Characteristic fuction and Central Limit Theorem - examples

Further Topics in Probability

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Example 1. Show that the characteristic function of the $\mathcal{N}(\mu, \sigma^2)$ distribution is $\mathrm{e}^{-\sigma^2 t^2/2 + i\mu t}$.

Example 2. Show that the characteristic function of the Cauchy(b, a) distribution is $\mathrm{e}^{itb - a|t|}$.

Example 3. Show that the Uniform(-1, 1) distribution cannot be obtained as the difference of two i.i.d. random variables.

Example 4. An astronomer measures the unknown distance μ of an astronomical object. He performs n i.i.d. measurements each with mean μ and standard deviation 2 lightyears. How large should n be to have ± 0.5 lightyears accuracy with at least 95% probability?

Example 5. Work out the above estimation using Chebyshev's inequality on the sample mean $\bar{X}$. How does your bound compare to 62?

Example 6. The number of students who apply to a course is $X \sim \mathrm{Poi}(100)$. Estimate the probability that more than 120 apply.