

UNIVERSITY OF BRISTOL

School of Mathematics

MATH 11300
PROBABILITY 1

Mock Exam Paper for January 2016

1 hour and 30 minutes

**This paper contains two sections: Section A and Section B.
Each section should be answered in a separate answer book.**

Section A contains FIVE questions and Section B contains TWO questions
All SEVEN answers will be used for assessment.

Calculators are not permitted in this examination.
Statistical tables will be provided.

On this examination, the marking scheme is indicative and is intended only as a guide to the
relative weighting of the questions.

Do not turn over until instructed.

Section A: Short Questions

Please explain all your answers, show your computations.

- A1. A pair of seagulls have two chicks in the nest, call them A and B . A has 90% chance of survival and B also has 90% chance of survival. However, these events cannot be considered independent.
- (a) **(3 marks)** Define a sample space Ω for this situation.
 - (b) **(5 marks)** Prove that the probability that both chicks survive is at least 80%. (*Again: survival of A and B cannot be considered independent!*)
- A2. **(8 marks)** Pick any of the numbers 1, 2, ..., 20 with equal chance, and define the events $E := \{\text{the chosen number is even}\}$, $F := \{\text{the chosen number is divisible by 3}\}$. Are E and F independent? Why or why not?
- A3. **(8 marks)** Manufacturing of a die went wrong, and the sides "3" or "4", respectively, have half the chance of the sides "1", "2", "5", or "6", respectively. Determine the expected value and the standard deviation of the number shown on this die.
- A4. My Grandma's phone company charges 1p per minute, and the duration of the call will be rounded up to the next whole minute. When she starts chatting with her old friend, the length of the call is exponentially distributed with an average duration of 10 minutes.
- (a) **(4 marks)** Find the distribution of the charge, in pence, for one chat over the phone with the old friend.
 - (b) **(4 marks)** Find the expected amount to pay after one such call.
- A5. A street has n street lamps, and it can be divided into $n - 1$ segments between the neighboring lamps. Such a segment is said to be dark, if both lamps at its ends are defective. Each lamp is defective independently with probability p .
- (a) **(3 marks)** Are the segments dark independently of each other? Why or why not?
 - (b) **(5 marks)** Find the expected number of dark segments on this street.

Section B

Please explain all your answers, show your computations.

- B1. Alice and Bob (both aged 5) roll one fair die each separately, they don't even see each other. Each of them stops the first time they see number "6" appearing on their own die.
- (a) **(6 marks)** What is the expected number of rolls Alice makes?
 - (b) **(18 marks)** They come to us very excited: they tell us that they rolled their dice the same number of times! But they won't tell us what this number was. With this piece of information, what is the distribution of the number of rolls Alice made?
 - (c) **(6 marks)** Still with this piece of information, now what is the expected number of rolls Alice made?
- B2. Find the density of the sum of
- (a) **(15 marks)** two independent $\text{Exponential}(\lambda)$ random variables,
 - (b) **(15 marks)** an $\text{Exponential}(\lambda)$ and an independent $\text{Exponential}(\mu)$ random variable ($\lambda \neq \mu$).

End of examination.