Dependent Double Branching Annihilating Random Walk

Joint with
Attila László Nagy

Márton Balázs

University of Bristol

SPA 2014, 29 July, 2014.

Non-attractivity and the second class particle

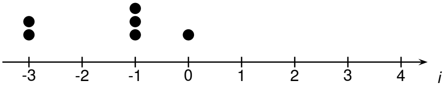
A mean field model

Positive recurrence

Two words on the proof

Conservative IPS

Think e.g. zero range:



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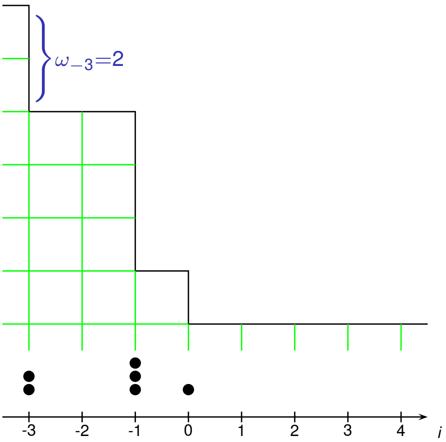

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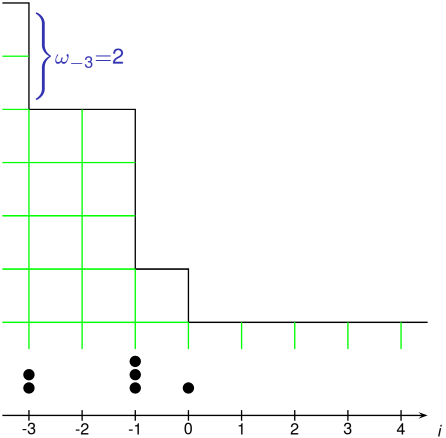

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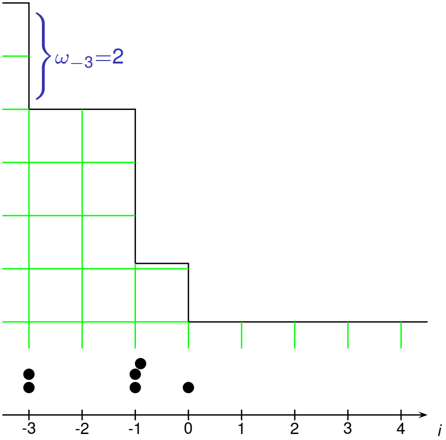

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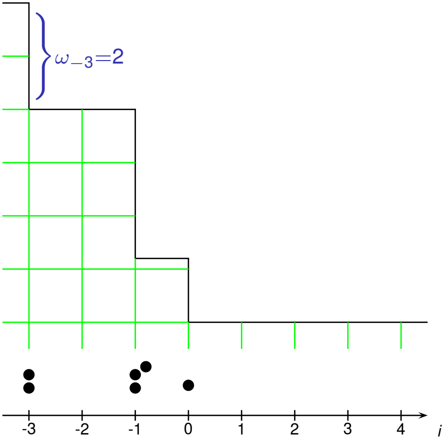

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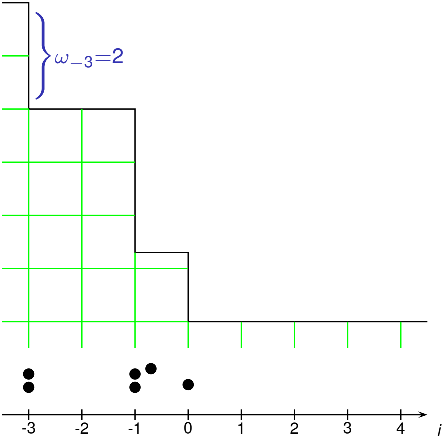

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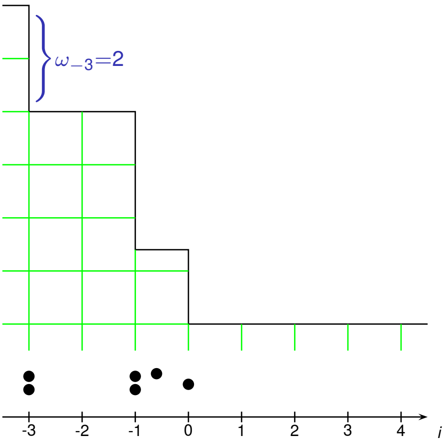

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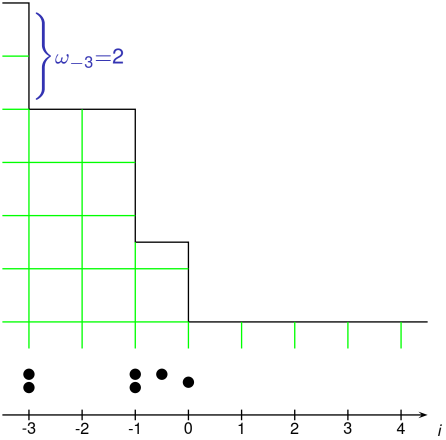

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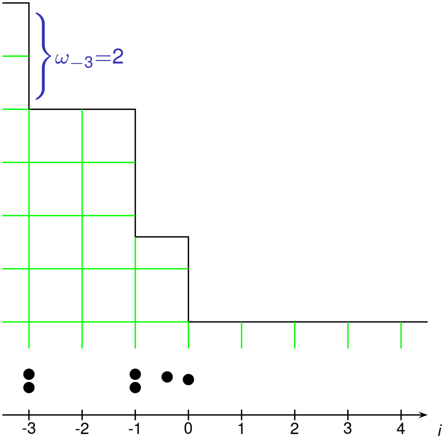

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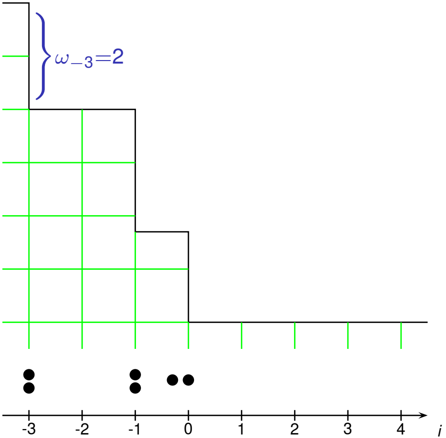

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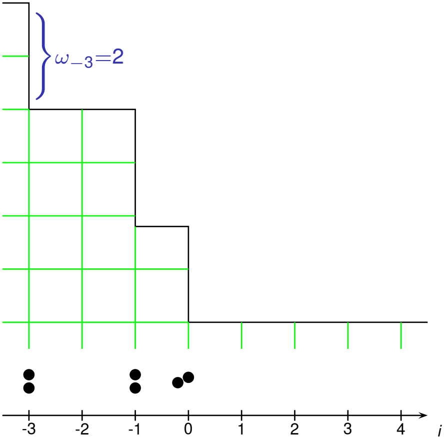

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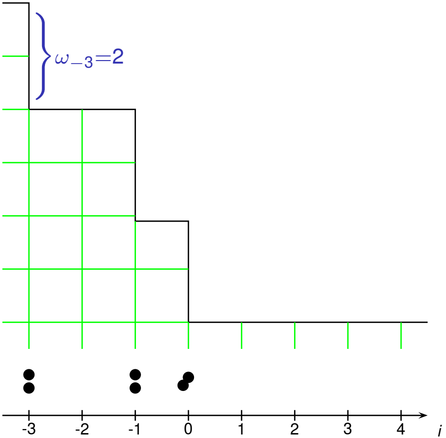

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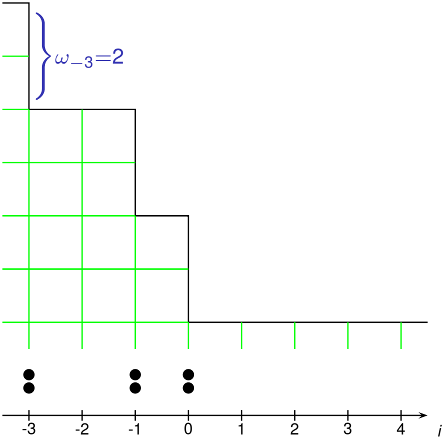

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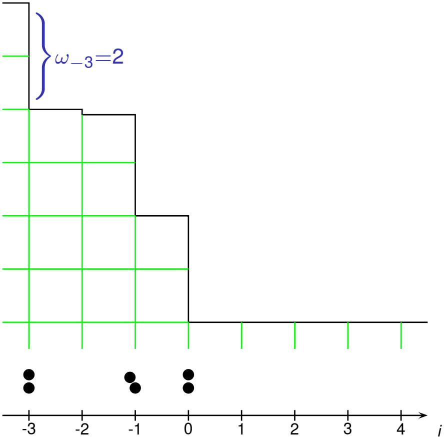

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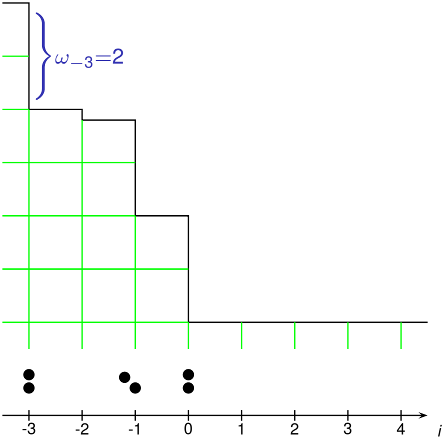

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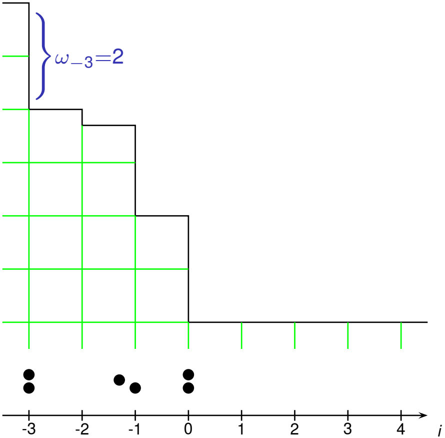

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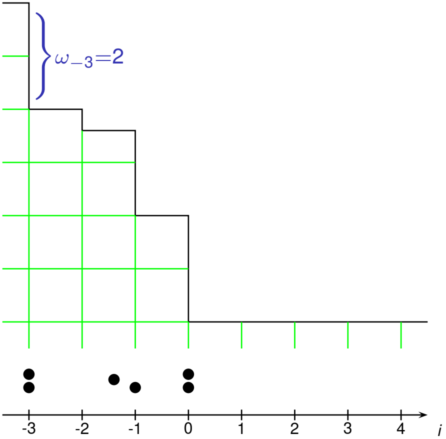

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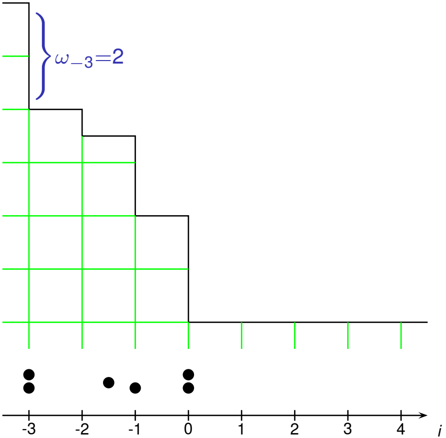

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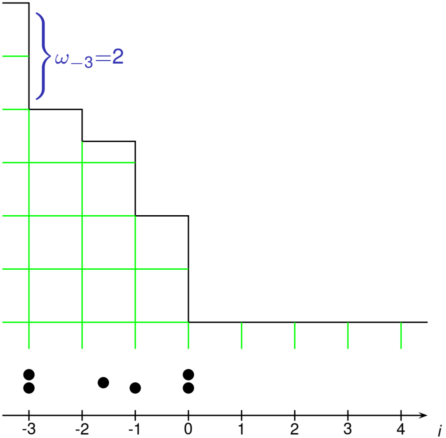

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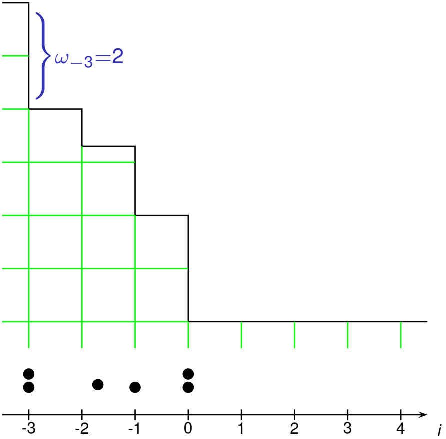

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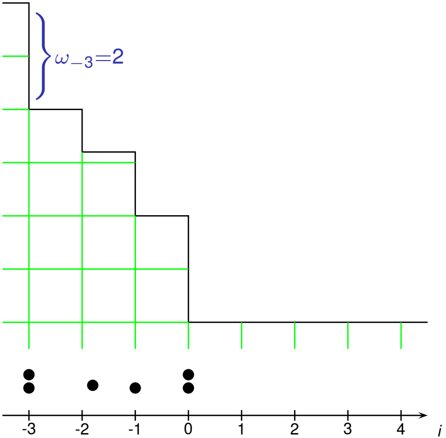

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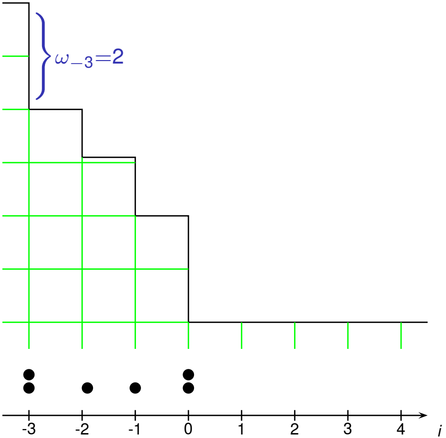

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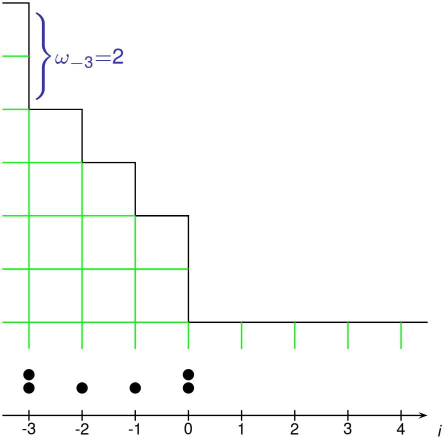

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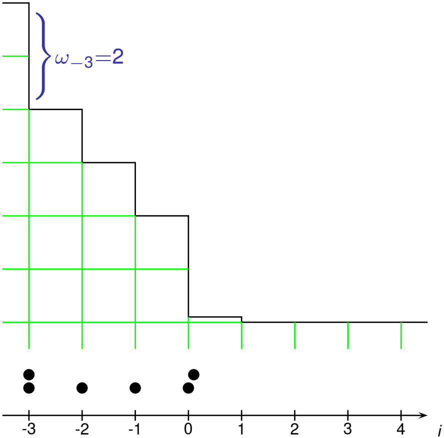

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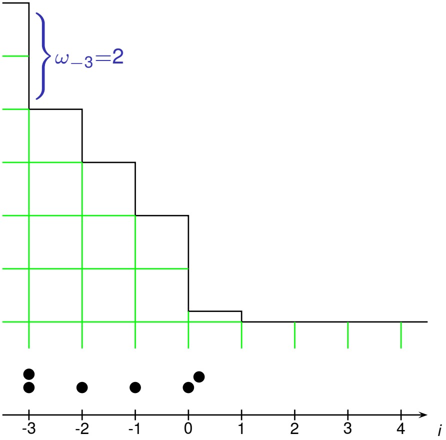

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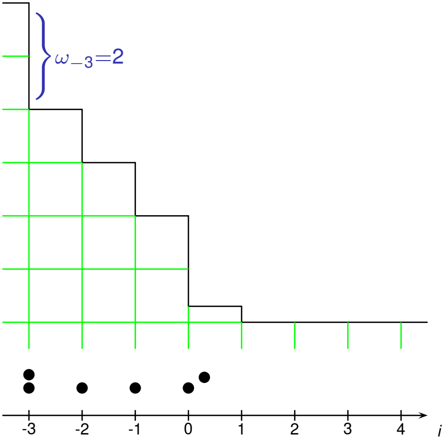

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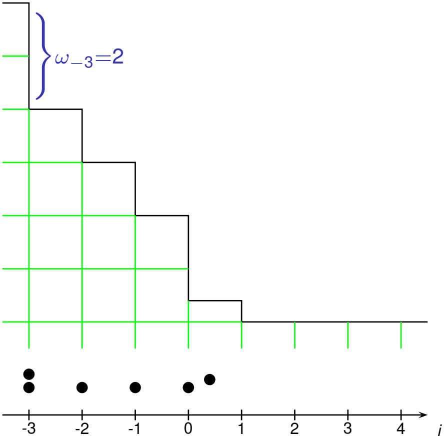

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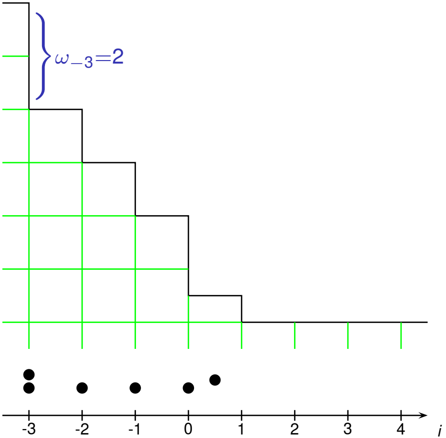

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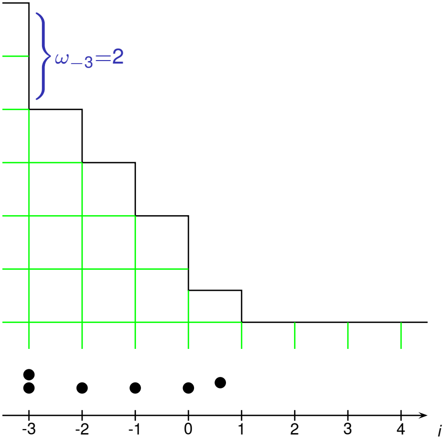

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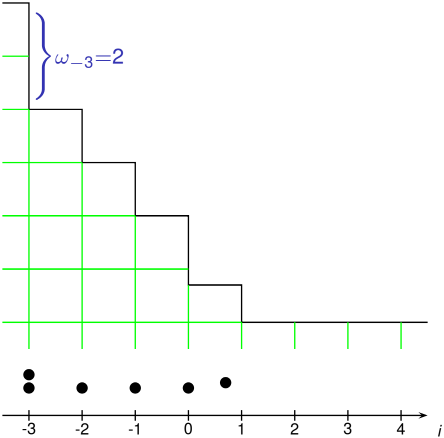

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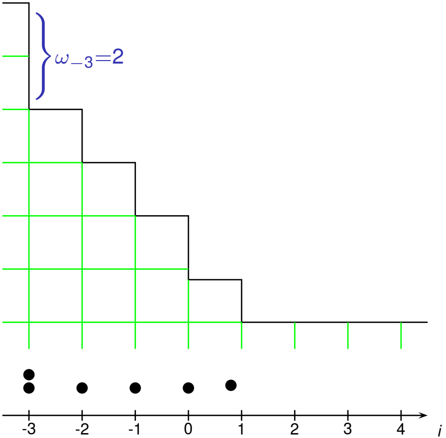

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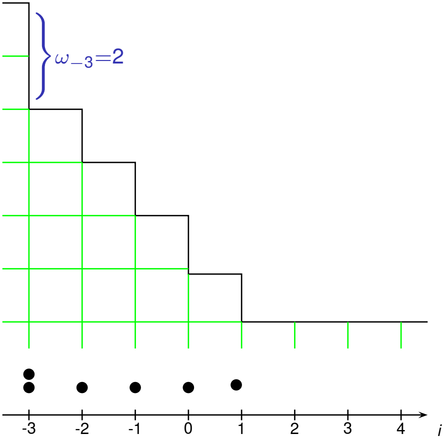

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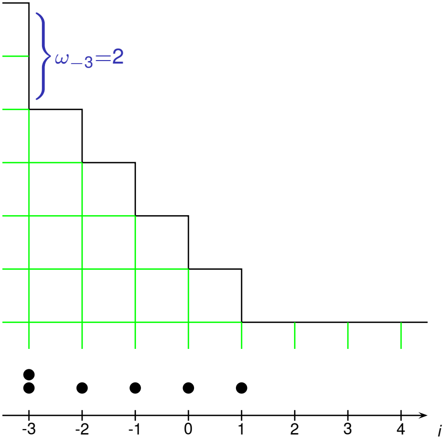

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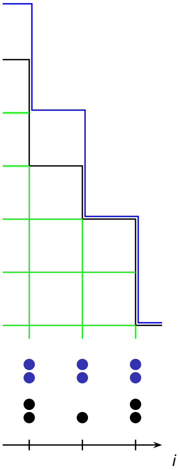
The second class particle: attractive case

States ω and ω only differ at one site.



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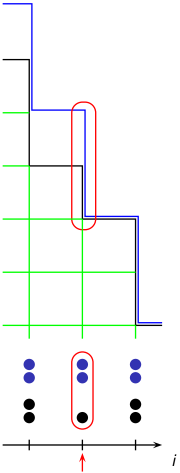

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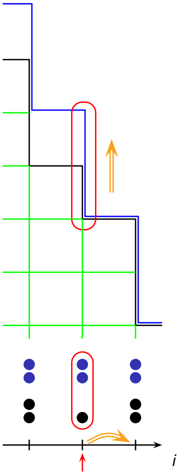

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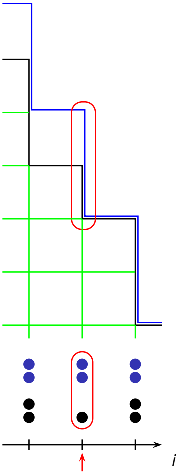

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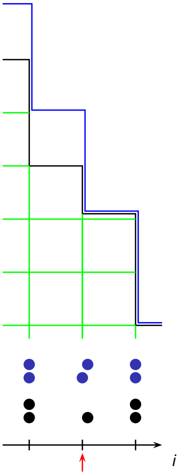

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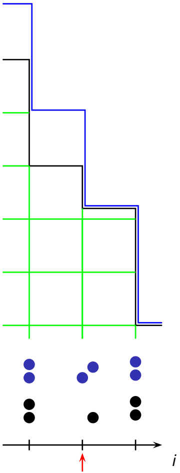

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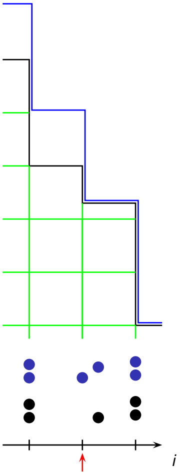

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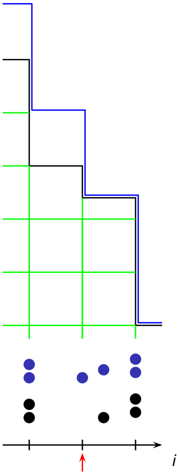

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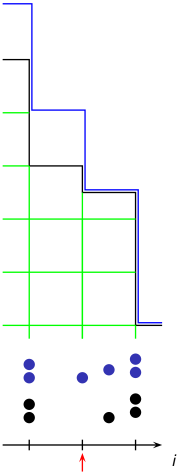

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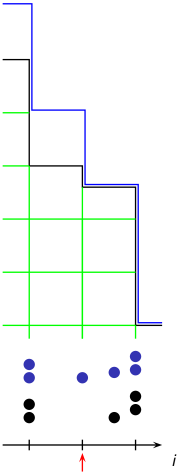

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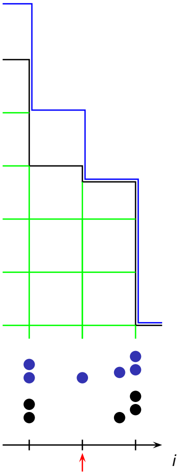

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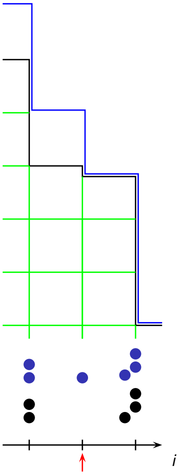

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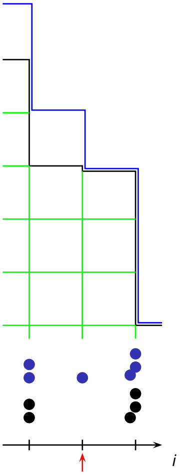

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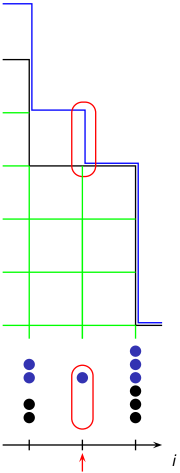

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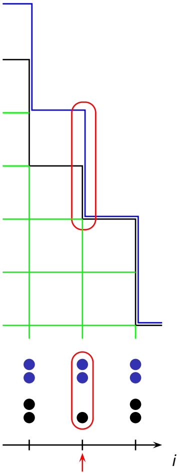

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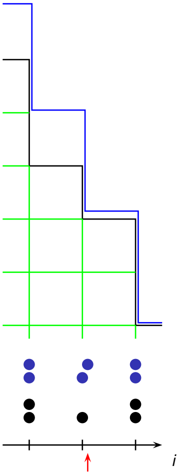

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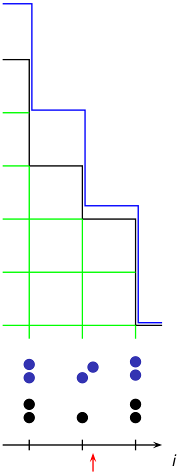

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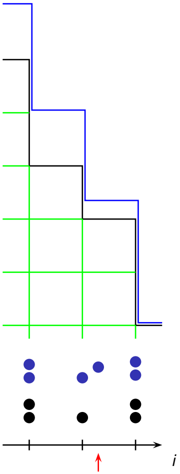

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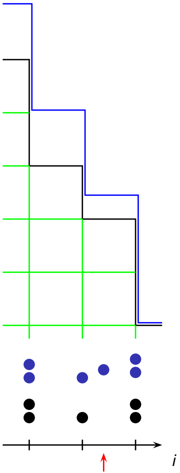

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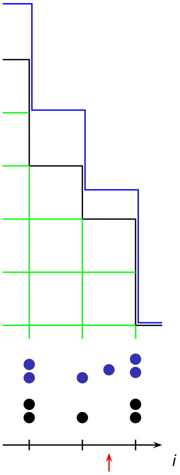

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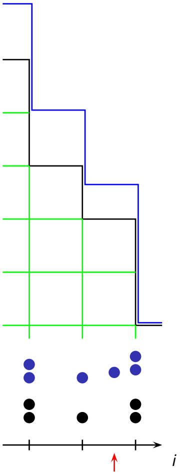

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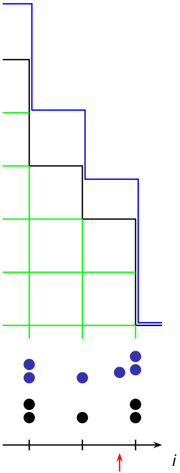
with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.



Growth on the right:
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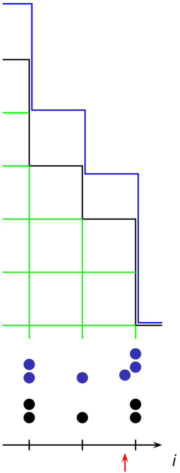

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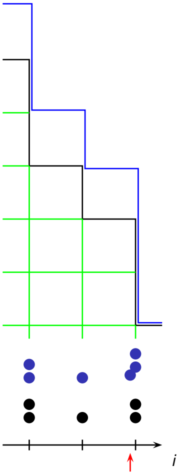

Growth on the right:
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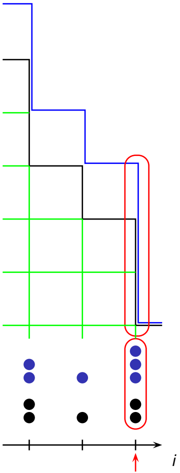
with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.

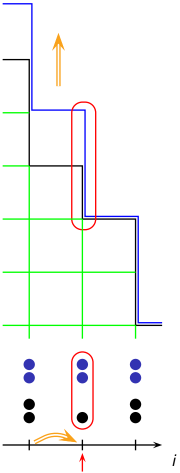


Growth on the right:
rate≤rate
with rate-rate:

The second class particle: attractive case

States ω and ω only differ at one site.

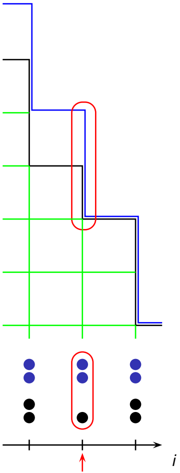
Growth on the left:
rate$\geq$rate



The second class particle: attractive case

States ω and ω only differ at one site.

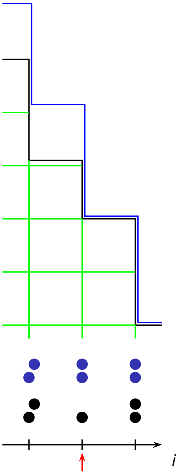
Growth on the left:
rate≥rate
with rate:



The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:
rate≥rate
with rate:



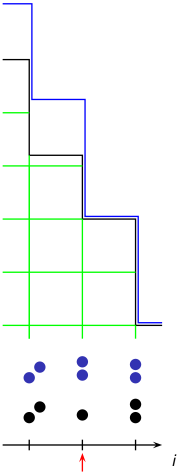
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

rate≥rate

with rate:



The second class particle: attractive case

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Growth on the left:
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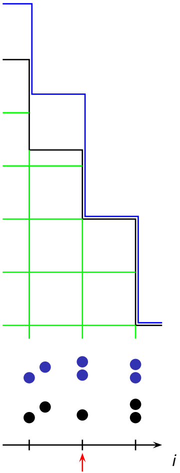

The second class particle: attractive case

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Growth on the left:
rate≥rate
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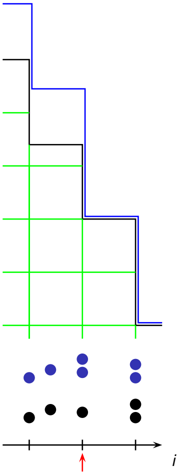

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rate≥rate
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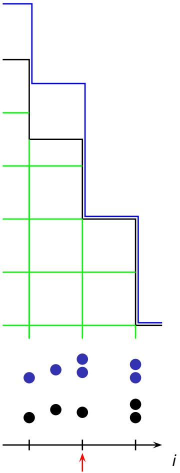

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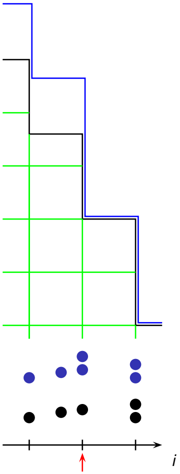

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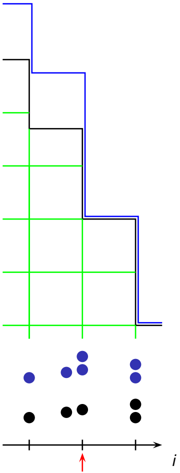

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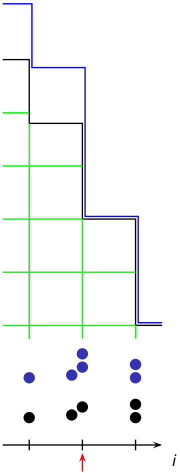

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rate≥rate
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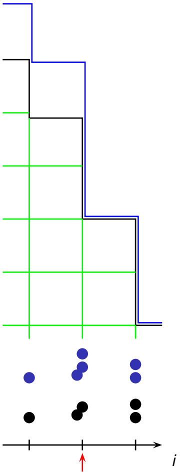

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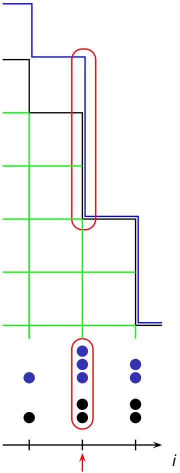
rate$\geq$rate
with rate:



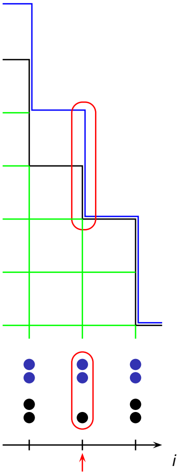
The second class particle: attractive case

States ω and ω only differ at one site.

Growth on the left:

rate$\geq$rate

with rate-rate:



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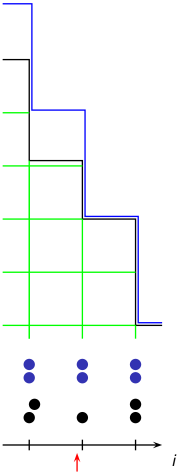

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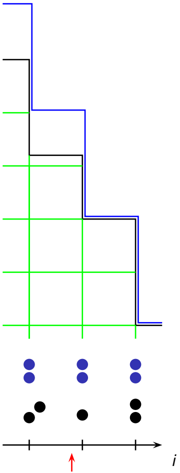

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Growth on the left:
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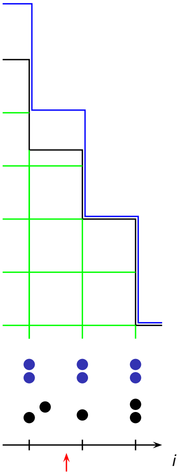

The second class particle: attractive case

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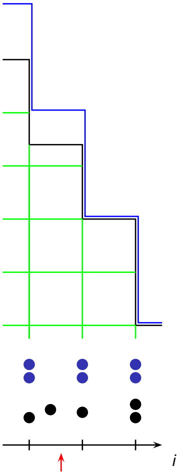
Growth on the left:
rate$\geq$rate
with rate-rate:



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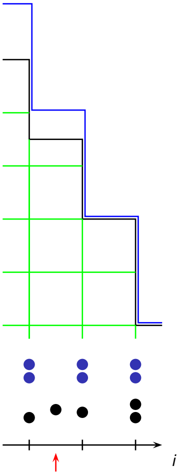

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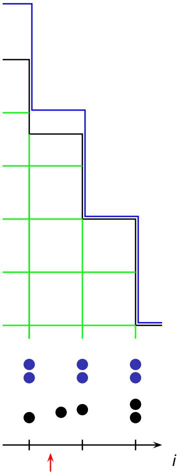
Growth on the left:

rate$\geq$rate

with rate-rate:



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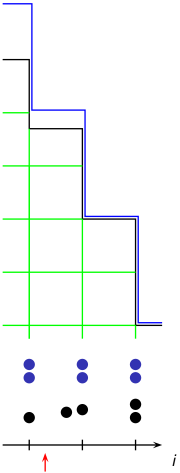

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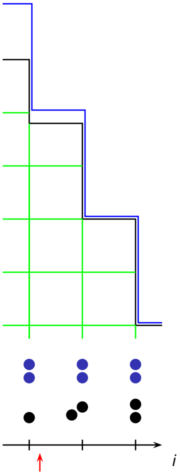

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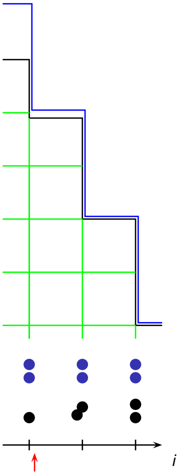

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Growth on the left:

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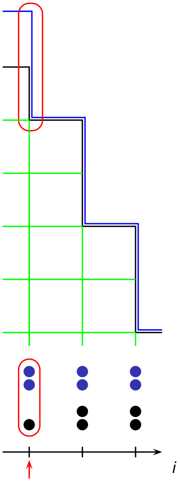
with rate-rate:



The second class particle: attractive case

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Growth on the left:
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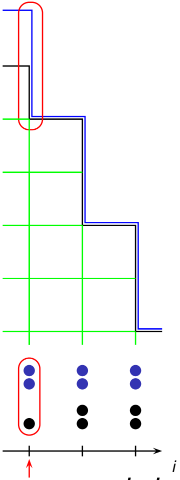
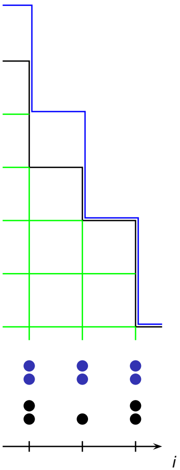
with rate-rate:

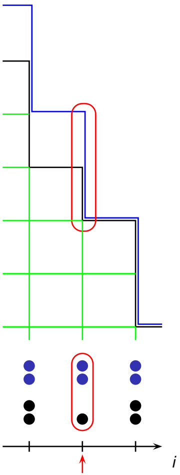


A single discrepancy↑, the *second class particle*, is conserved.

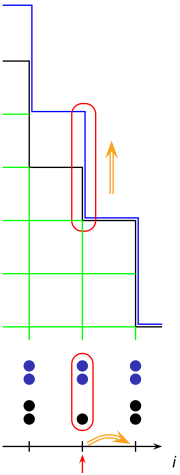
The second class particle: non-attractive case



The second class particle: non-attractive case

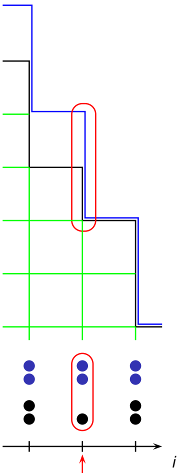


The second class particle: non-attractive case



Growth on the right:
rate>rate

The second class particle: non-attractive case

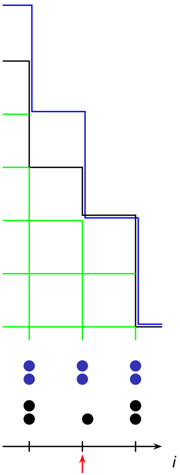


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

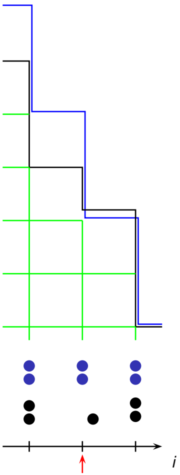


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

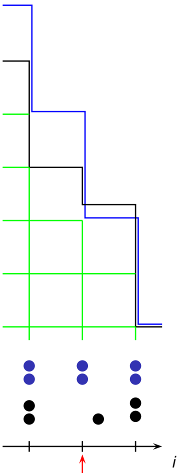


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

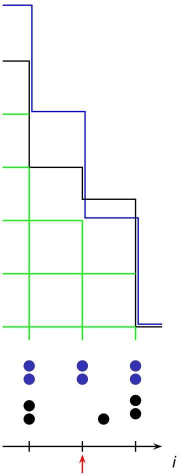


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

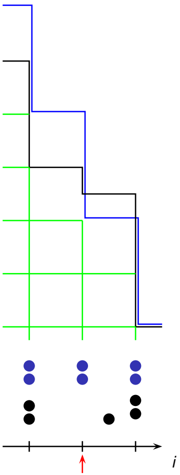


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

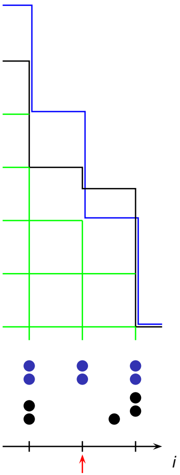


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

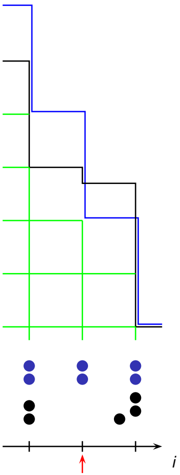


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

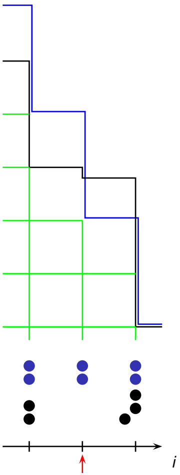


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

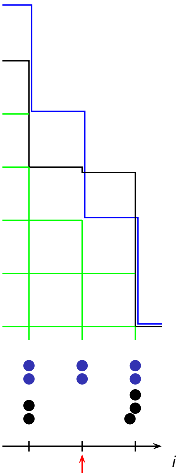


Growth on the right:

rate>rate

rate-rate:

The second class particle: non-attractive case

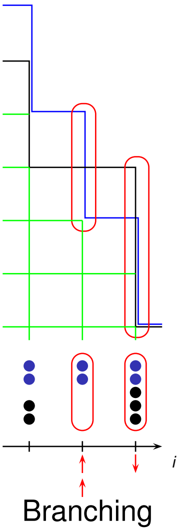


Growth on the right:

rate>rate

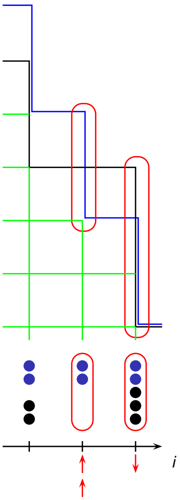
rate-rate:

The second class particle: non-attractive case

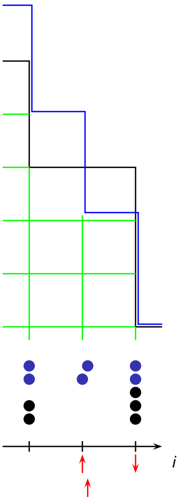


Growth on the right:
rate>rate
rate-rate:

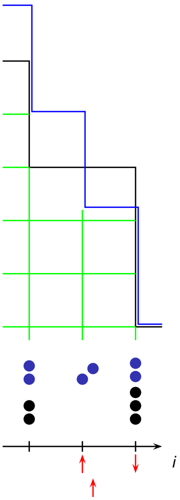
The second class particle: non-attractive case



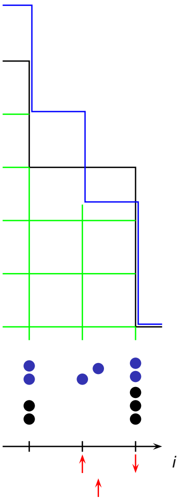
The second class particle: non-attractive case



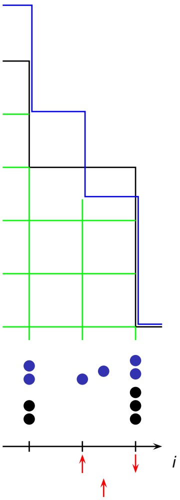
The second class particle: non-attractive case



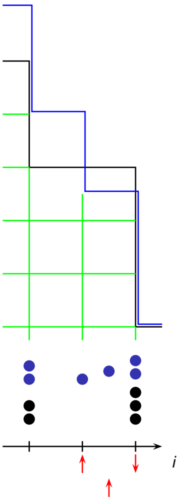
The second class particle: non-attractive case



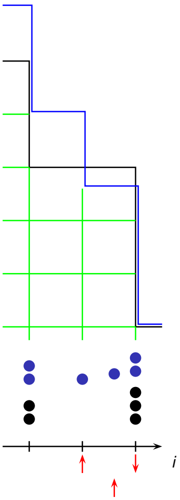
The second class particle: non-attractive case



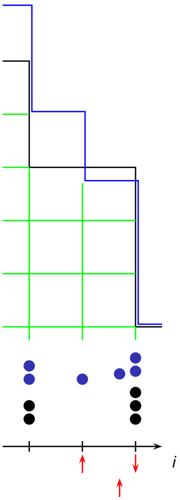
The second class particle: non-attractive case



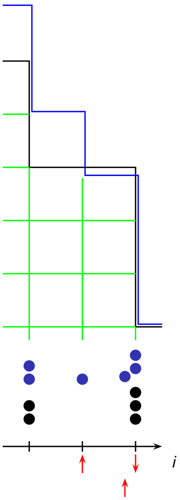
The second class particle: non-attractive case



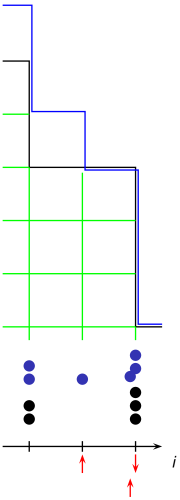
The second class particle: non-attractive case



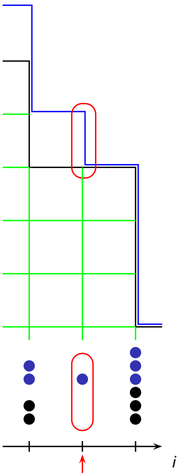
The second class particle: non-attractive case



The second class particle: non-attractive case



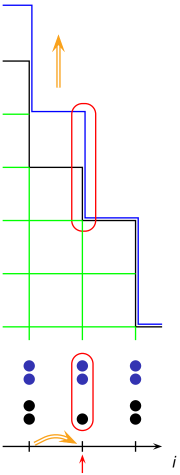
The second class particle: non-attractive case



Annihilating

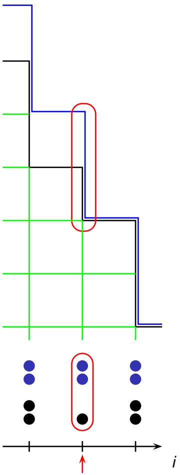
The second class particle: non-attractive case

Growth on the left:
rate$<$rate



The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:

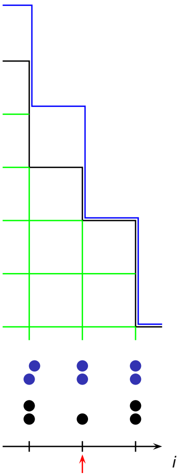


The second class particle: non-attractive case

Growth on the left:

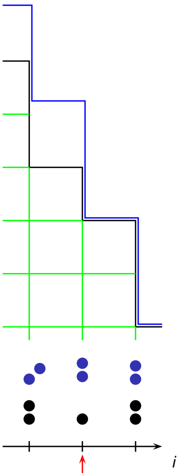
rate<rate

with rate-rate:



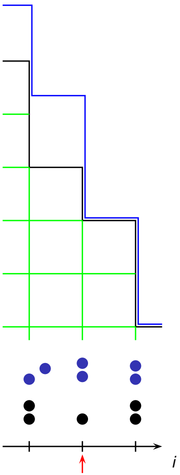
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



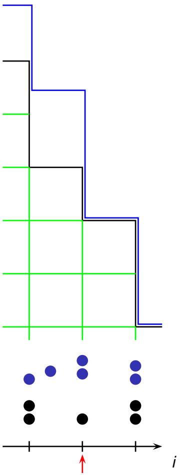
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:

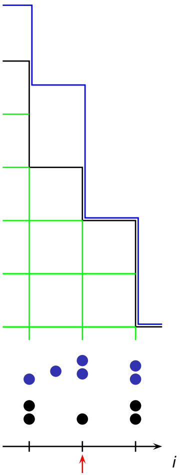


The second class particle: non-attractive case

Growth on the left:

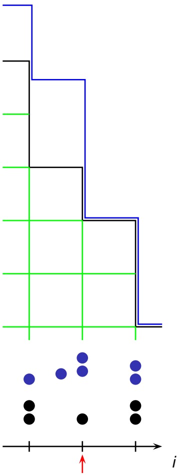
rate<rate

with rate-rate:



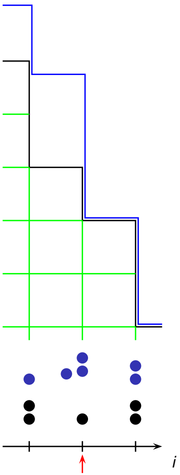
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



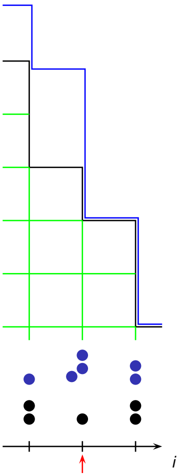
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



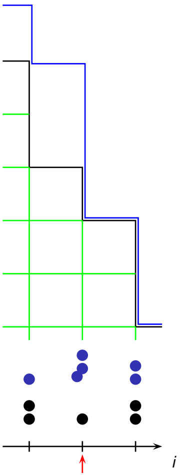
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



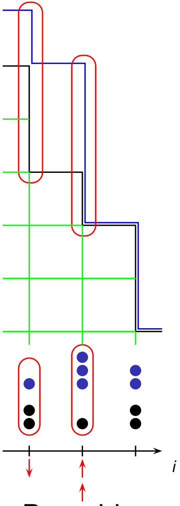
The second class particle: non-attractive case

Growth on the left:
rate<rate
with rate-rate:



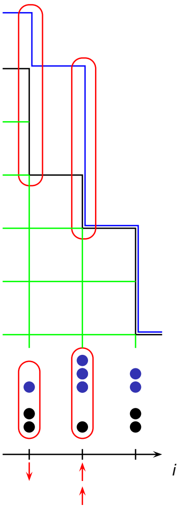
The second class particle: non-attractive case

Growth on the left:
rate$<$rate
with rate-rate:

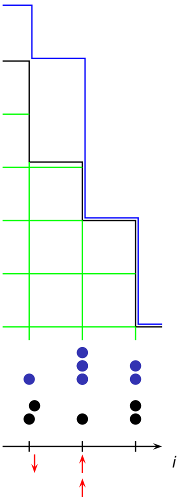


Branching

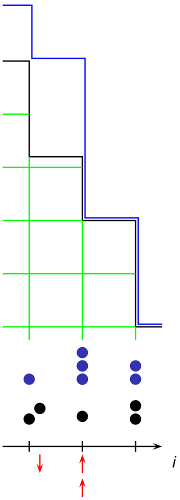
The second class particle: non-attractive case



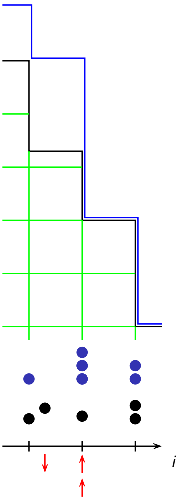
The second class particle: non-attractive case



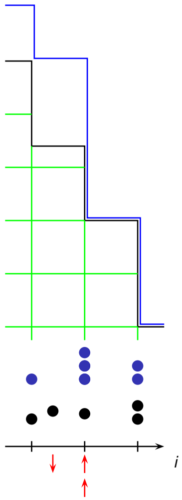
The second class particle: non-attractive case



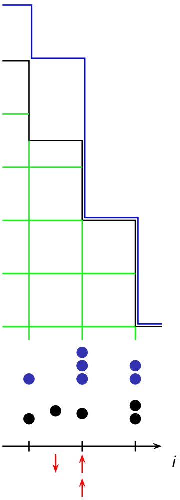
The second class particle: non-attractive case



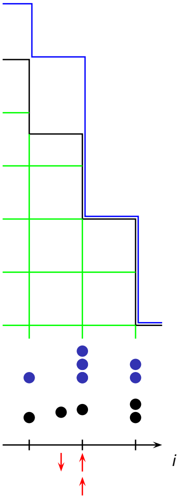
The second class particle: non-attractive case



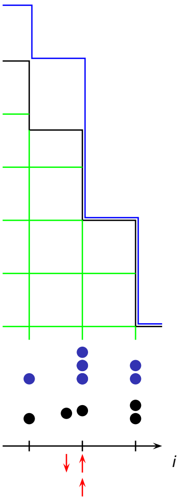
The second class particle: non-attractive case



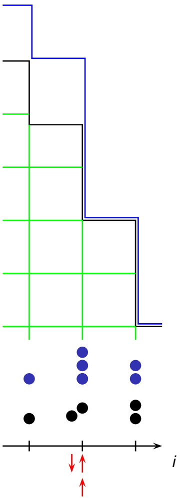
The second class particle: non-attractive case



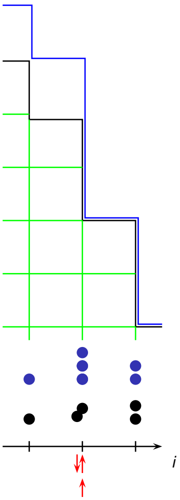
The second class particle: non-attractive case



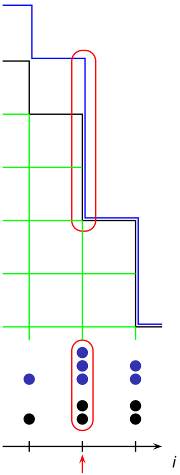
The second class particle: non-attractive case



The second class particle: non-attractive case



The second class particle: non-attractive case



Annihilating

The second class particle: non-attractive case

We are facing a

- nearest neighbour
- parity conserving
- branching
- annihilating process
- on the dynamic background of first class particles.

The aim is to control the number of $\uparrow$ and $\downarrow$'s. Idea from Bálint Tóth.

$\rightsquigarrow$ `homog2.avi`

A mean field model

A model we can say something about:

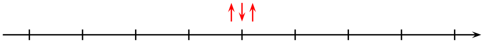
A mean field model

A model we can say something about:

Branching

A mean field model

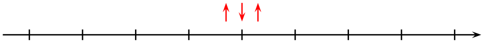
A model we can say something about:



Branching

A mean field model

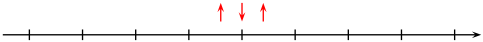
A model we can say something about:



Branching

A mean field model

A model we can say something about:



Branching

A mean field model

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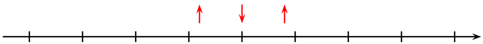

Branching

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A mean field model

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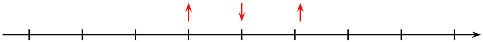
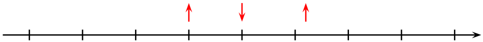


Jump

A mean field model

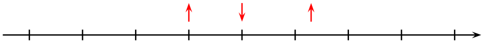
A model we can say something about:



Jump

A mean field model

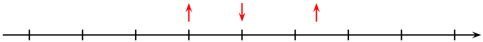
A model we can say something about:



Jump

A mean field model

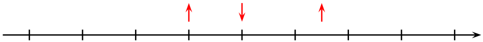
A model we can say something about:



Jump

A mean field model

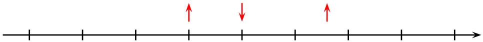
A model we can say something about:



Jump

A mean field model

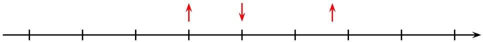
A model we can say something about:



Jump

A mean field model

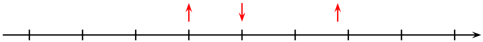
A model we can say something about:



Jump

A mean field model

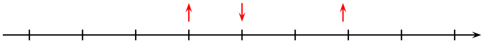
A model we can say something about:



Jump

A mean field model

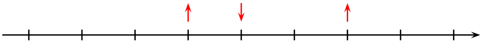
A model we can say something about:



Jump

A mean field model

A model we can say something about:



Jump

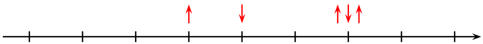
A mean field model

A model we can say something about:

Branching

A mean field model

A model we can say something about:



Branching

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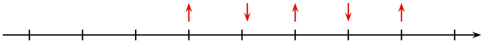
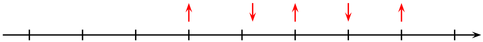


Jump with annihilation

A mean field model

A model we can say something about:



Jump with annihilation

A mean field model

A model we can say something about:

Jump with annihilation

A mean field model

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Jump with annihilation

A mean field model

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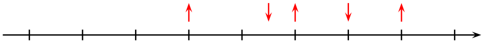

Jump with annihilation

A mean field model

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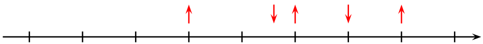
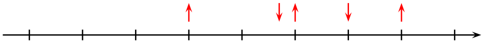


Jump with annihilation

A mean field model

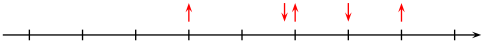
A model we can say something about:



Jump with annihilation

A mean field model

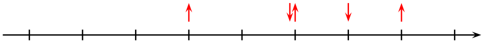
A model we can say something about:



Jump with annihilation

A mean field model

A model we can say something about:



Jump with annihilation

A mean field model

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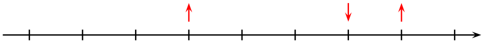
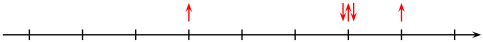


Jump with annihilation

A mean field model

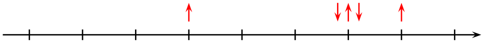
A model we can say something about:



Branching with annihilation

A mean field model

A model we can say something about:



Branching with annihilation

A mean field model

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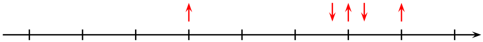
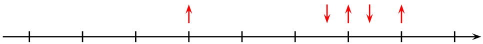


Branching with annihilation

A mean field model

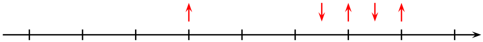
A model we can say something about:



Branching with annihilation

A mean field model

A model we can say something about:



Branching with annihilation

A mean field model

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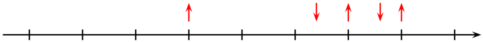

Branching with annihilation

A mean field model

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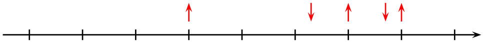

Branching with annihilation

A mean field model

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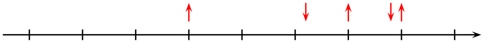
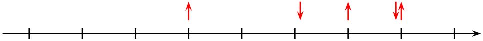


Branching with annihilation

A mean field model

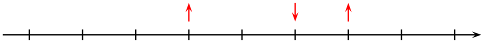
A model we can say something about:



Branching with annihilation

A mean field model

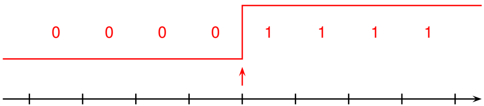
A model we can say something about:



Branching with annihilation

A mean field model

A model we can say something about:



A mean field model

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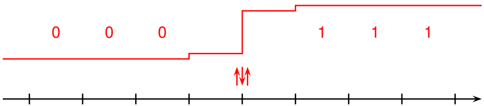
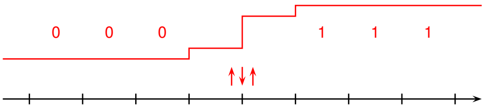


Branching: exclusion

A mean field model

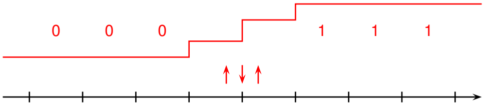
A model we can say something about:



Branching: exclusion

A mean field model

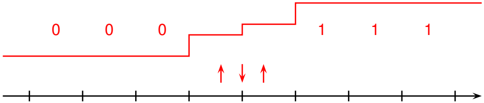
A model we can say something about:



Branching: exclusion

A mean field model

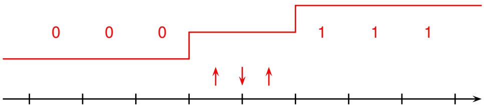
A model we can say something about:



Branching: exclusion

A mean field model

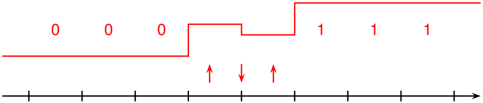
A model we can say something about:



Branching: exclusion

A mean field model

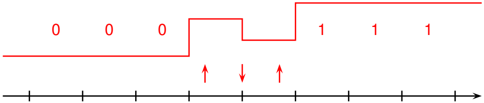
A model we can say something about:



Branching: exclusion

A mean field model

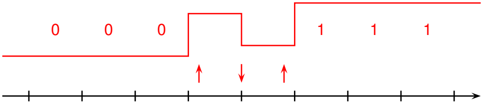
A model we can say something about:



Branching: exclusion

A mean field model

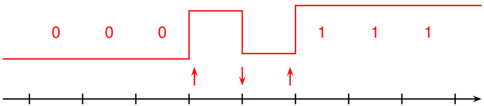
A model we can say something about:



Branching: exclusion

A mean field model

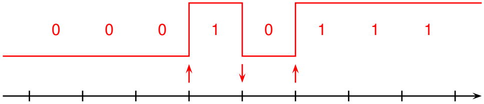
A model we can say something about:



Branching: exclusion

A mean field model

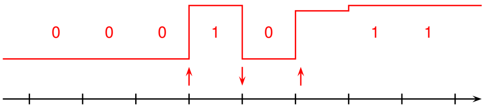
A model we can say something about:



Branching: exclusion

A mean field model

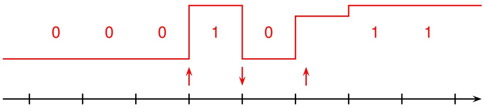
A model we can say something about:



Jump: flip (voting)

A mean field model

A model we can say something about:



Jump: flip (voting)

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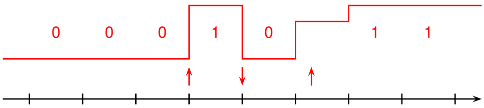

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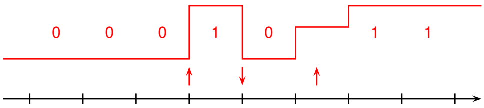

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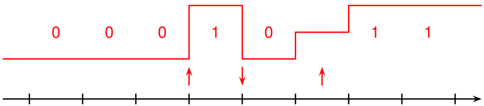

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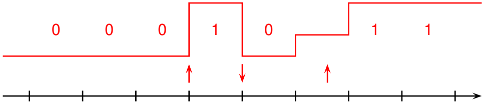

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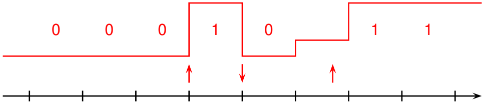

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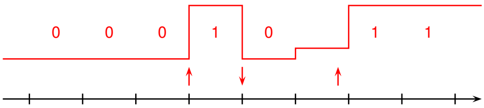

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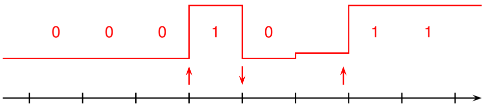

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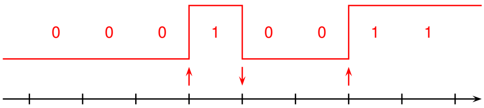

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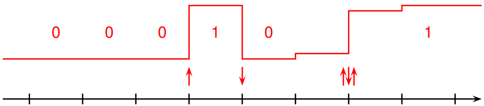

Branching: exclusion

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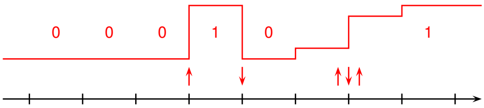

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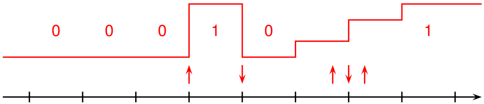

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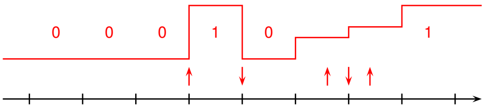

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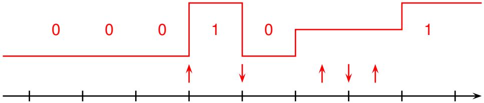

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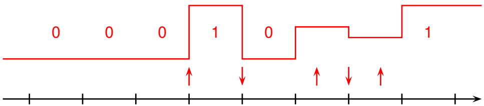

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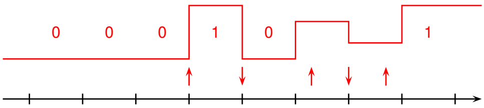

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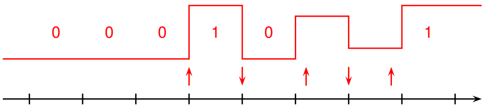

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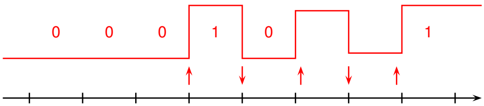

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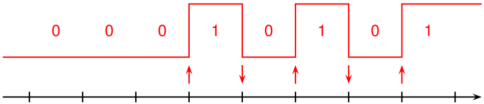

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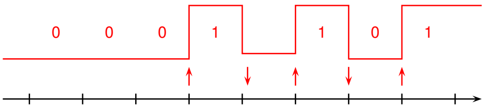

Jump with annihilation: flip (voting)

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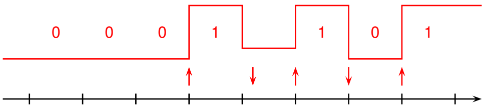

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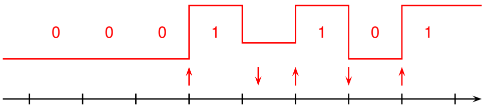

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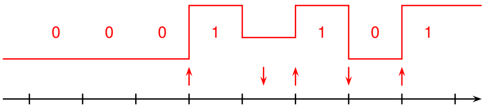

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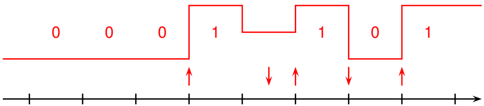

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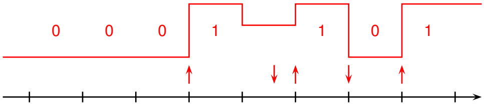

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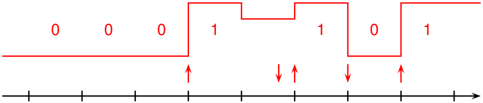

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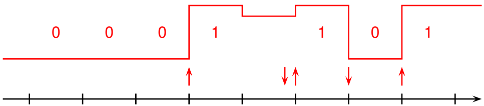

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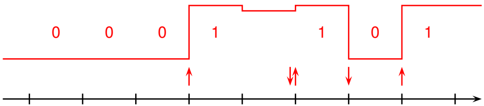

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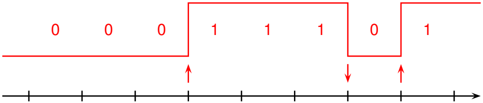

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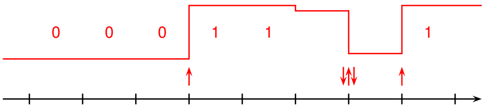

Branching with annihilation: exclusion

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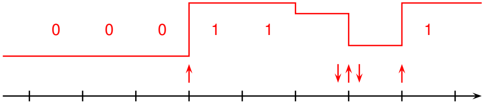

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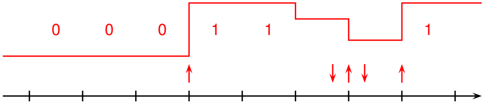

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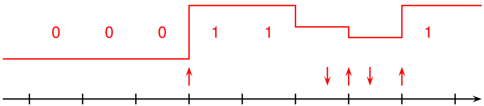

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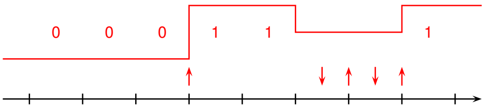

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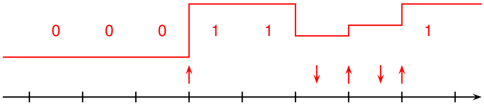

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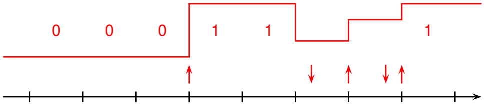

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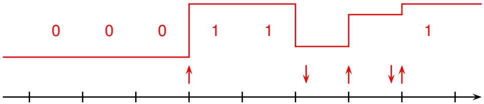

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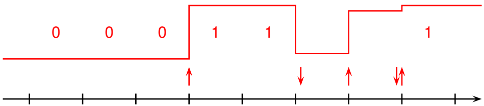

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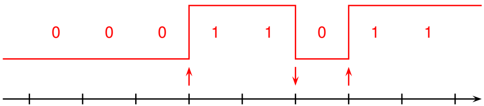

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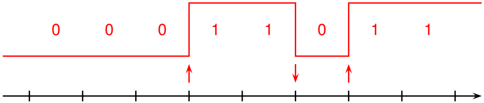

Branching with annihilation: exclusion

- Double branching-annihilating random walks (DBARW)

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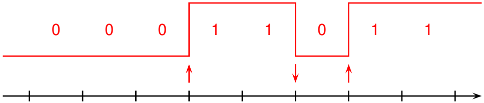

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- ↑'s and ↓'s always alternate;

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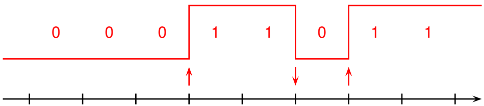

Branching with annihilation: exclusion

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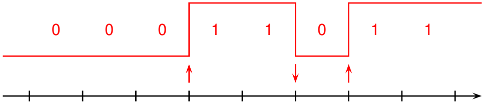

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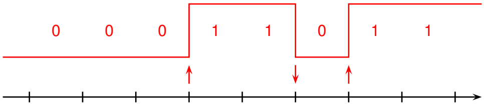

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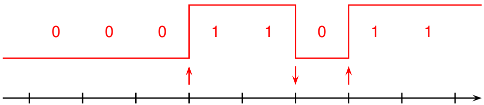
Question: Is the process, as seen by the leftmost ↑, recurrent?

DBARW



First instance of DBARW we could find in the literature: A. Sudbury '90. Positive recurrence: V. Belitsky, P.A. Ferrari, M.V. Menshikov and S.Y. Popov '01; A. Sturm and J.M. Swart '08. *Results are very sensitive to the details of branching.*

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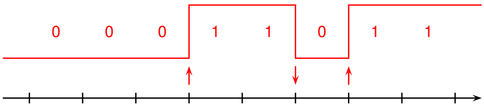

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But: true second class particles interact (*common background of first class particles*).

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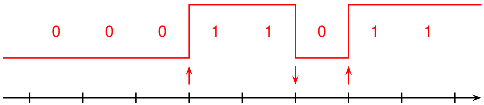

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But: true second class particles interact (*common background of first class particles*).

$\rightsquigarrow$ Repeat the Sturm-Swart proof with configuration dependent jump rates. Jump rates can depend on the whole configuration.

DBARW

Conditions on the jumping and branching rates:

- Translation invariance.

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- Bounds on the branching rates.
- Bounds on the difference for branching rates of $\uparrow$'s and $\downarrow$'s.
- Weak dependence on particles far away.
- No repulsion in the jumping rates between particles. (*A bit of repulsion locally is still OK.*)

Positive recurrence

Theorem

Then, starting from a single $\uparrow$*:*

- *The process takes finitely many steps in finite time (construction).*

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- *The width of the process has all moments finite.*
- *The process as seen from the leftmost $\uparrow$ is positive recurrent.*
- *The stationary distribution sees a finite expected number of particles.*
- *(Extension of all this to non nearest neighbour symmetric branching.)*

An example

- Branching rates: constant.
- Jump rate to the right:

$$\frac{1}{2} + \sum_{\text{particle on right}} \frac{1}{\text{distance}^{\alpha}},$$

jump rate to the left:

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$\alpha > 1$.

Unfortunately we do not seem to be there yet... This is not covered at the moment.

Another example

- Branching rates: constant.
- Jump rate to the right:

$$\frac{1}{2} + \sum_{\text{gaps } L_i \text{ on the right}} \frac{1}{L_i^\alpha}$$

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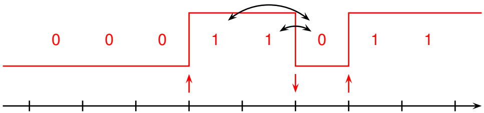
This one is fine.

Two words on the proof



Main tool 1: the number of inversions, i.e., wrongly ordered 1-0 pairs.
If there are too many of them, the generator is negative of the number of these pairs.

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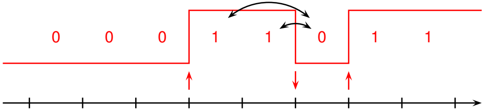
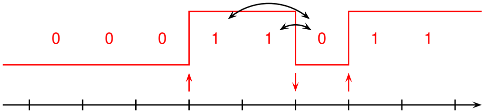


<u>Main tool 1:</u> the number of inversions, i.e., wrongly ordered 1-0 pairs.
If there are too many of them, the generator is negative of the number of these pairs.

<u>Main tool 2:</u> if the process is not tight, then on the long run there cannot be any finite number of particles:

$$\frac{1}{T}\int_0^T \mathbf{P}\{\text{number}(t) < N\}\,\mathrm{d}t \to 0 \qquad (\forall N).$$

Two words on the proof



Main tool 1: the number of inversions, i.e., wrongly ordered 1-0 pairs.
If there are too many of them, the generator is negative of the number of these pairs.

Main tool 2: if the process is not tight, then on the long run there cannot be any finite number of particles:

$$\frac{1}{T}\int_0^T \mathbf{P}\{\text{number}(t) < N\}\,\mathrm{d}t \to 0 \qquad (\forall N).$$

Thank you.