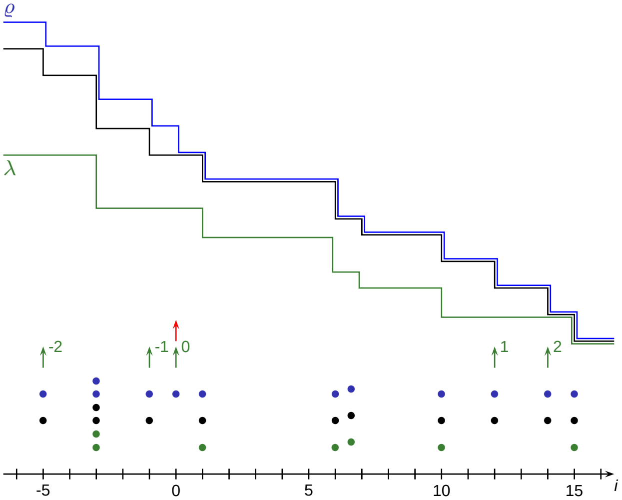

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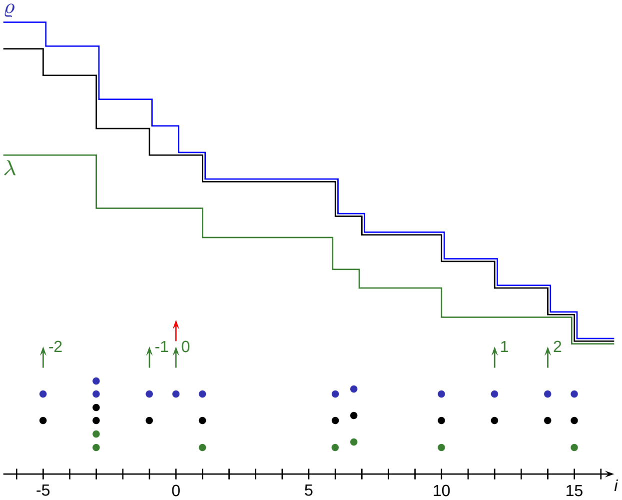

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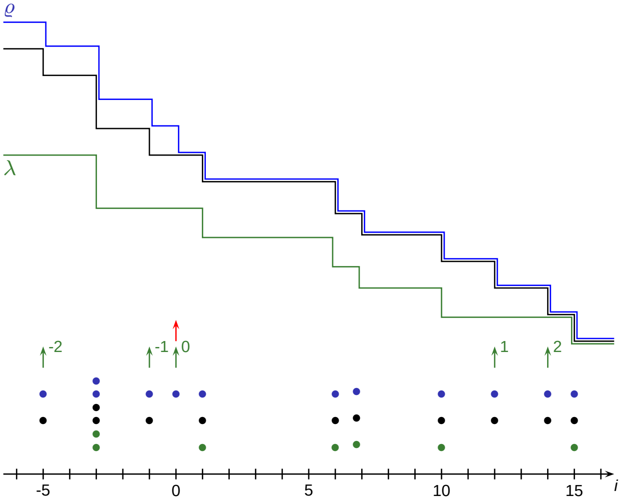

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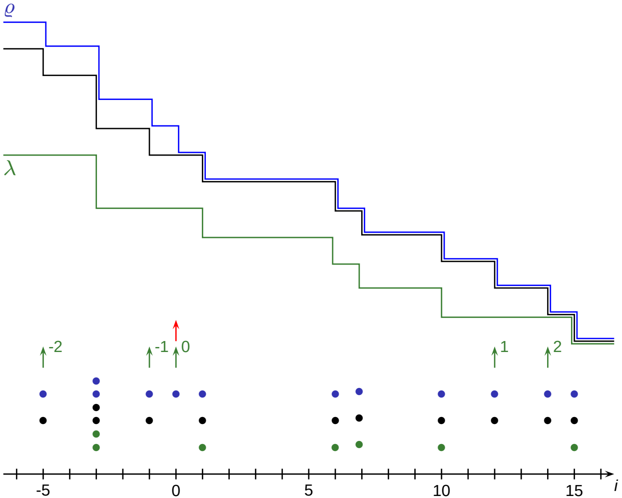

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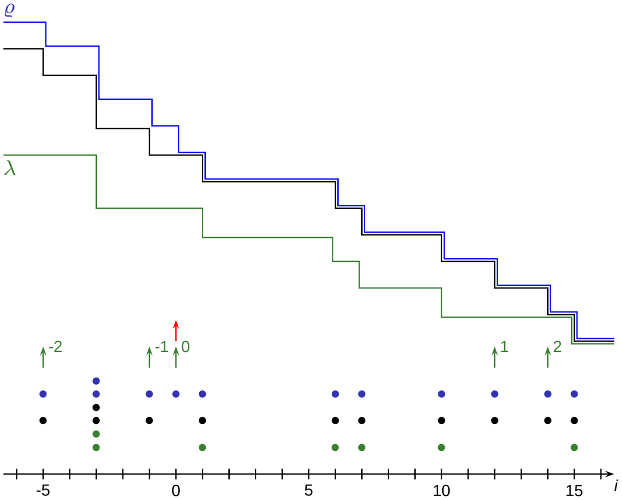

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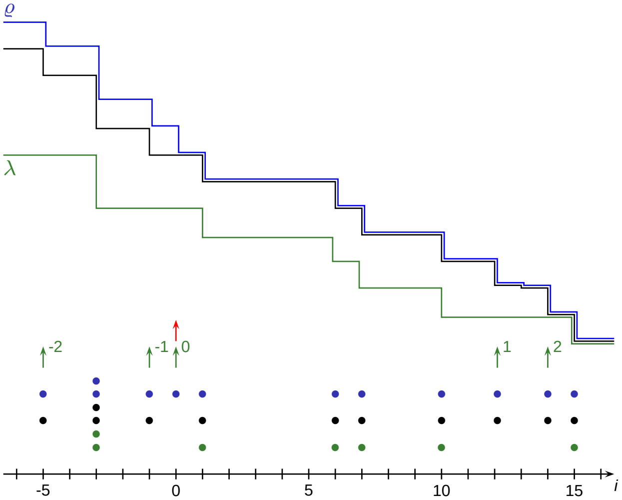

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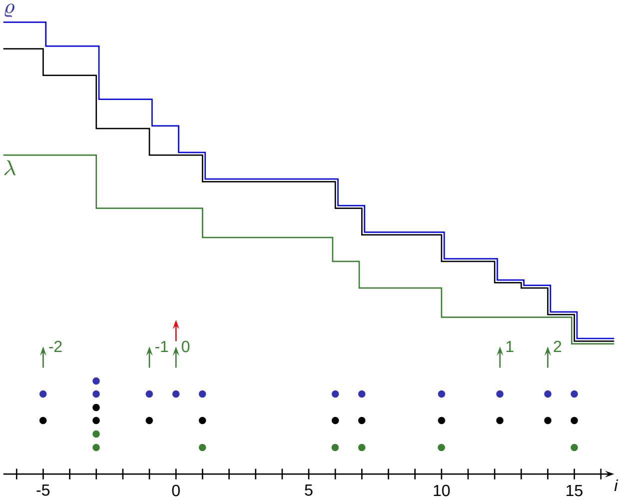

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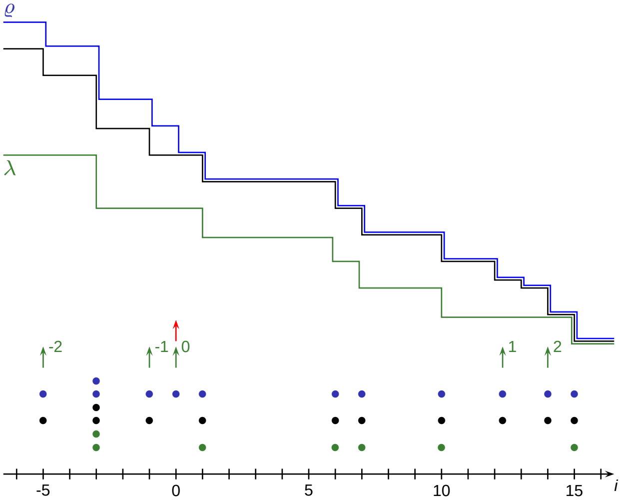

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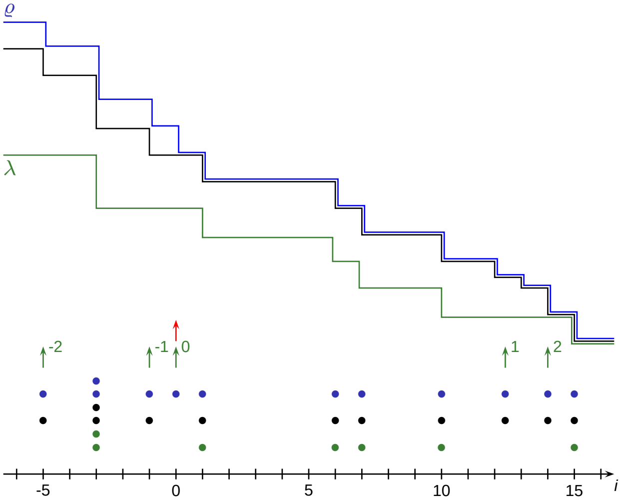

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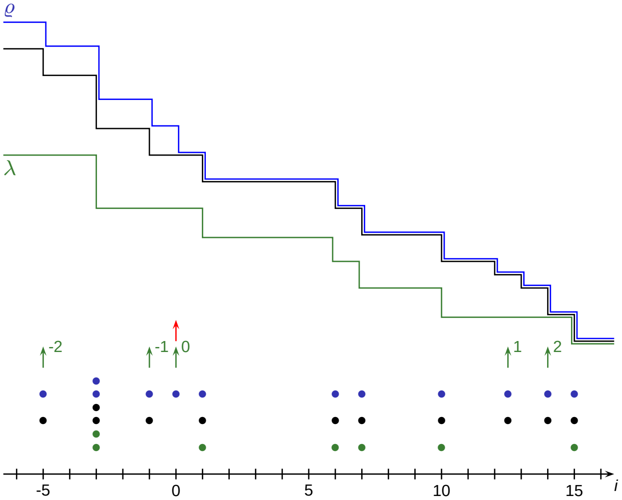

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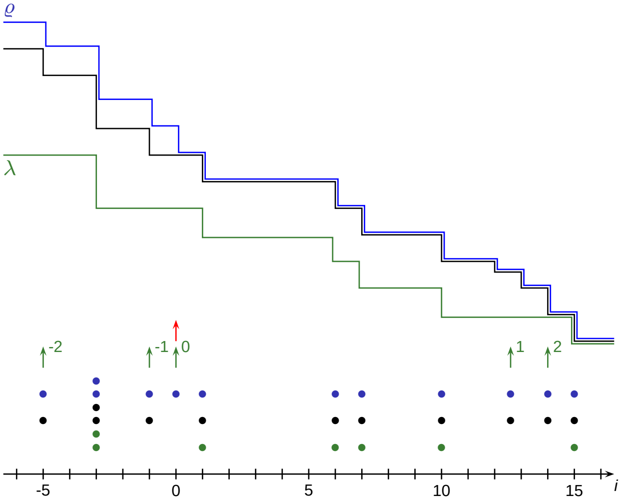

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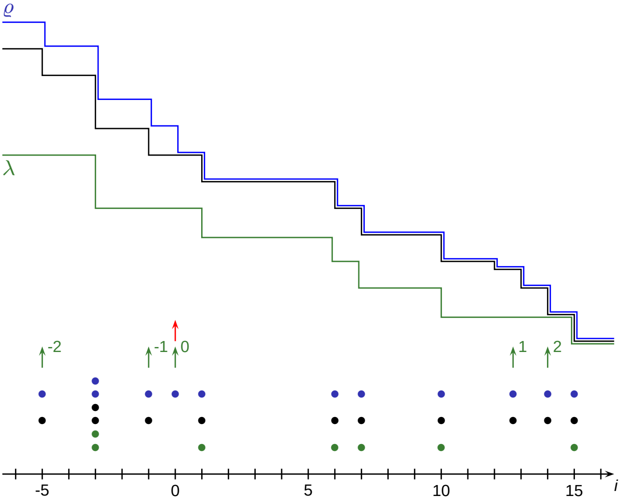

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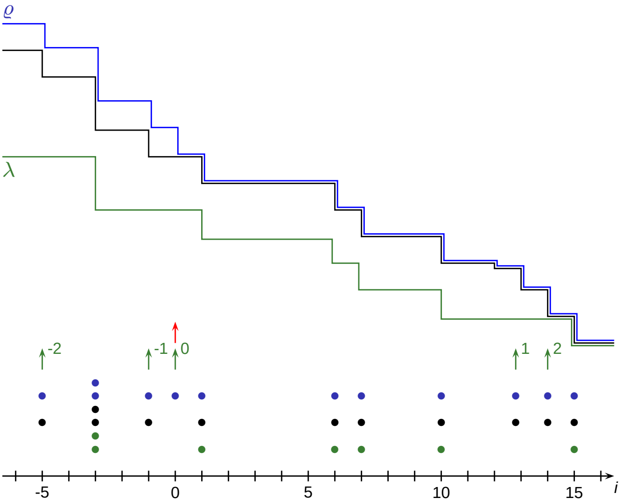

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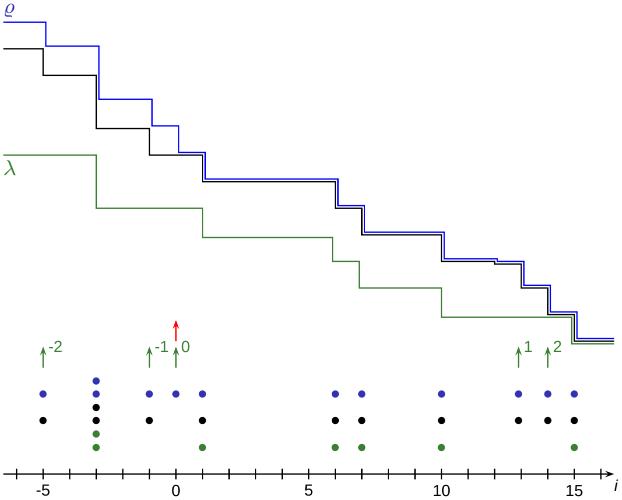

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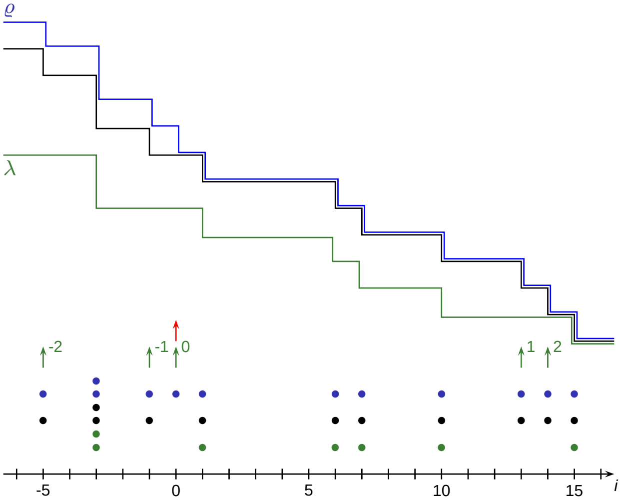

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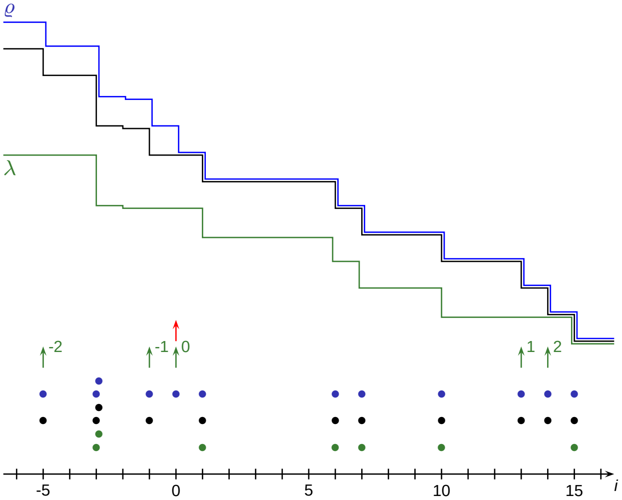

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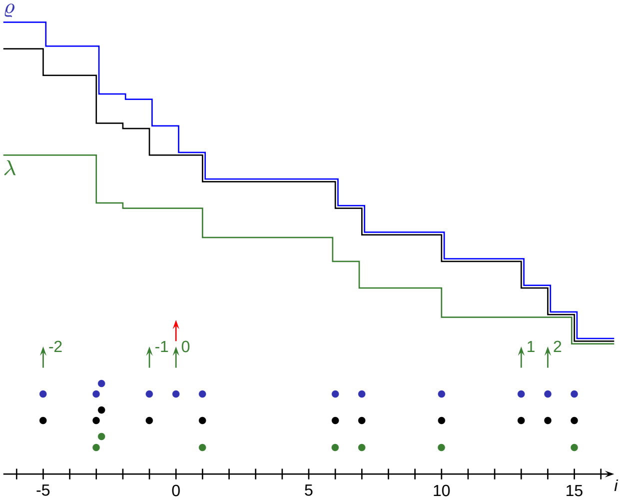

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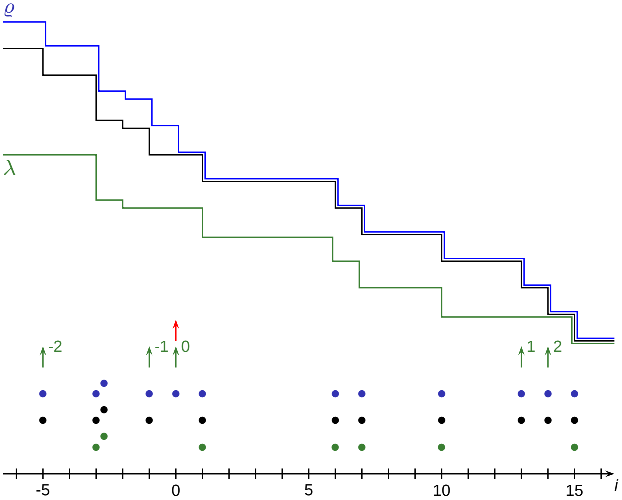

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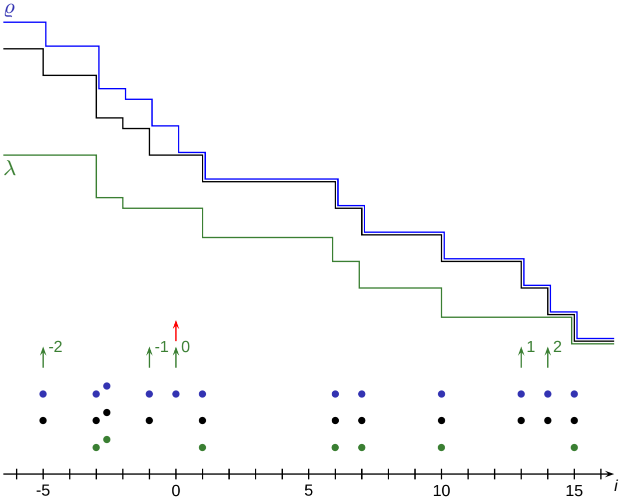

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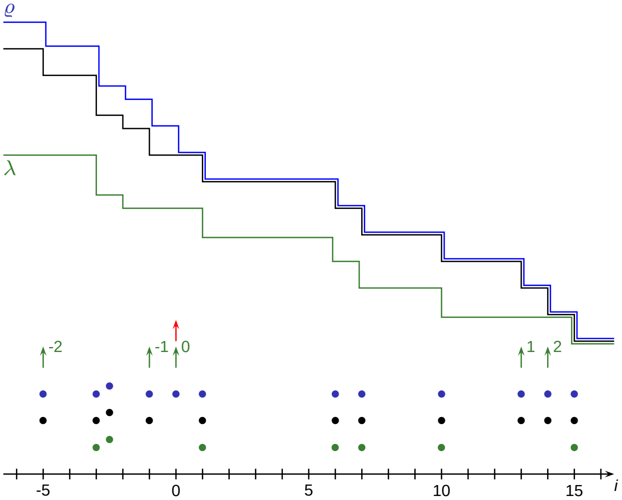

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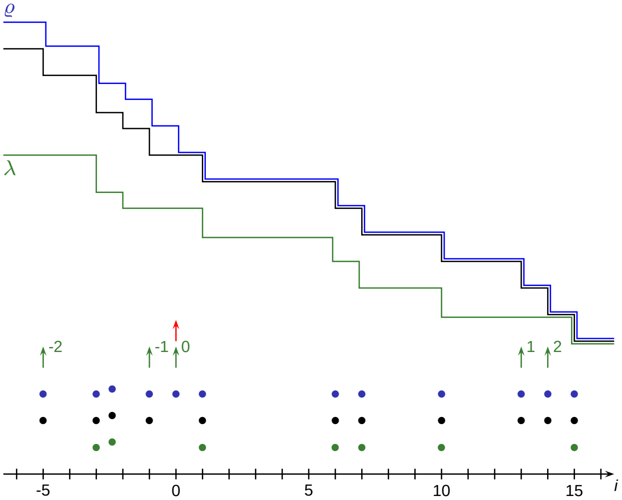

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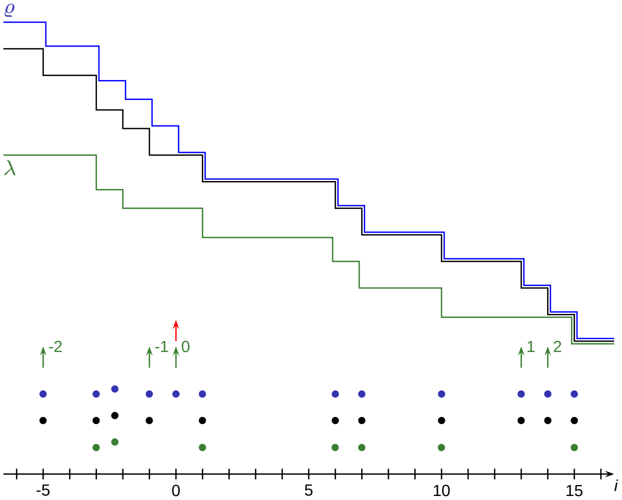

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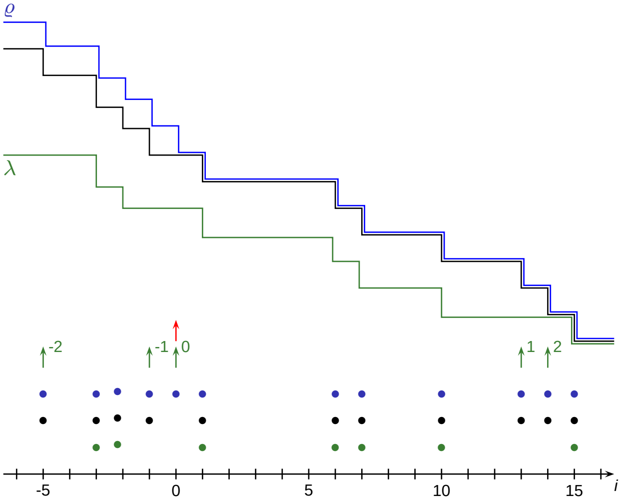

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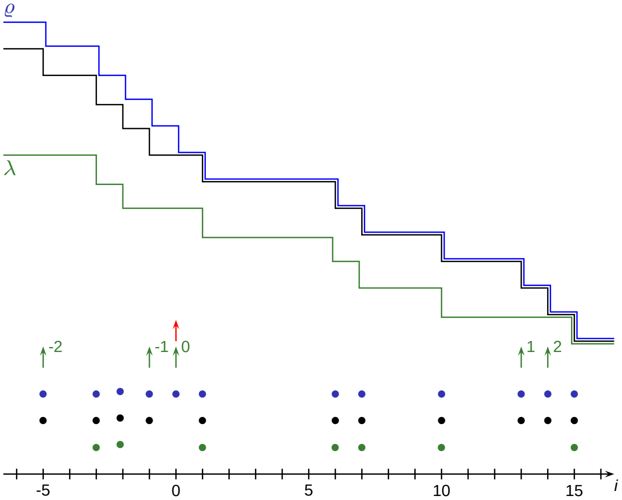

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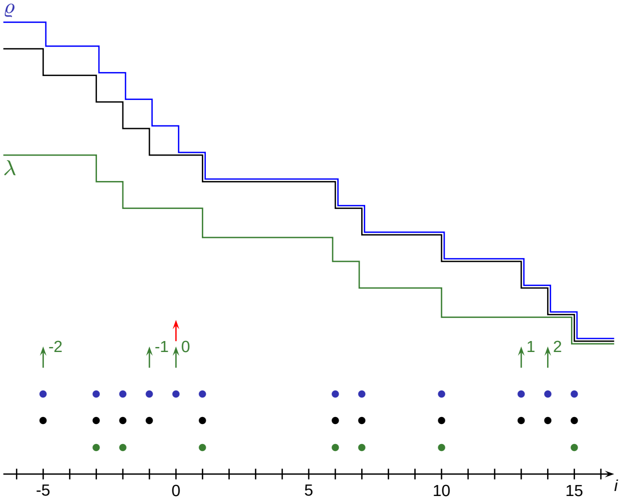

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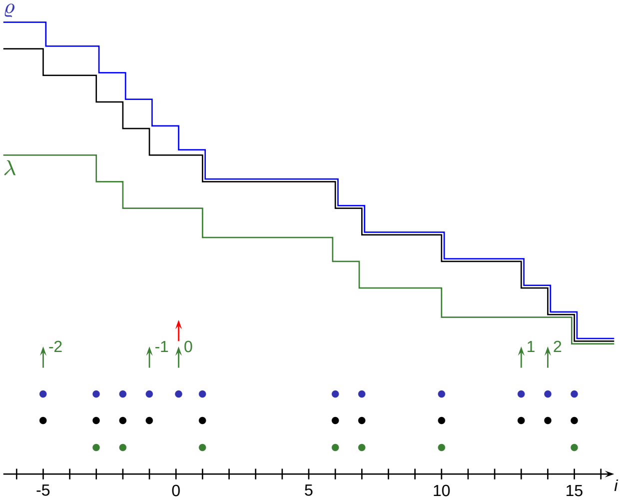

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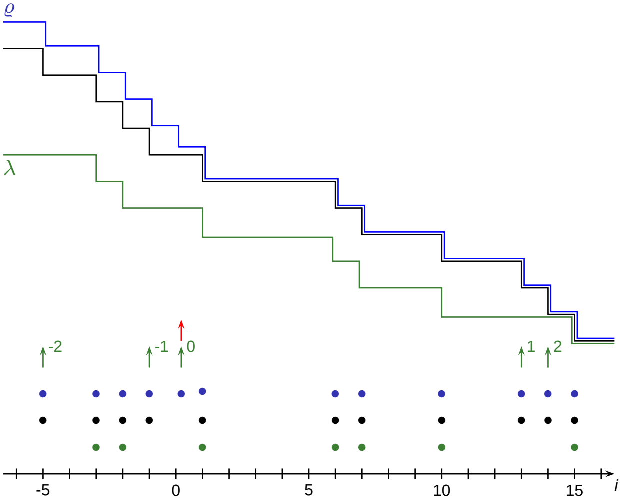

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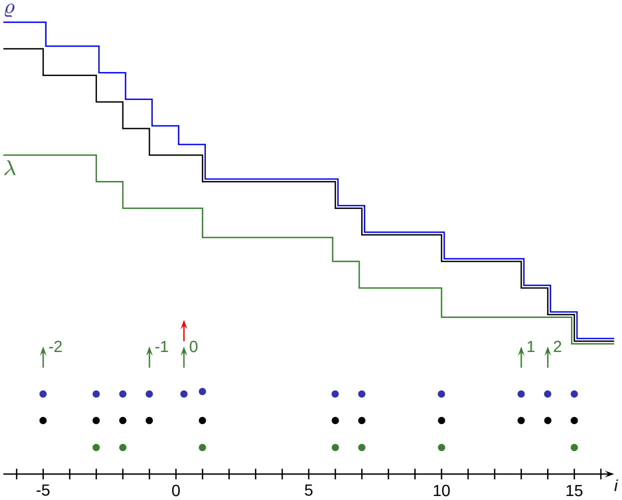

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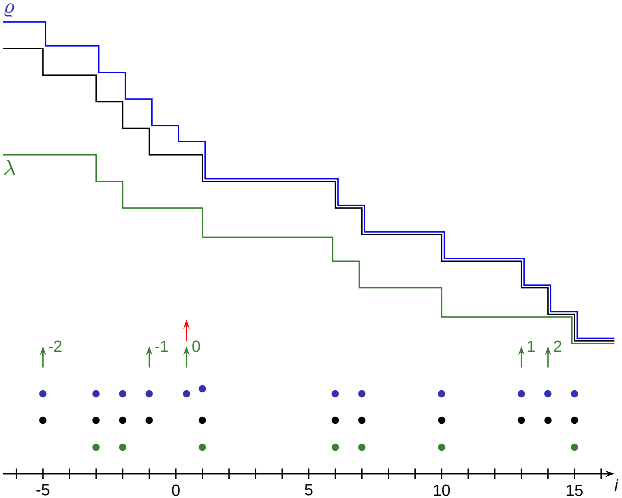

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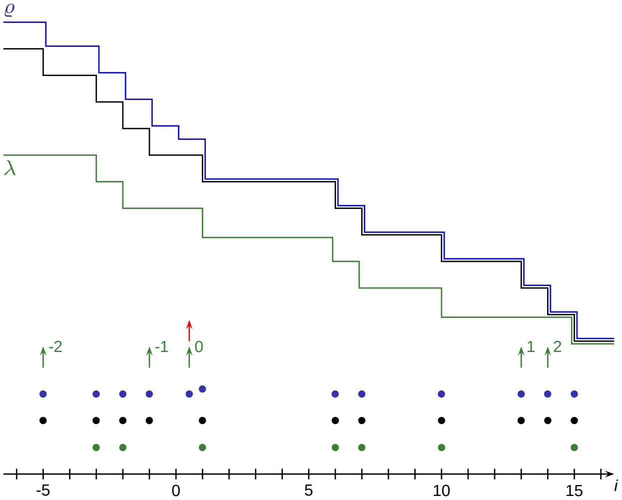

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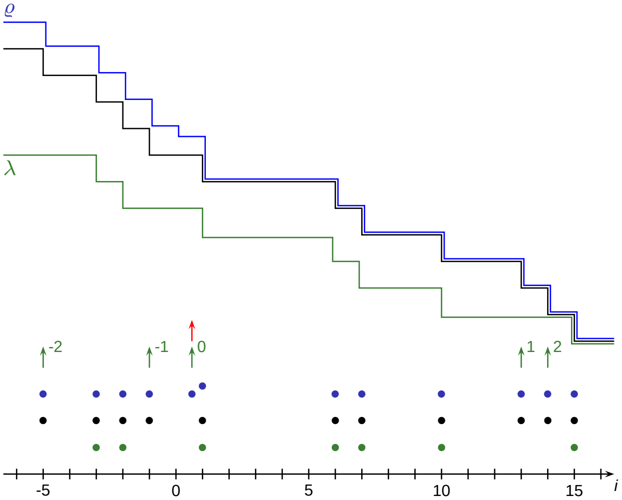

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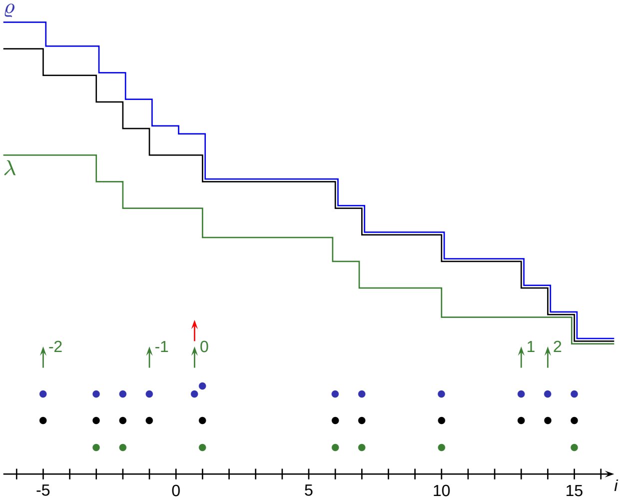

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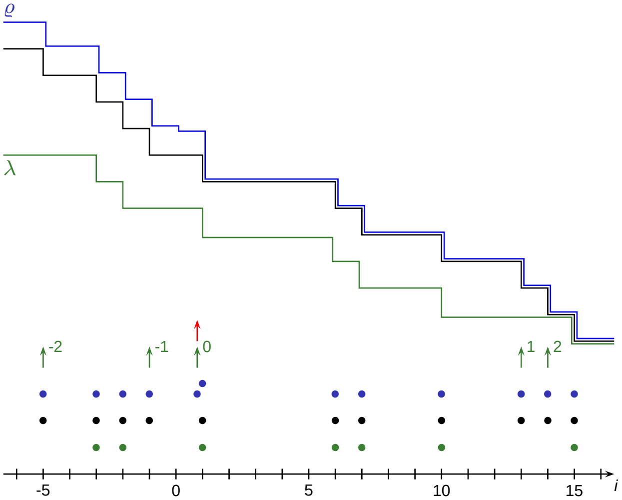

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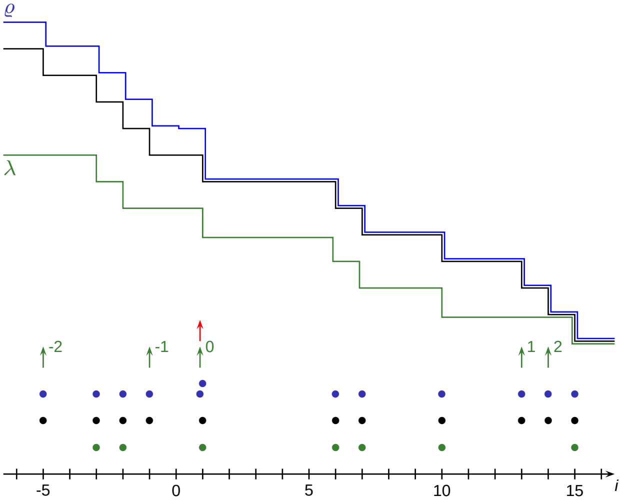

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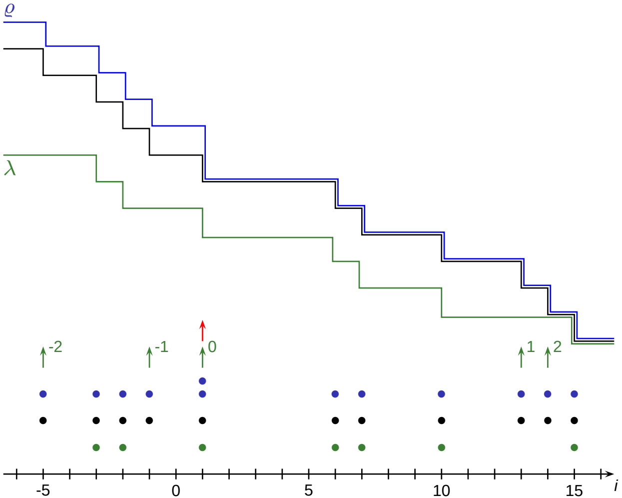

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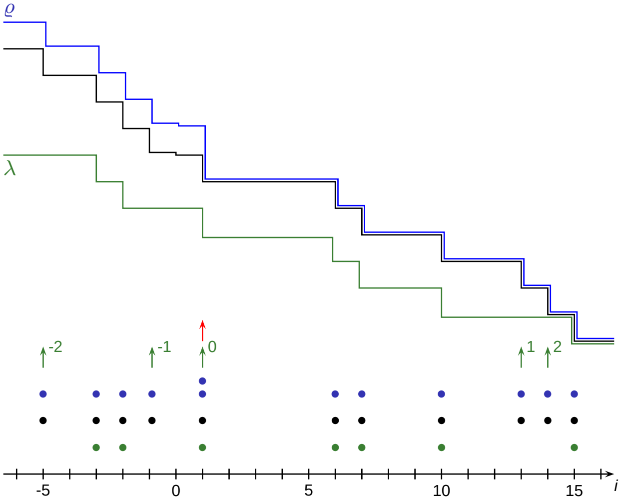

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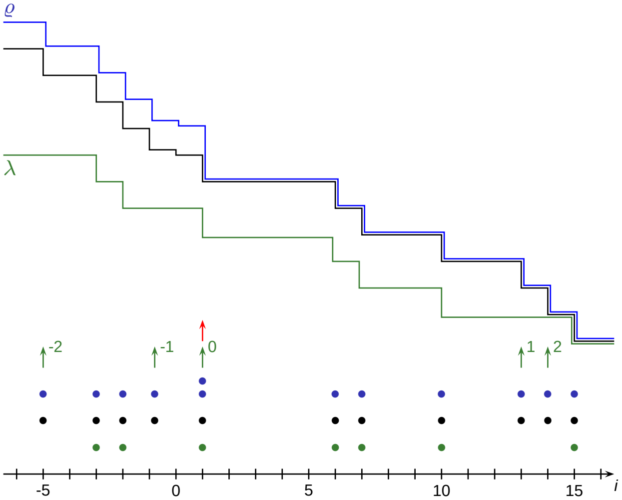

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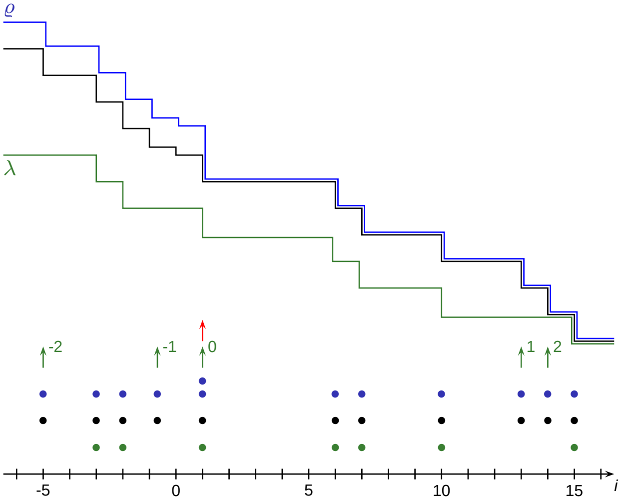

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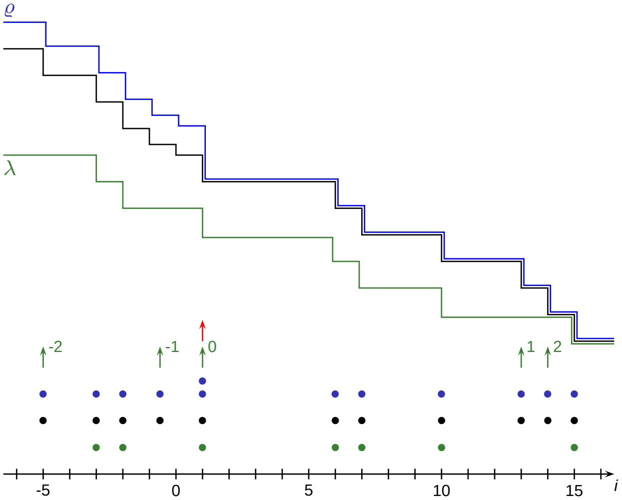

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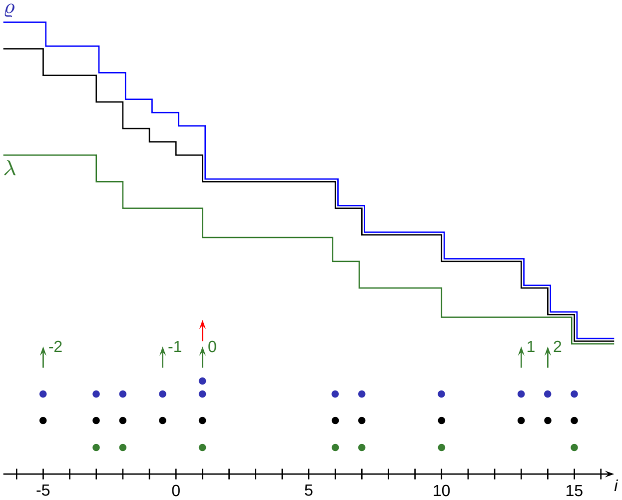

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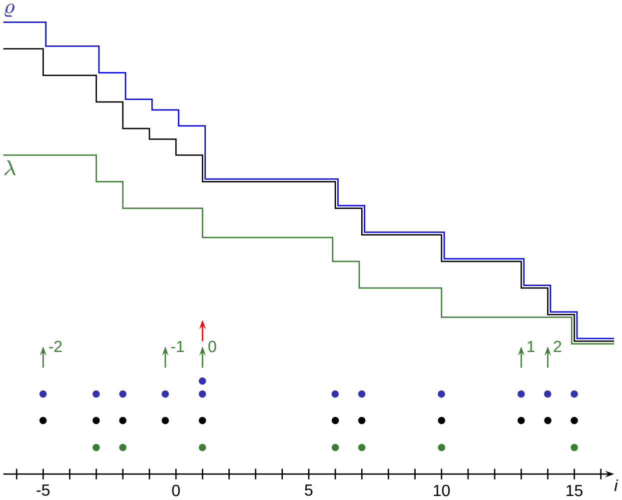

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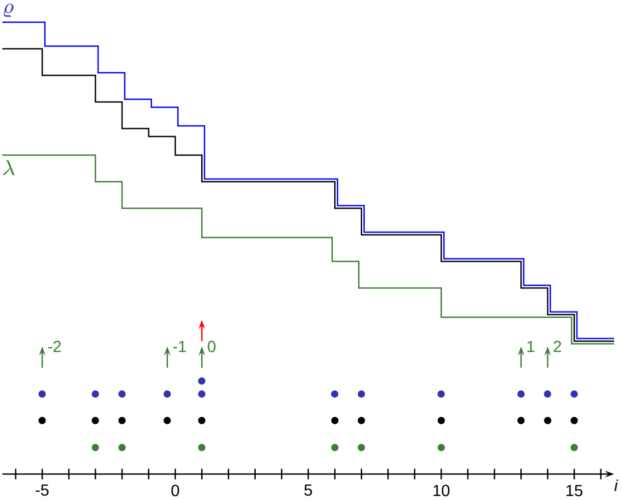

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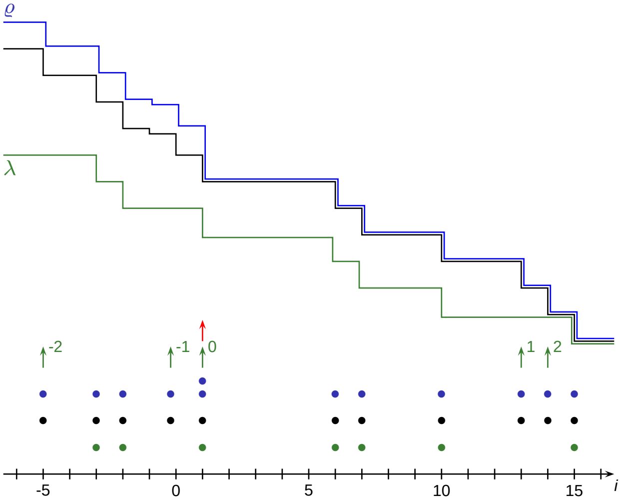

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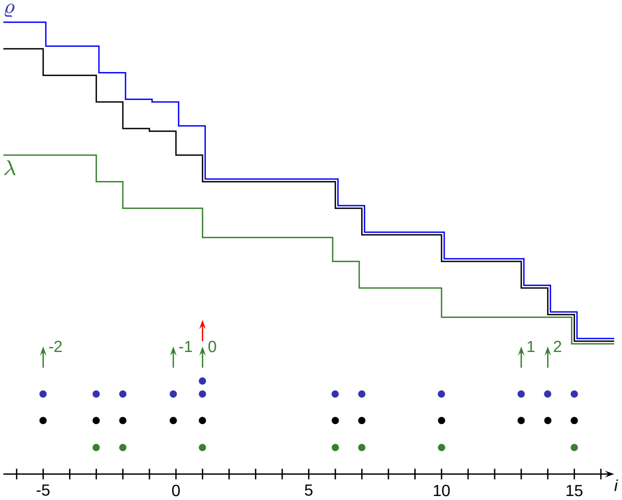

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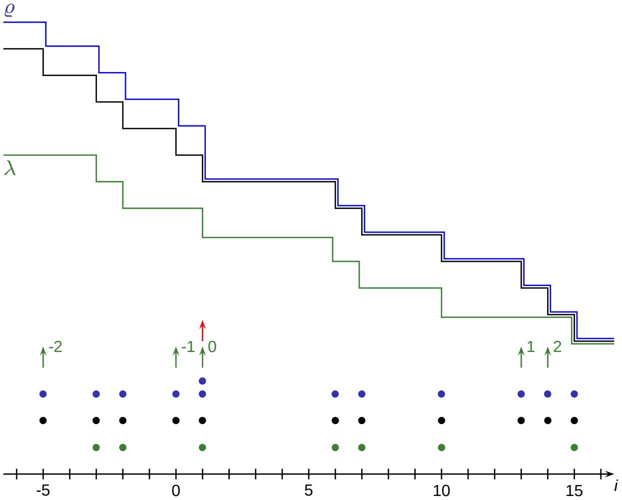

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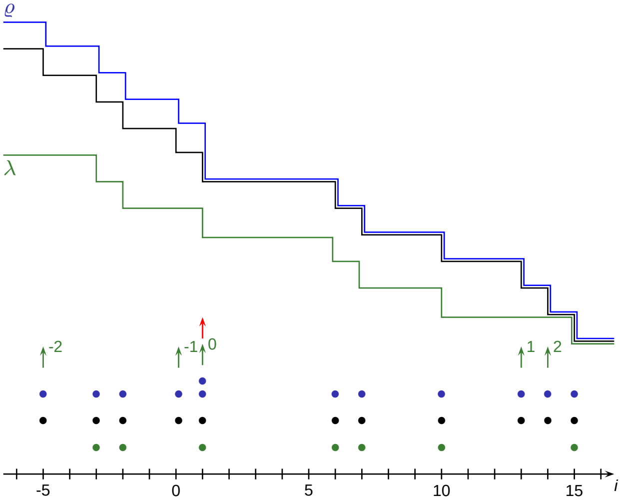

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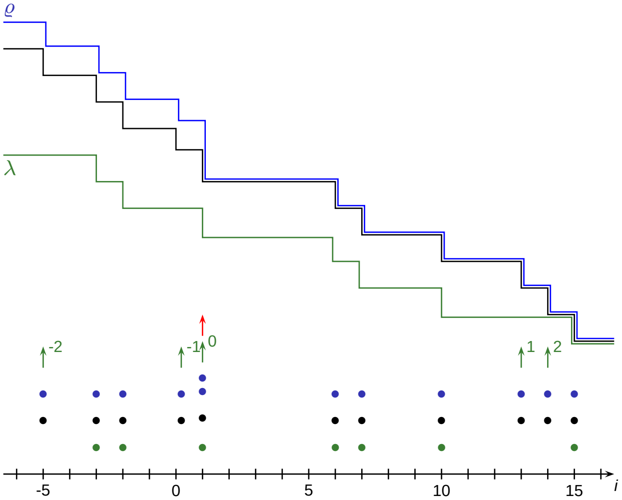

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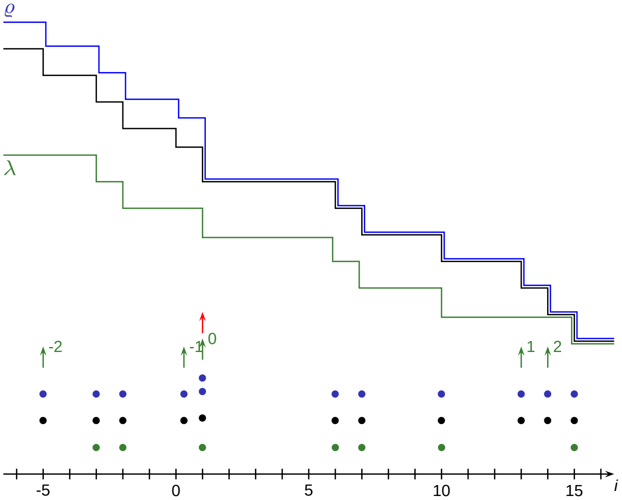

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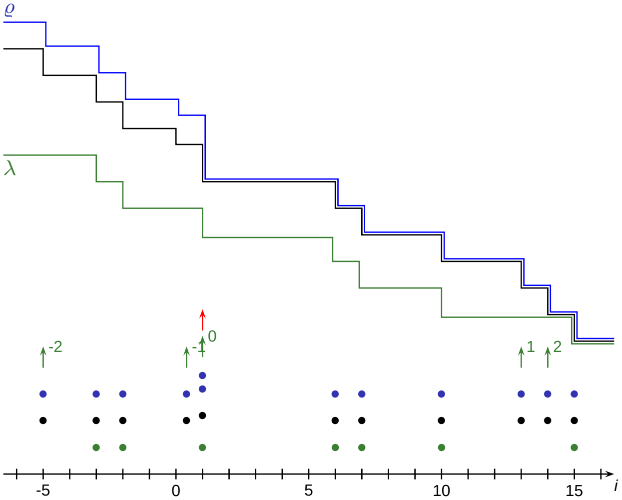

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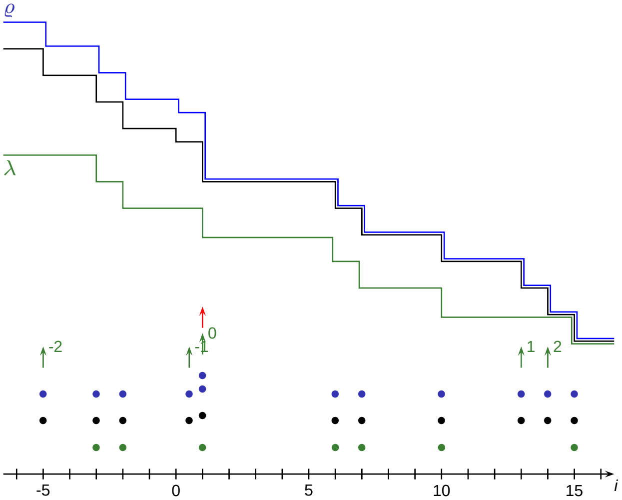

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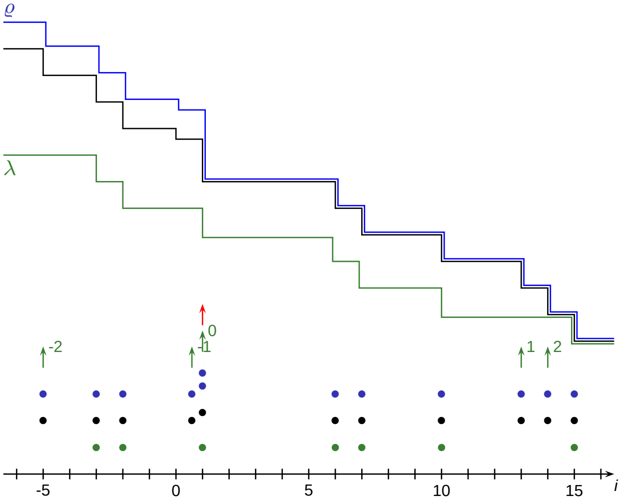

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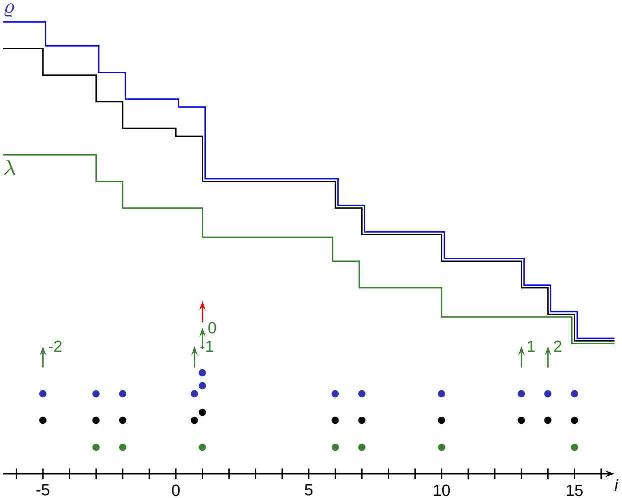

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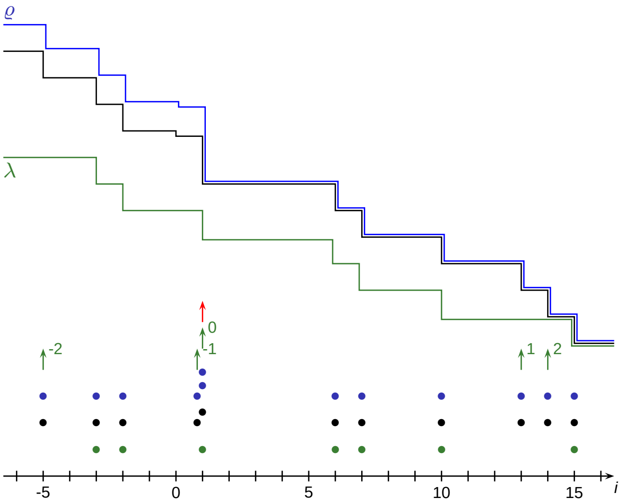

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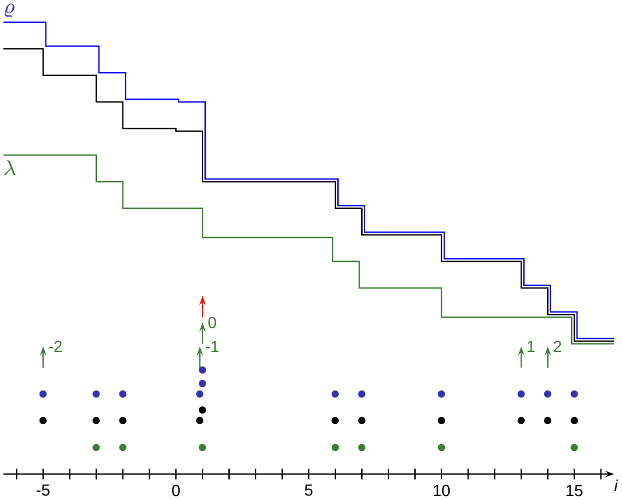

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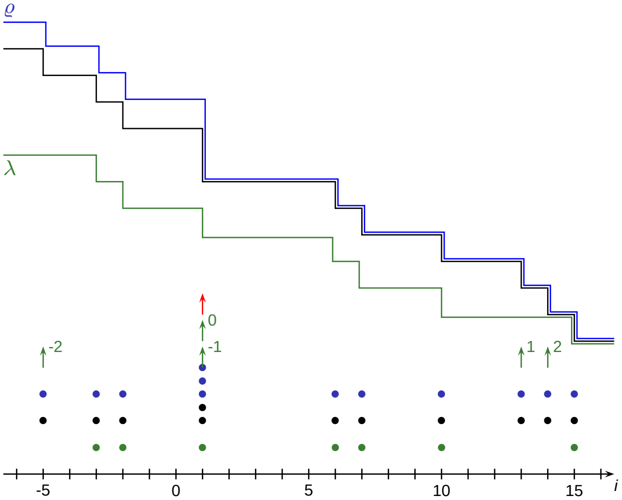

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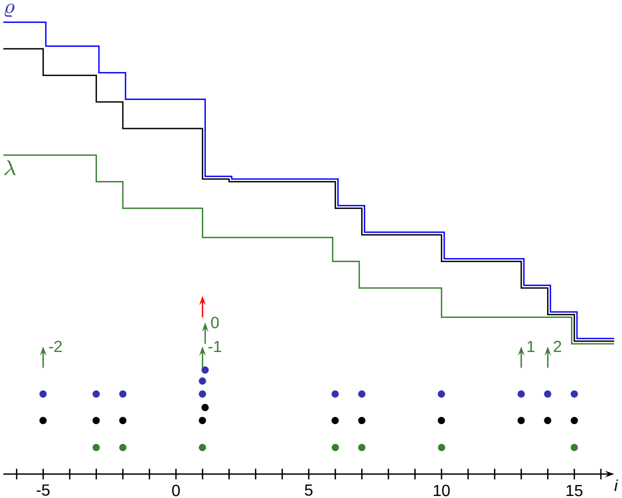

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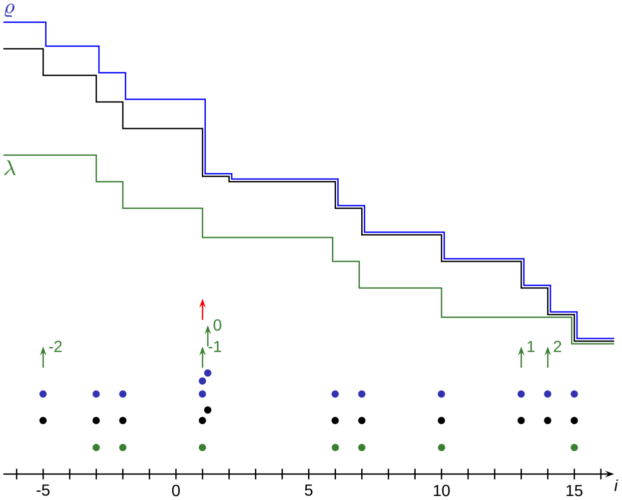

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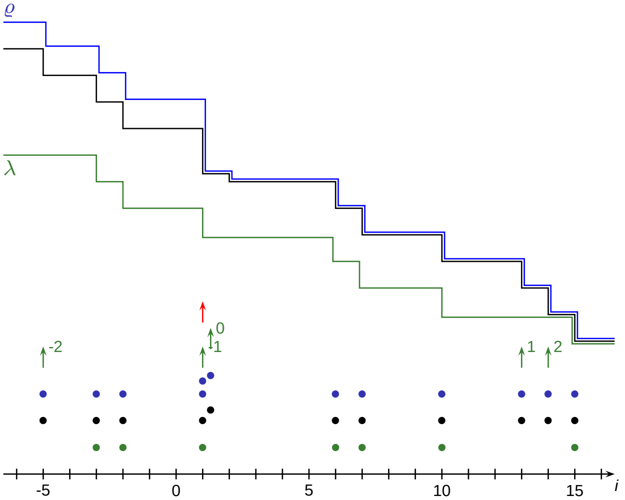

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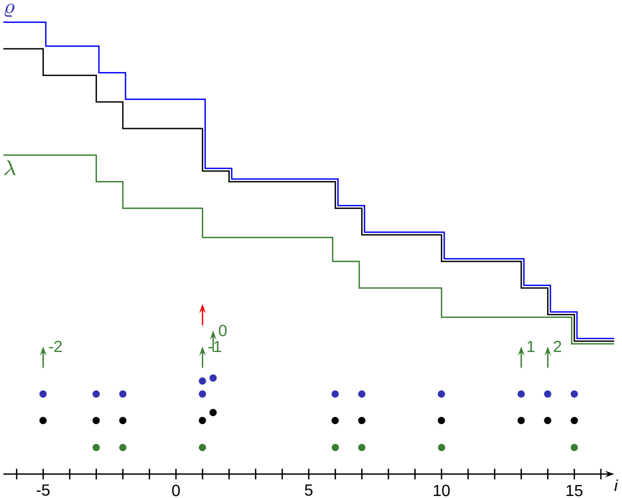

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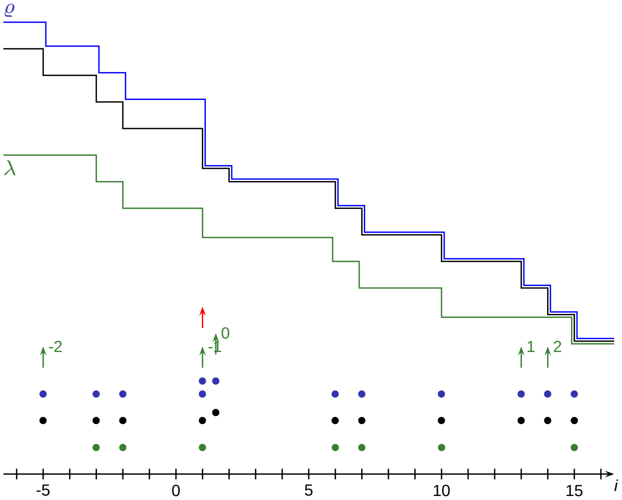

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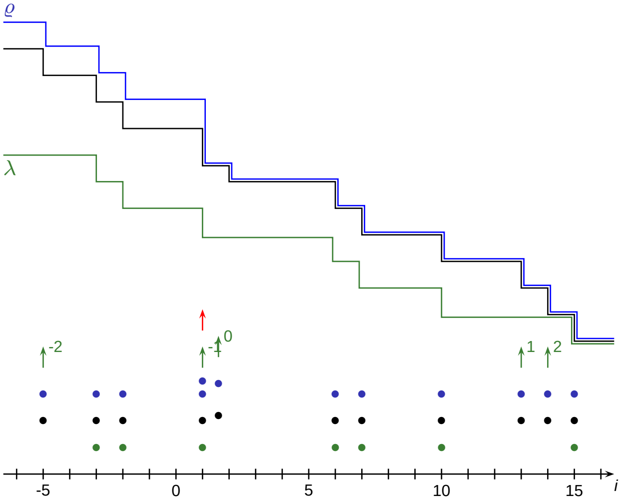

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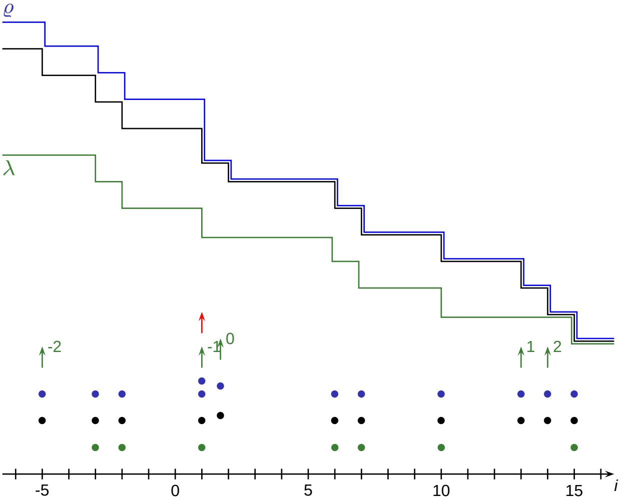

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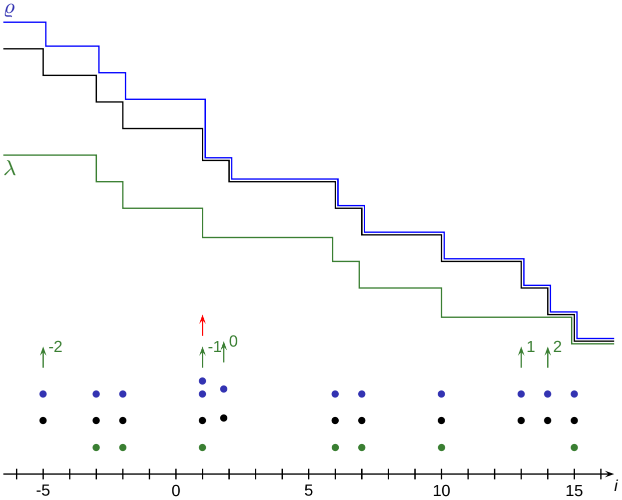

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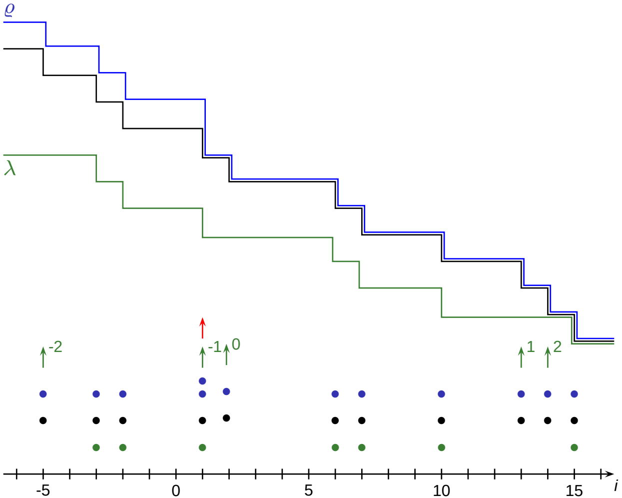

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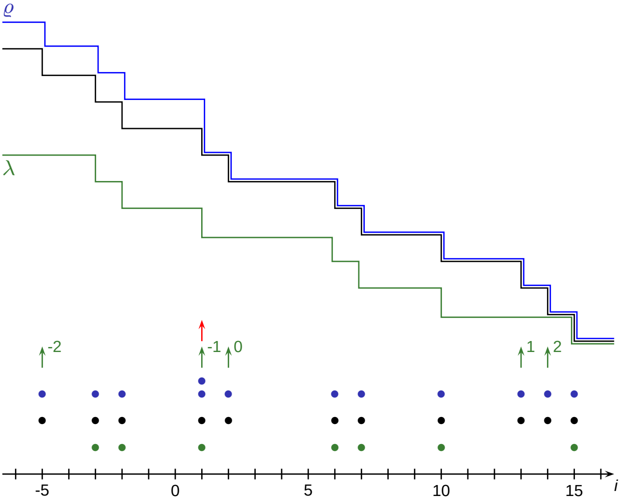

Couple three processes, and $X(t)$ to $Q(t)$.
We'll assume $Q(t) \leq X(t)$ can be achieved.

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Once we have $Q(t) \geq X(t)$ (convex) or $Q(t) \leq X(t)$ (concave),

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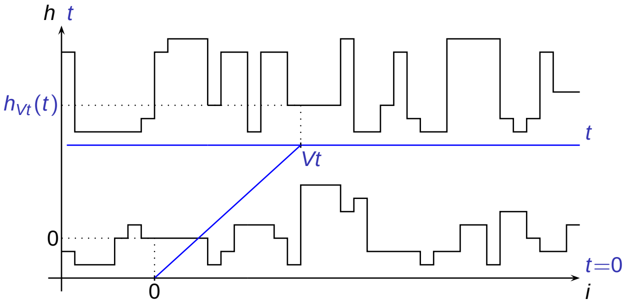

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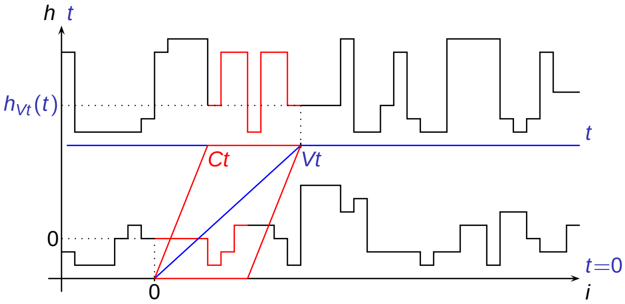

Initial fluctuations are transported along the characteristics on this scale.

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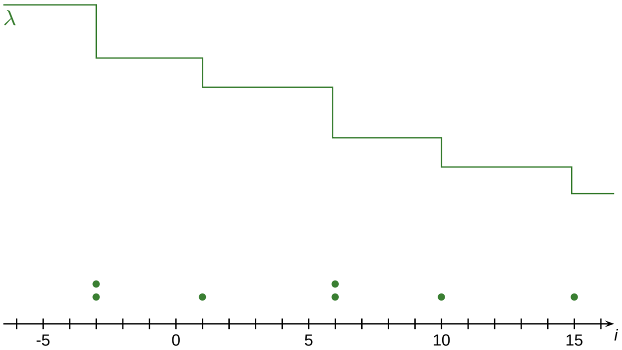
There are limit distribution results for TASEP by Johansson 2000, Prähofer and Spohn 2001, Ferrari and Spohn 2006. Their methods give limit distributions as well, but are very model-dependent: they rewrite the model as a determinantal process, and perform asymptotic analysis of the determinants.

Many second class particles



Second class particle current: difference in growth.

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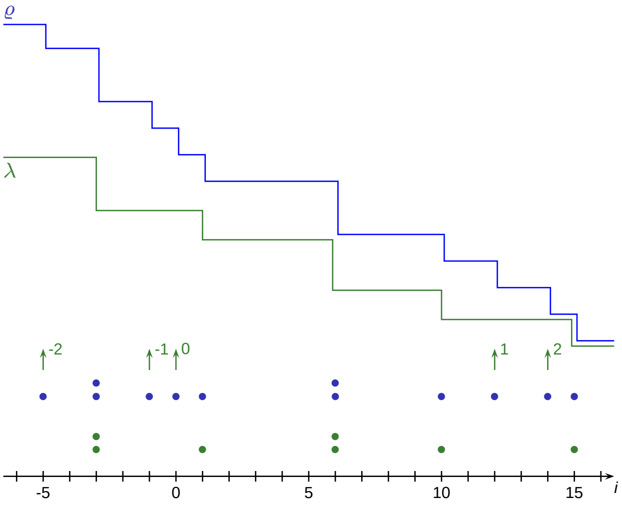

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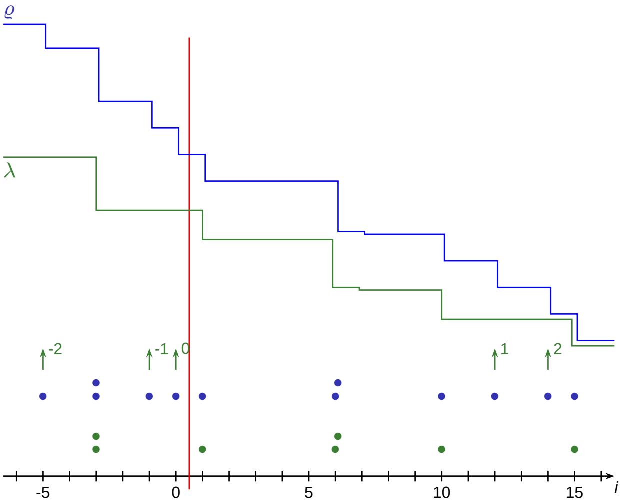

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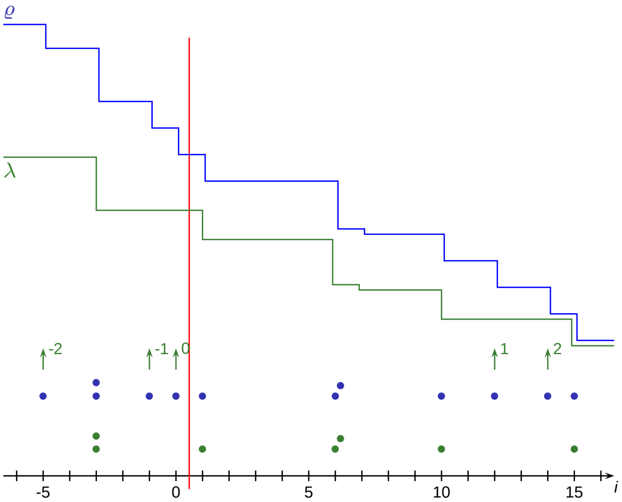

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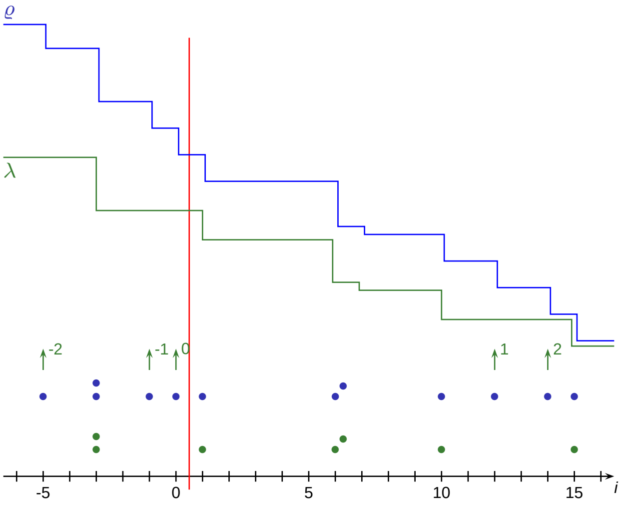

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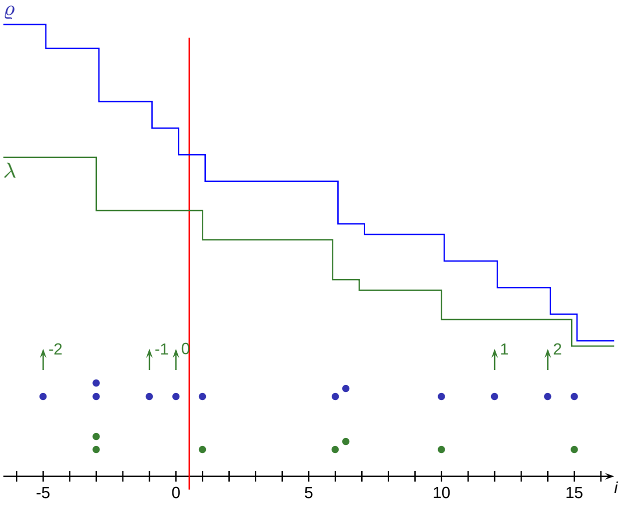

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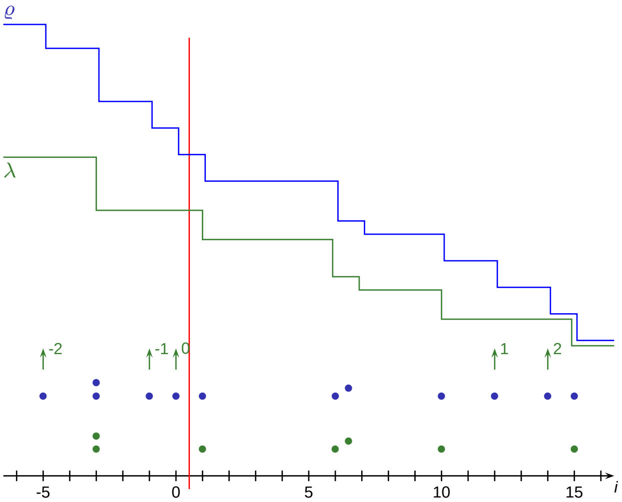

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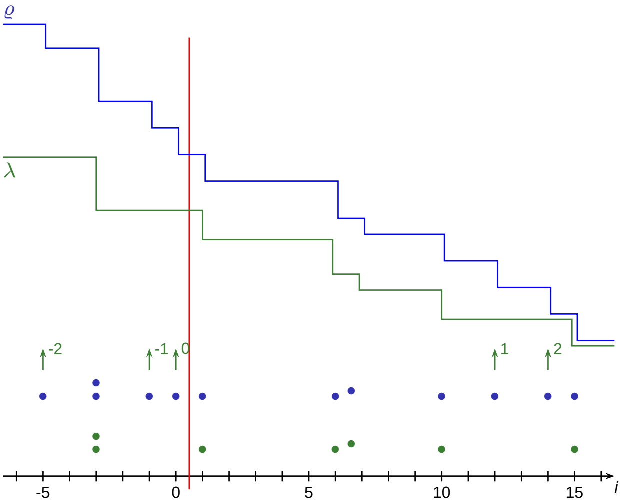

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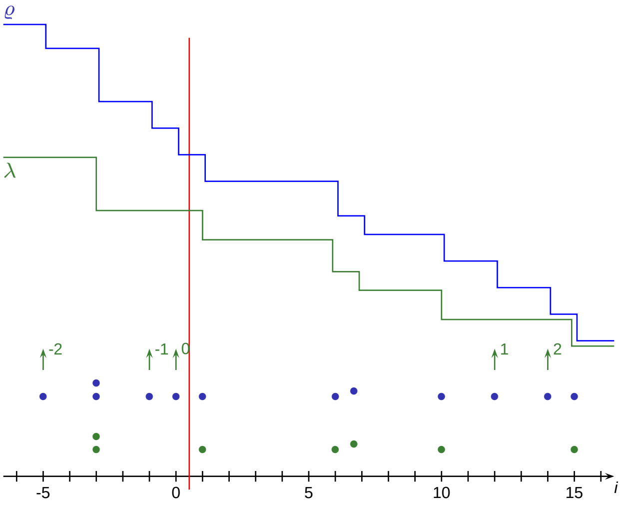

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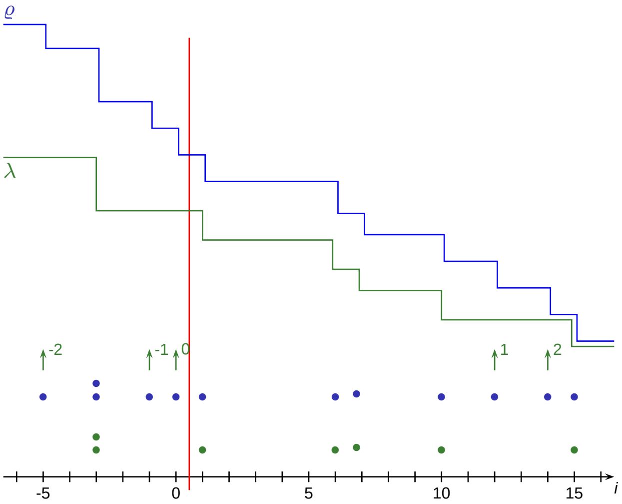

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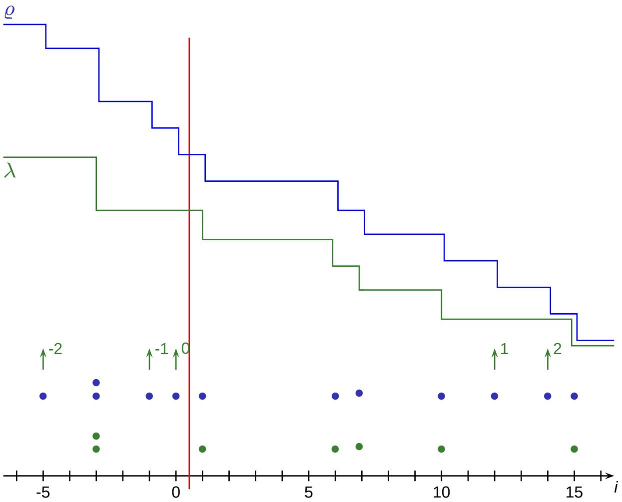

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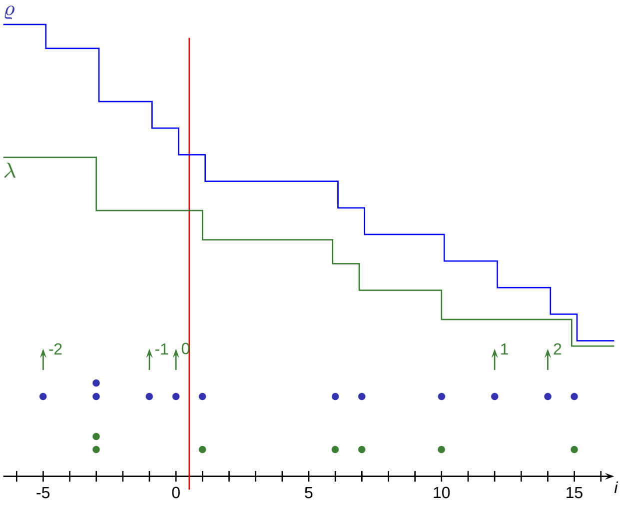

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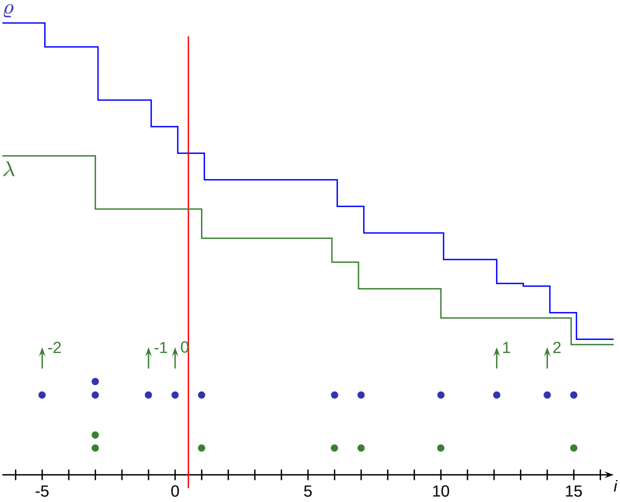

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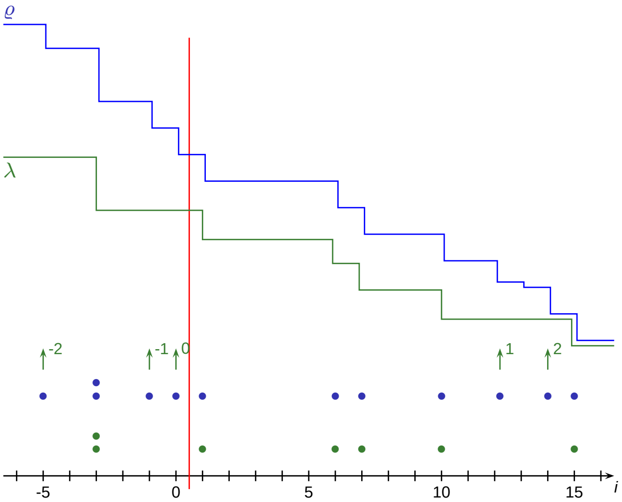

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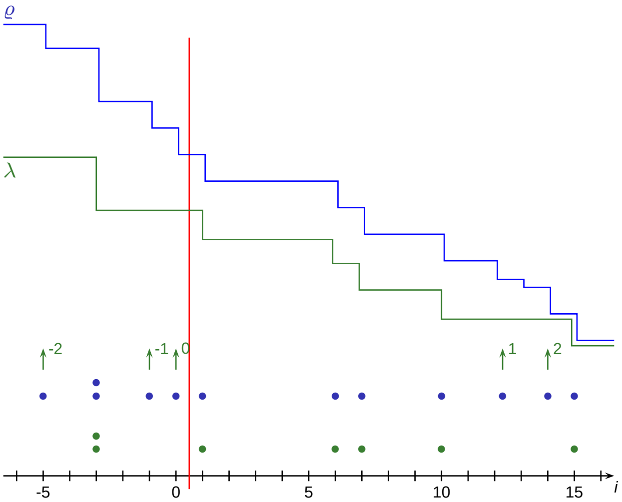

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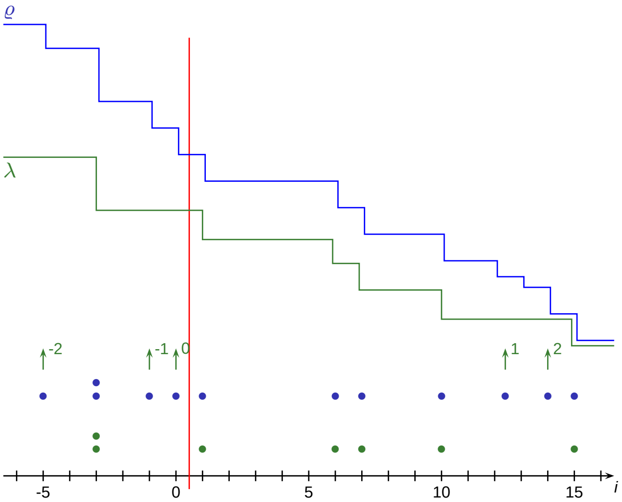

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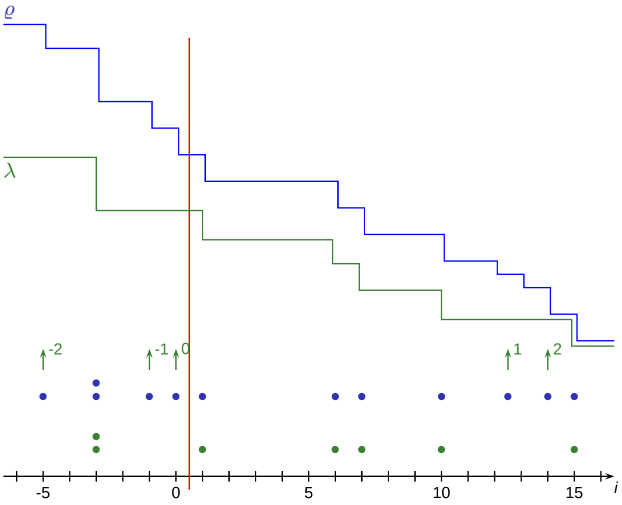

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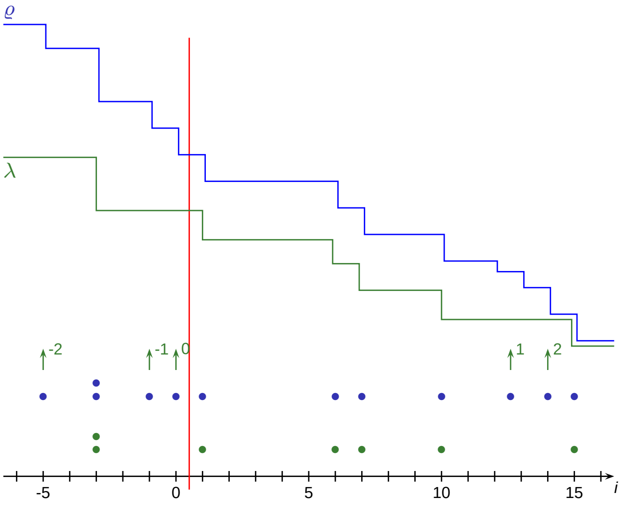

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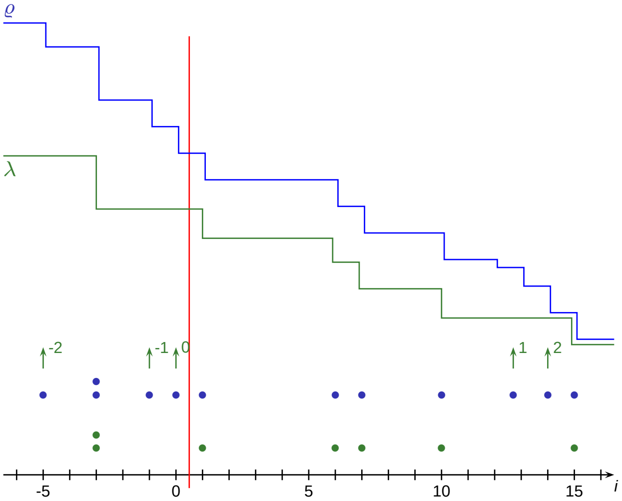

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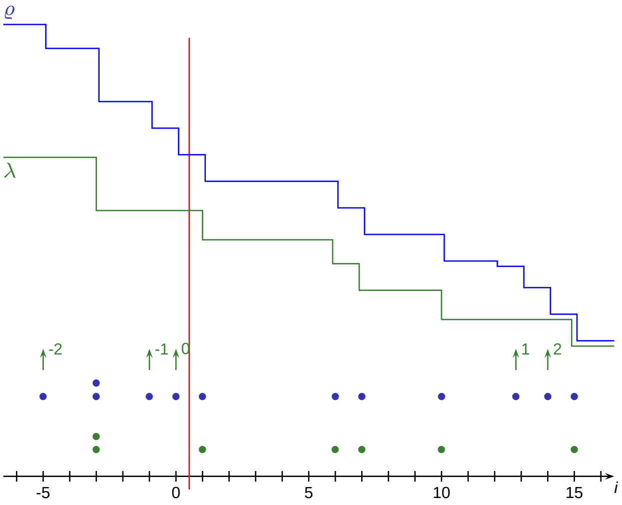

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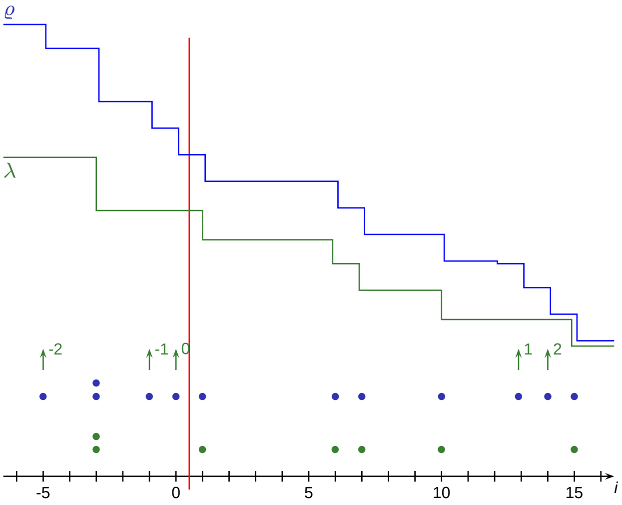

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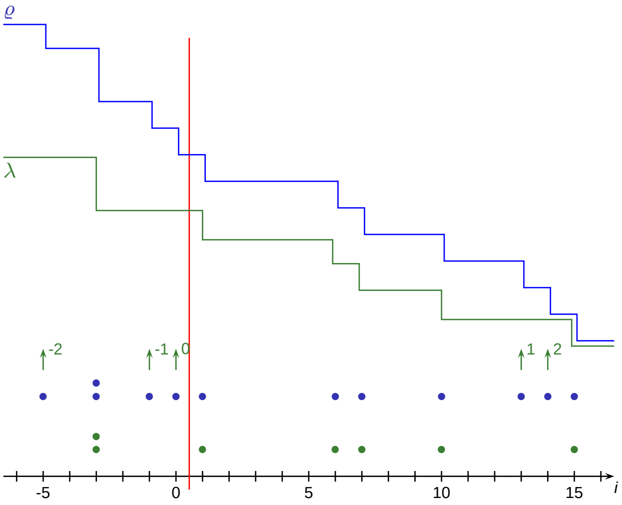

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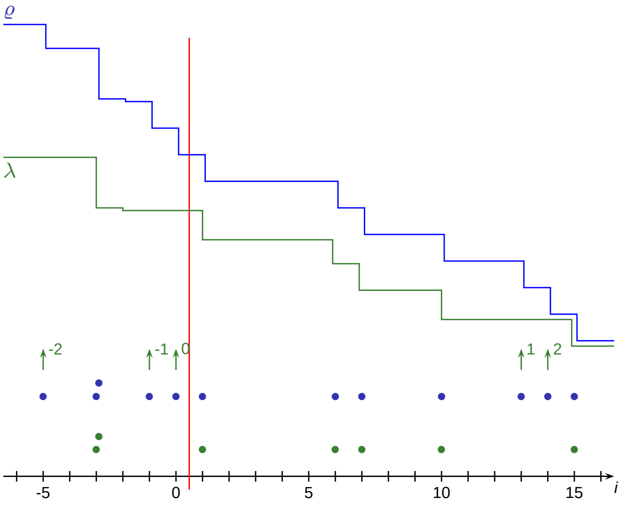

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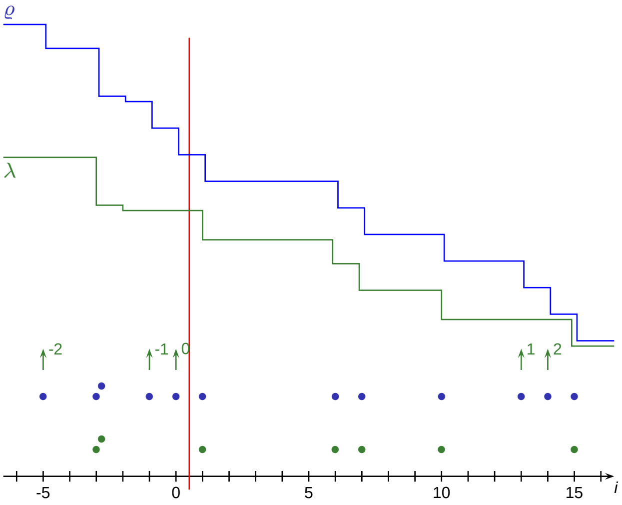

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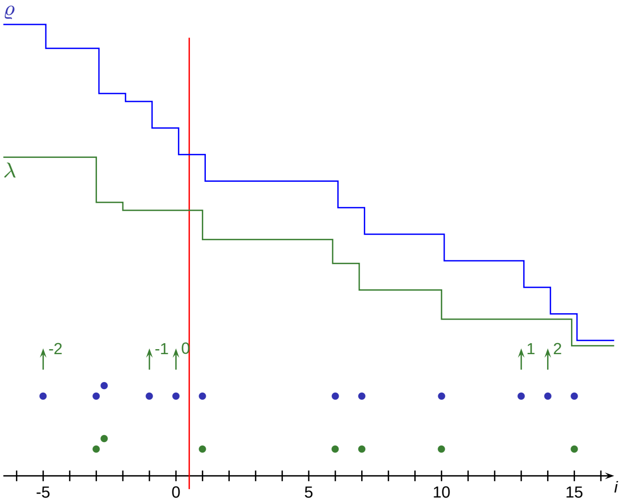

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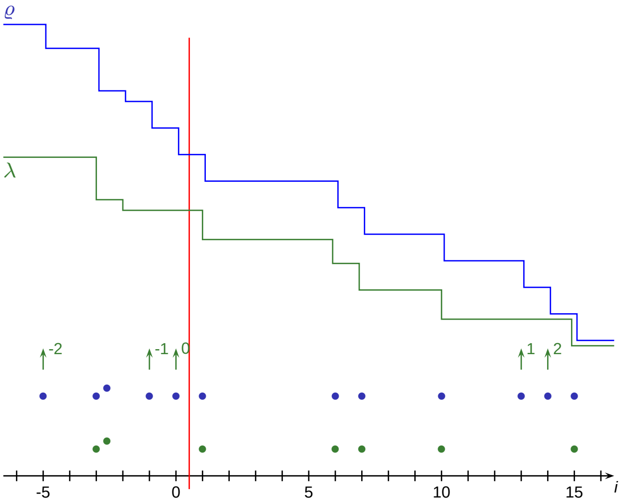

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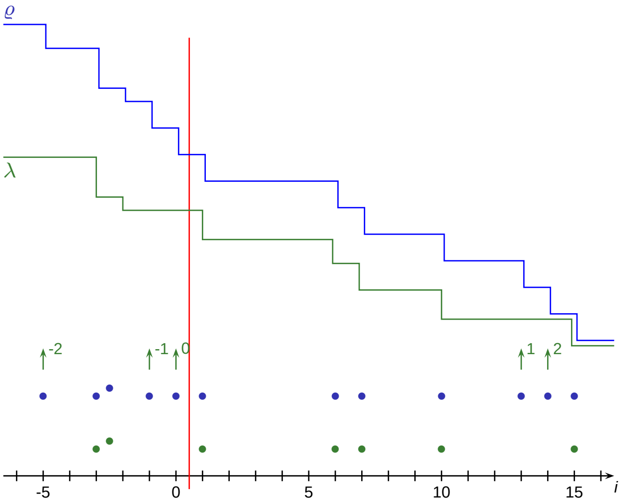

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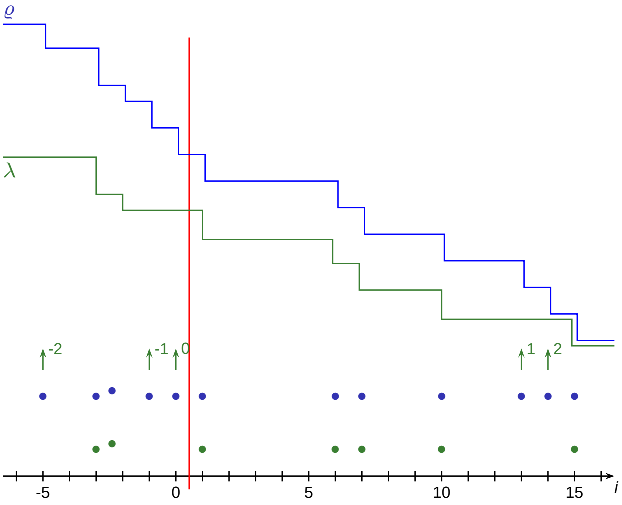

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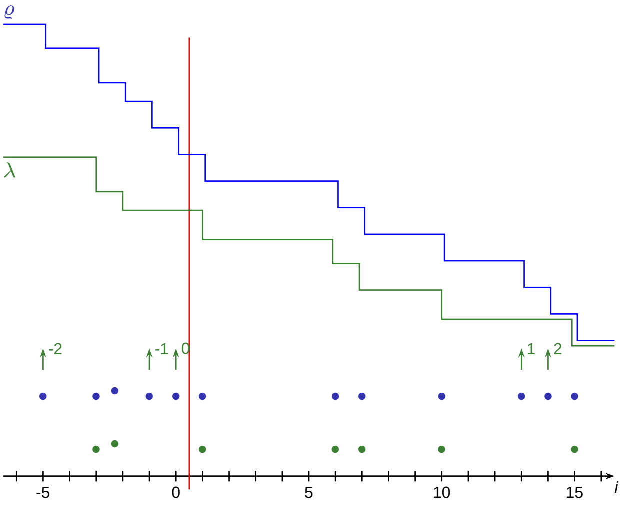

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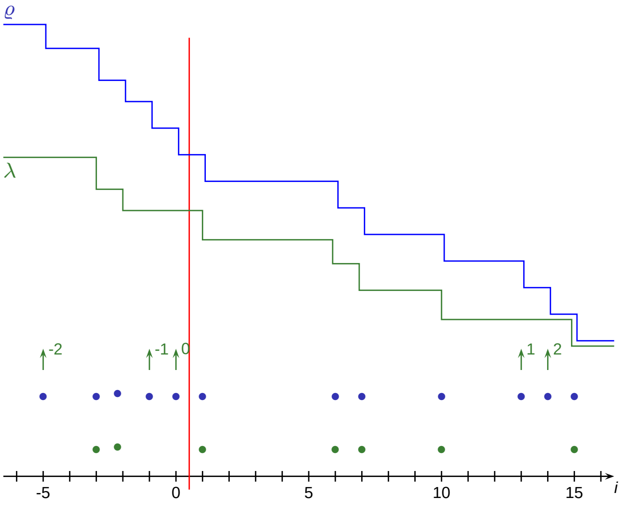

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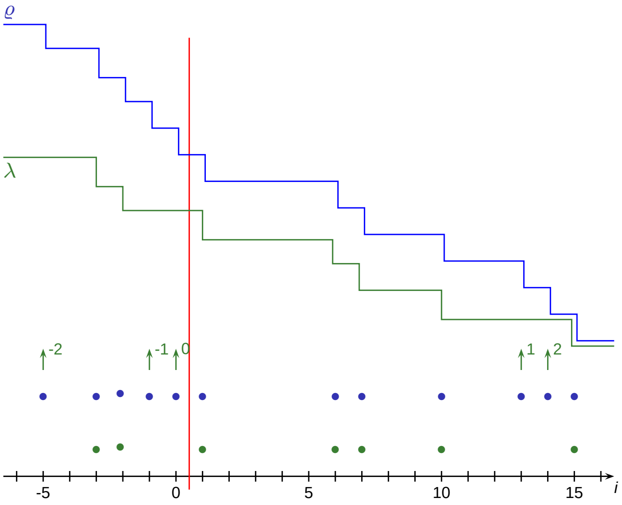

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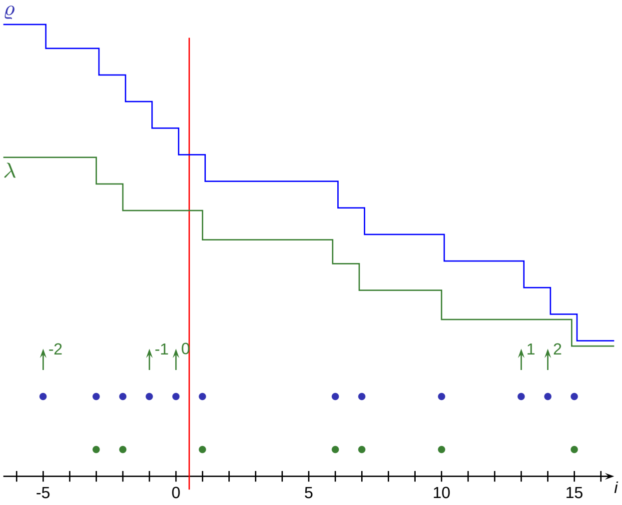

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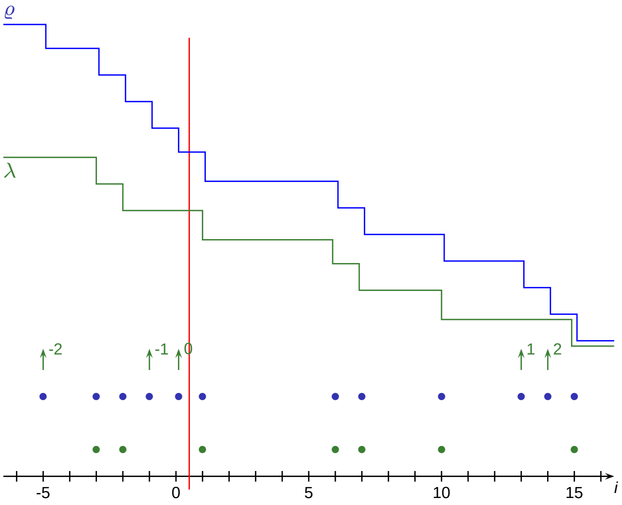

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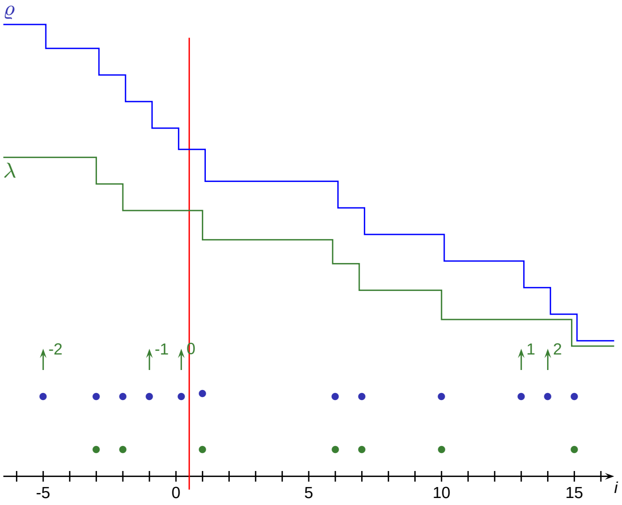

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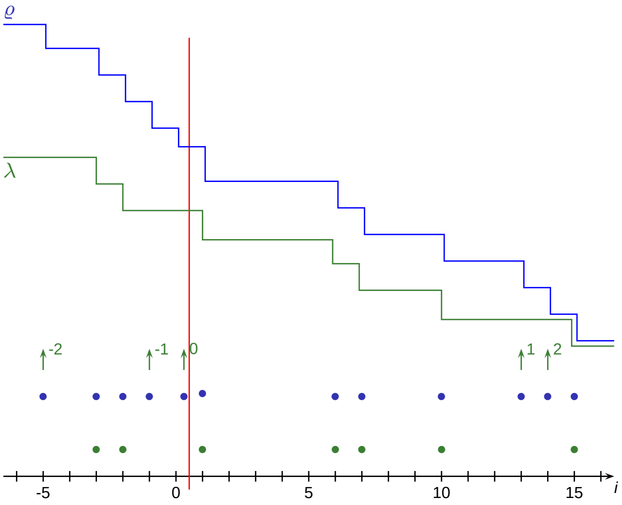

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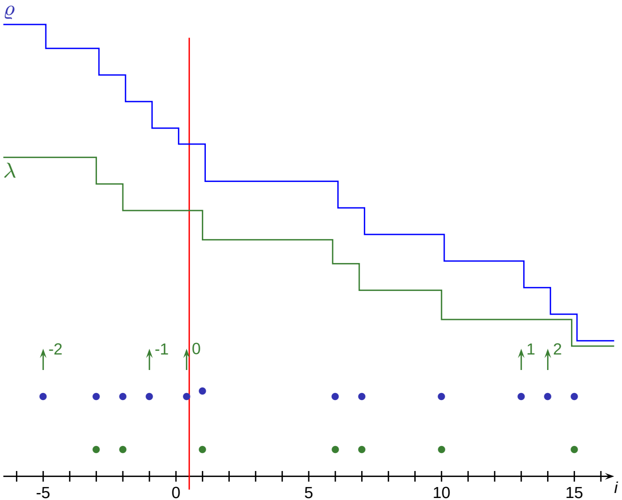

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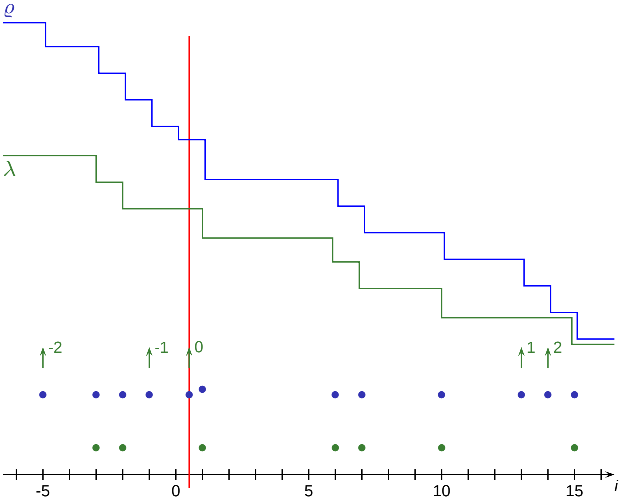

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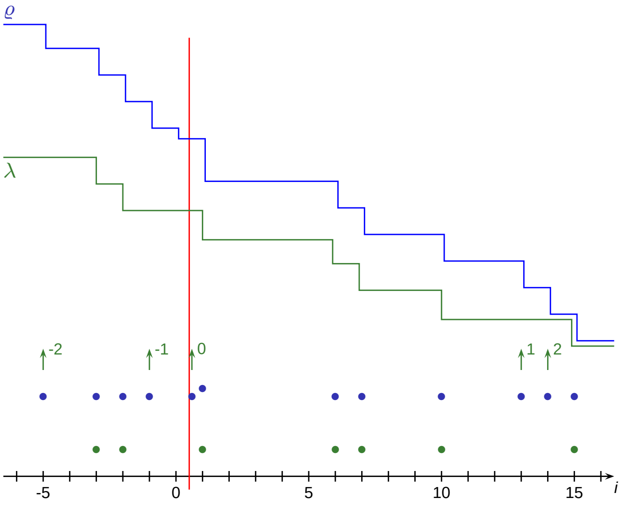

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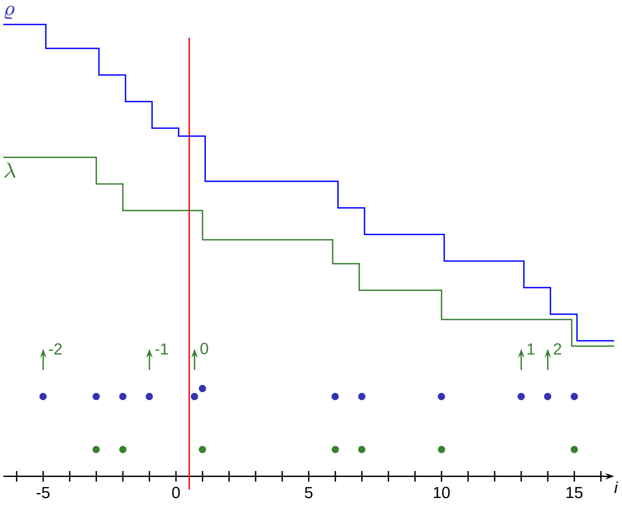

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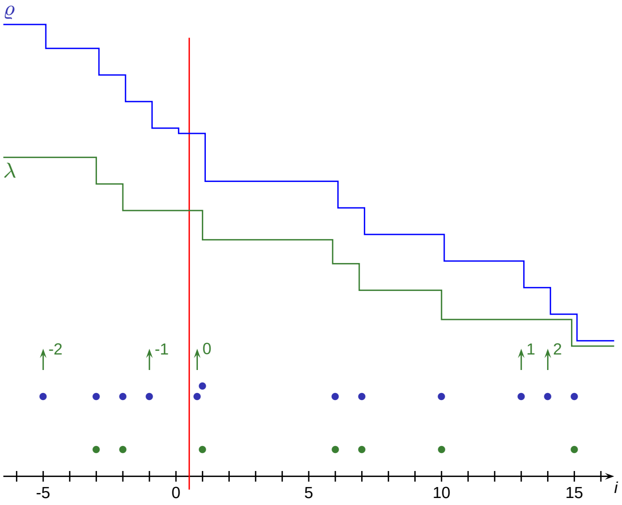

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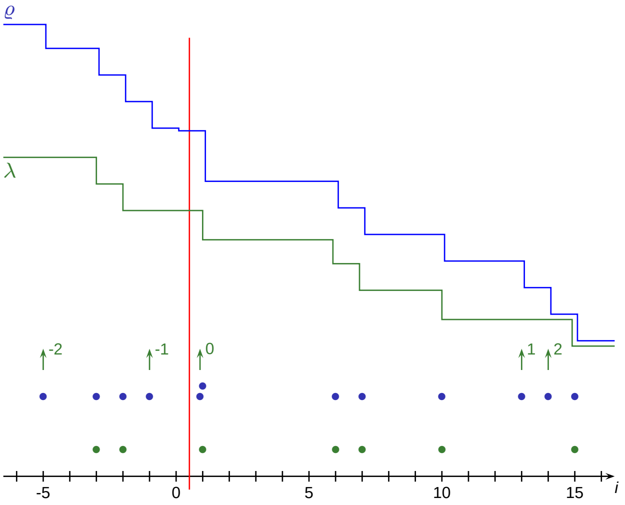

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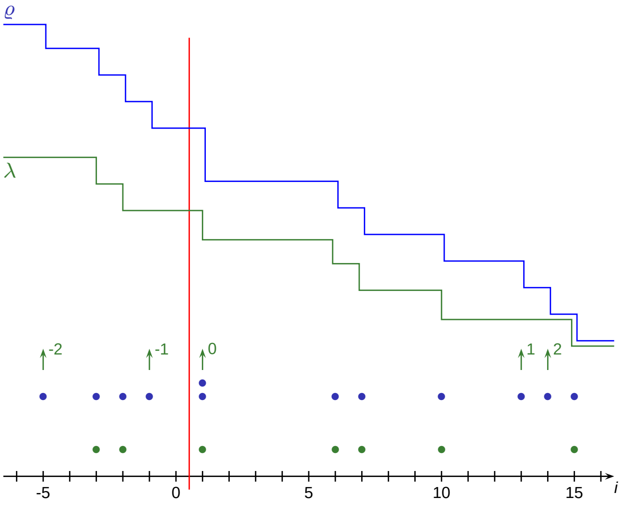

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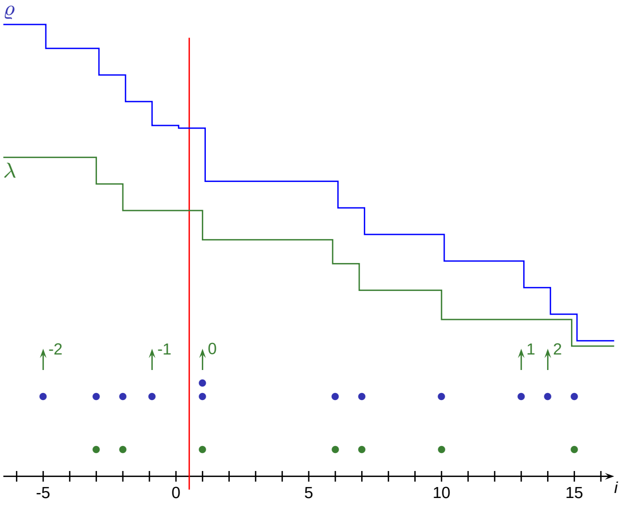

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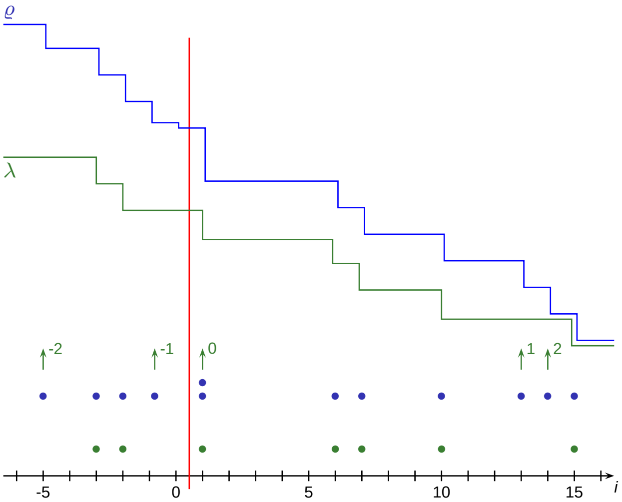

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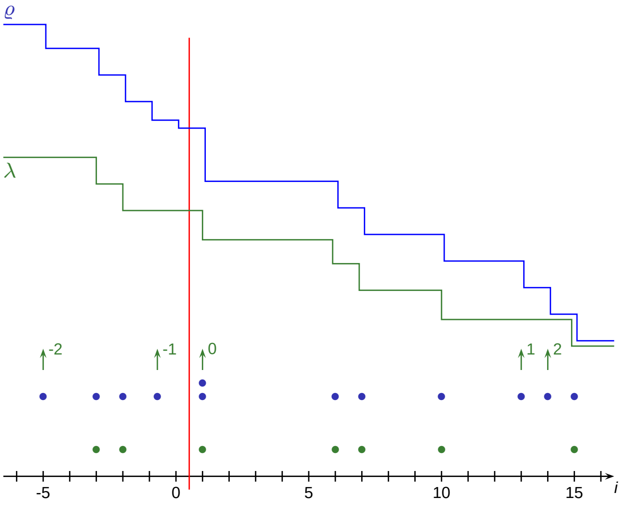

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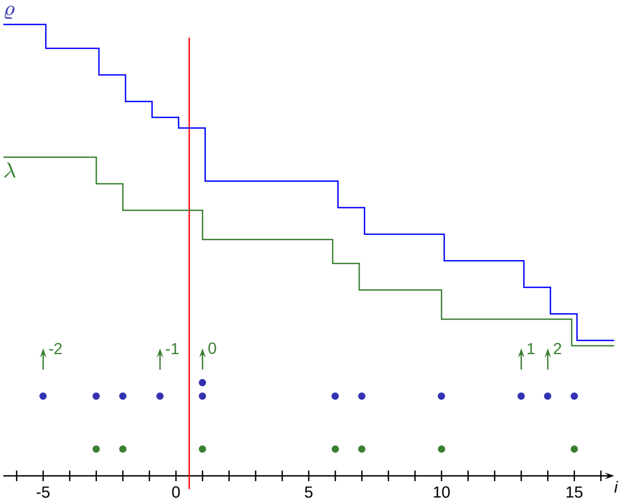

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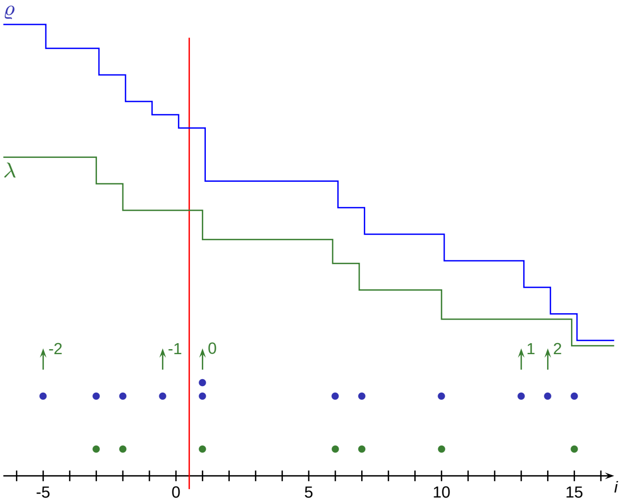

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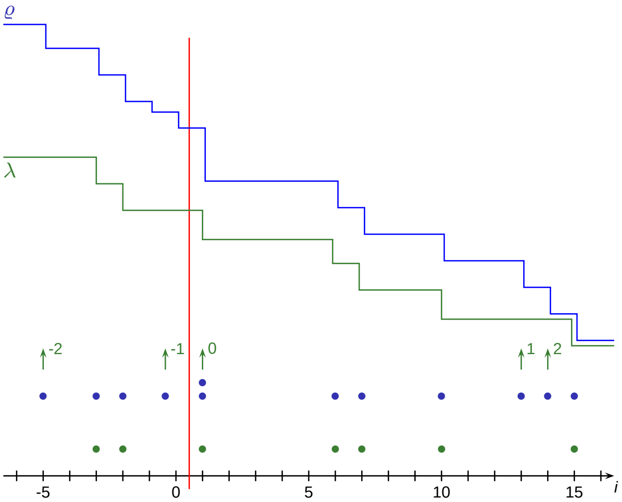

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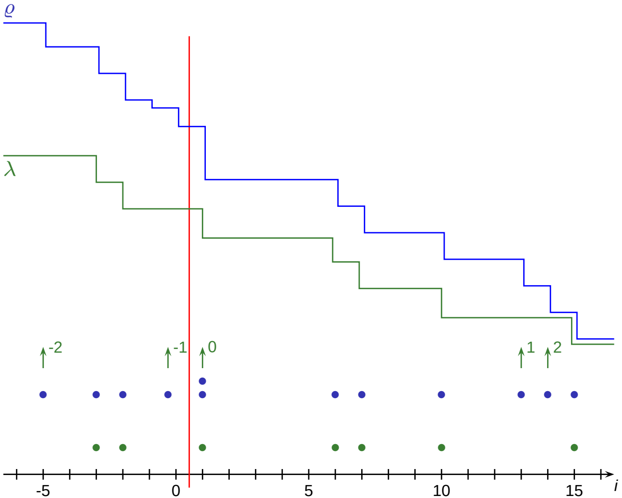

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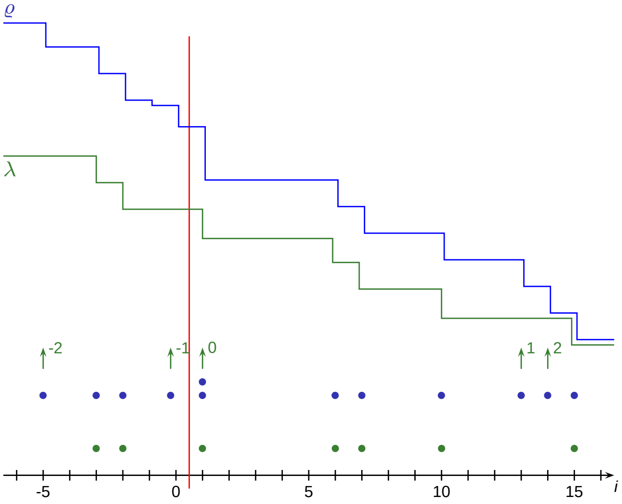

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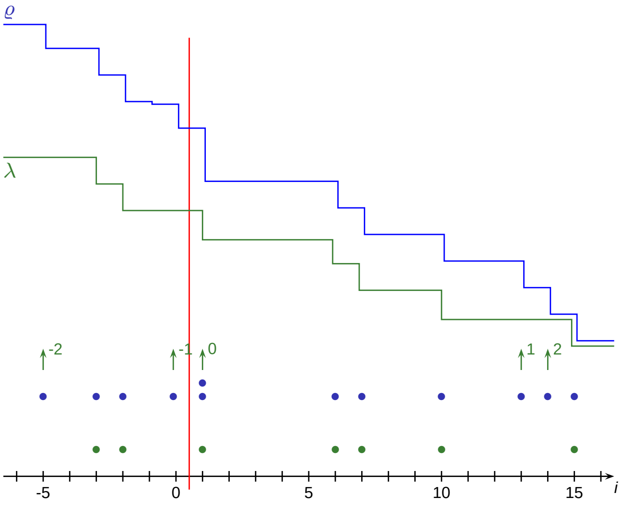

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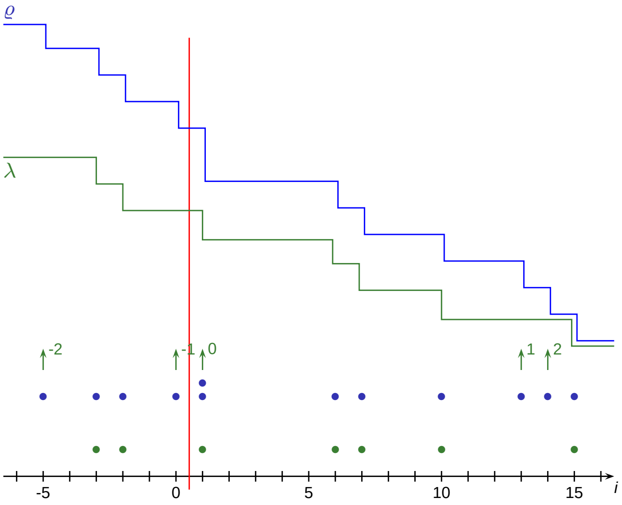

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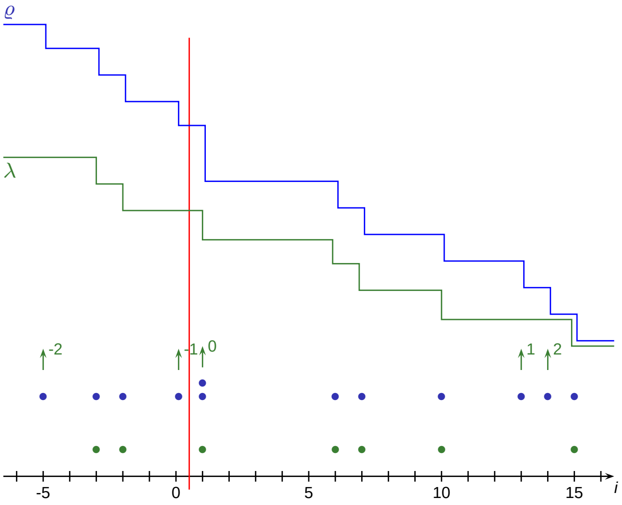

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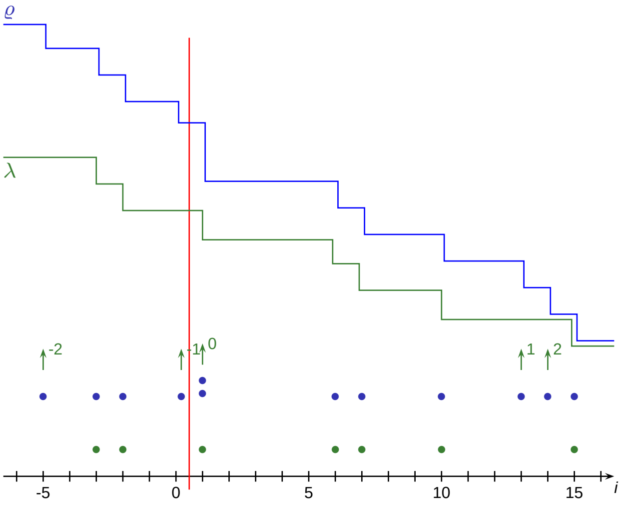

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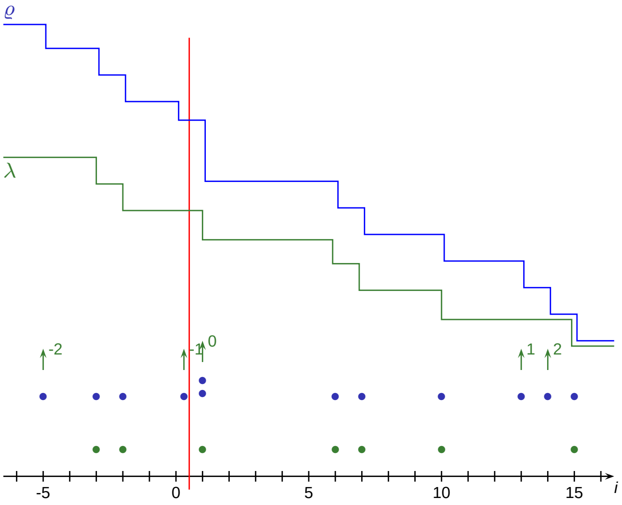

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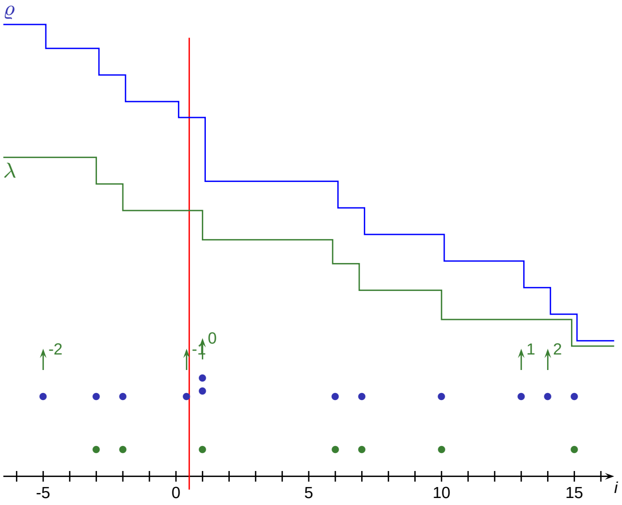

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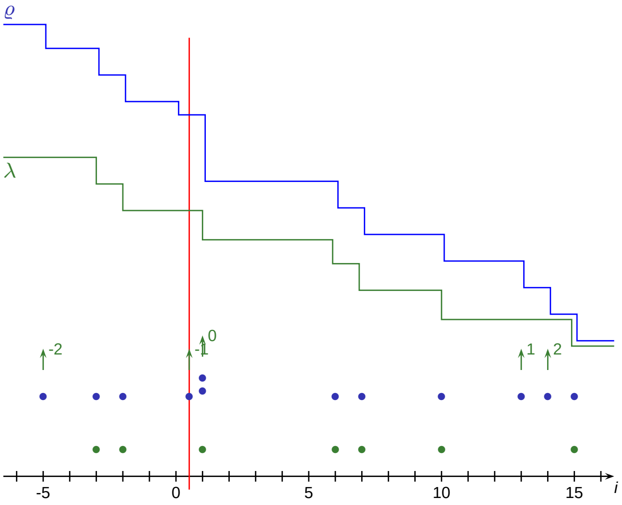

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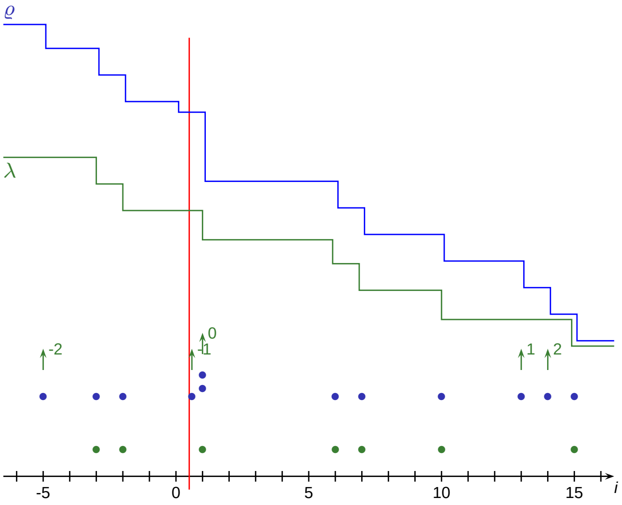

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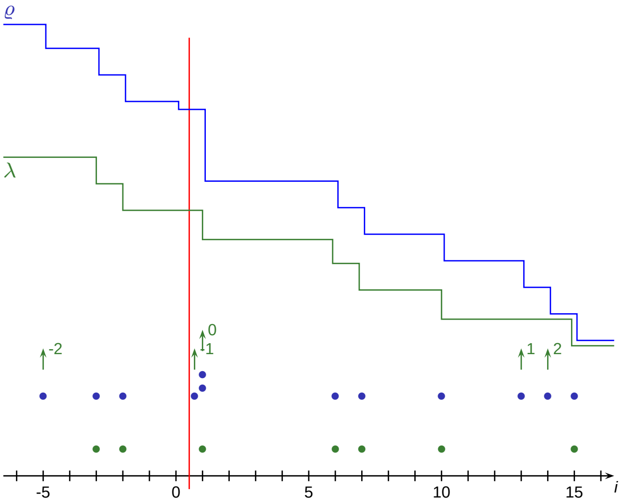

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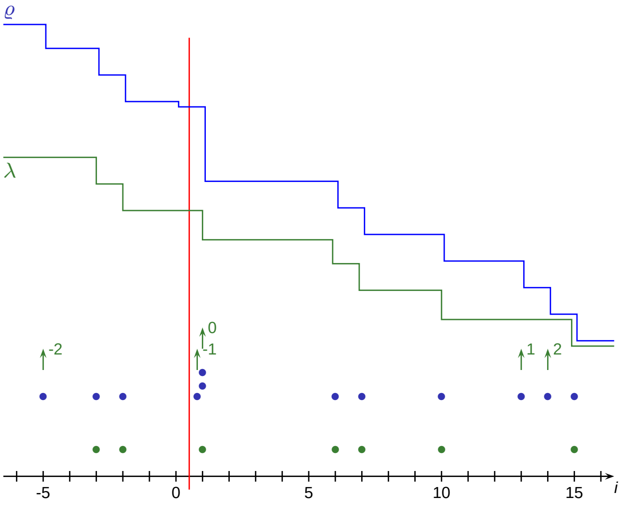

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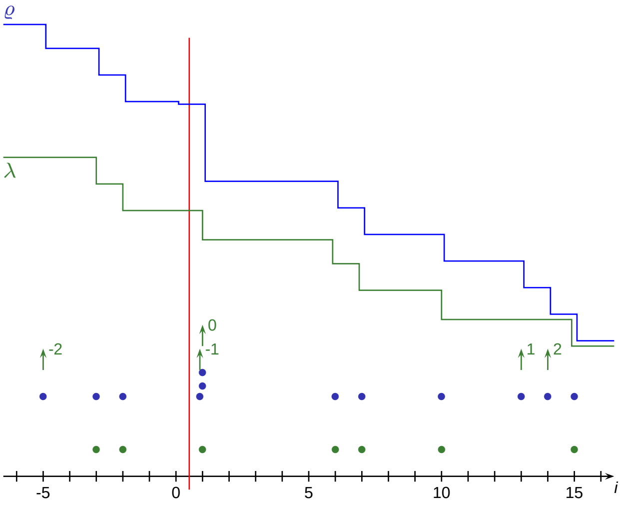

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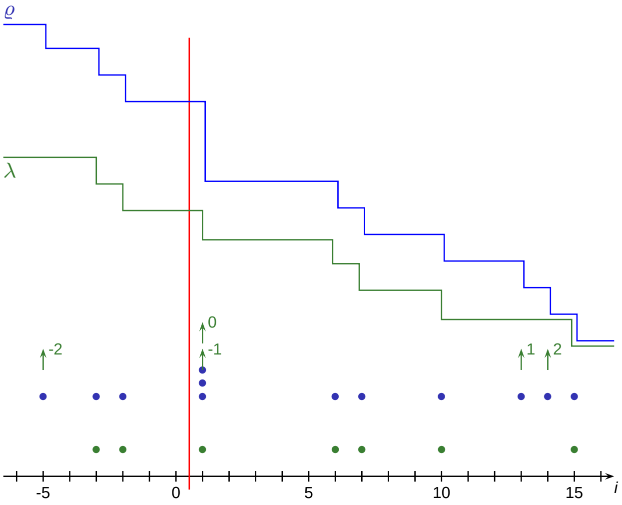

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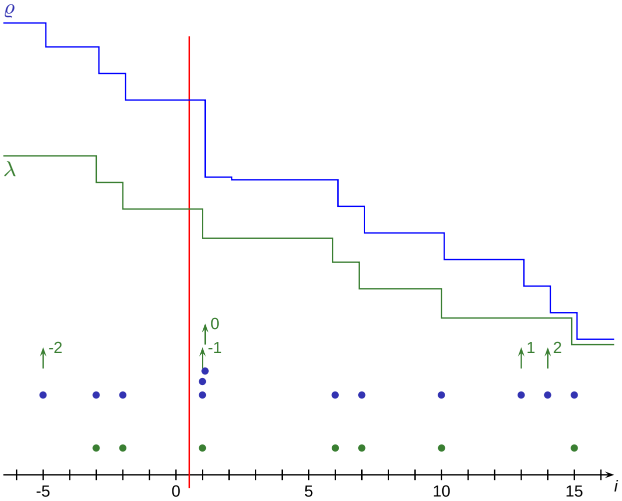

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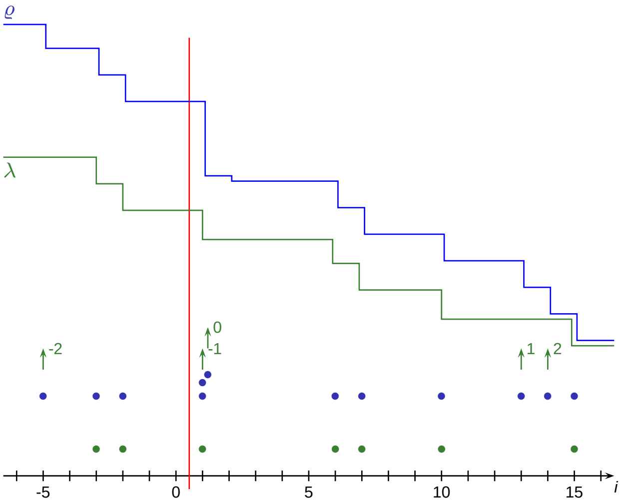

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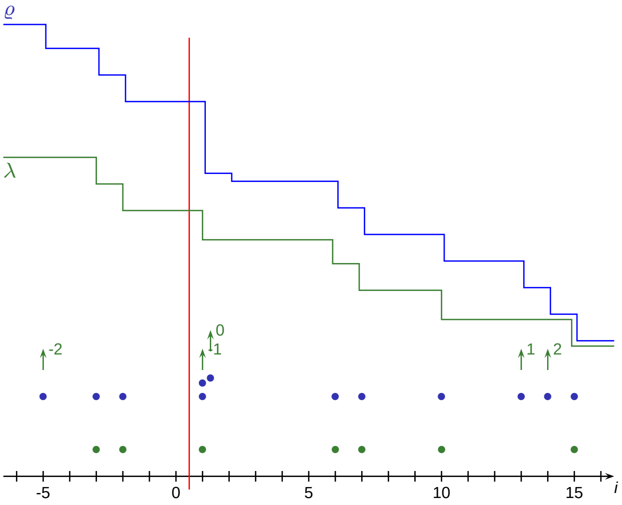

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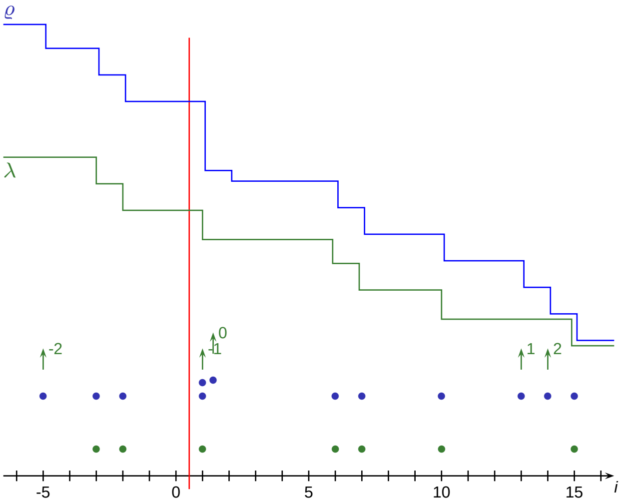

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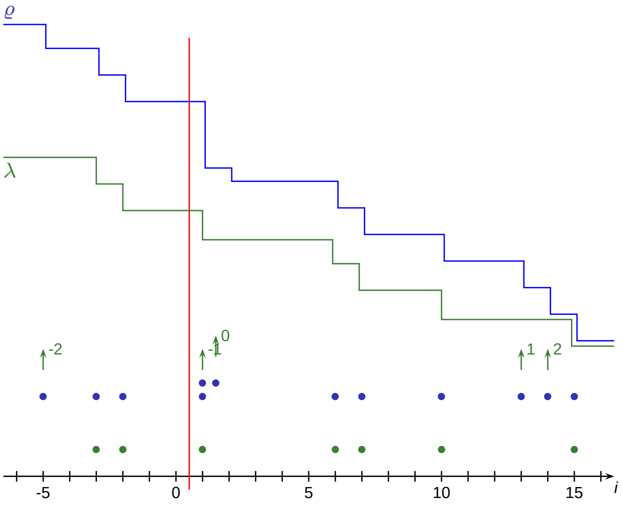

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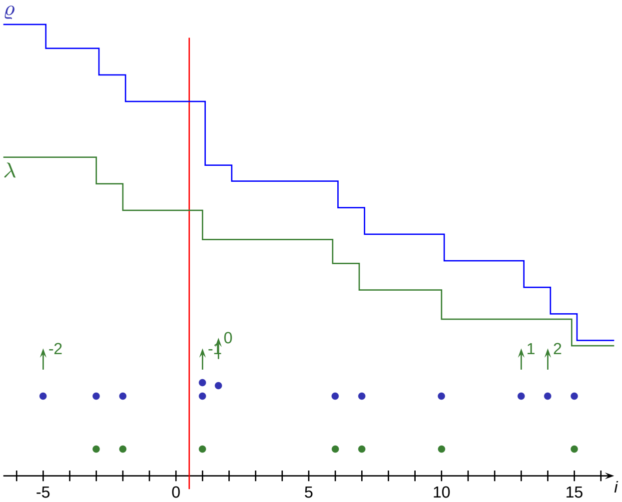

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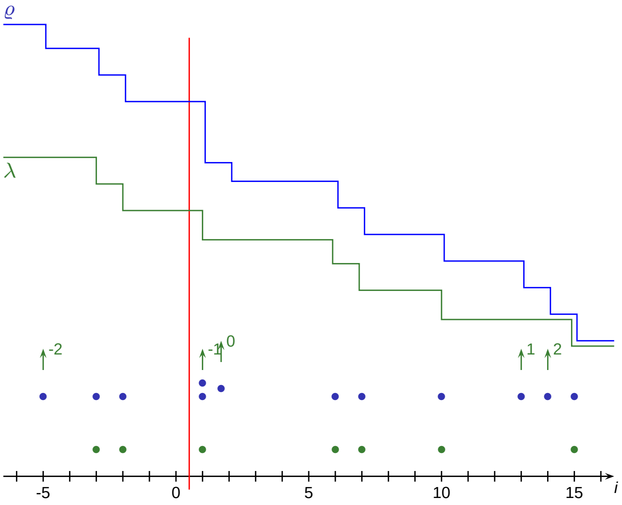

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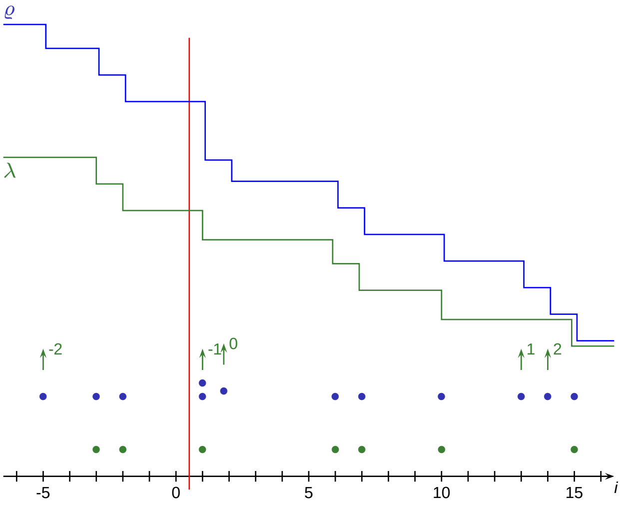

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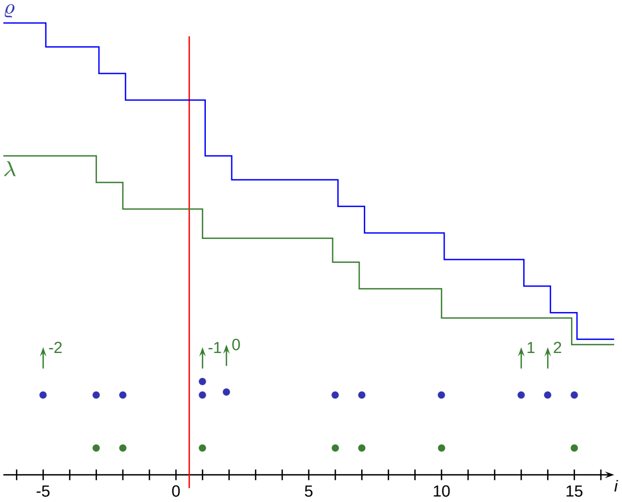

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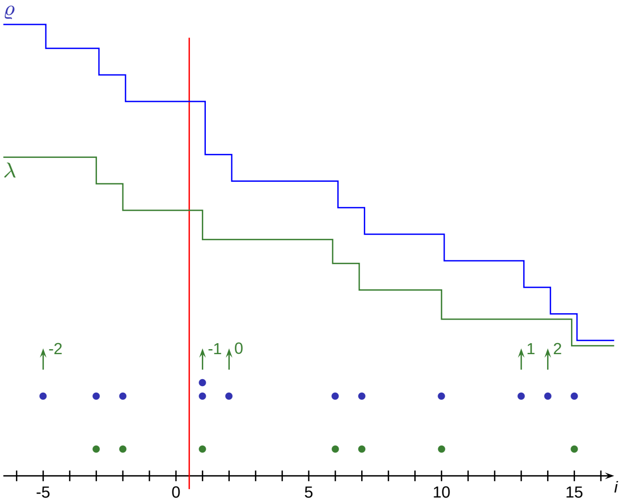

Second class particle current: difference in growth.

Upper bound

$\mathbf{P}\{Q(t)$ is too large$\}$

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Upper bound

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$$\le \mathbf{P}\{\text{too many } \uparrow\text{'s have crossed } Ct\}$$
$$\le \mathbf{P}\{h_{Ct}(t) - h_{Ct}(t) \text{ is too large}\}.$$

Upper bound

$$
\begin{aligned}
\mathbf{P}\{Q(t) \text{ is too large}\} &\le \mathbf{P}\{X(t) \text{ is too large}\}\\
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Centering $h_{Ct}(t) - h_{Ct}(t)$ brings in a second-order Taylor-expansion of $H(\varrho)$.

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The computations result in (remember $\mathbf{E}(Q(t)) = Ct$)

$$\mathbf{P}\{Q(t) - Ct \ge u\} \le c \cdot \frac{t^2}{u^4} \cdot \mathbf{Var}(h_{Ct}(t)).$$

Upper bound

Theorem (B. - Seppäläinen; also ideas from B. Tóth, H. Spohn and M. Prähofer)

Started from (almost) equilibrium,

$$\mathbf{Var}(h_{Ct}(t)) = c \cdot \mathbf{E}|Q(t) - C \cdot t|$$

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Hence proceed with

$$\mathbf{P}\{Q(t) - Ct \geq u\} \leq c \cdot \frac{t^2}{u^4} \cdot \mathbf{Var}(h_{Ct}(t))$$
$$= c \cdot \frac{t^2}{u^4} \cdot \mathbf{E}|Q(t) - C \cdot t|.$$

Upper bound

With

$$\widetilde{Q}(t) := Q(t) - Ct \qquad \text{and} \qquad E := \mathbf{E}|\widetilde{Q}(t)|,$$

we have (with a similar lower deviation bound)

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Claim: this already implies the $t^{2/3}$ upper bound:

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that is, $E^3 \le c \cdot t^2$.

$$\begin{aligned} \mathbf{Var}(h_{Ct}(t)) &\overset{\text{Thm}}{=} \text{const.} \cdot \mathbf{E}|Q(t) - Ct| \\ &= \text{const.} \cdot E \le c \cdot t^{2/3}. \end{aligned}$$

■

Lower bound

In the upper bound, the relevant orders were

$$u \text{ (deviation of } Q(t)) \sim t^{2/3}, \qquad \varrho - \lambda \sim t^{-1/3}.$$

The lower bound works with similar arguments: compare models of which the densities differ by $t^{-1/3}$, and use connections between $Q(t), X(t)$ and heights.

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The critical feature in both the upper bound and lower bound was $Q(t) \geq X(t)$ (convex) or $Q(t) \leq X(t)$ (concave).

Coupling results

Model	$H(\varrho)$ is	Feature?	$t^{2/3}$ law

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TASEP			

Coupling results

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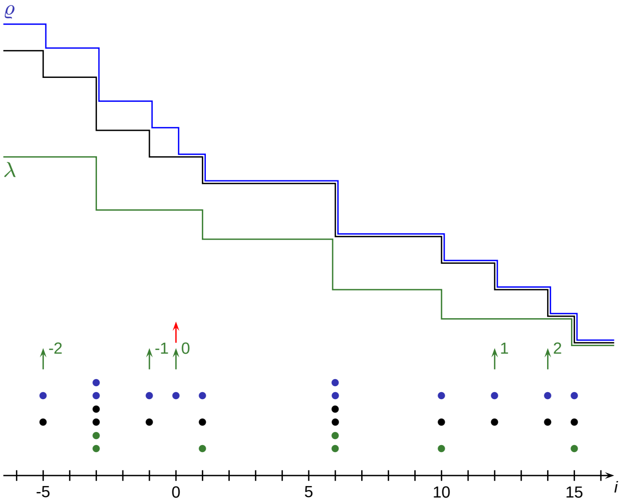
The critical feature: $Q(t) \leq X(t)$



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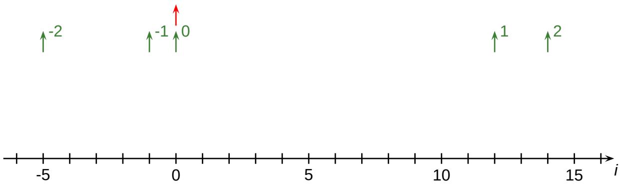

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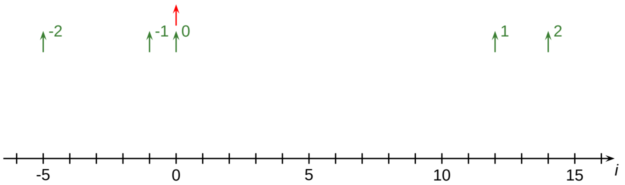

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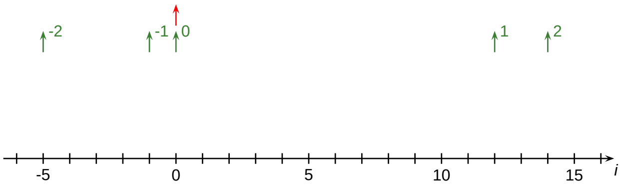

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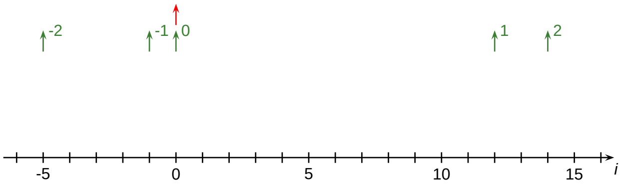

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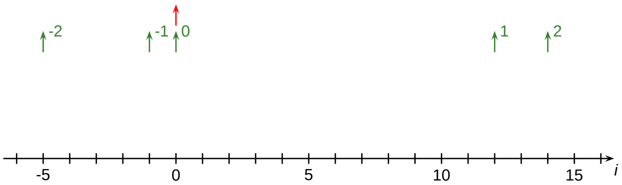

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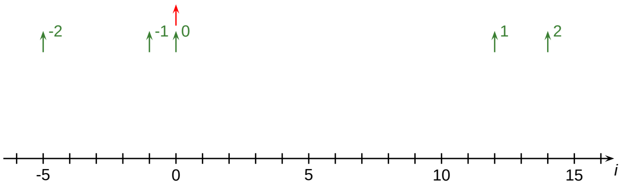

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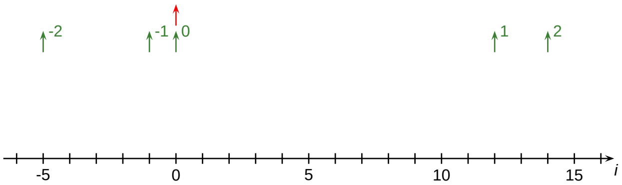

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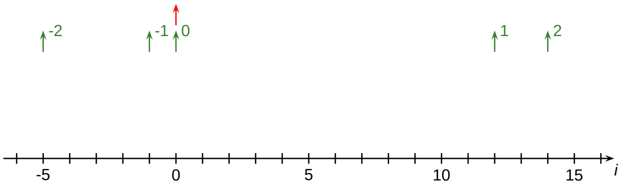

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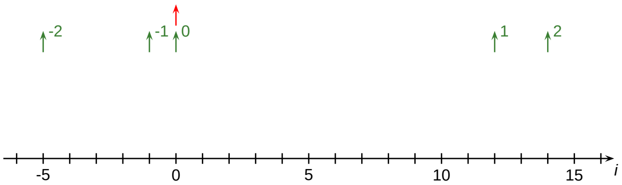

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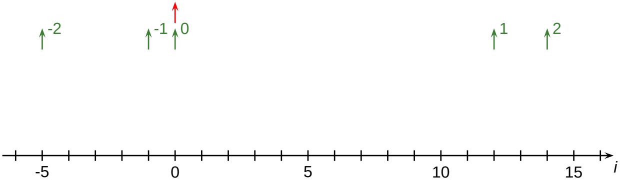

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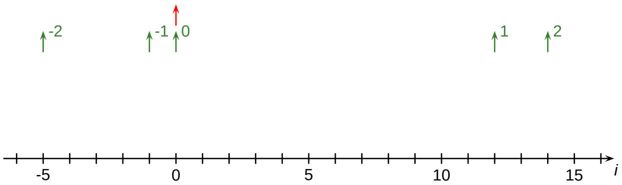

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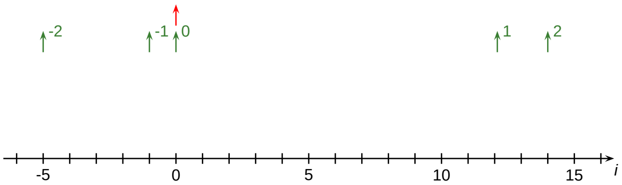

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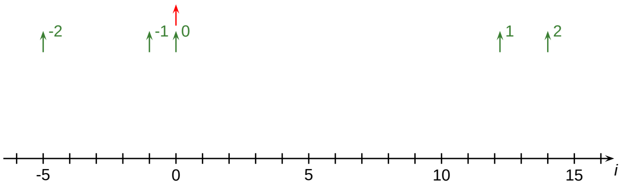

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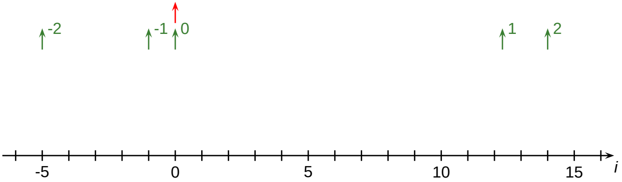

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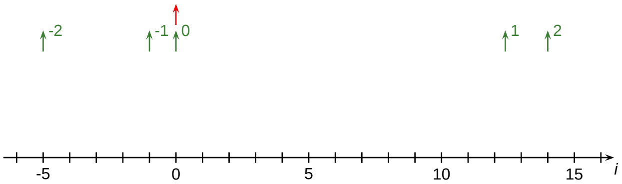

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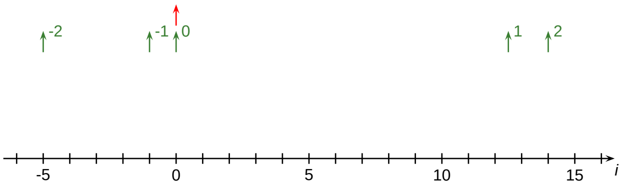

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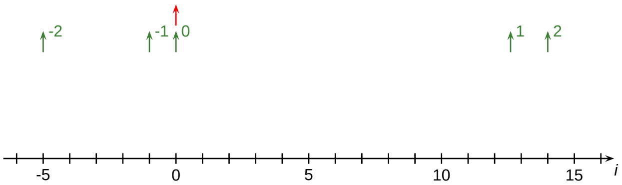

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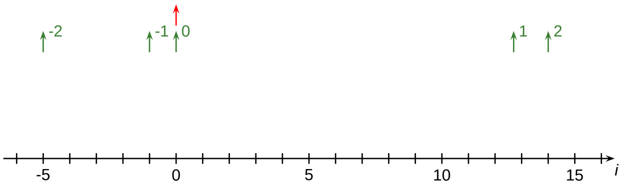

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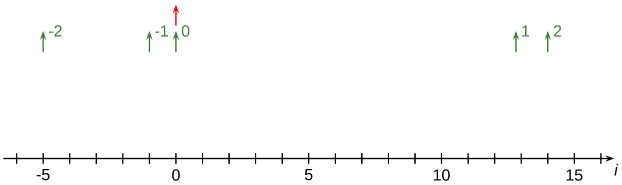

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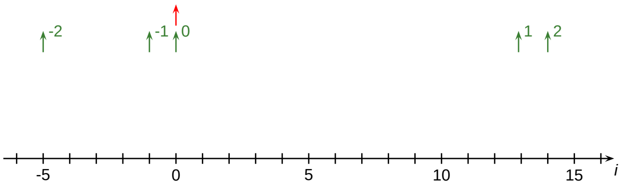

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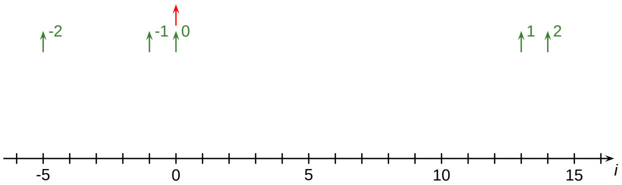

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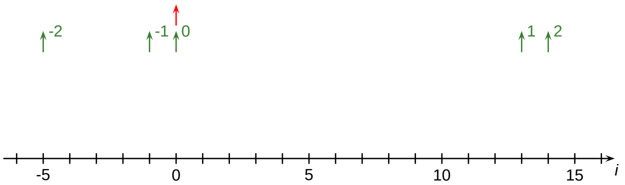

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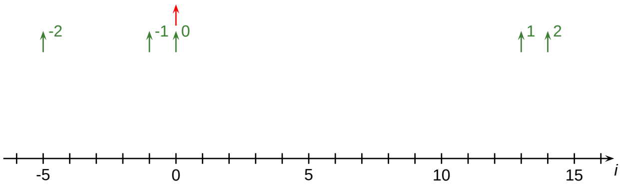

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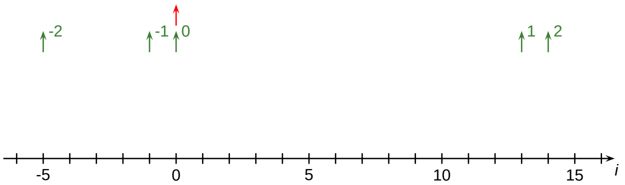

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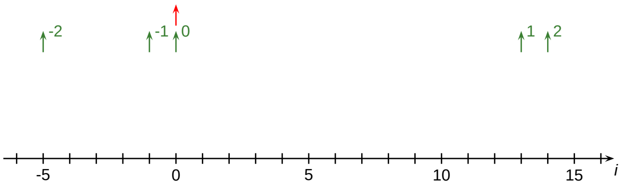

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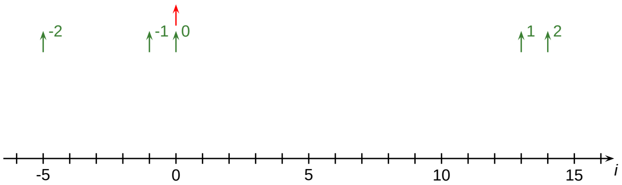

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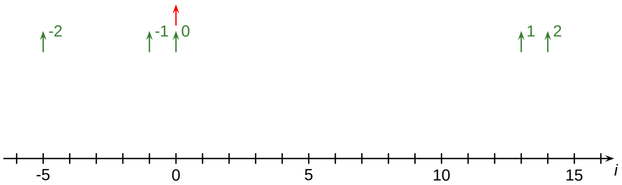

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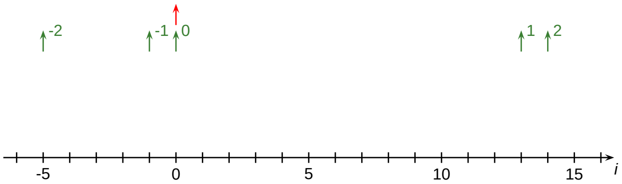

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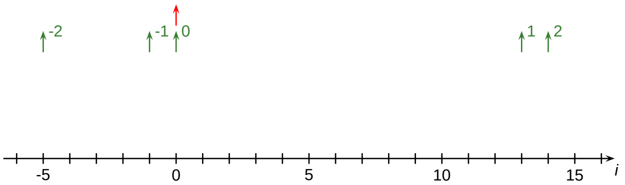

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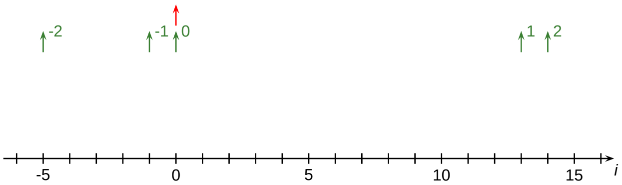

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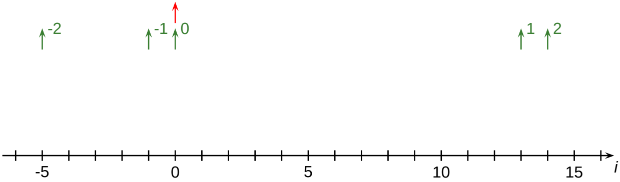

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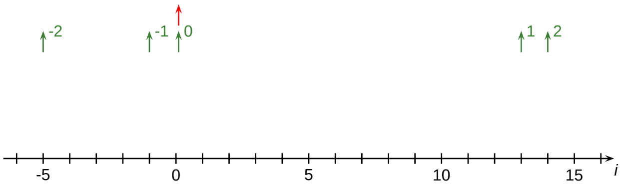

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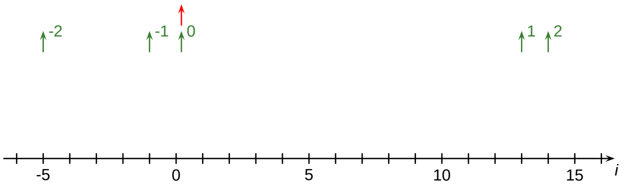

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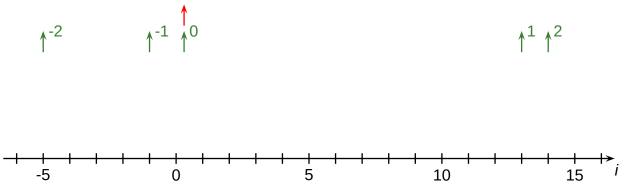

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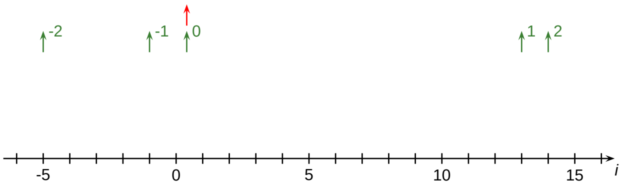

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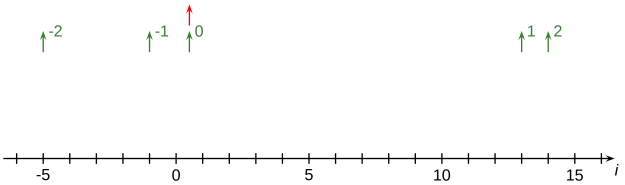

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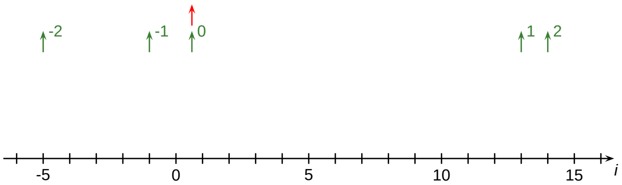

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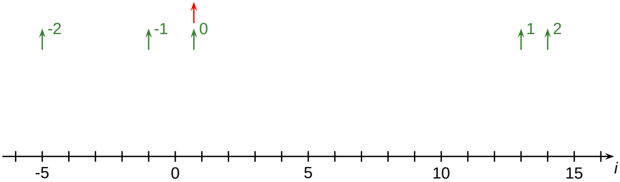

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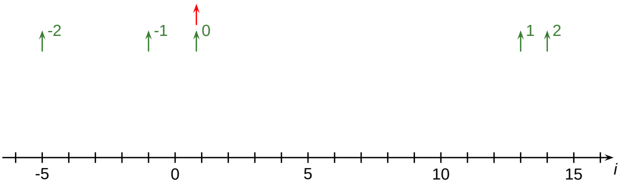

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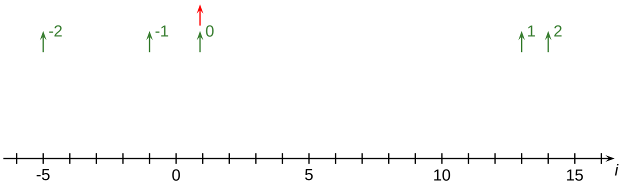

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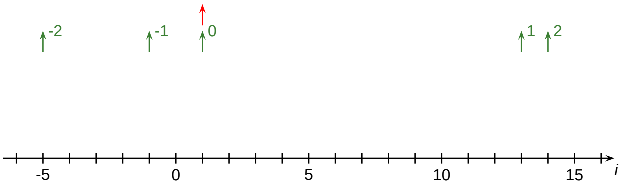

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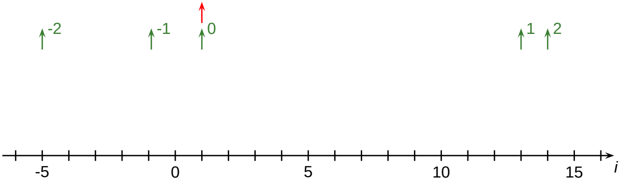

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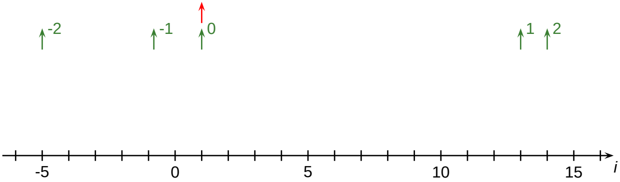

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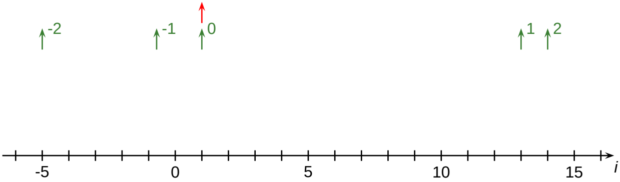

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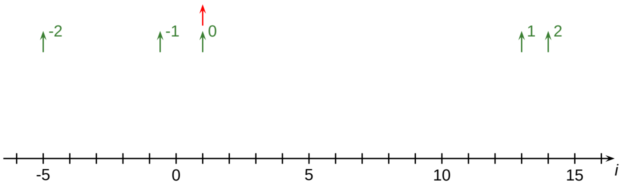

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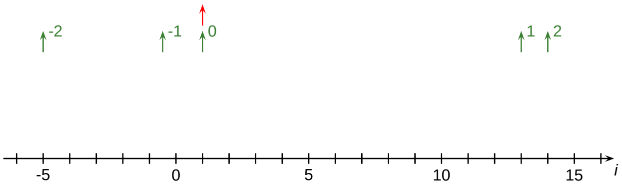

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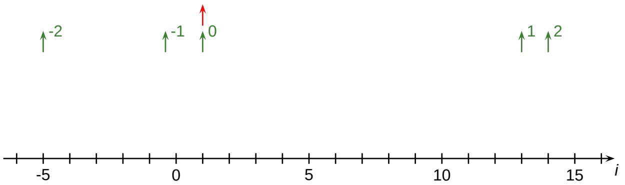

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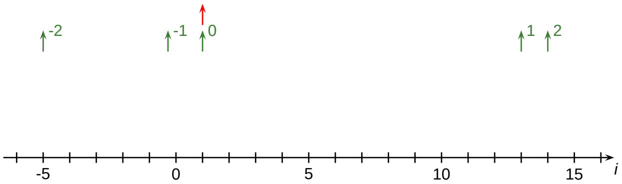

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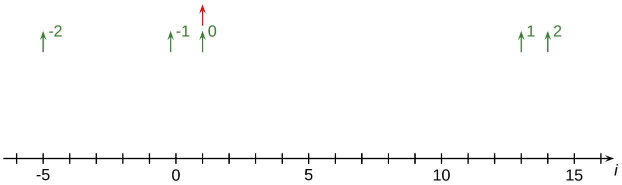

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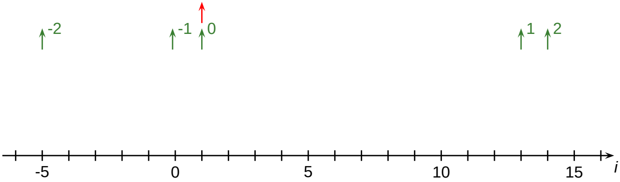

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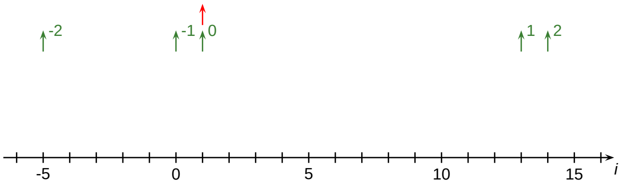

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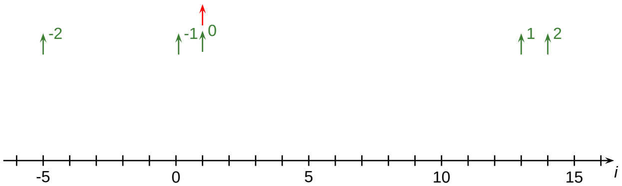

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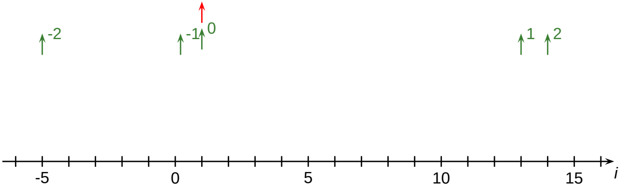

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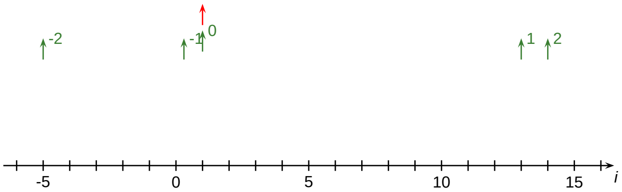

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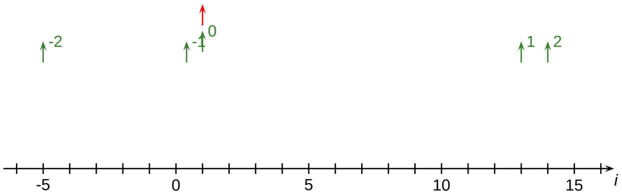

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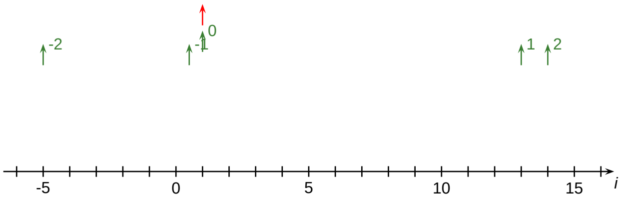

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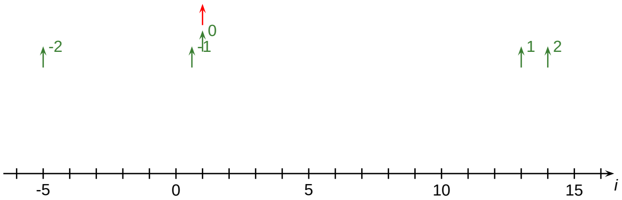

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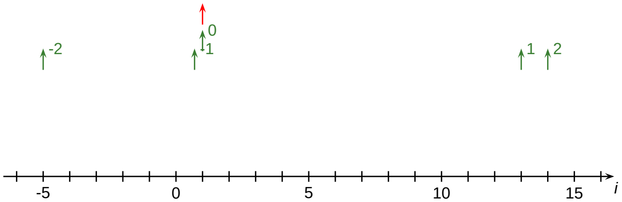

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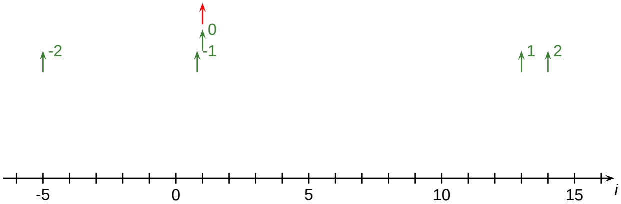

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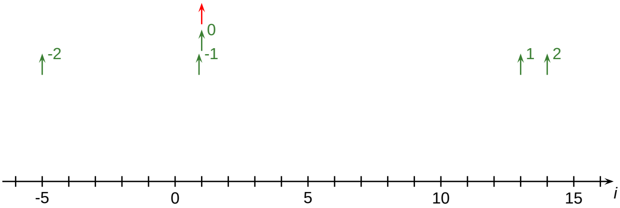

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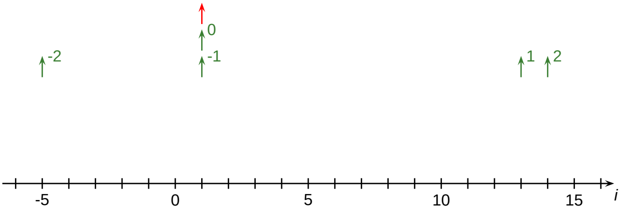

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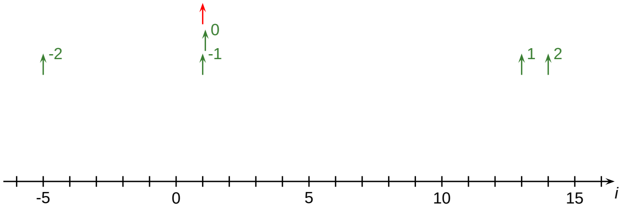

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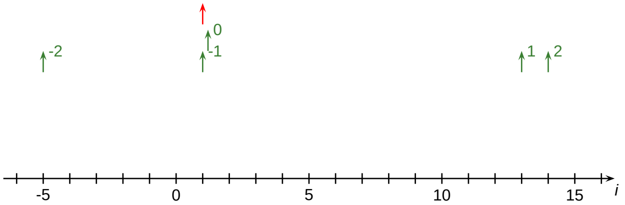

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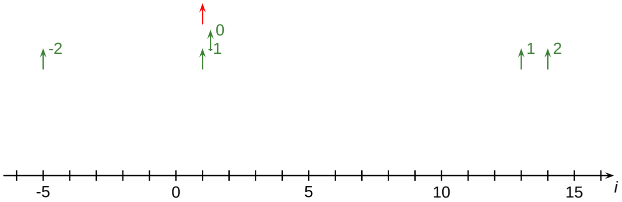

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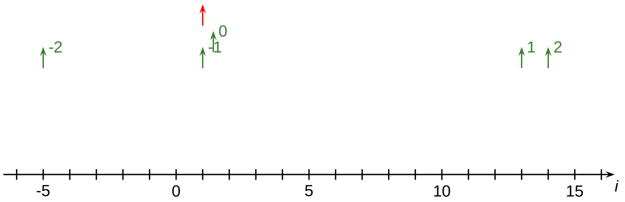

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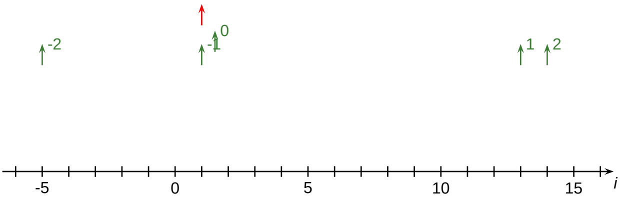

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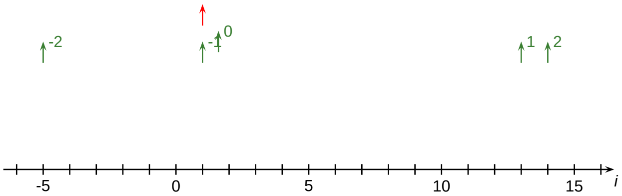

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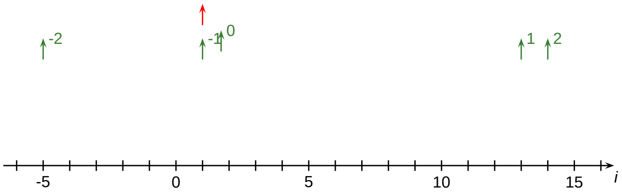

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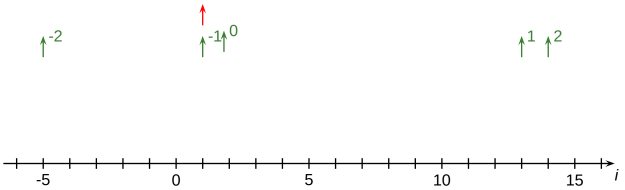

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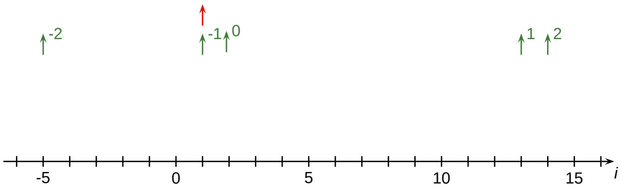

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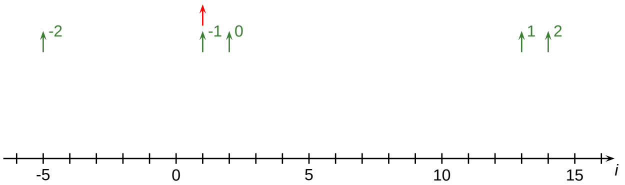

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Generalizing requires finding more models with nice $m_Q(t)$ behavior, or handle less nice cases of $m_Q(t)$ processes. This is subject to future work.

Linear models

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In their cases, we have

$$\lim_{t\to\infty} \frac{\mathbf{Var}(h_{Ct}(t))}{t^{1/2}} = \dots,$$

even convergence of the finite-dimensional distributions of the $h_{Ct}(t)$ process to Gaussian limits is known (Seppäläinen 2005, Ferrari and Fontes 1998, B., Rassoul-Agha and Seppäläinen 2006).

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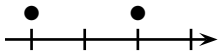
And there are attractive asymmetric models with nonlinear, nonconvex and nonconcave hydrodynamics:

- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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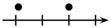
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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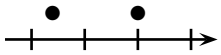
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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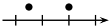
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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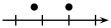
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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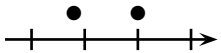
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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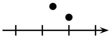
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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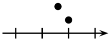
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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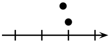
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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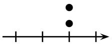
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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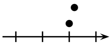
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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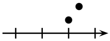
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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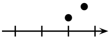
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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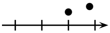
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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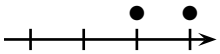
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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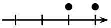
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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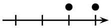
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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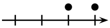
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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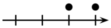
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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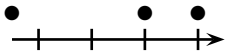
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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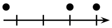
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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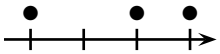
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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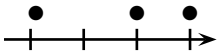
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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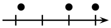
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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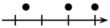
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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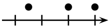
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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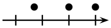
- 2-jump exclusion:  $H(\varrho)$ is a cubic polynomial;

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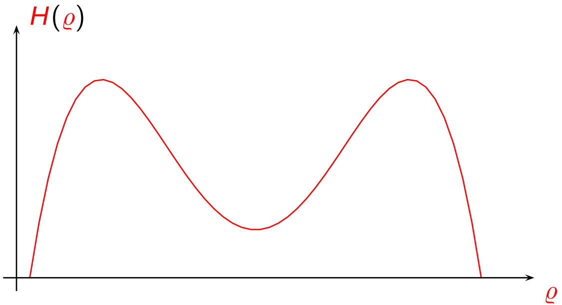
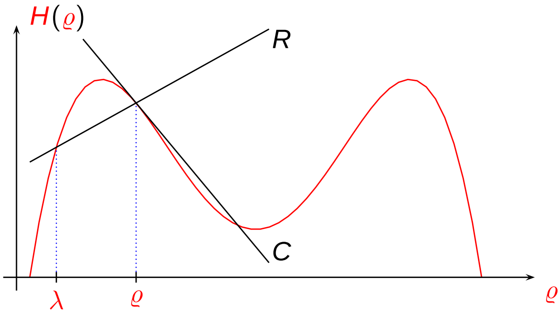
- A three-state process with variable rates (B. Tóth - I. Tóth).

Nonconvex, nonconcave



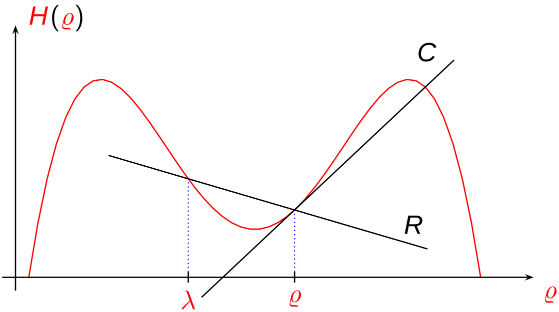
Nonconvex, nonconcave



$$C < R$$

$$Q(t) \overset{?}{\leq} X(t)$$

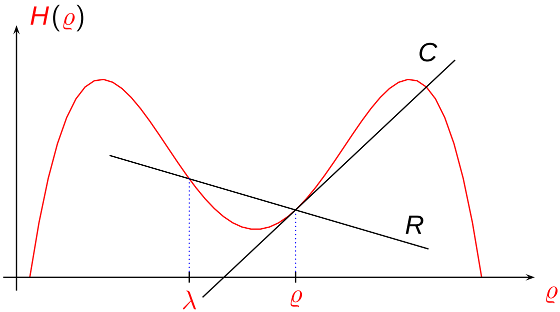
Nonconvex, nonconcave



$$C > R$$

$$Q(t) \overset{?}{\geq} X(t)$$

Nonconvex, nonconcave

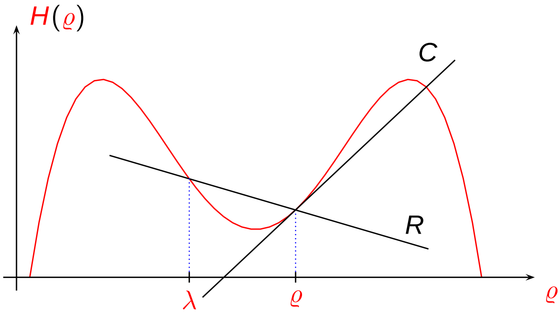


$$C > R$$

$$Q(t) \overset{?}{\geq} X(t)$$

Inequality changes with the density... ?

Nonconvex, nonconcave



$$C > R$$

$$Q(t) \overset{?}{\geq} X(t)$$

Inequality changes with the density... ?

Any coupling *must be very very tricky*.

Thank you.