Construction of the zero range process and a deposition model with superlinear growth rates

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Joint work with
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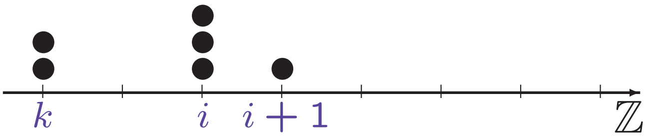
and
Timo Seppäläinen (UW-Madison)

Budapest, 2005

1. The zero range process and the bricklayers' process
2. Construction materials
3. Transferring the estimates
4. Results

1. The zero range process (ZR):

$\omega_i \in \mathbb{Z}^+$



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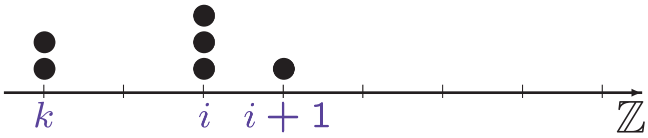

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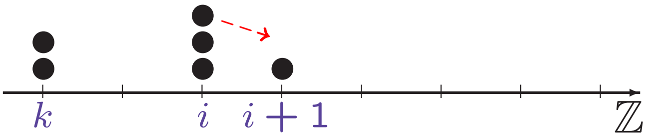

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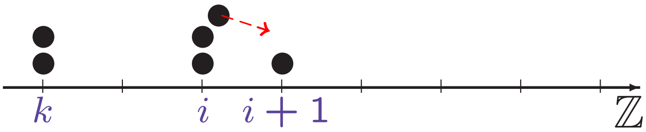

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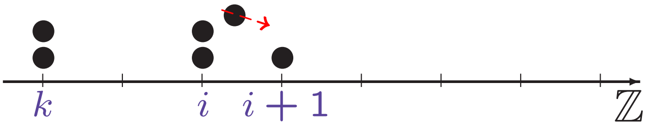

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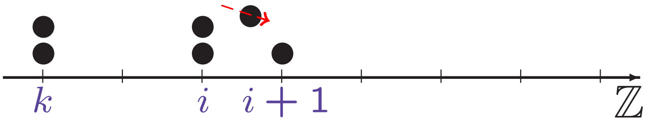

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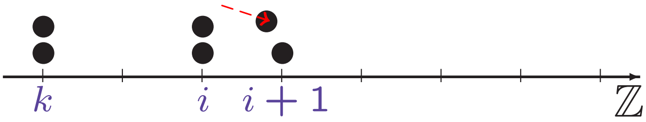

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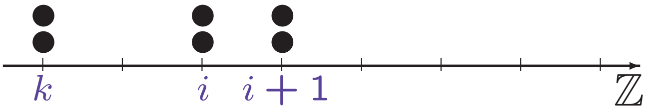

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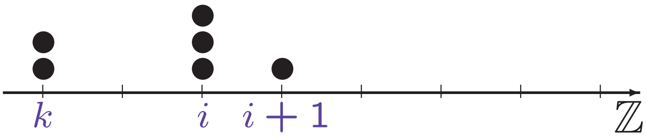

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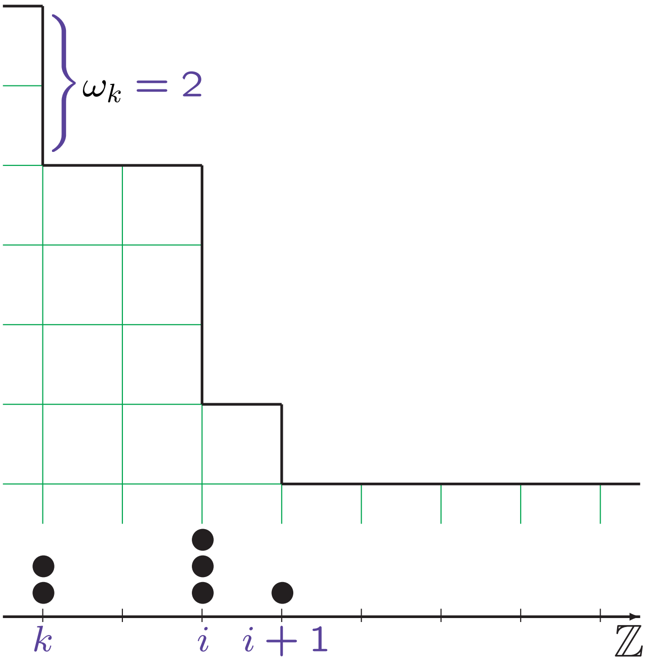

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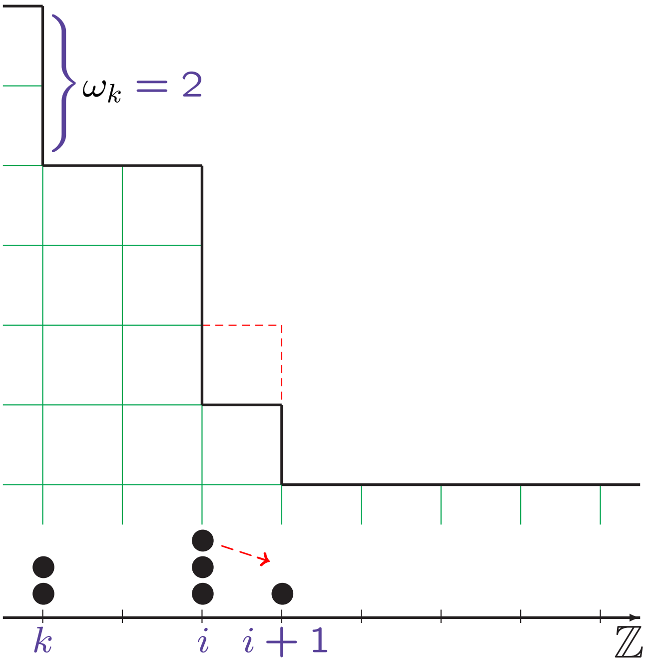

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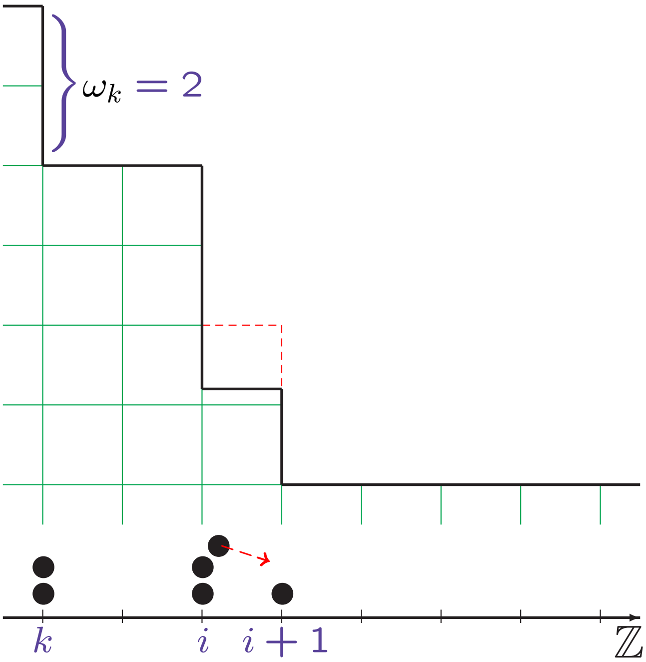

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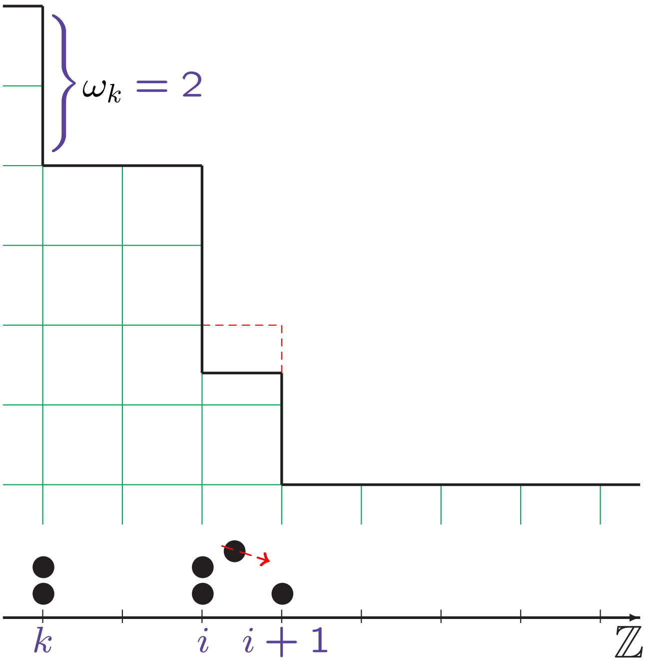

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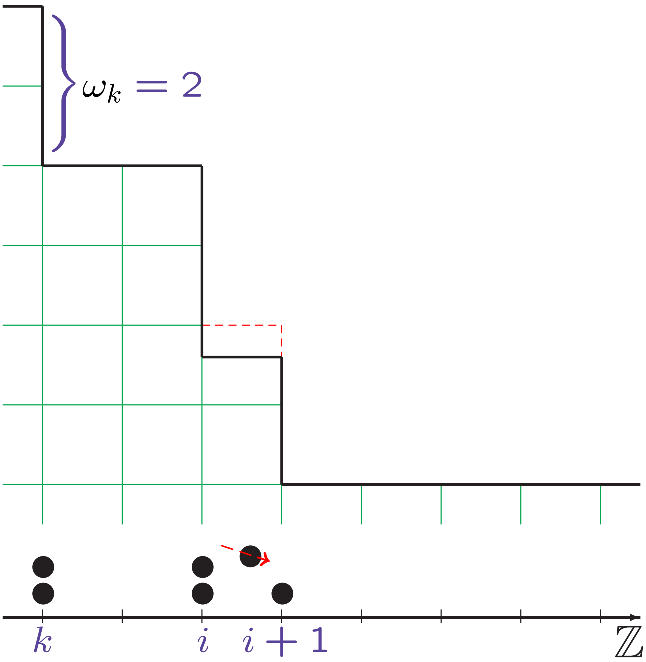

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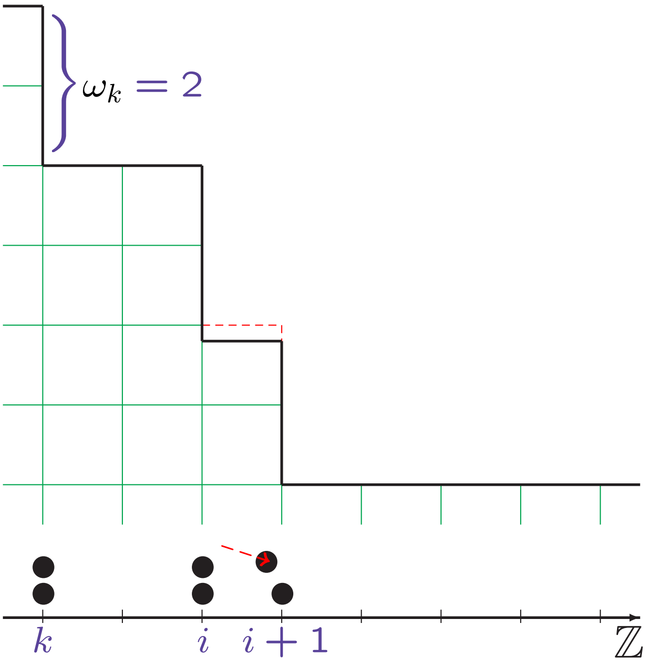

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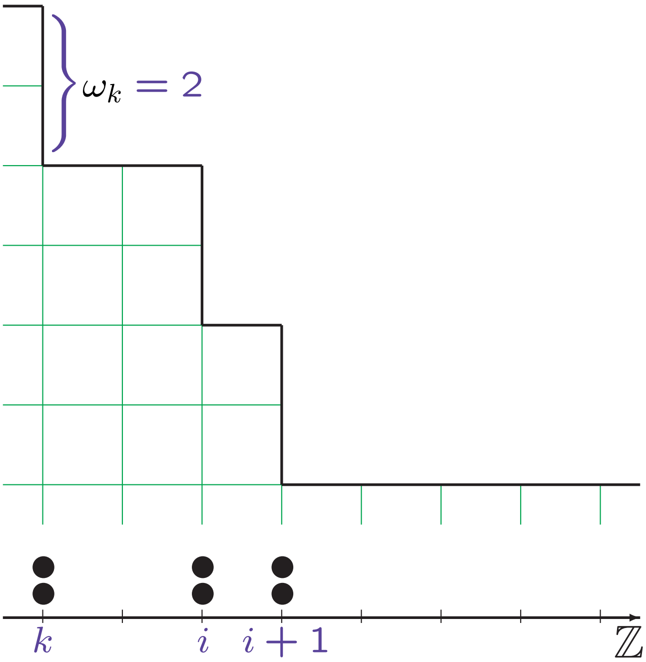

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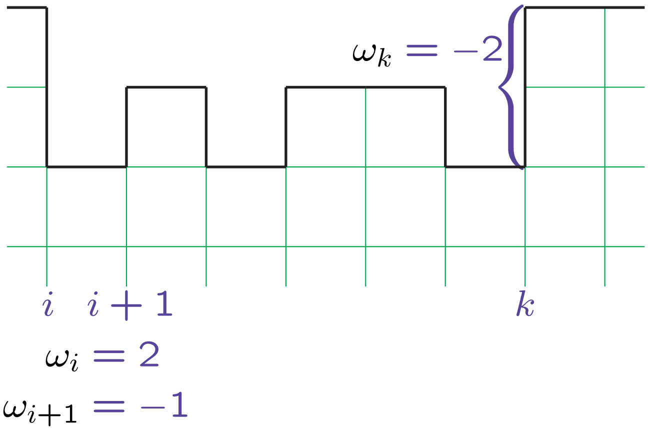
ω_i = negative discrete gradient



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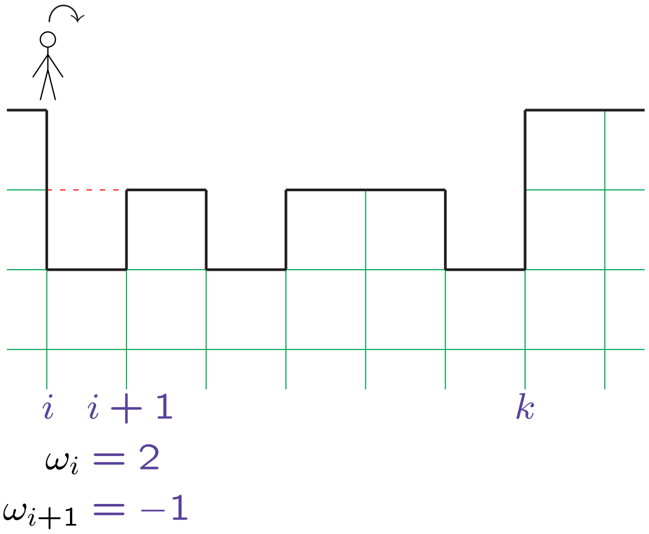

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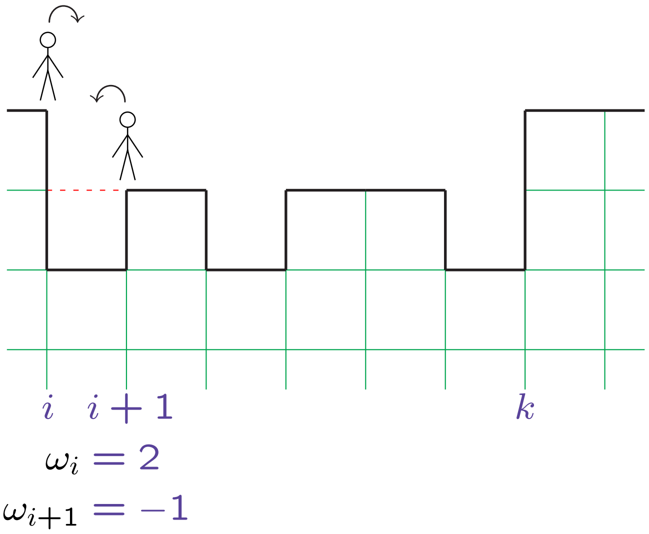

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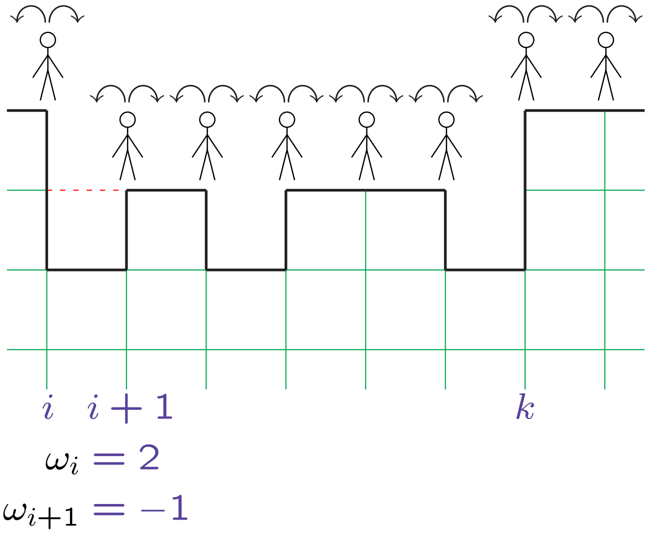

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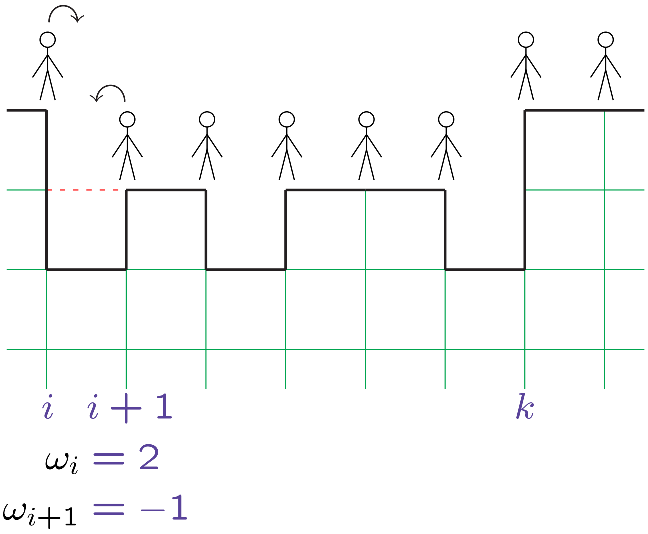

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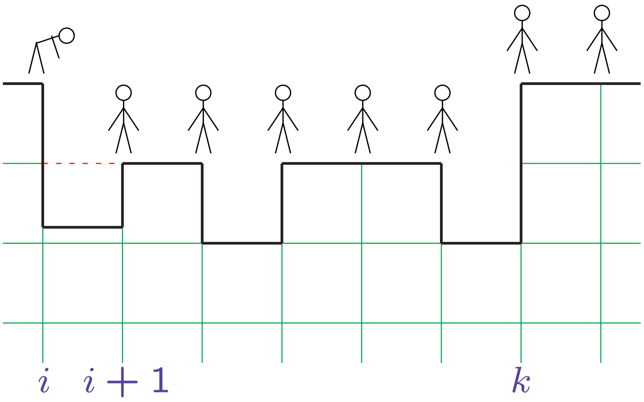

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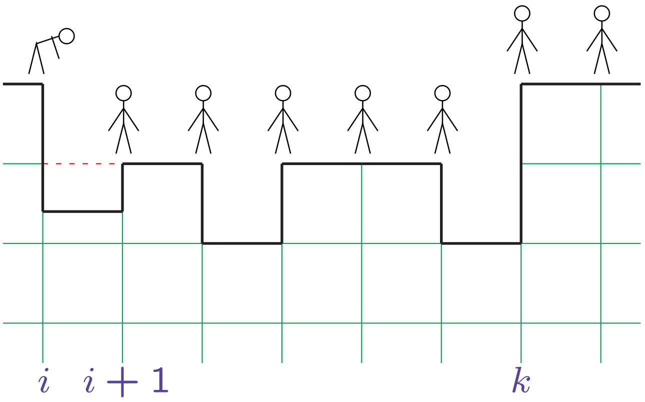

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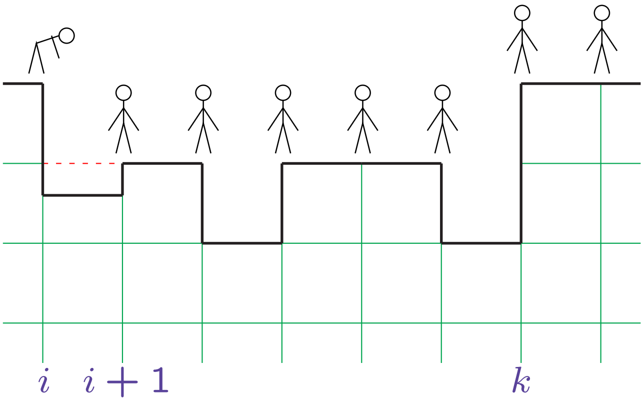

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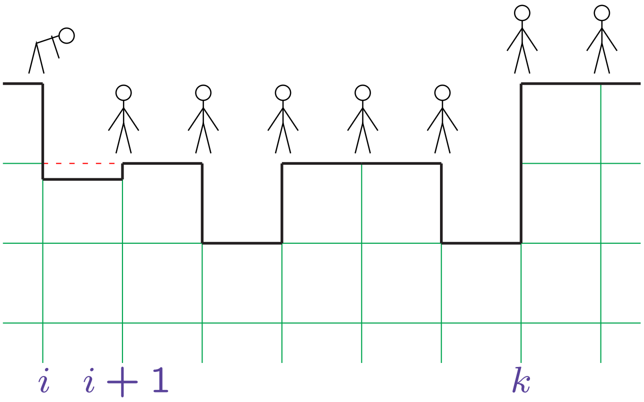

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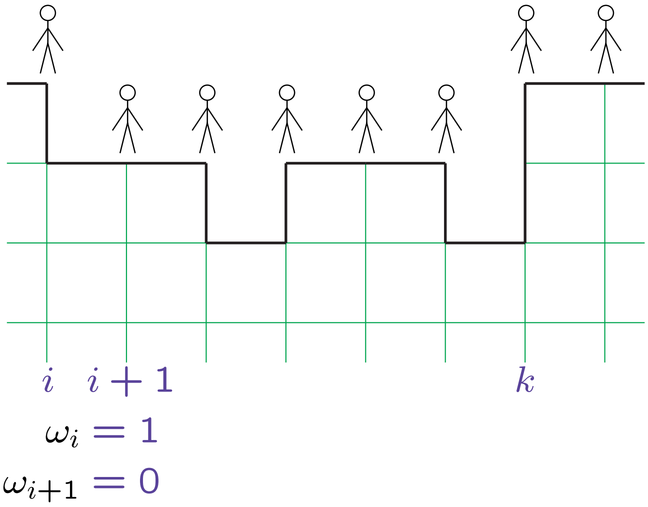

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Estimates used by Andjel do not work.

2. Construction materials

Equilibrium in finite volume

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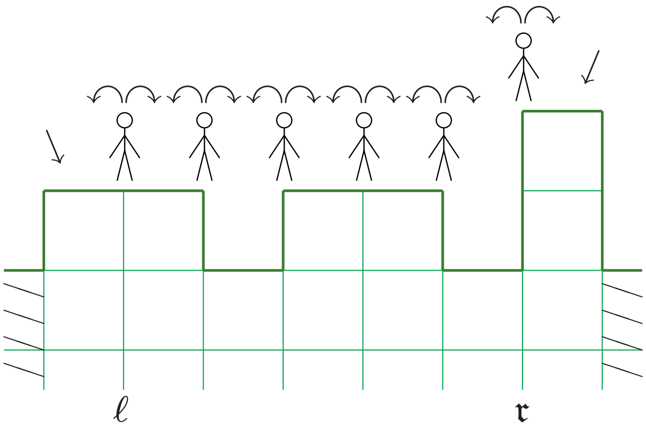

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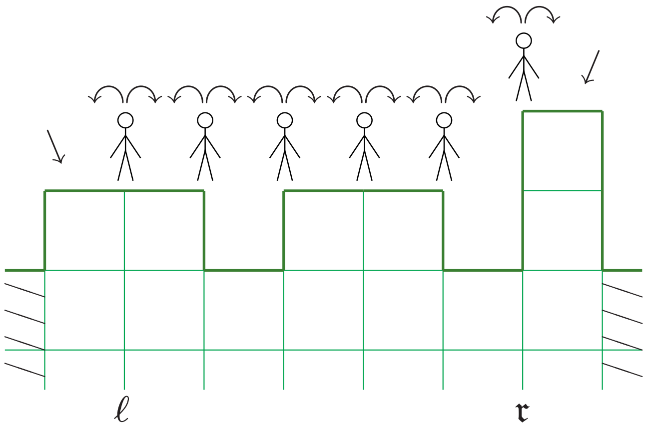

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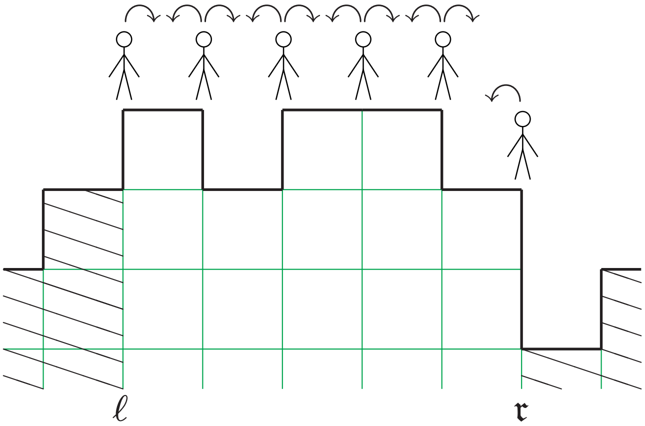

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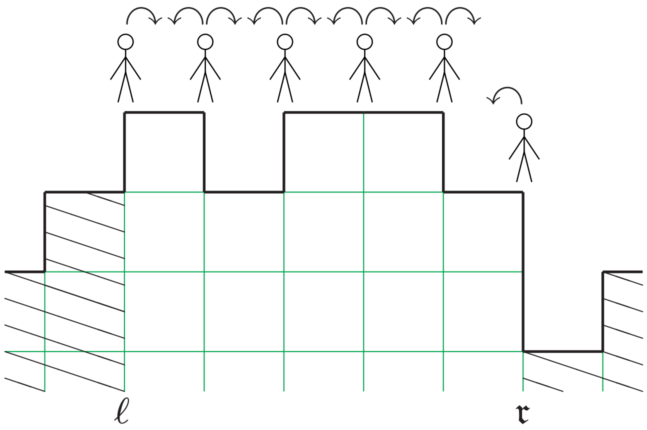

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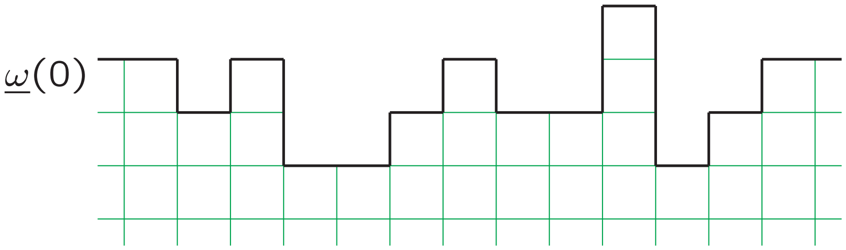
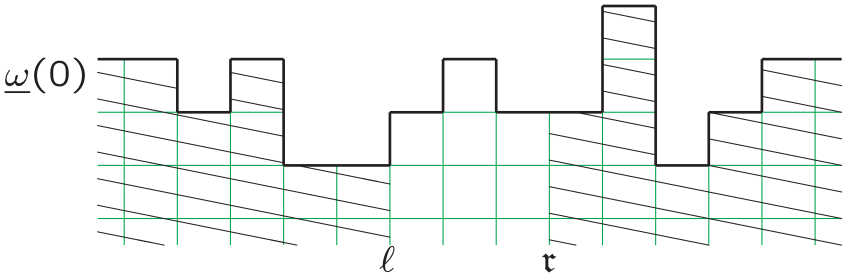
$\rightsquigarrow$ This process is far from equilibrium!



- Fix a state $\underline{\omega}(0) \in \widetilde{\Omega}$.



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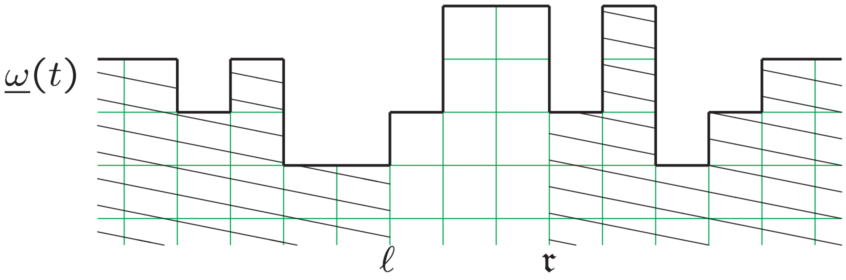

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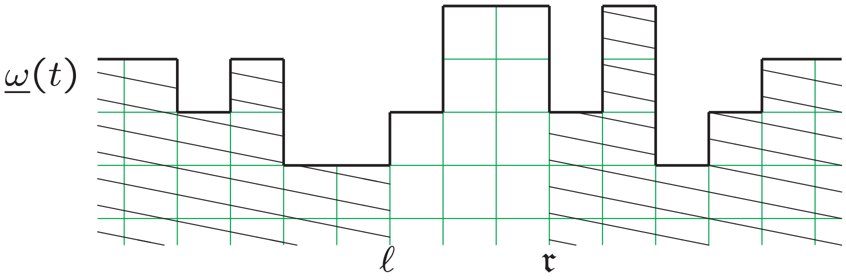

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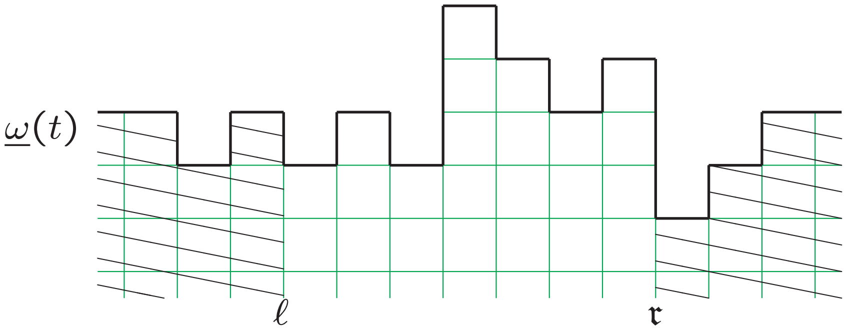

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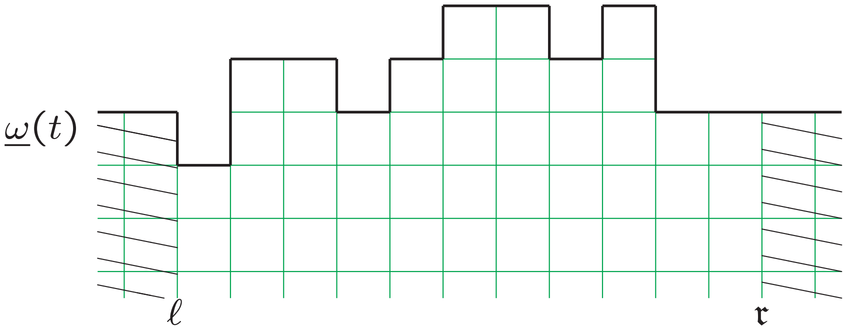

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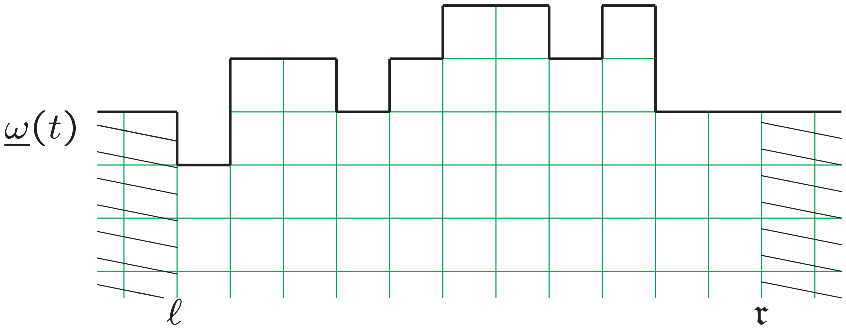

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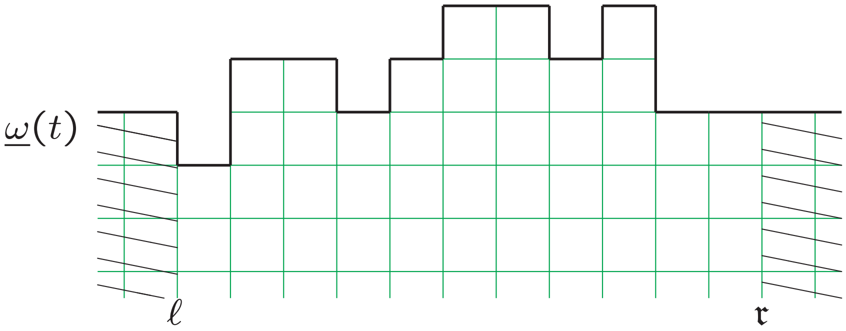

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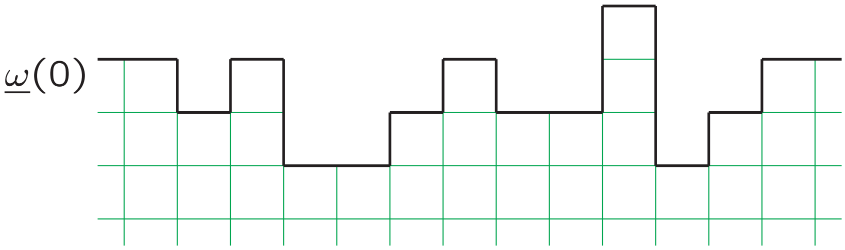

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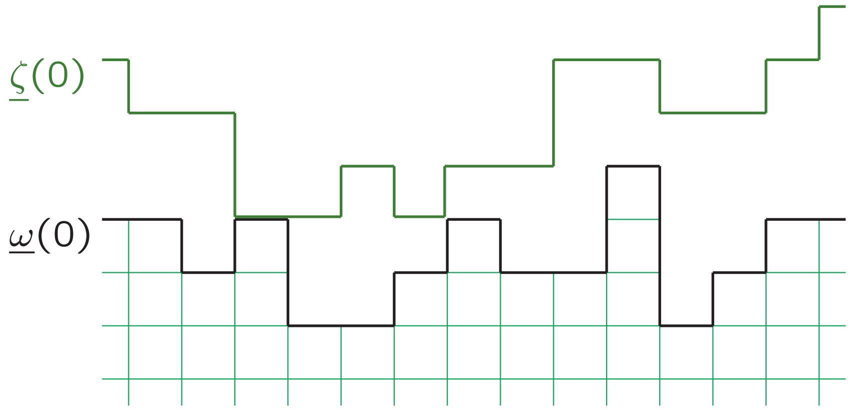

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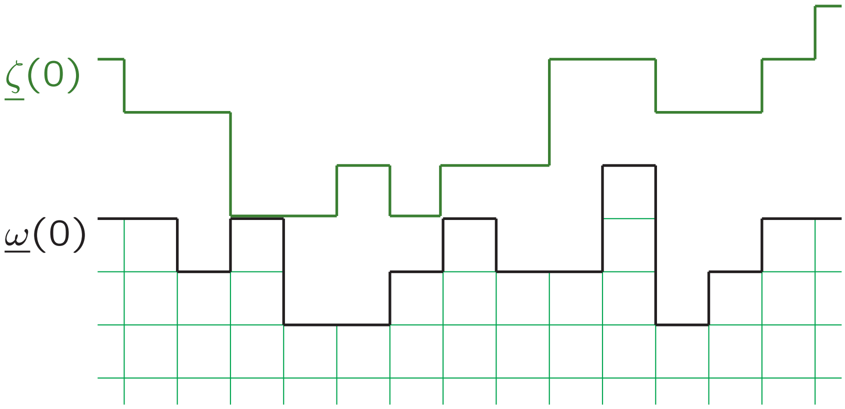

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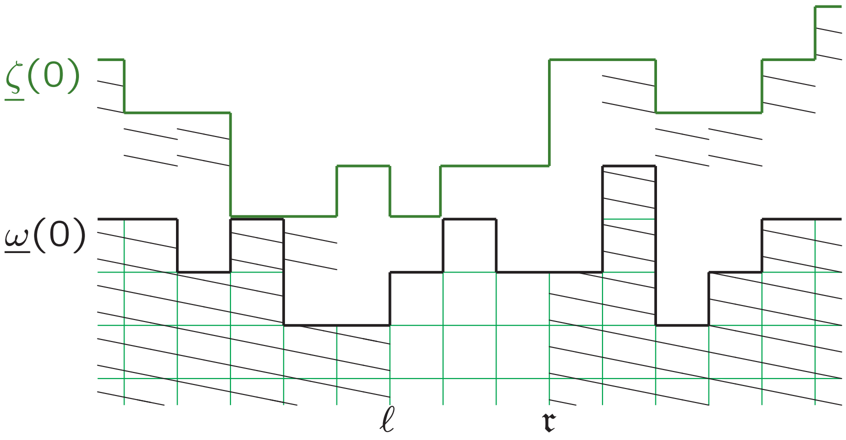

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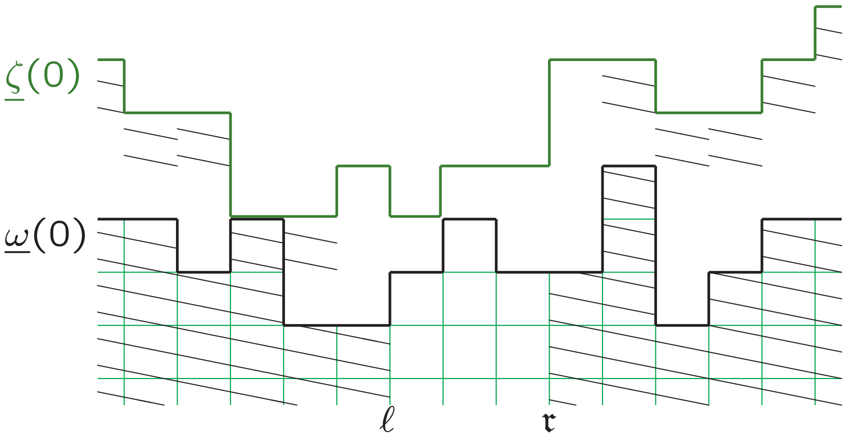

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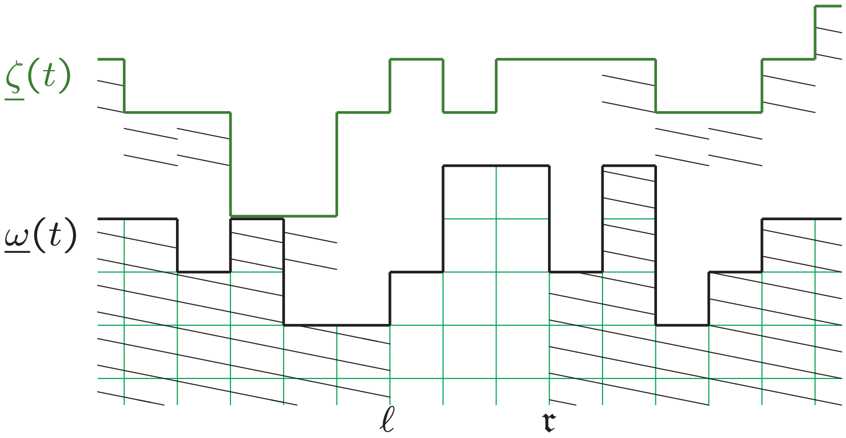

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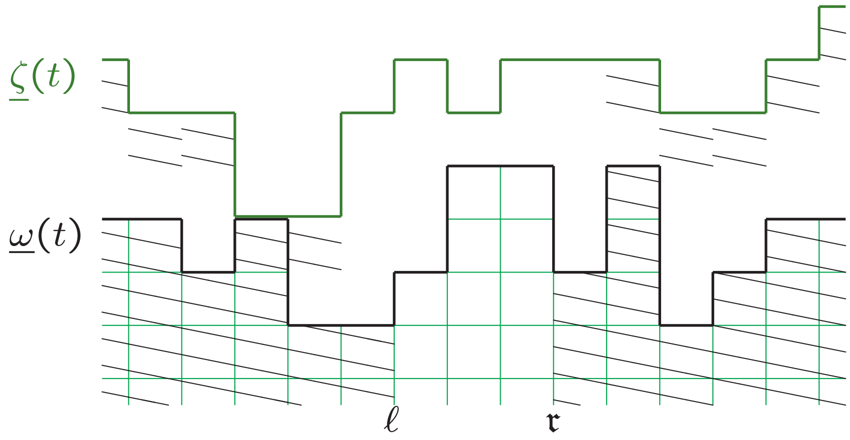

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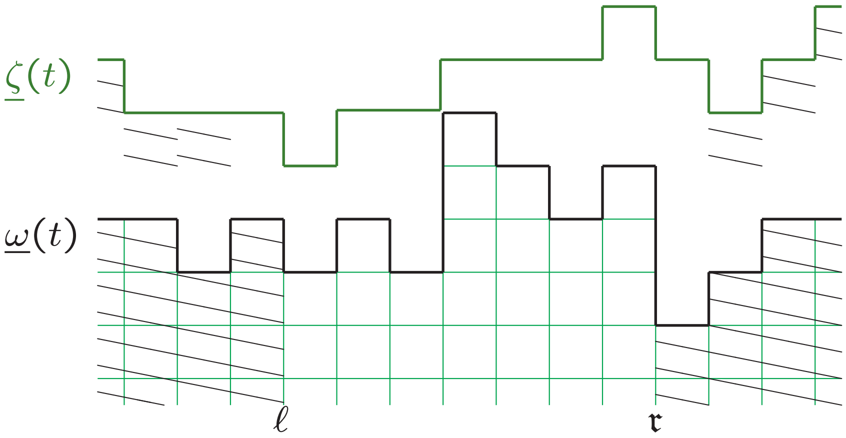

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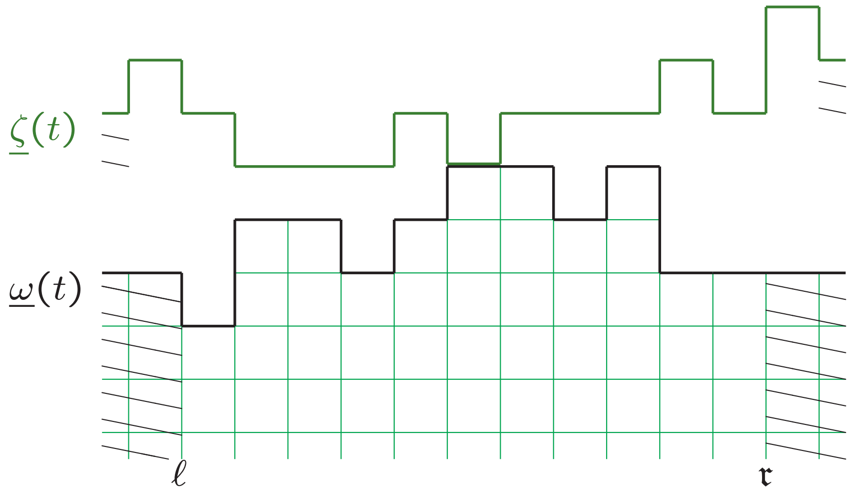

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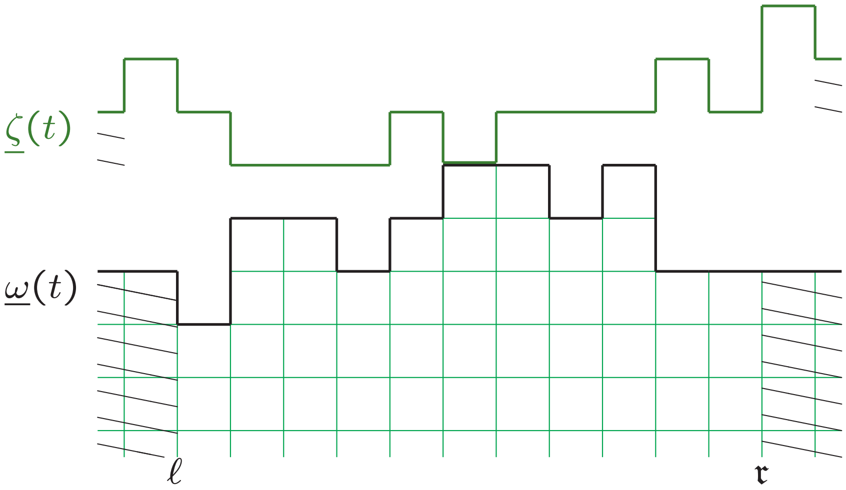

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$\rightsquigarrow$ Coupling 1: The *height* of a column of the monotone process is monotone in ℓ, $\mathfrak{r}$.
$\Rightarrow$ We have a limit of the monotone processes. Is the limit finite? Yes, it is.

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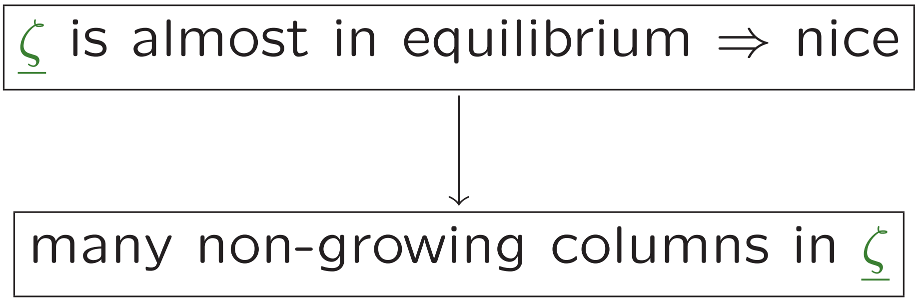

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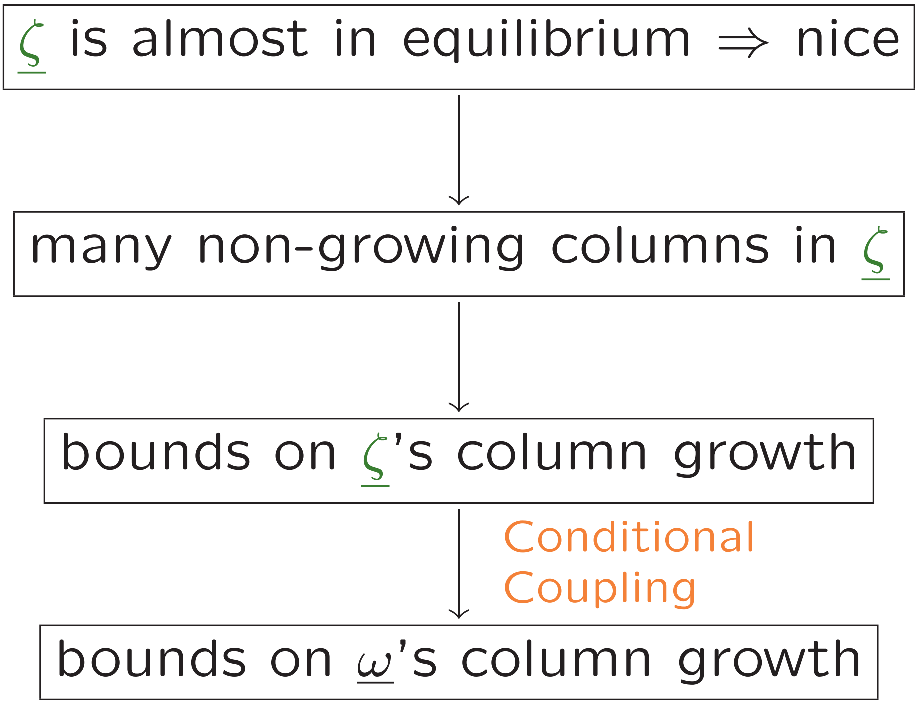

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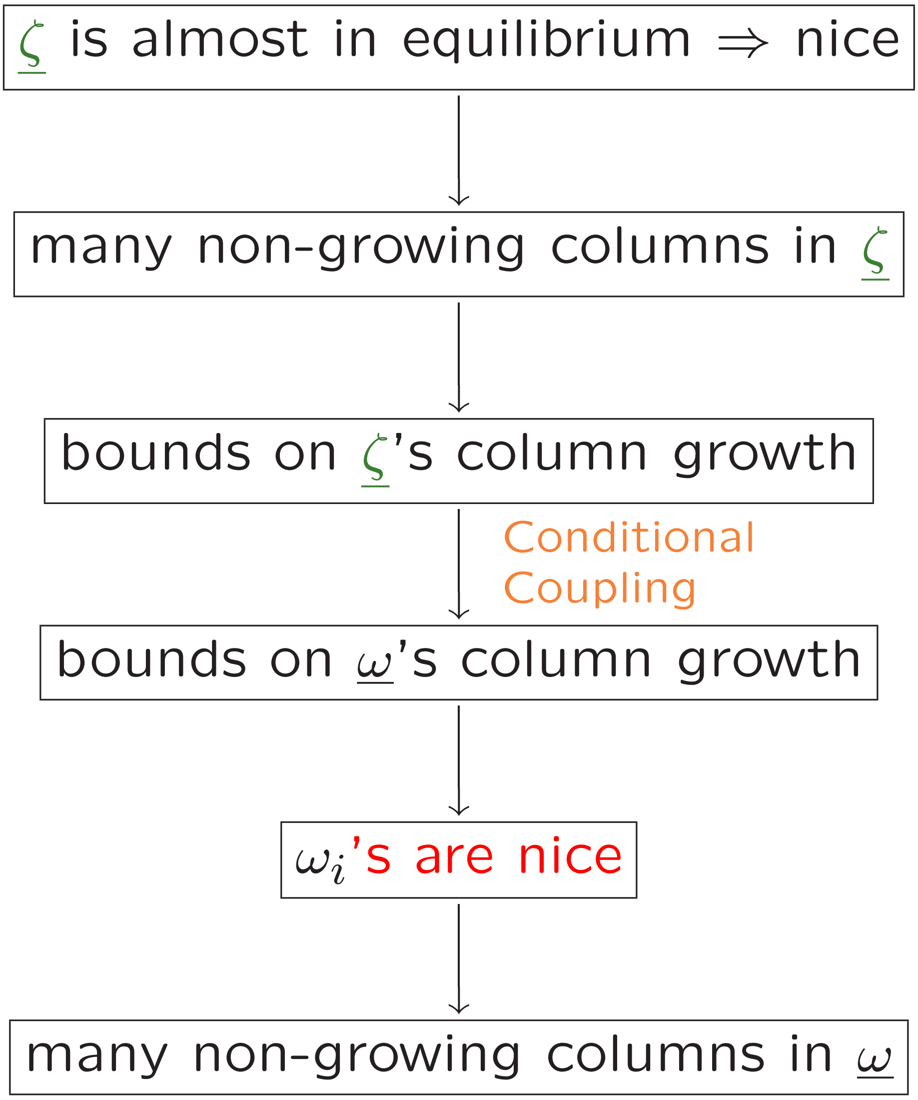

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bounds on disturbance propagation in $\underline{\omega}$

4. Results

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- $\rightsquigarrow$ The measure $\underline{\mu}^\theta$ is stationary for $\underline{\omega}(t)$.
  $\widetilde{\Omega}$ is $\underline{\mu}^\theta$-measure one.

- $\rightsquigarrow$ We have an $S(t)$ strongly continuous $\mathbb{L}^2_{\underline{\mu}^\theta}$-semigroup.

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$$S(t)\varphi(\underline{\omega}) = \varphi(\underline{\omega}) + \int\limits_0^t S(s)L\varphi(\underline{\omega})\,\mathsf{d}s$$

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$$\frac{\mathrm{d}}{\mathrm{d}t} S(t)\varphi(\underline{\omega})\bigg|_{t=0} = L\varphi(\underline{\omega})$$

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Our semigroup results do not seem to be enough for the usual Dirichlet-form proof of ergodicity.

Thank you.