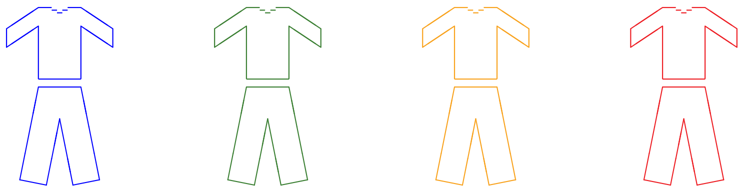
The four outfits and the fluctuations of the simple exclusion process



Márton Balázs
(University of Wisconsin - Madison)

Joint work with

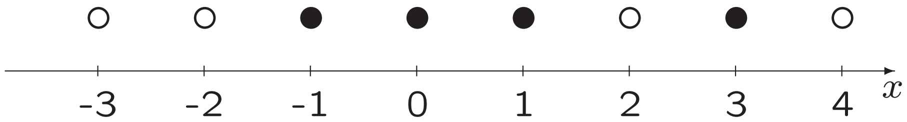
Eric Cator
(Delft University of Technology)

and

Timo Seppäläinen
(University of Wisconsin - Madison)

Ames, April 25

Outfit 1: Interacting particles



Bernoulli(ϱ) distribution

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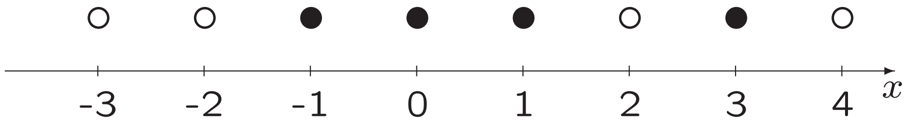

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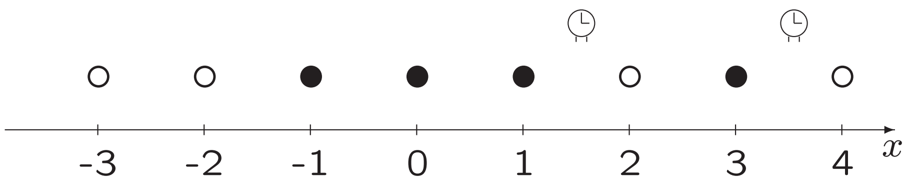

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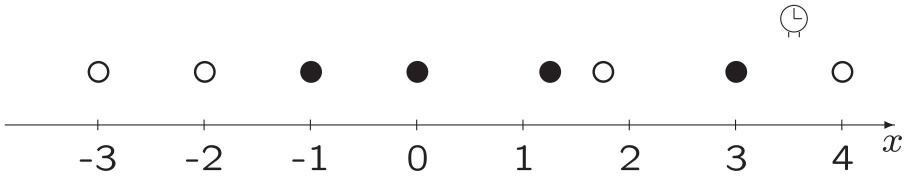

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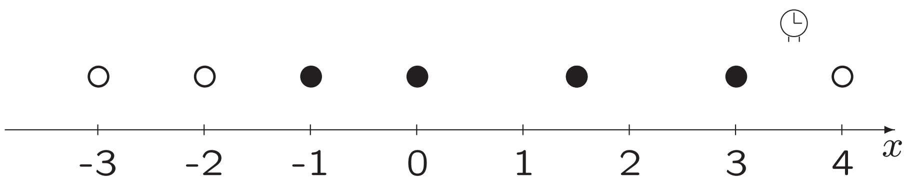

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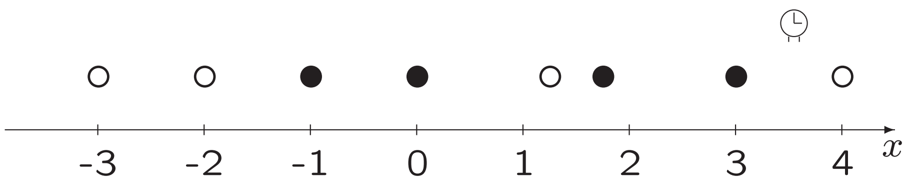

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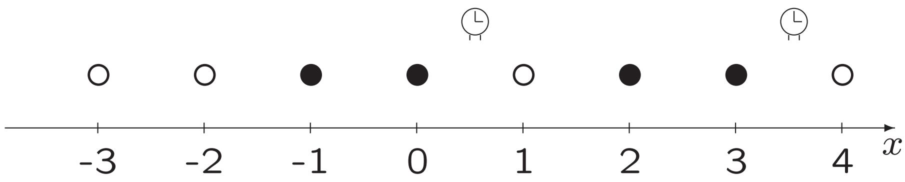

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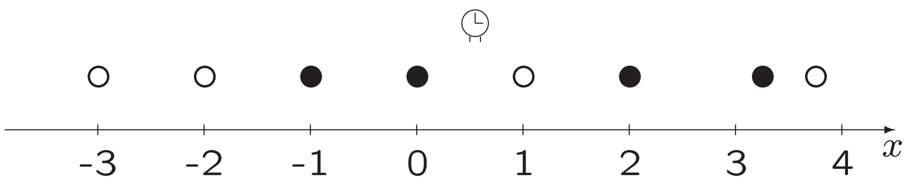

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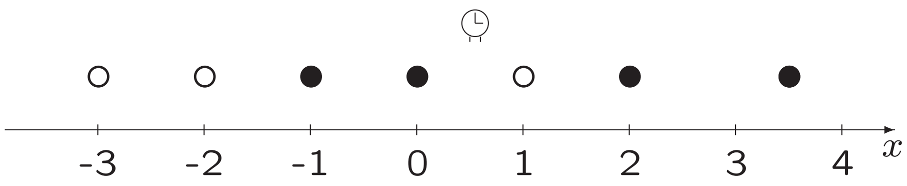

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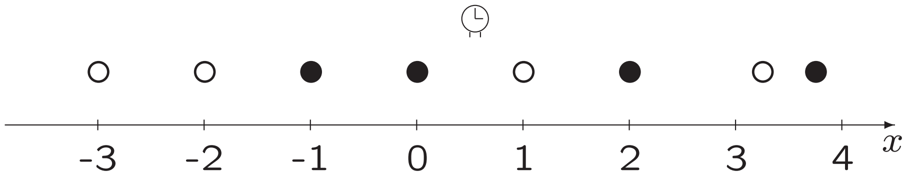

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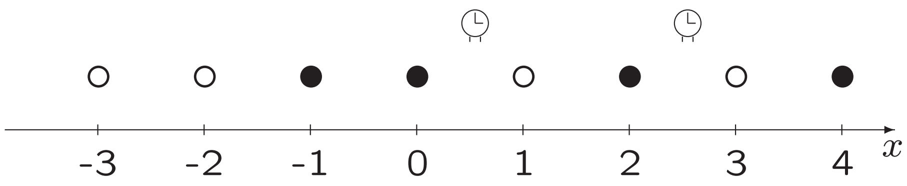

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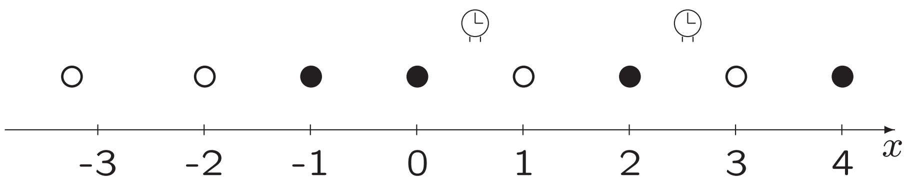

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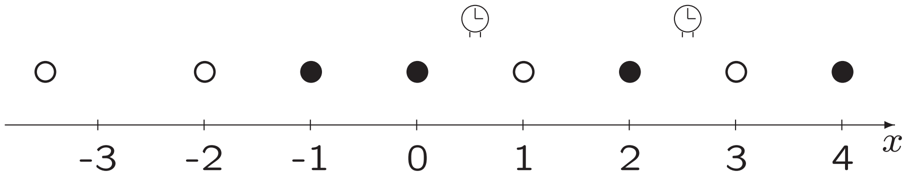

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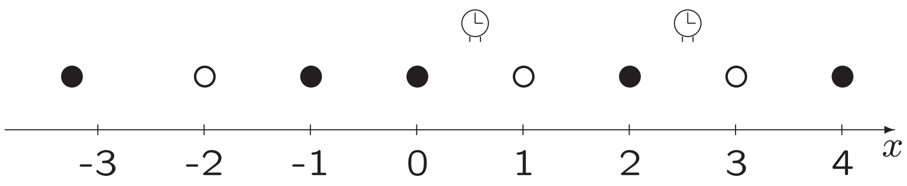

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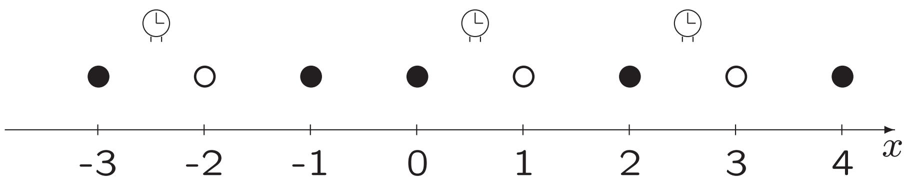

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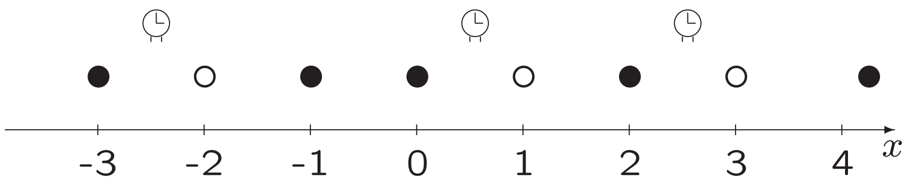

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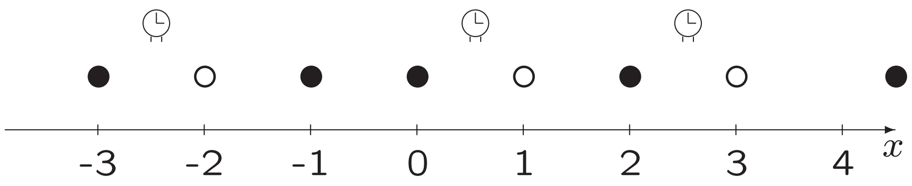

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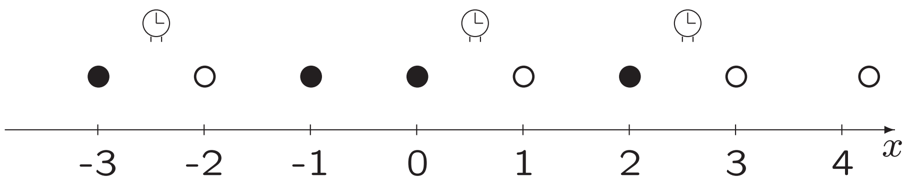

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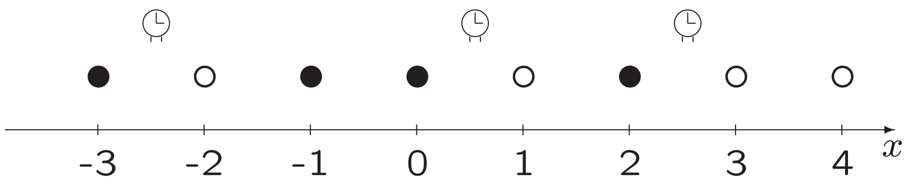

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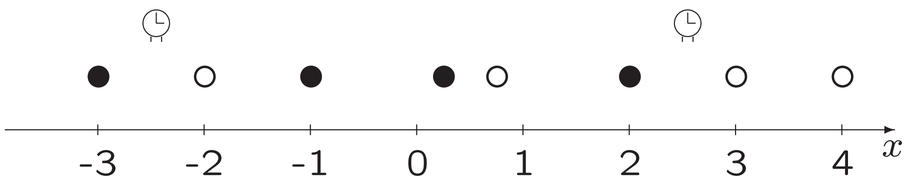

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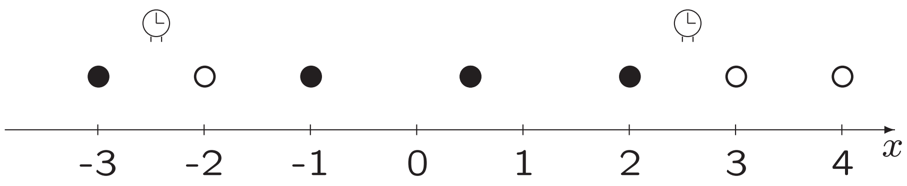

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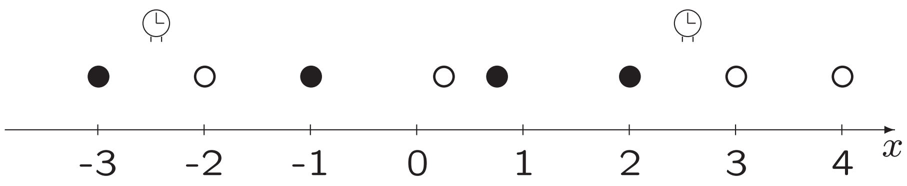

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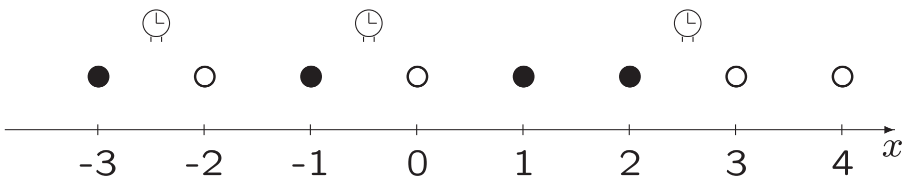

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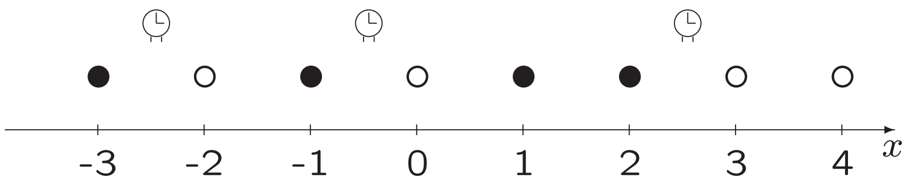

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Particles try to jump to the right, but block each other.

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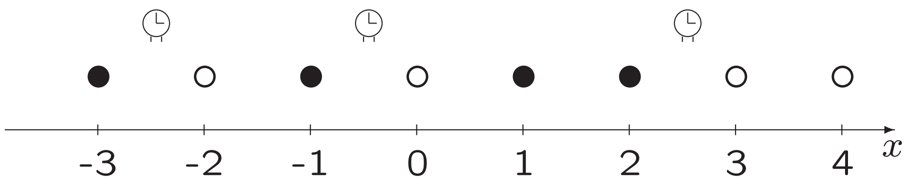

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The Bernoulli(ϱ) distribution is time-stationary for any ($0 \le \varrho \le 1$). Any translation-invariant stationary distribution is a mixture of Bernoullis.

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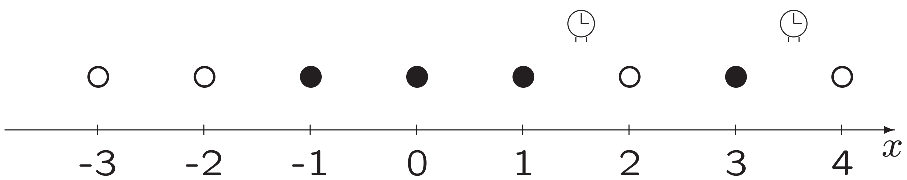
$\rightsquigarrow$ The characteristic speed $C(\varrho) := 1 - 2\varrho$. ($\varrho$ is constant along $\dot{X}(T) = C(\varrho)$.)

Outfit 2: Surface growth



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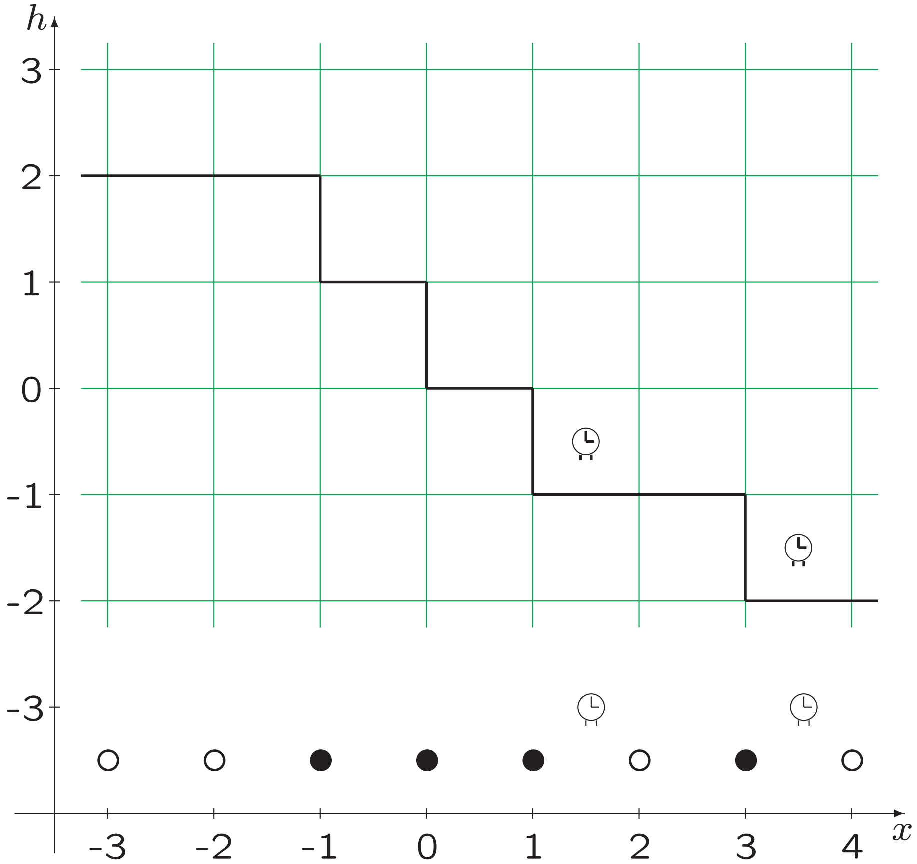

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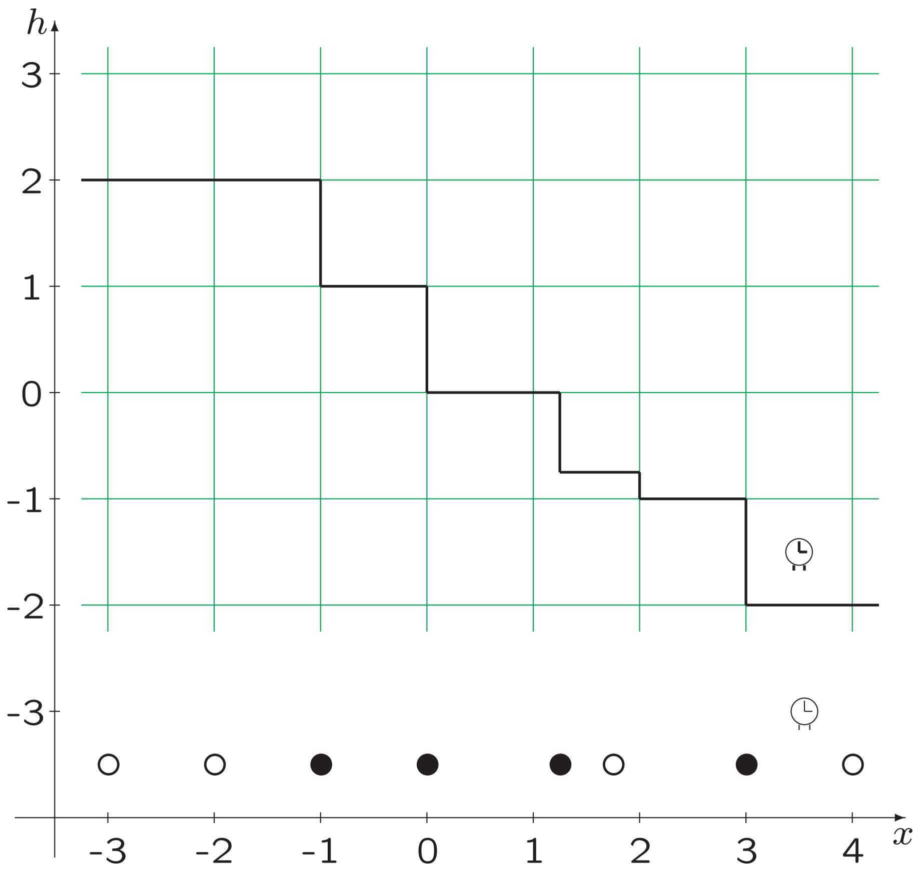

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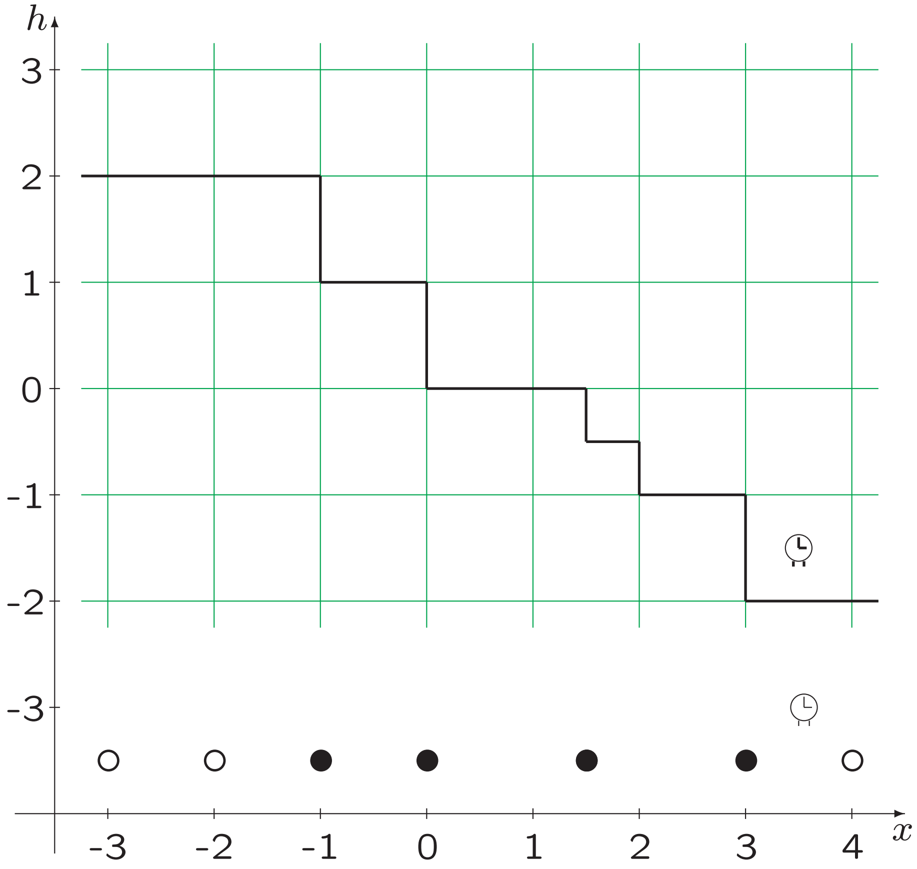

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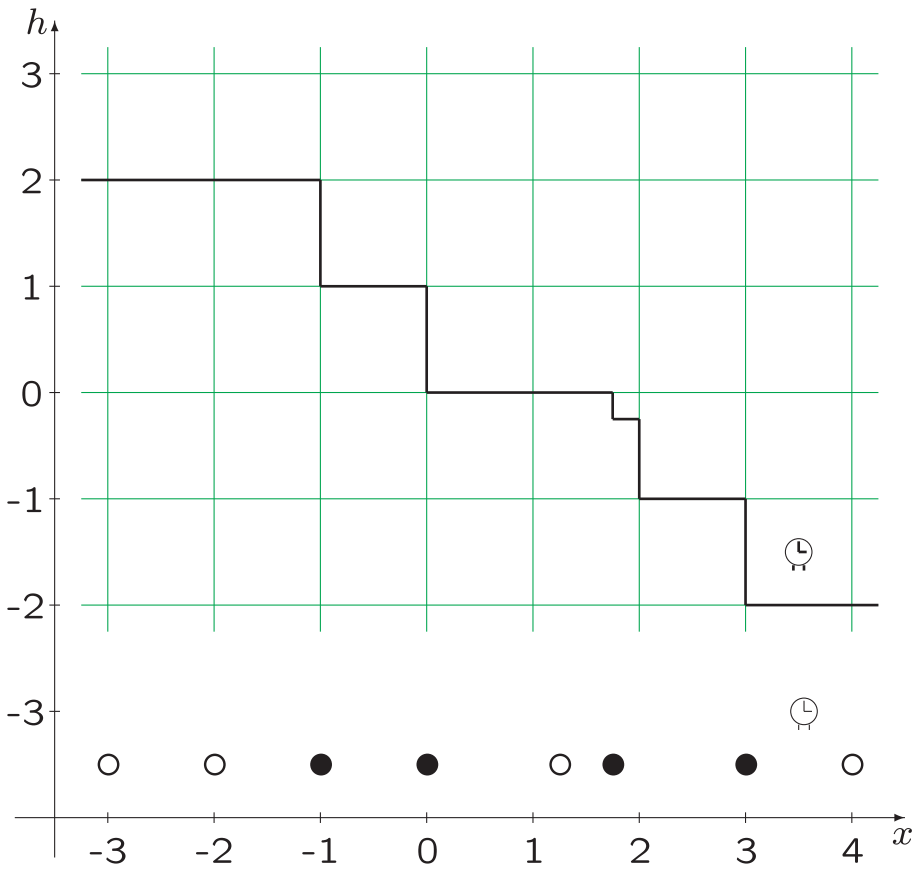

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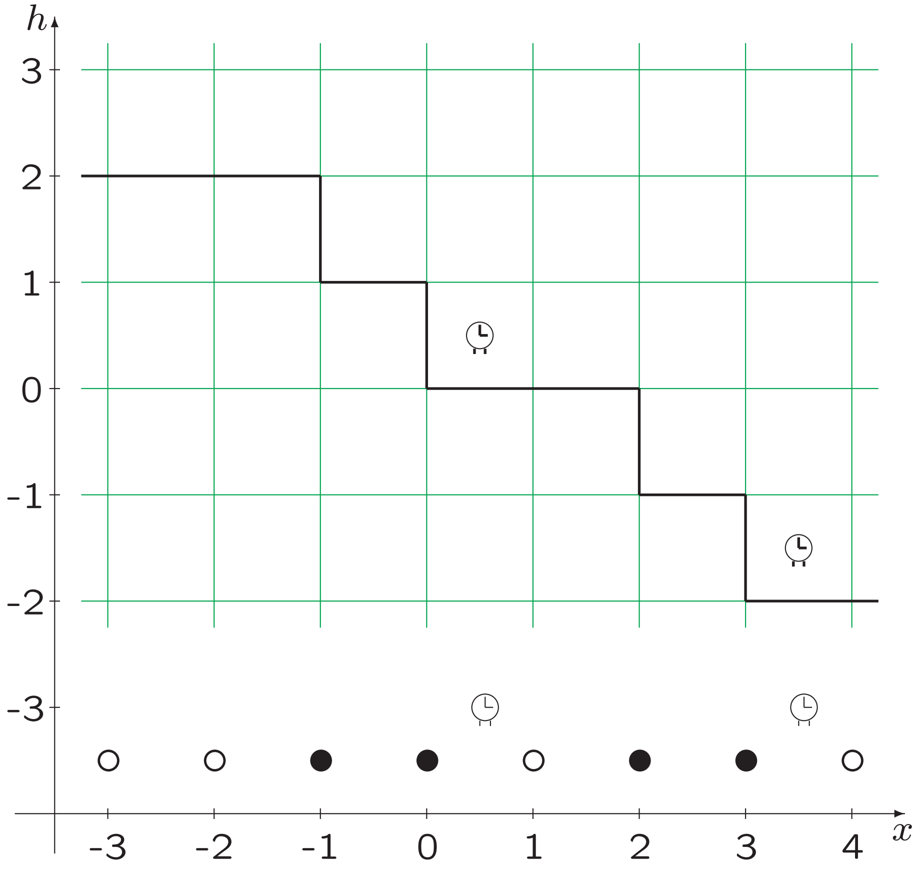

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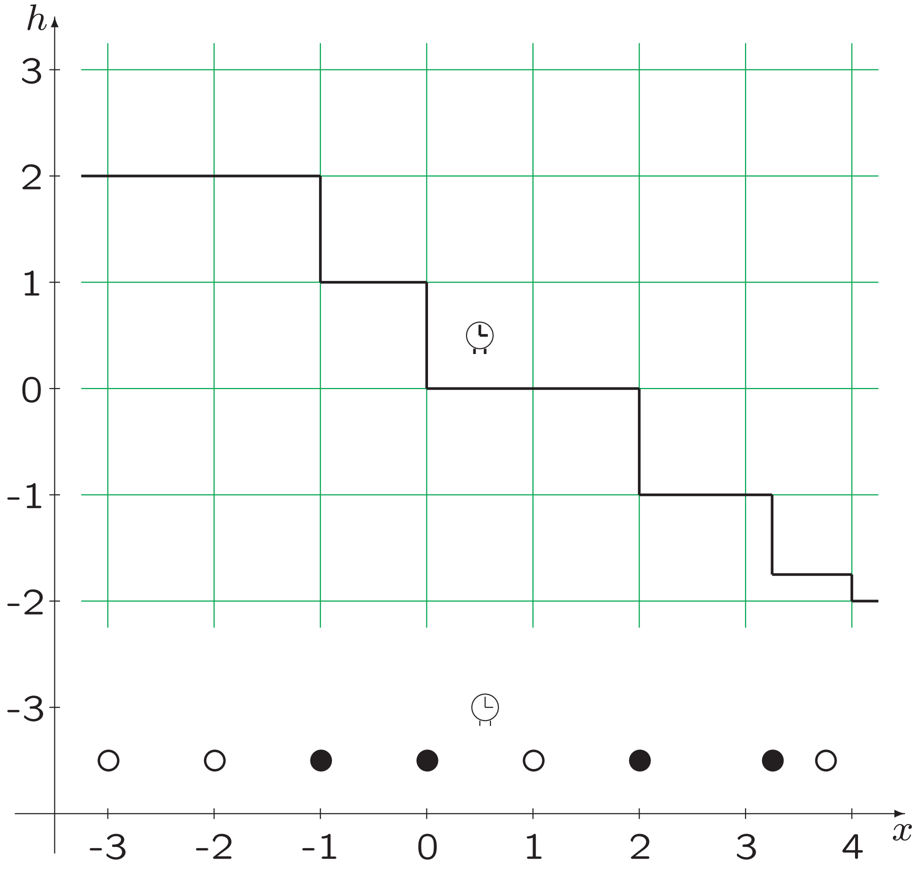

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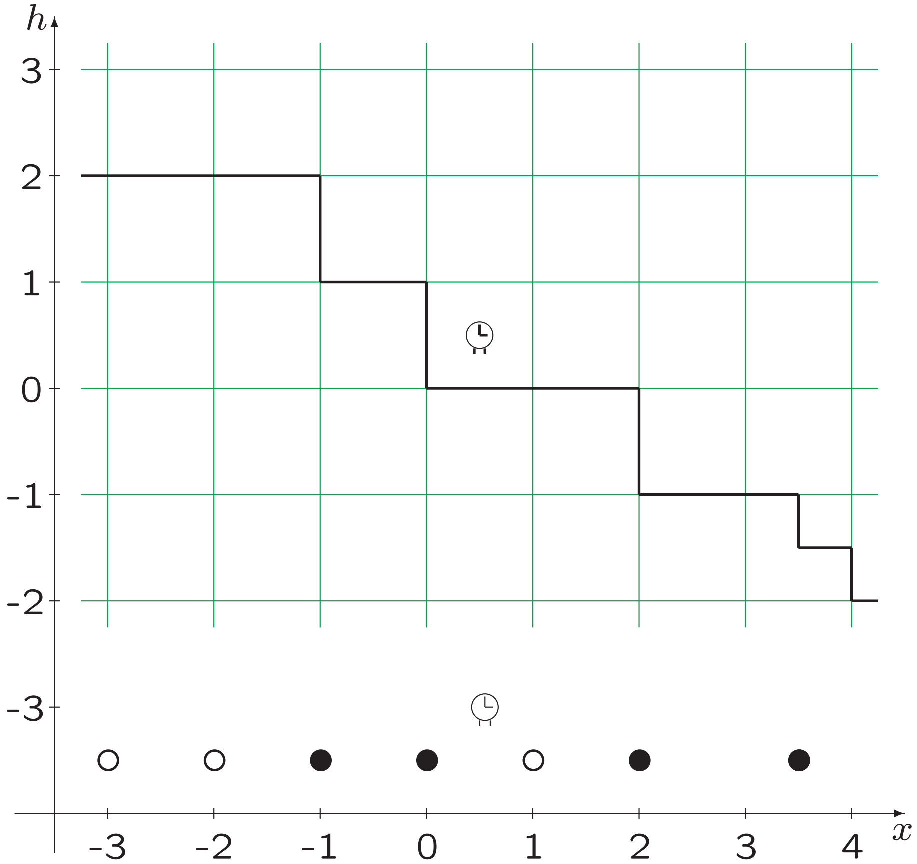

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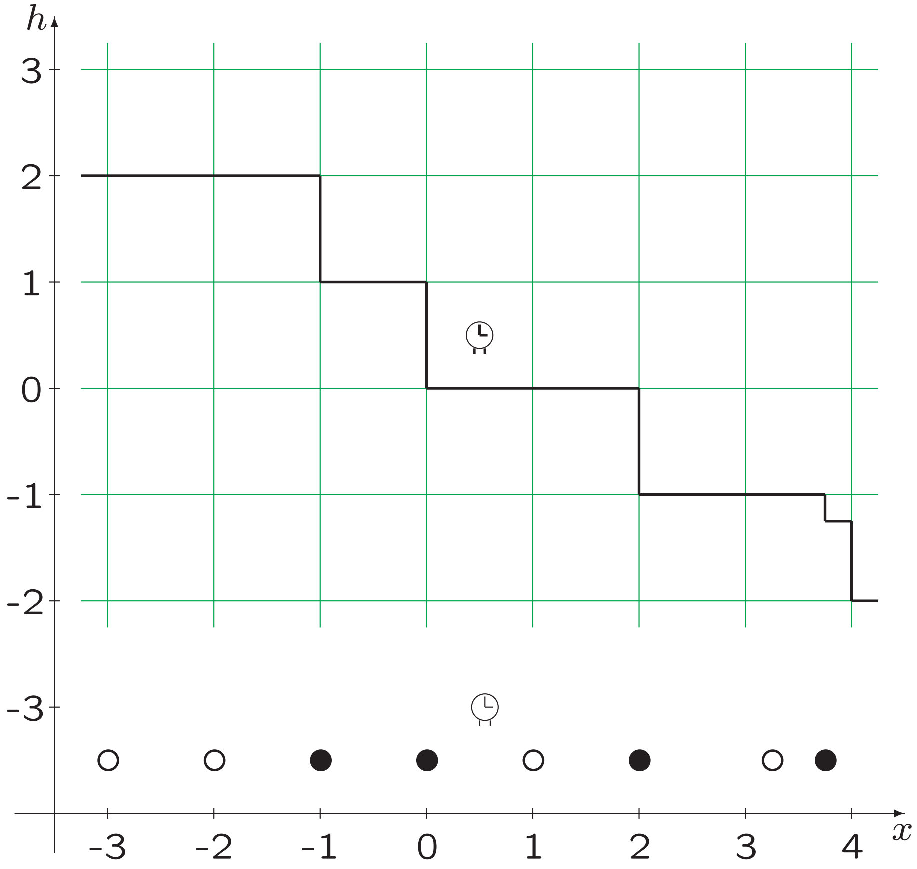

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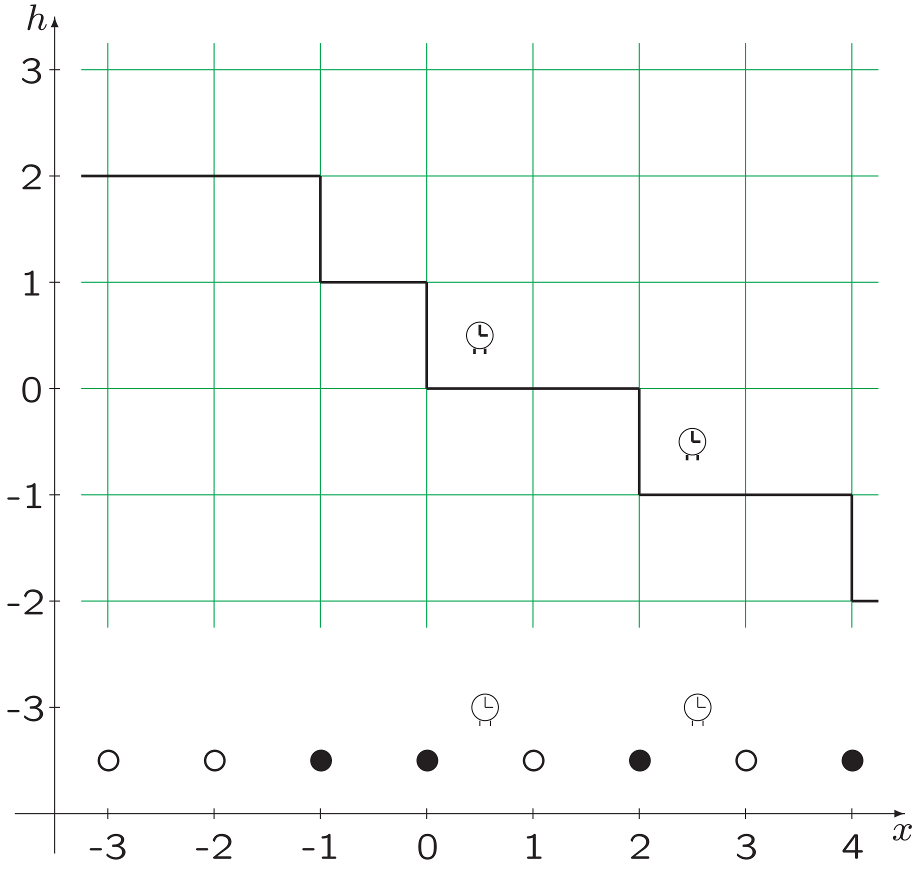

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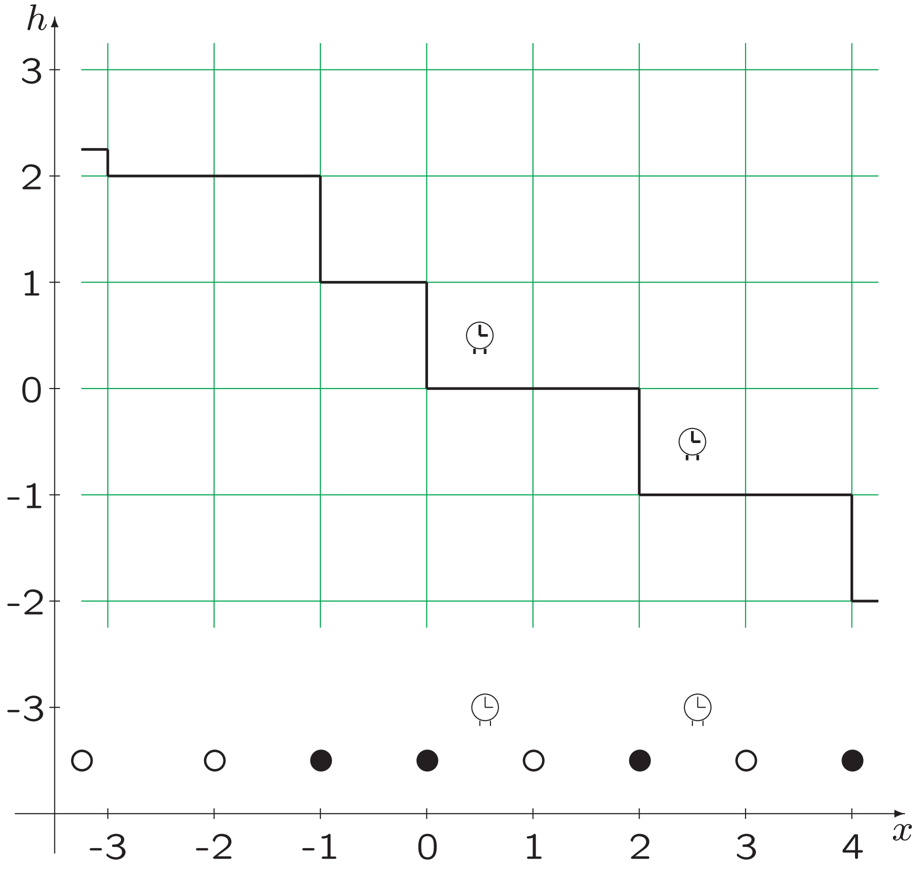

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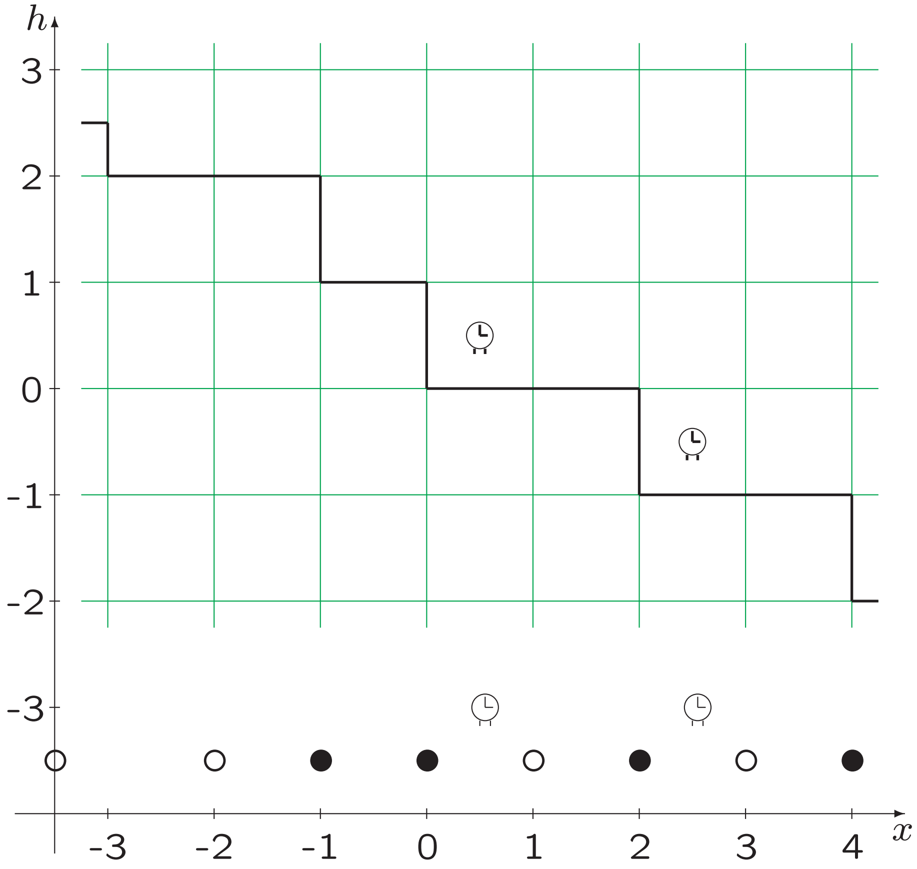

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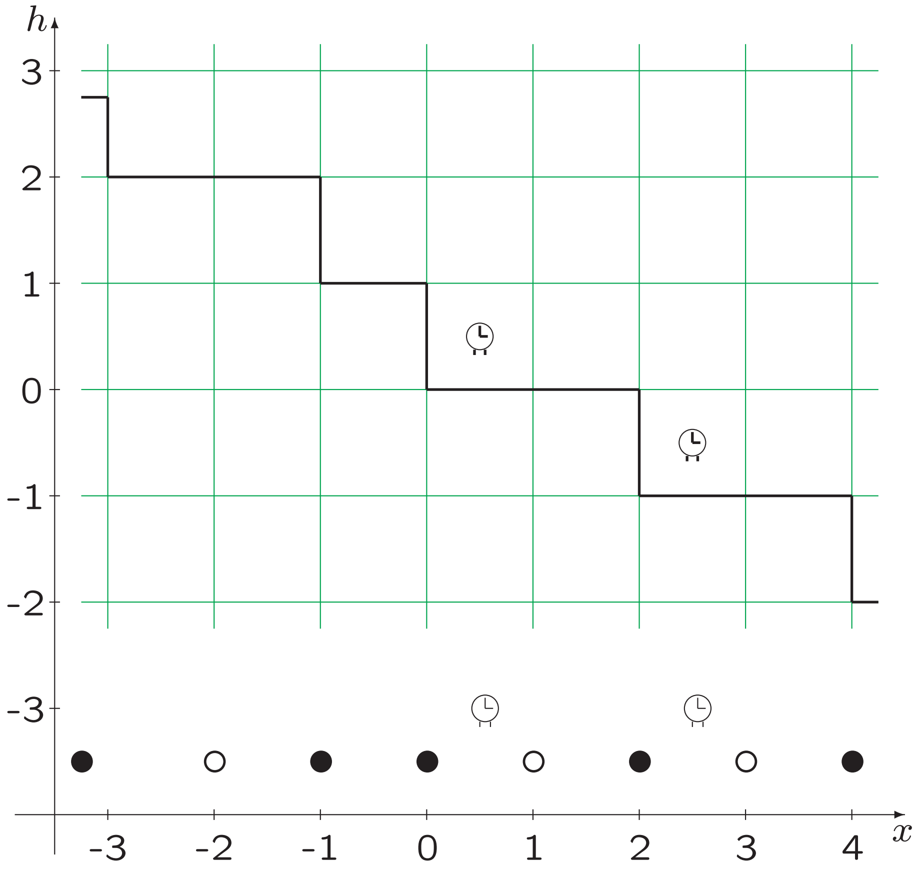

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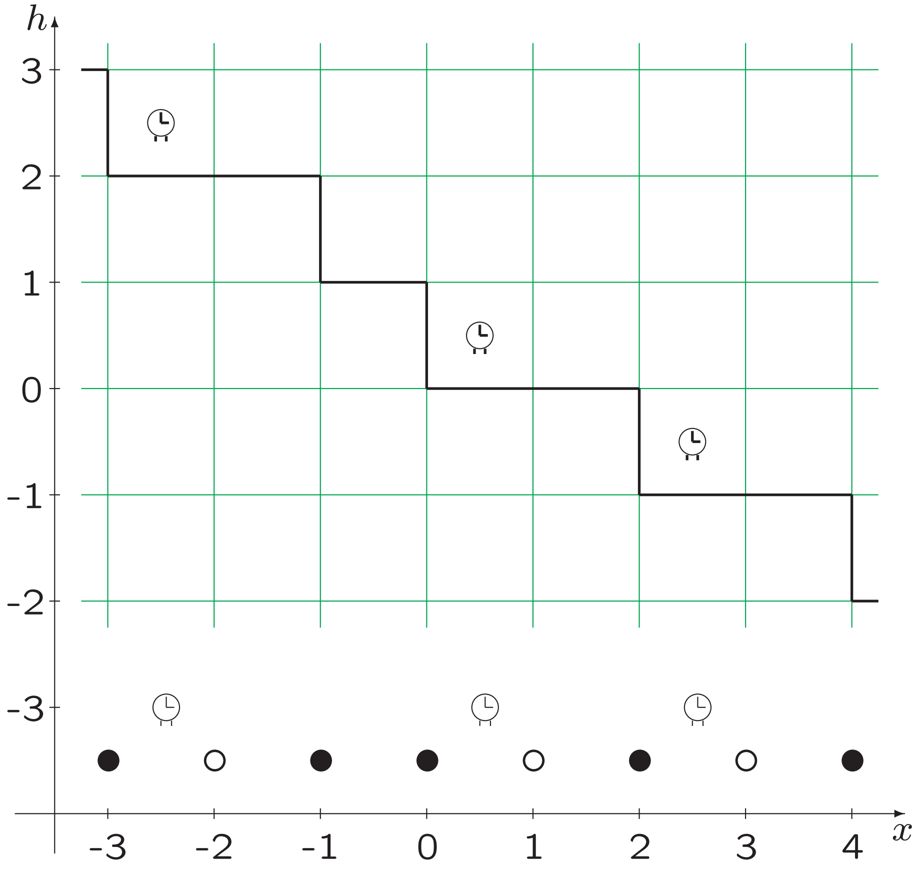

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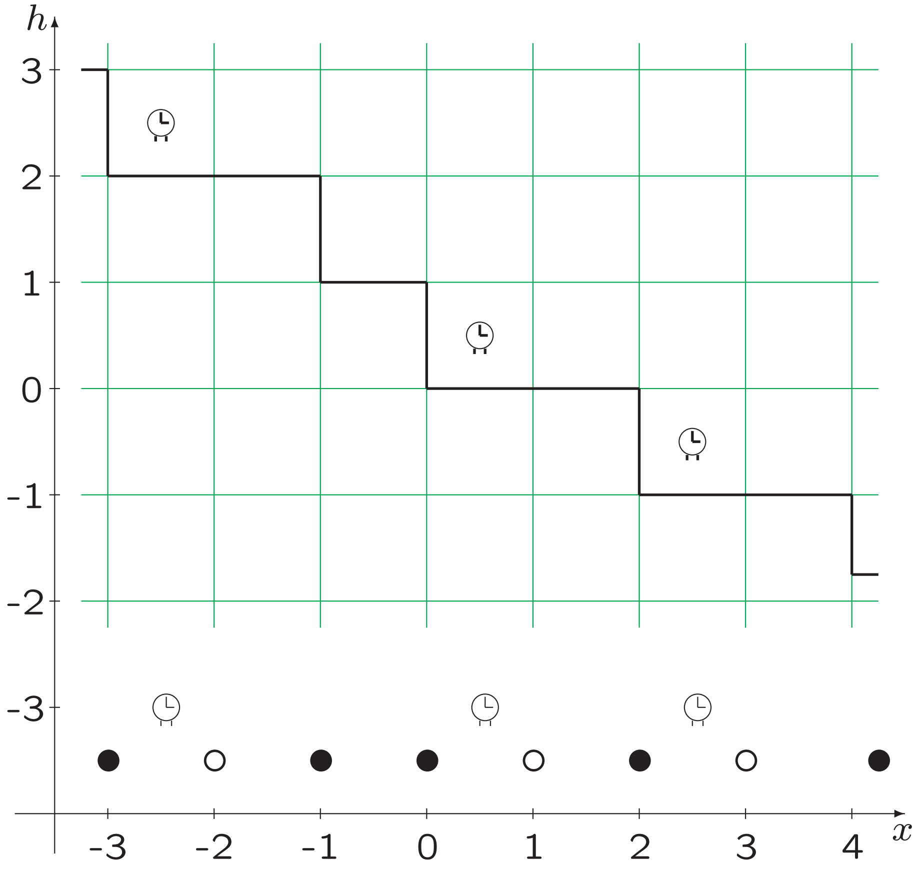

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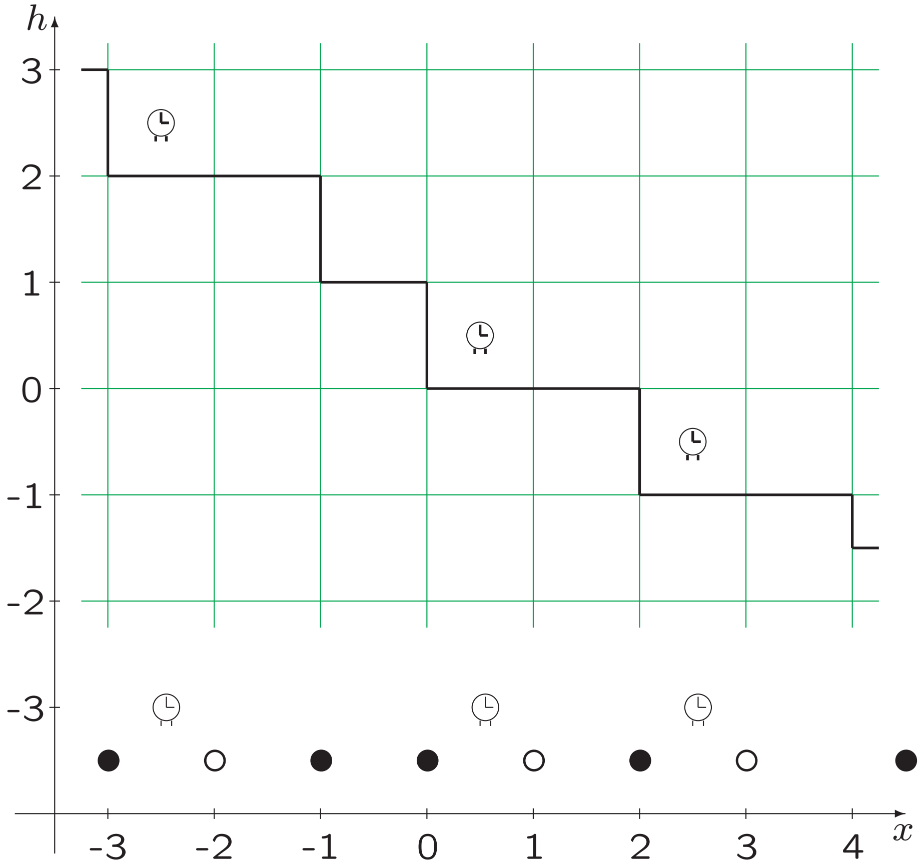

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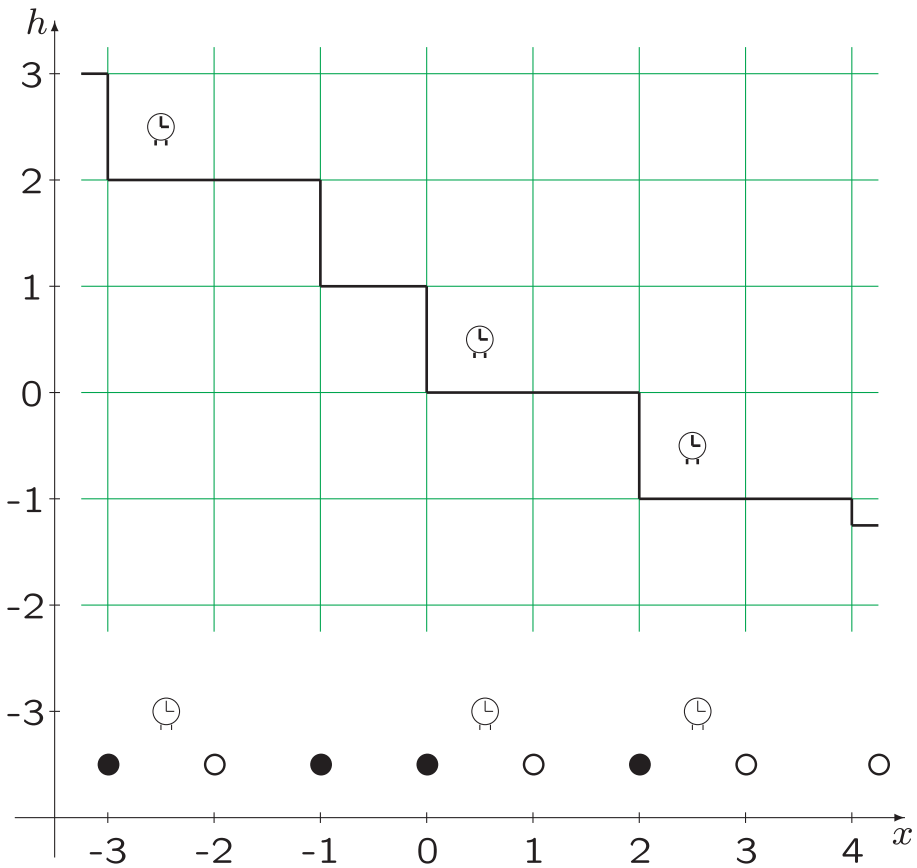

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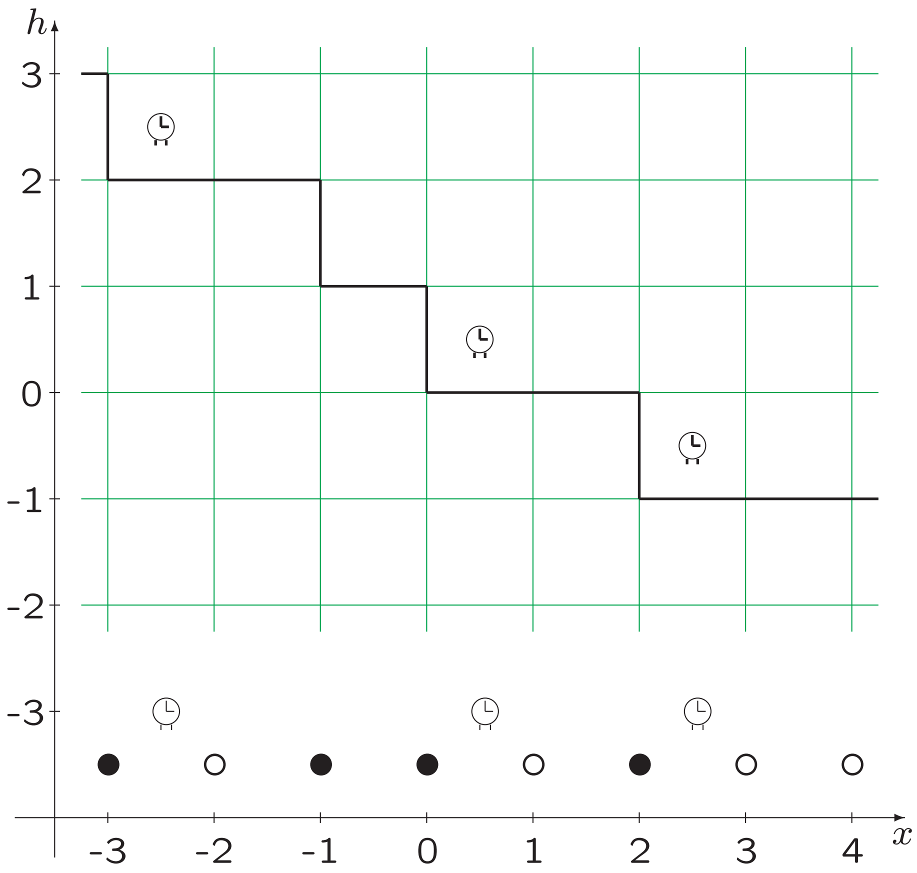

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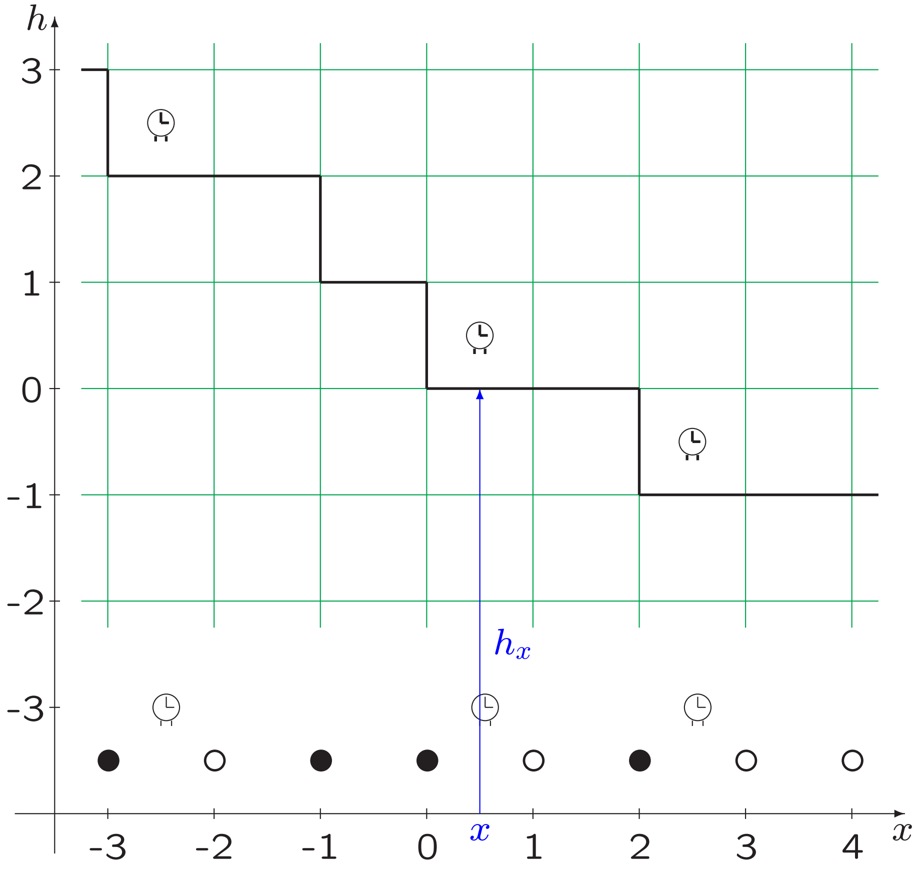

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$h_x(t)$ = height of the surface above x.

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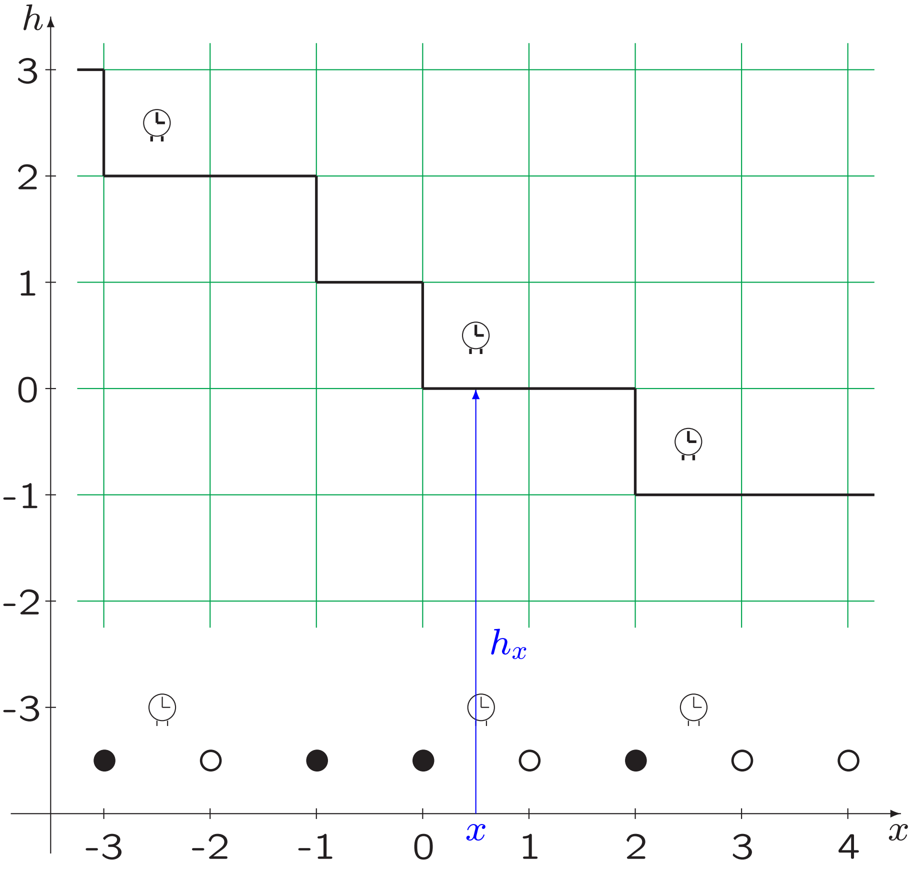

Bernoulli(ϱ) distribution

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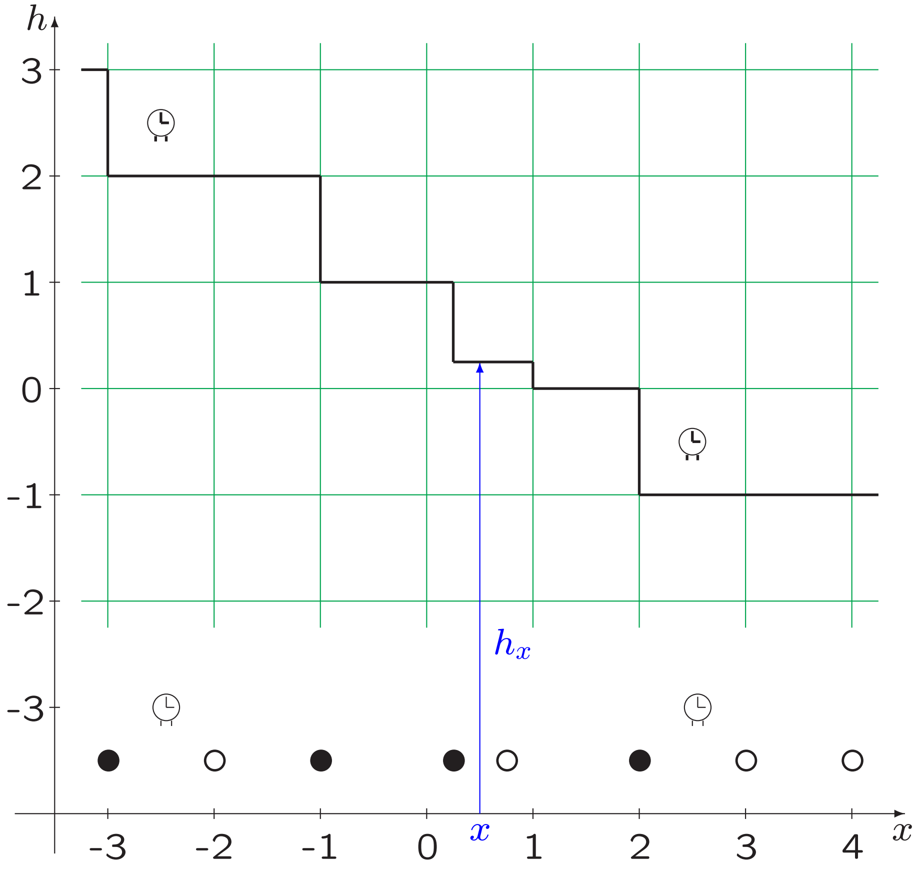

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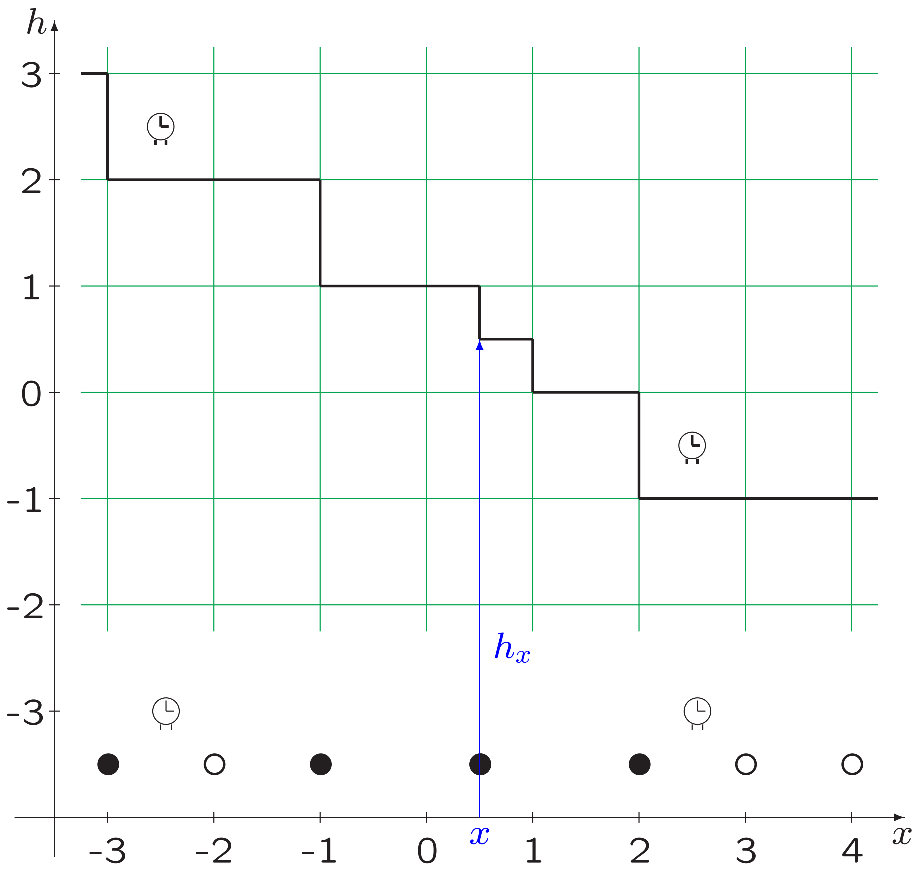

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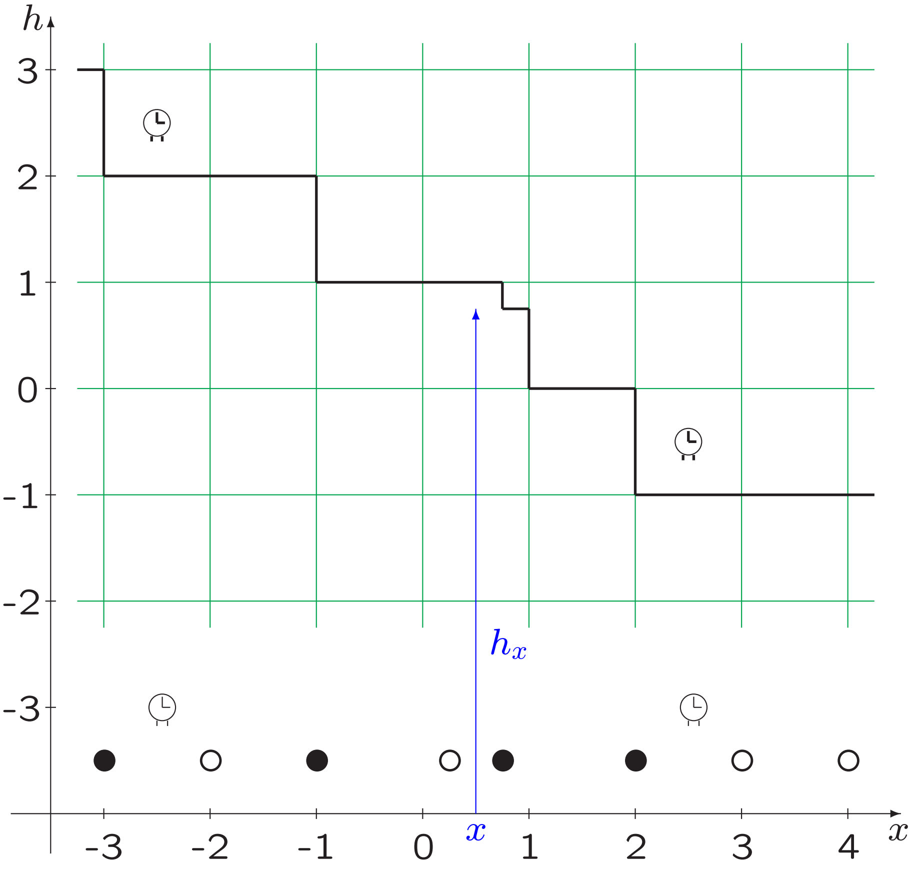

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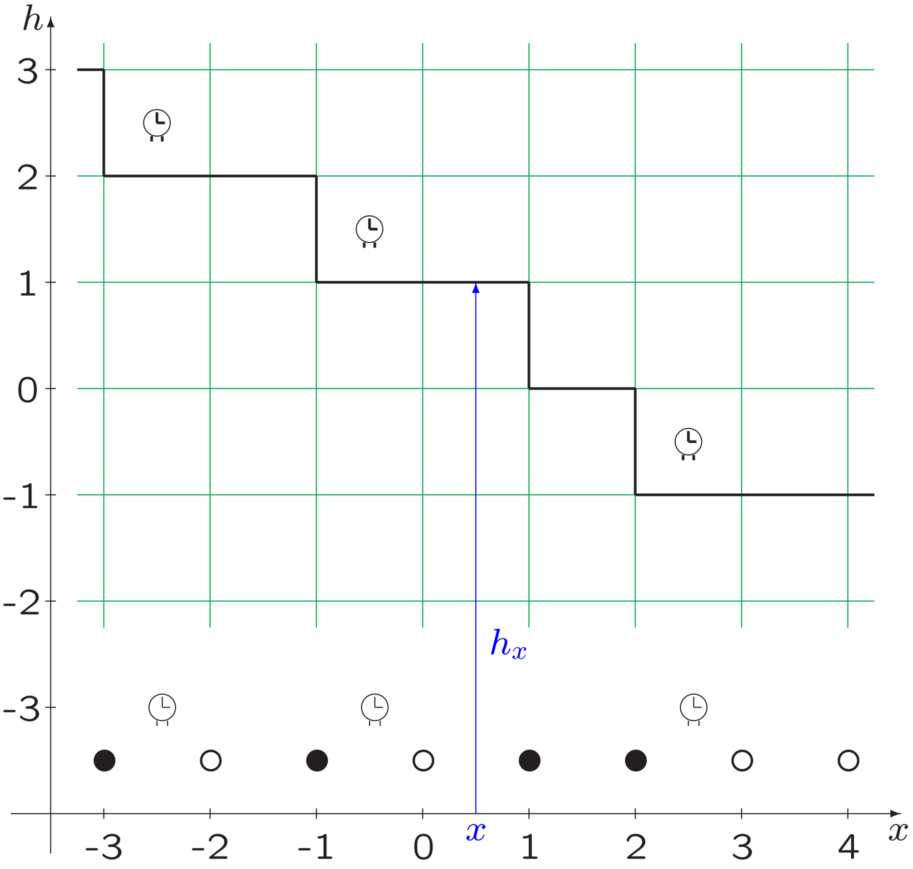

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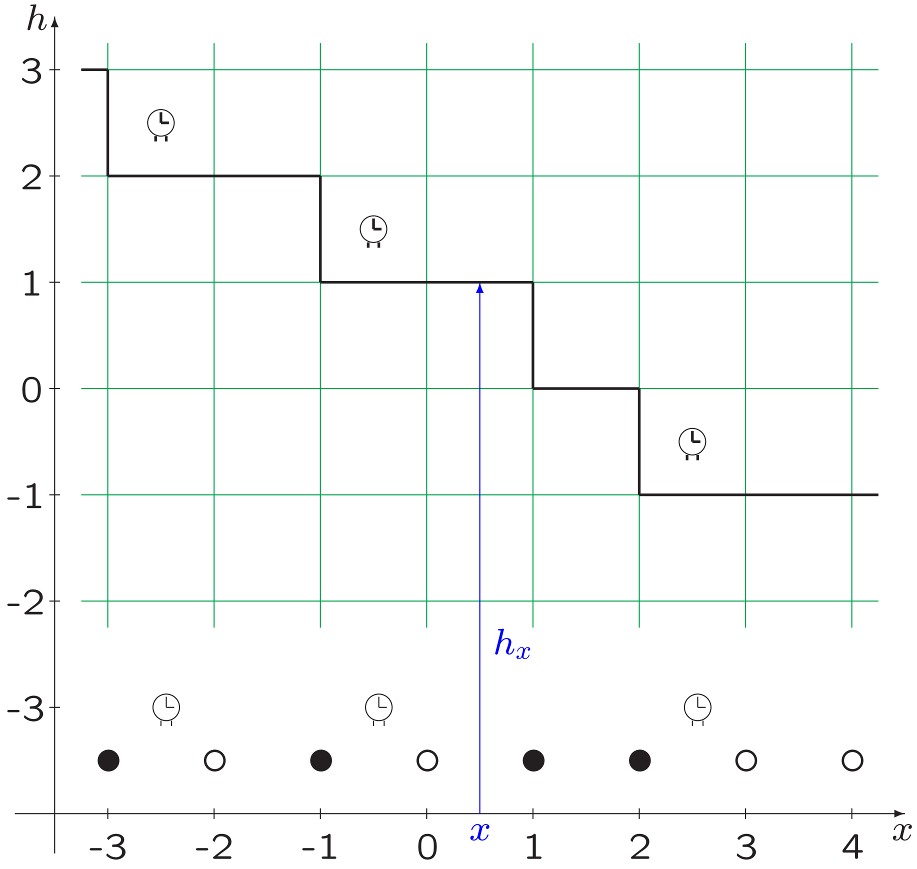

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$h_x(t)$ = height of the surface above x.

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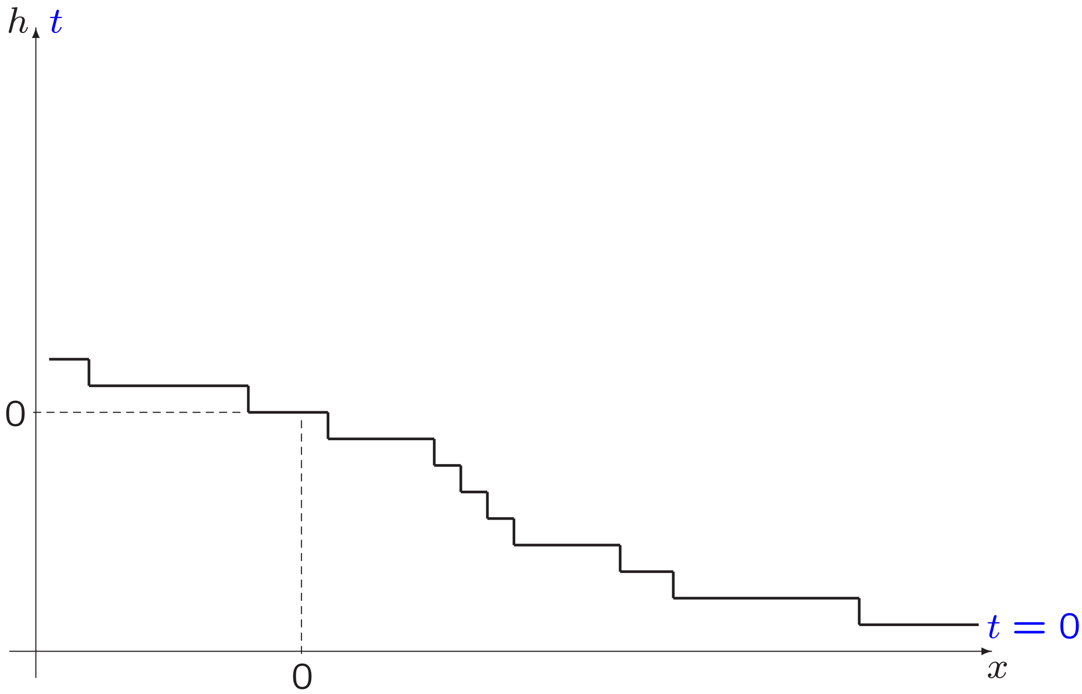
$h_{Vt}(t)$ = number of particles passed through the moving window at Vt $(V \in \mathbb{R})$.

Growth fluctuations



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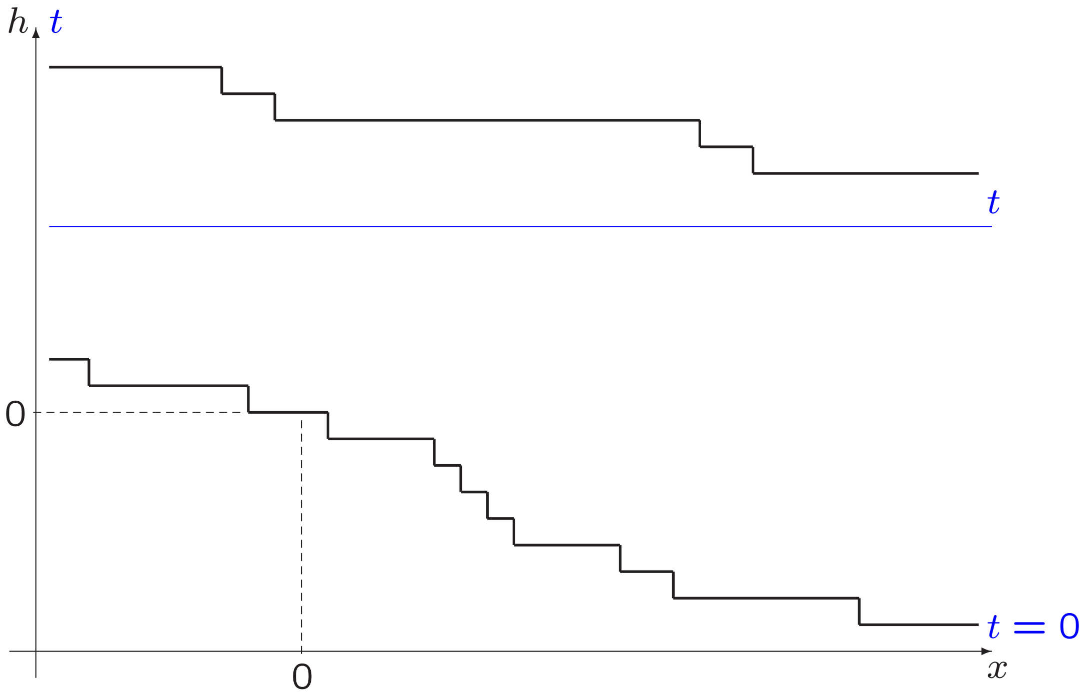

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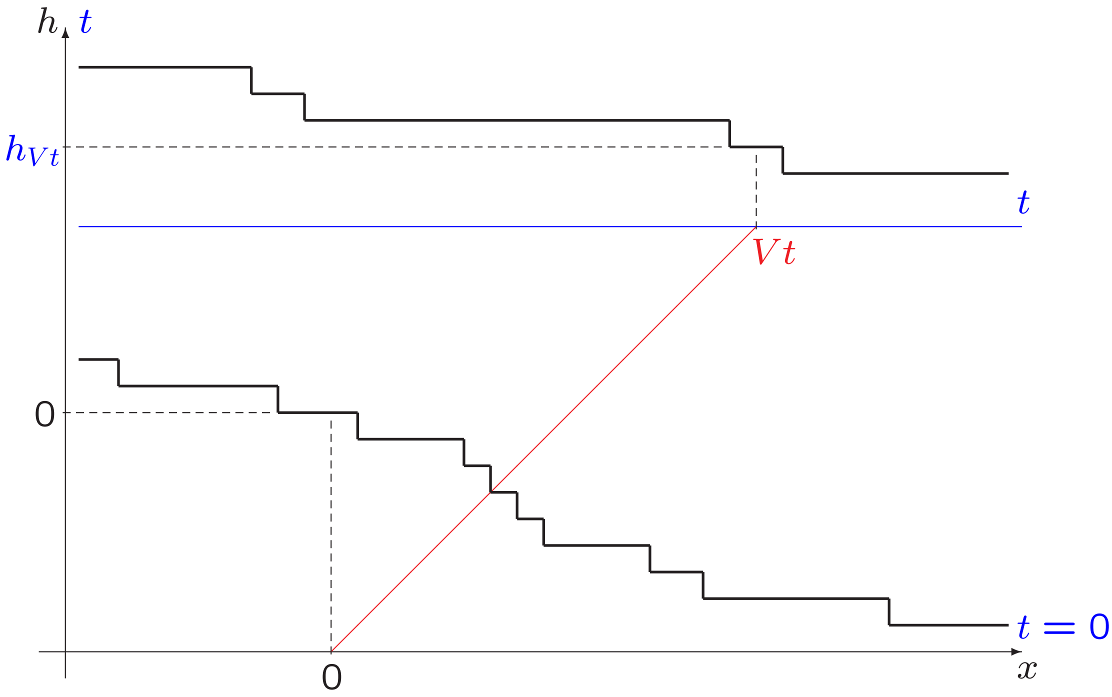

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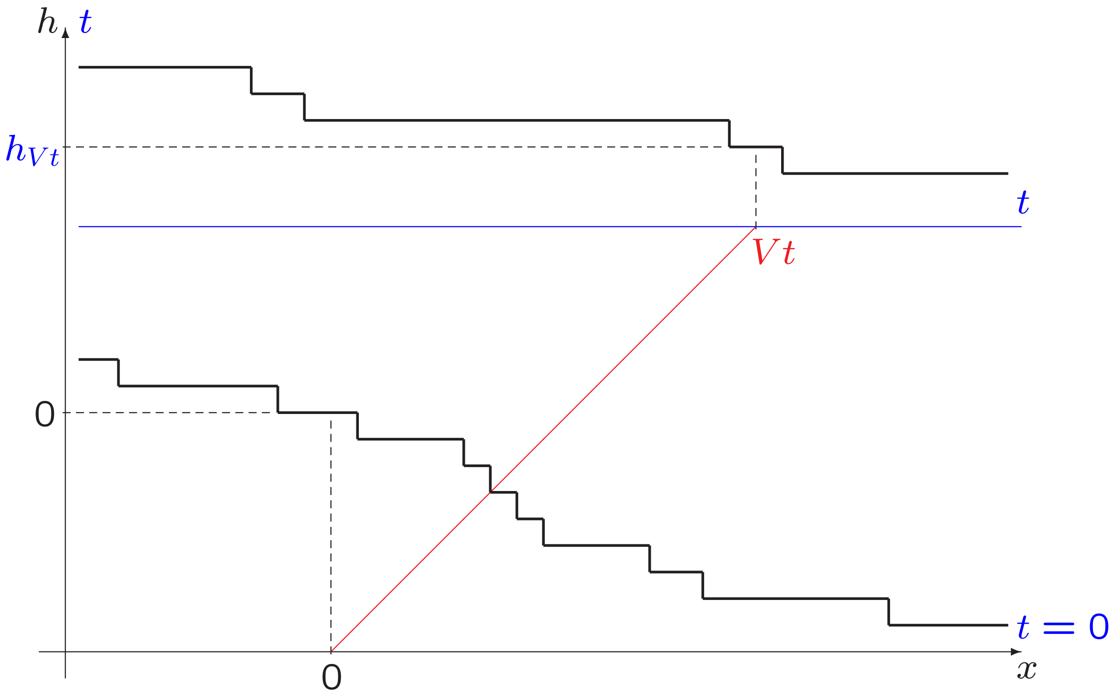

Ferrari - Fontes 1994:

$$\lim_{t\to\infty} \frac{\mathbf{Var}(h_{Vt}(t))}{t} = \text{const} \cdot |V - C(\varrho)|$$

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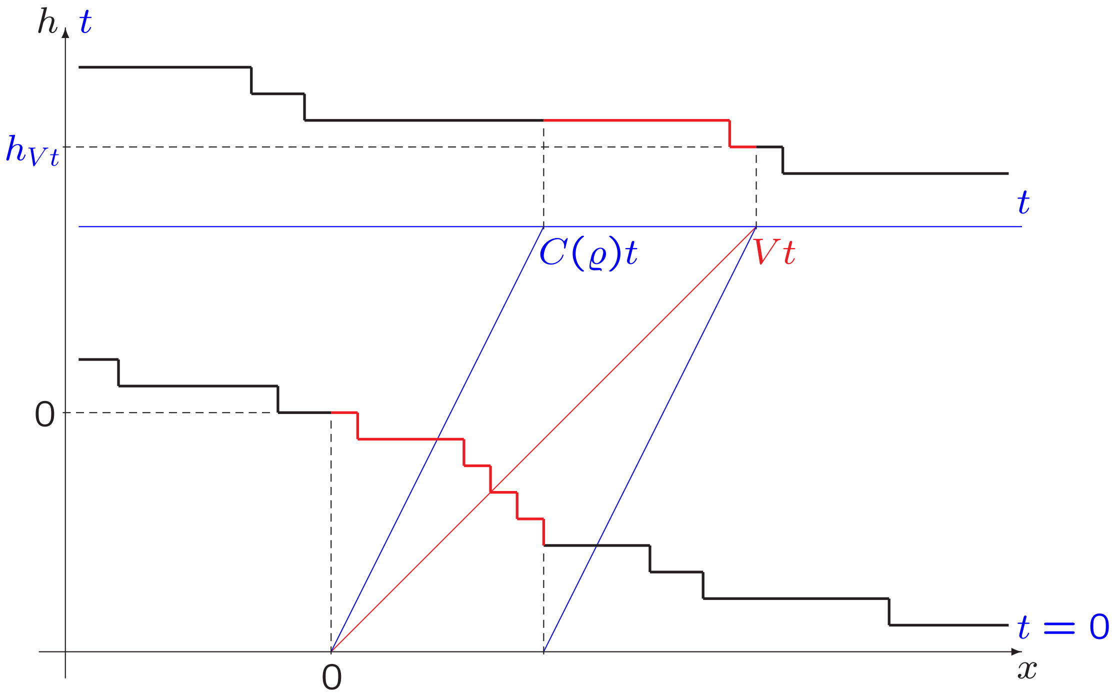

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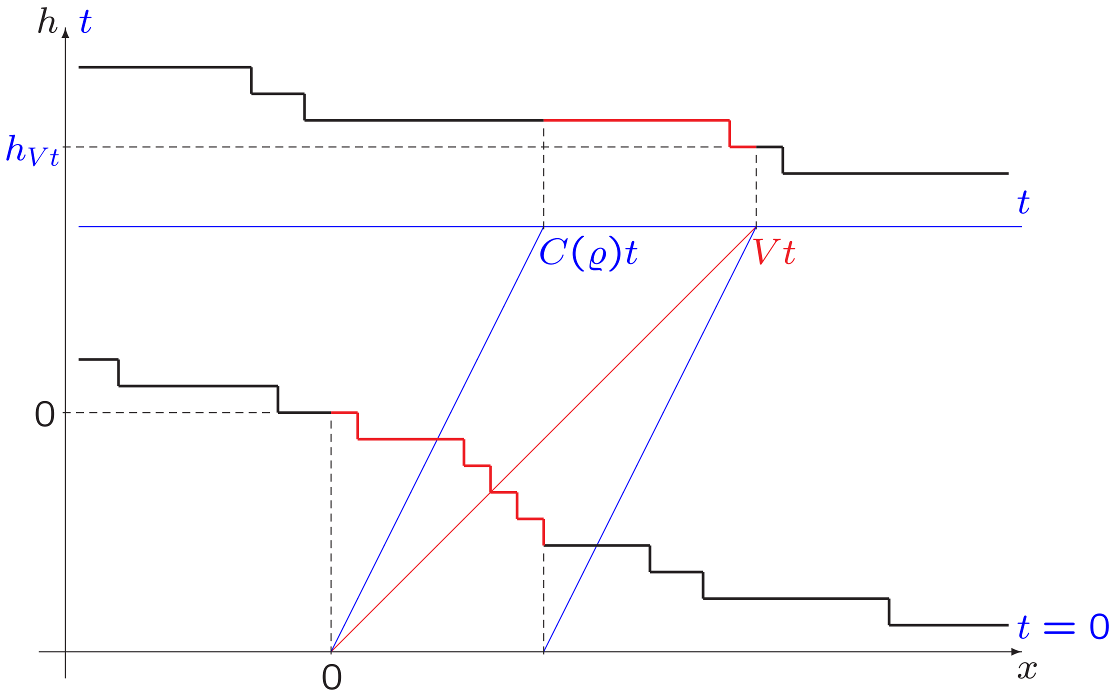

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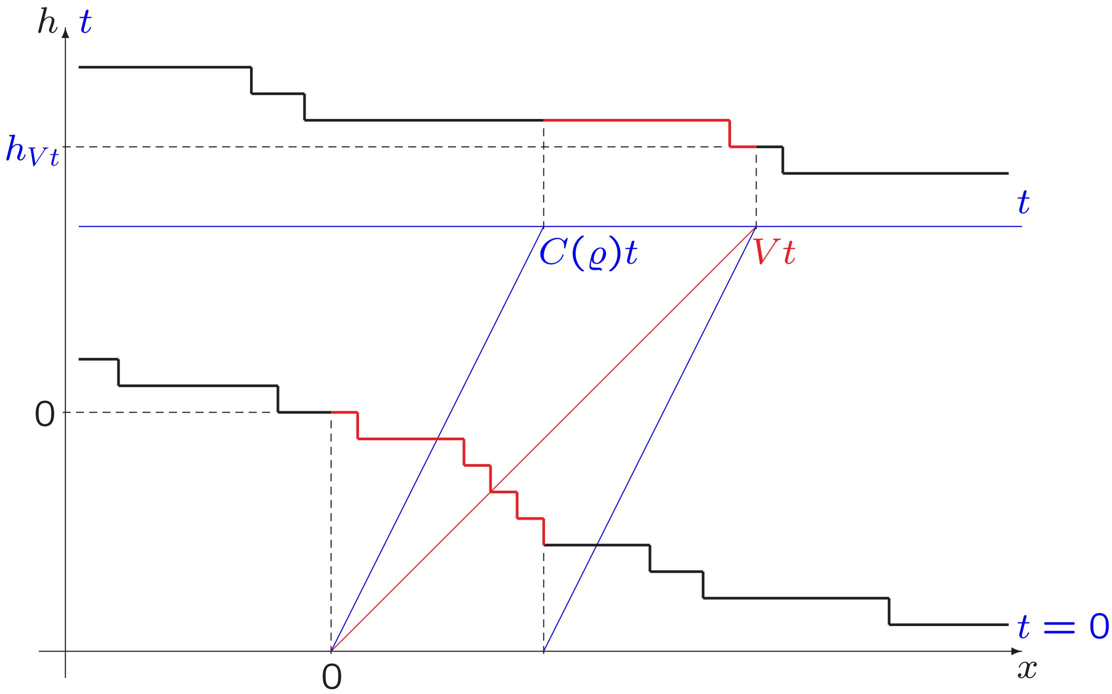

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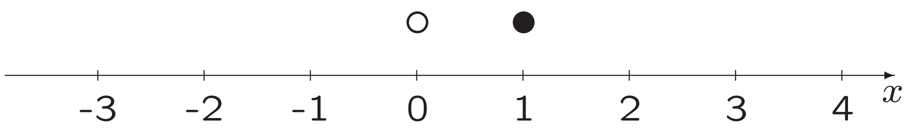
Conjecture:

$$\lim_{t\to\infty} \frac{\textbf{Var}(h_{C(\varrho)t}(t))}{t^{2/3}} = [\text{sg. non trivial}].$$

Outfit 3: Equilibrium queues



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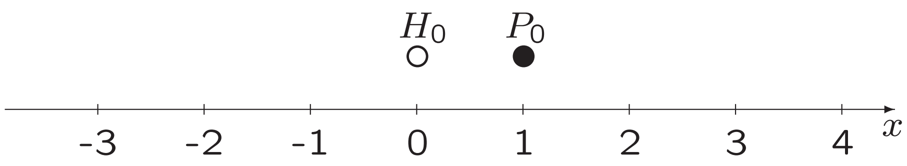

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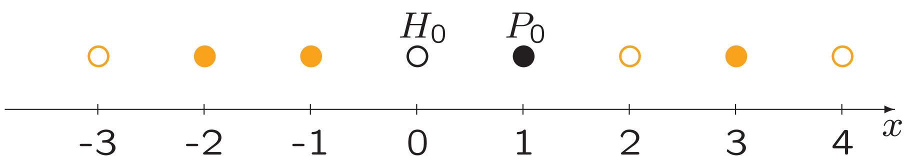

Bernoulli(ϱ) distribution

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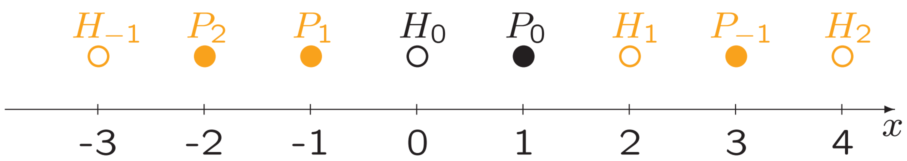

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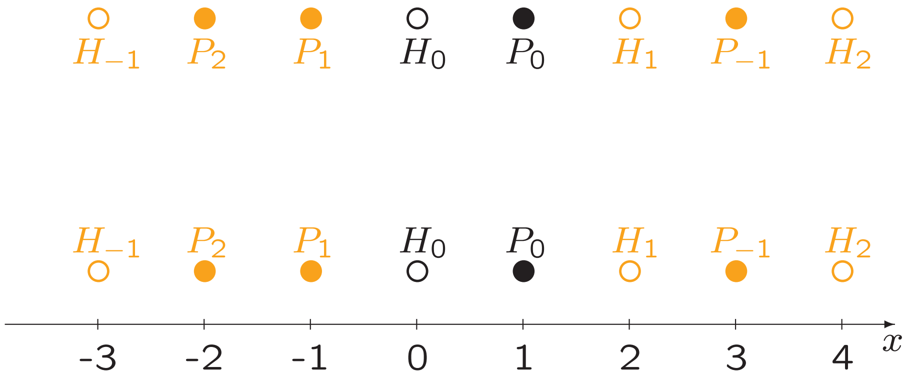

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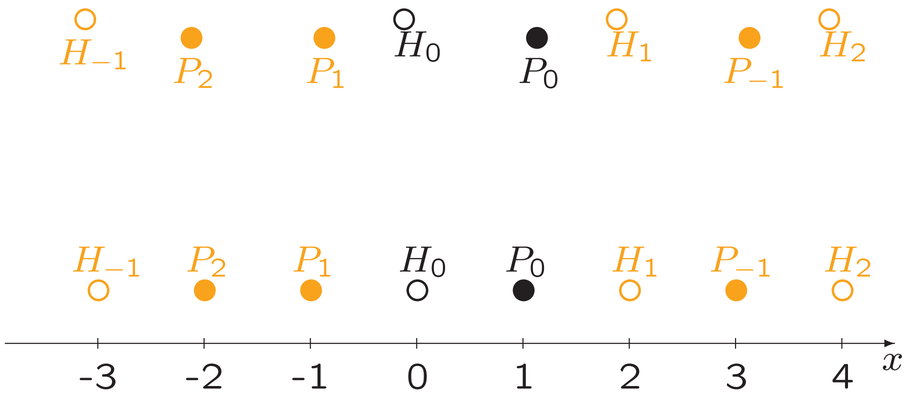

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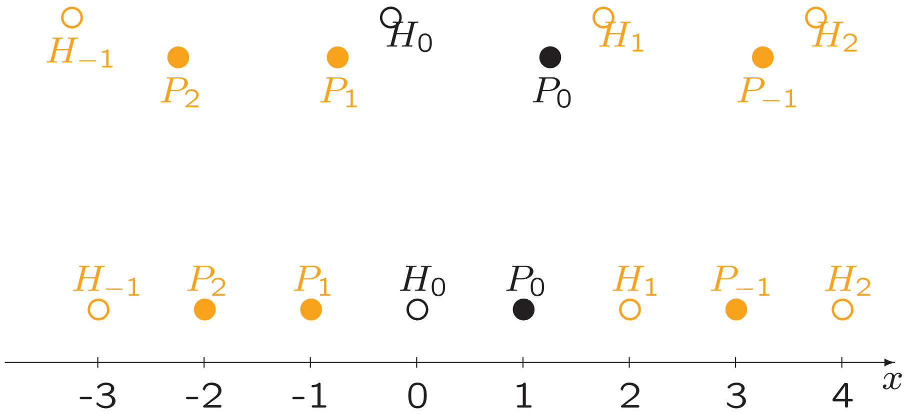

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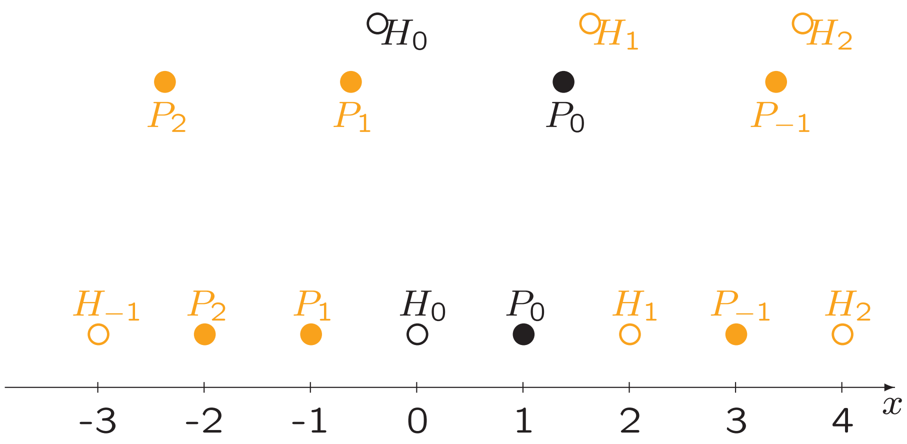

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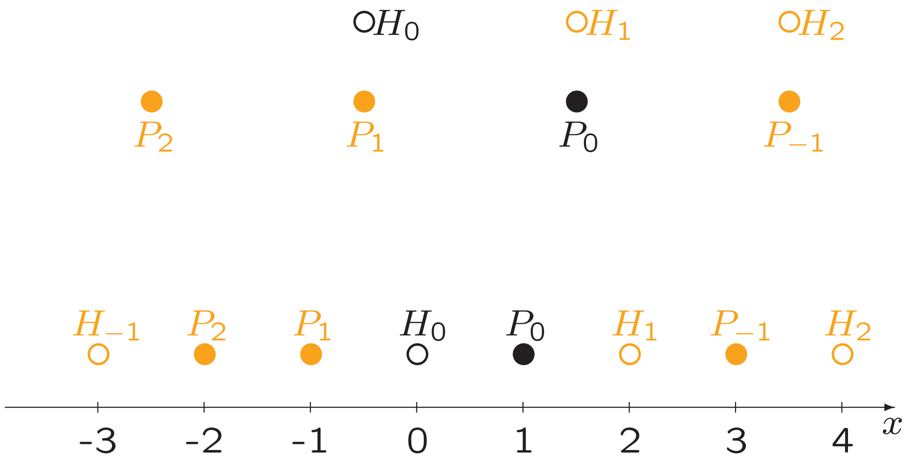

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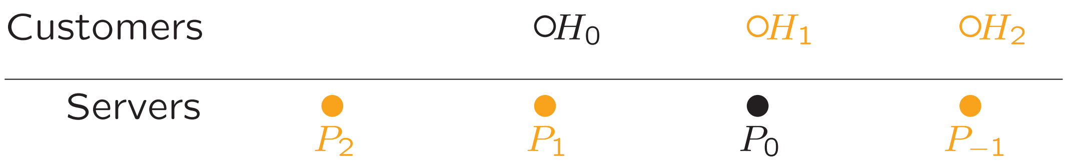

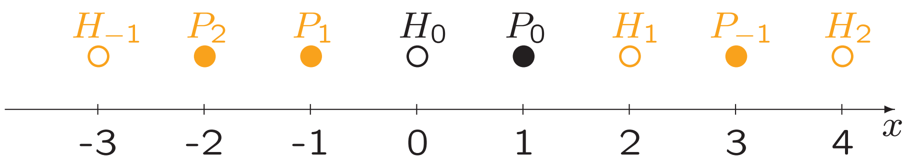

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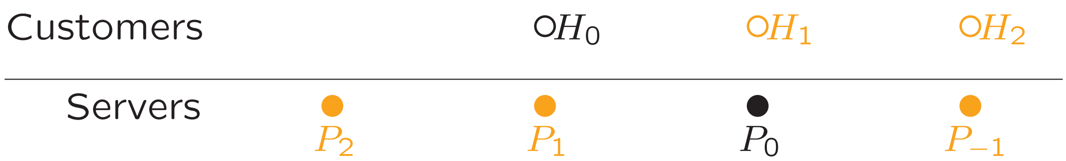

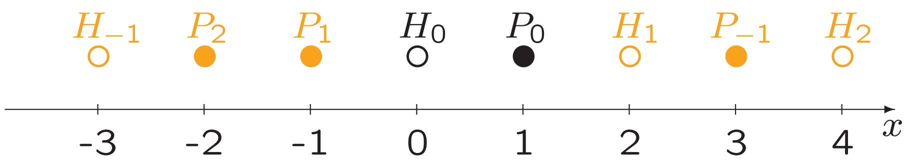

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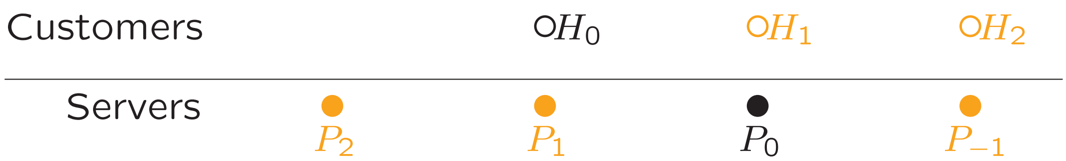

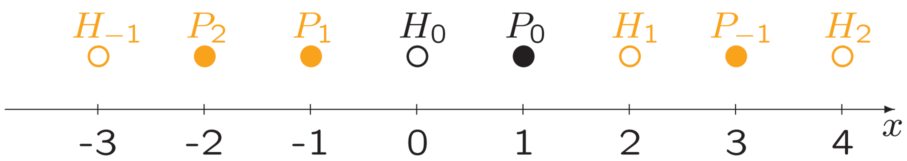

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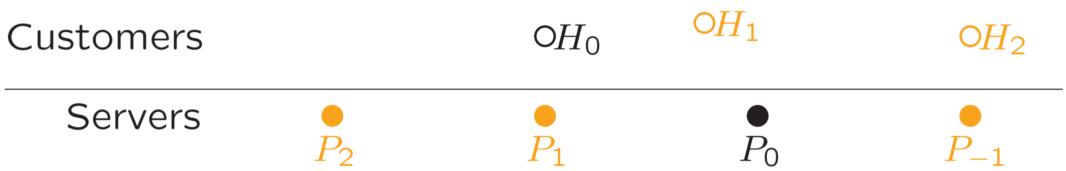

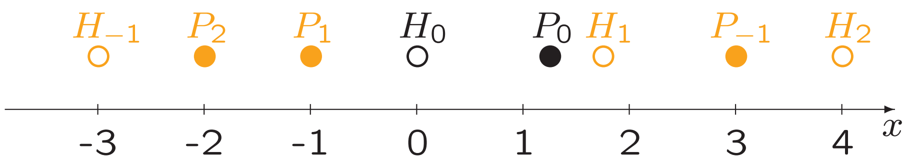

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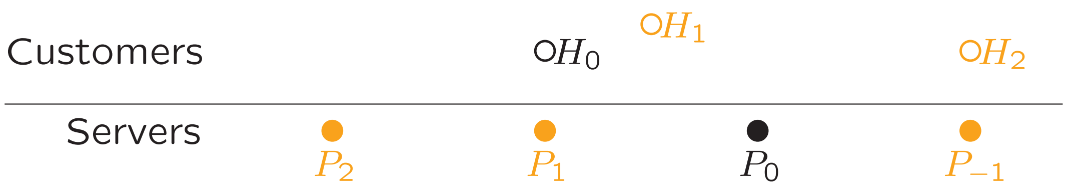

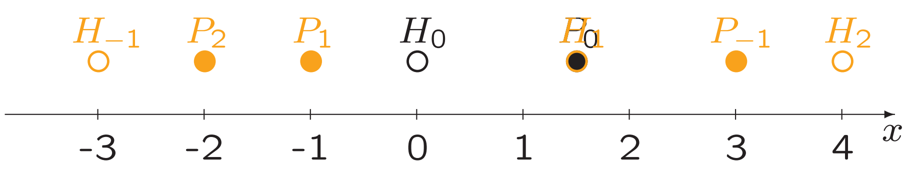

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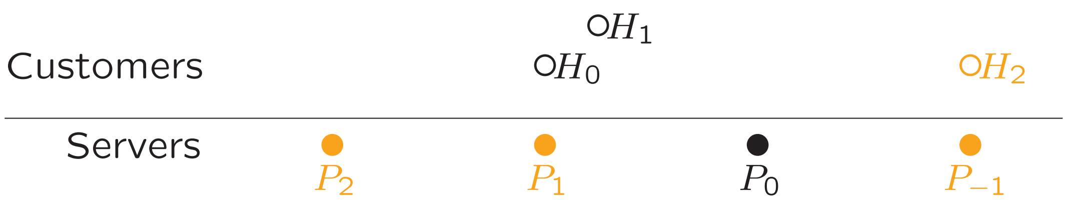

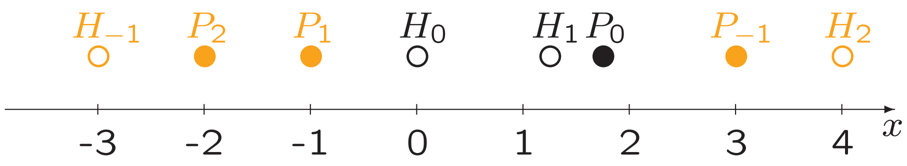

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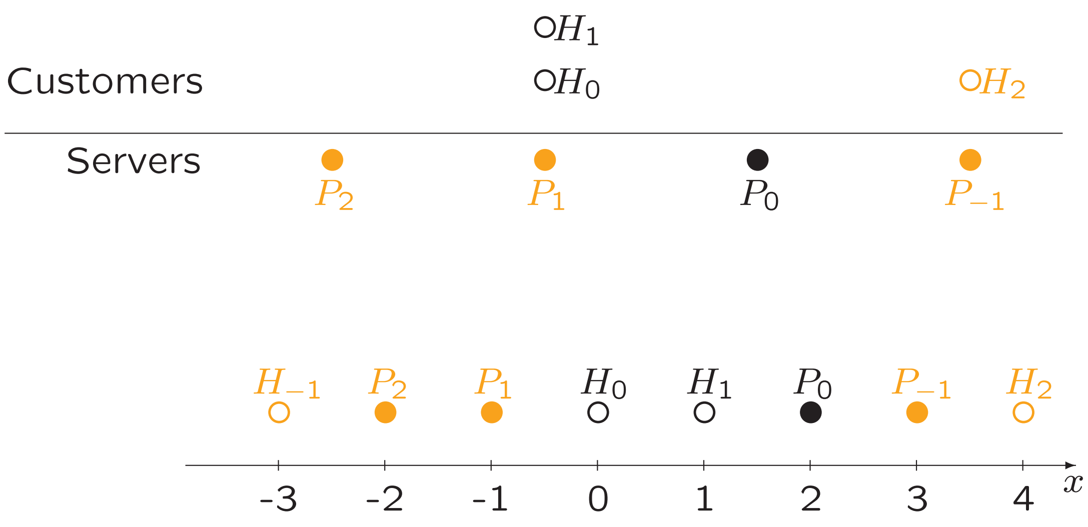

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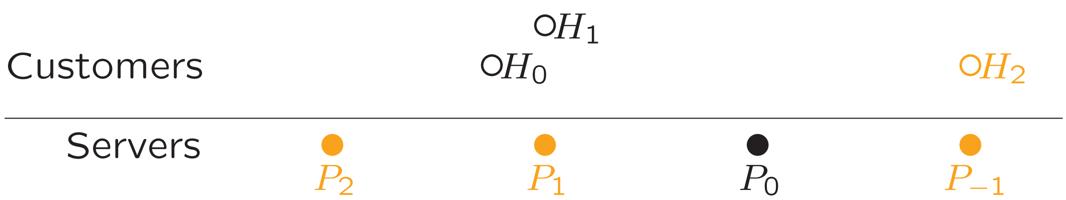

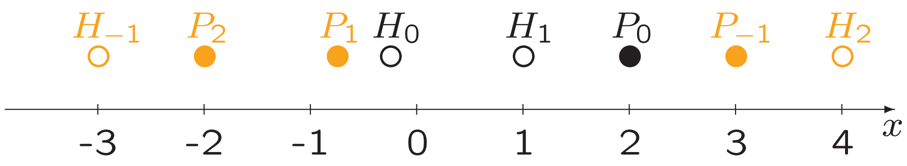

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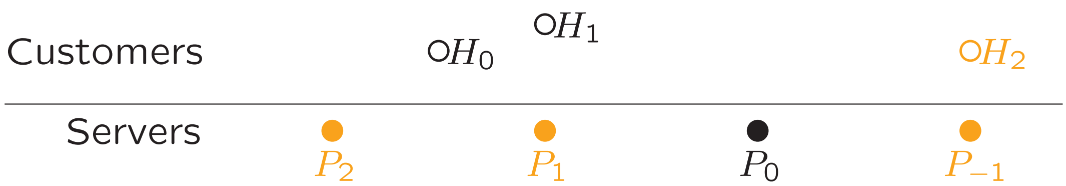

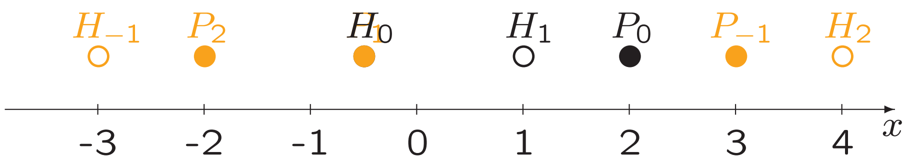

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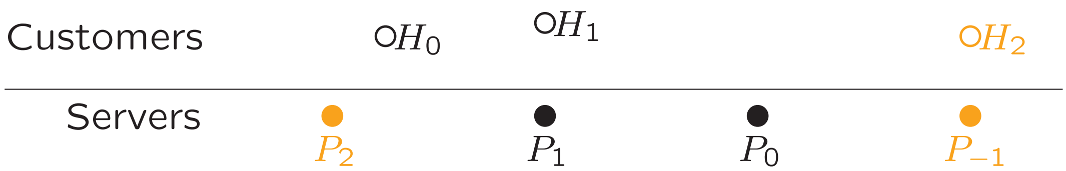

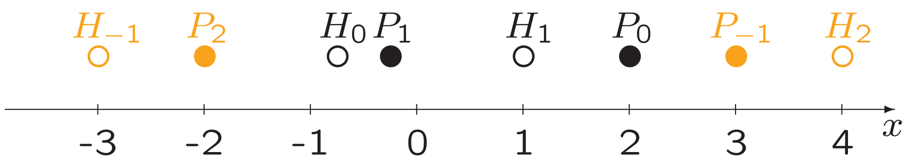

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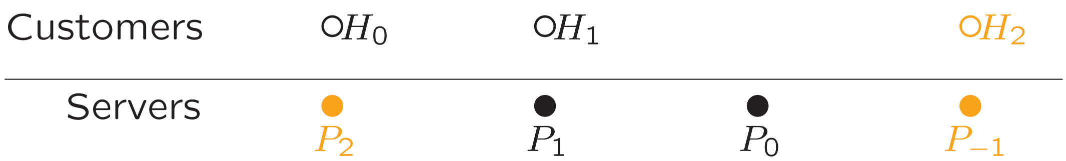

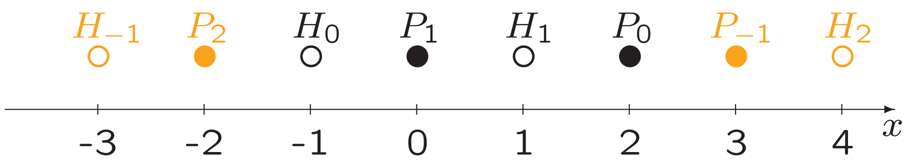

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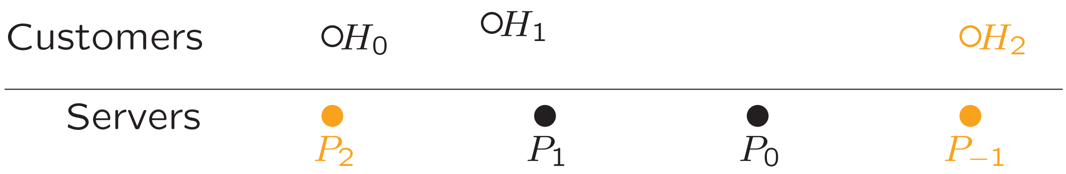

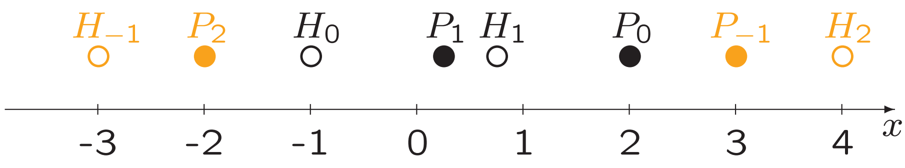

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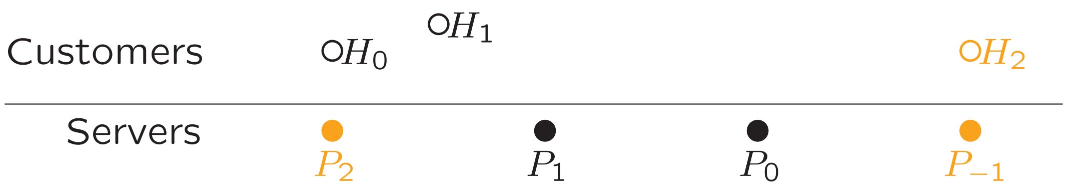

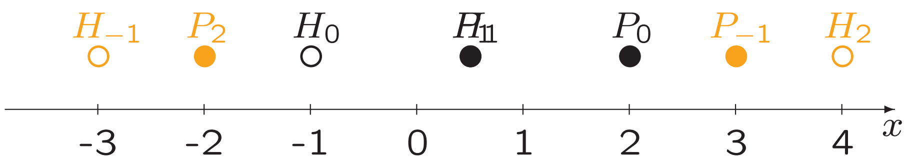

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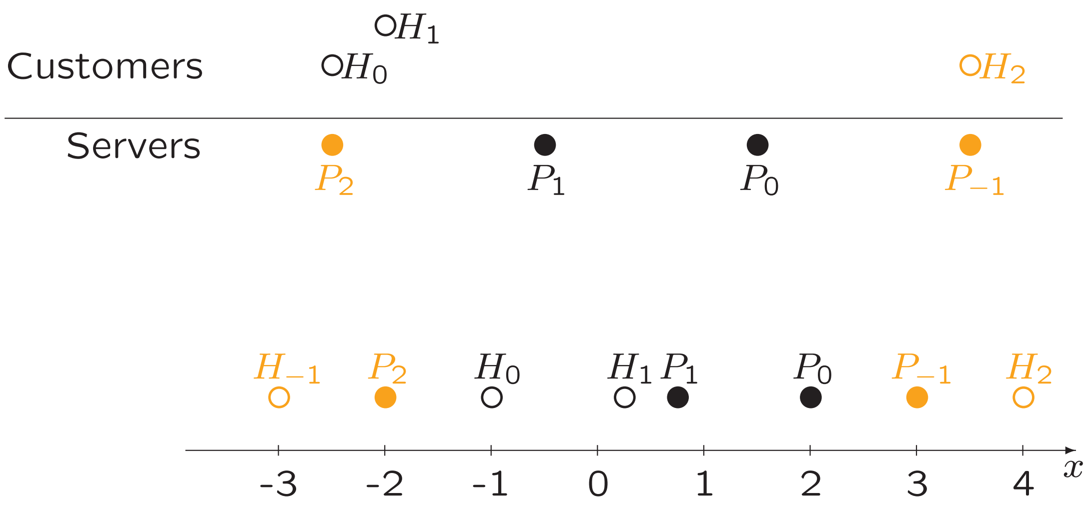

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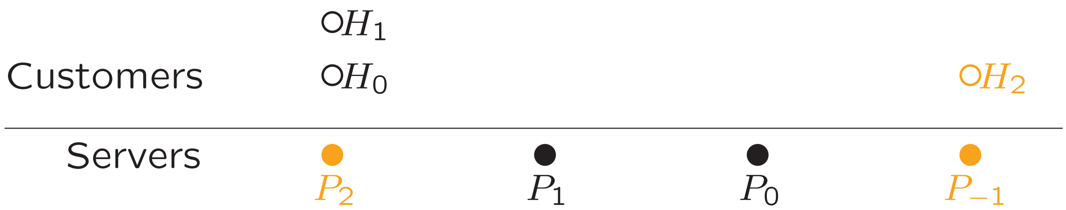

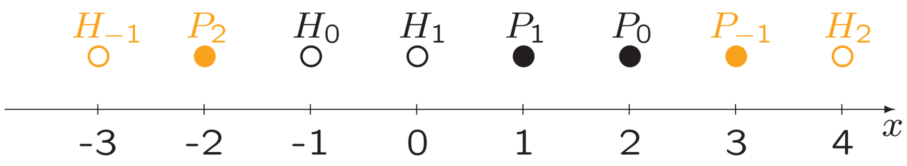

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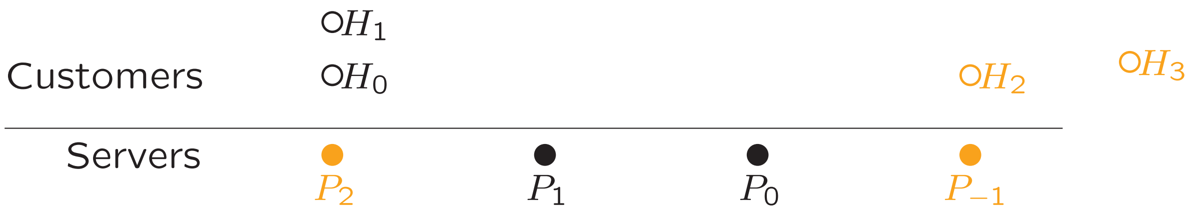

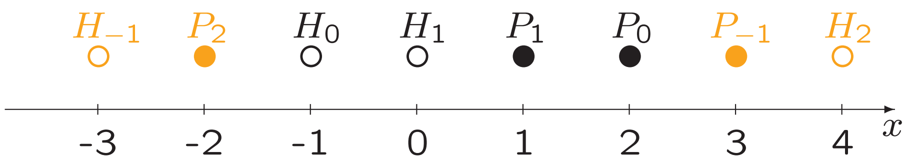

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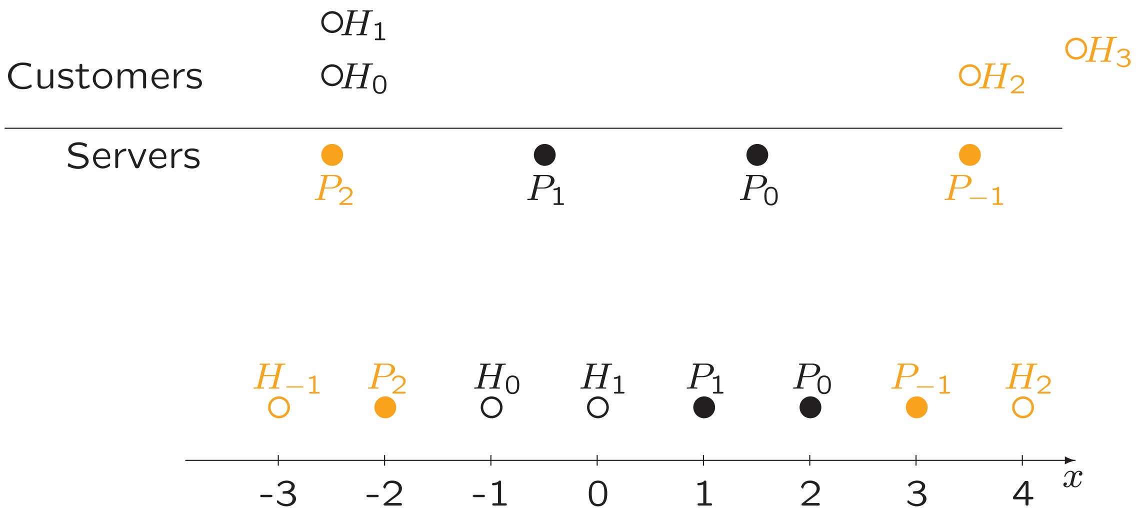

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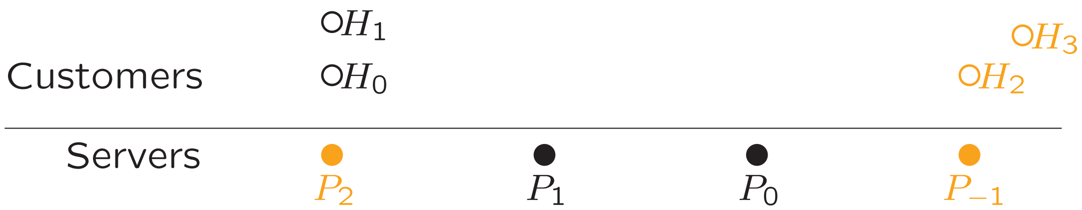

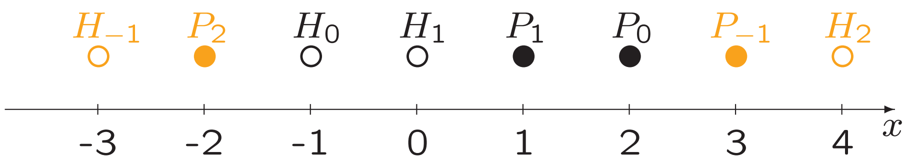

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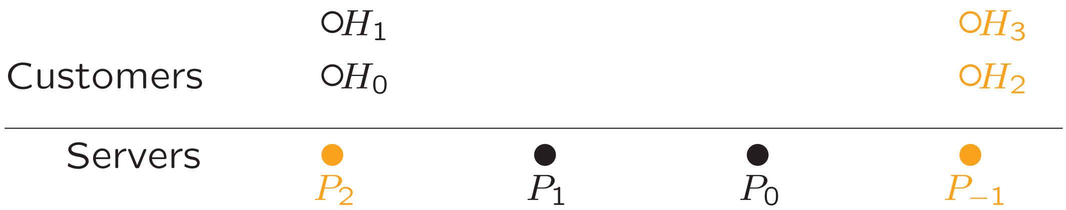
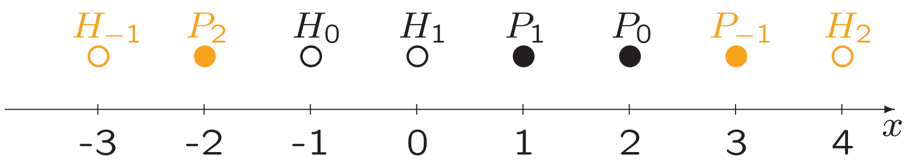
⇝ Equilibrium system of queues as seen right after H_0's jump.

Outfit 3: Equilibrium queues





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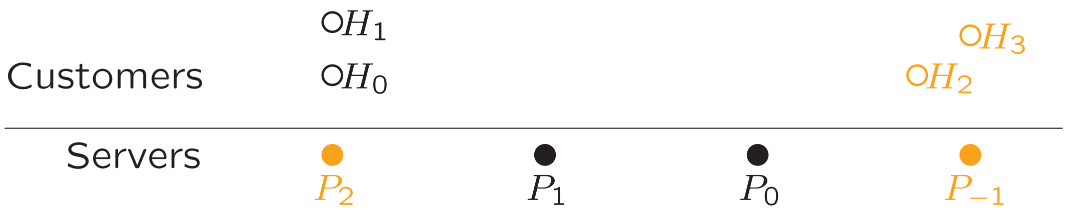

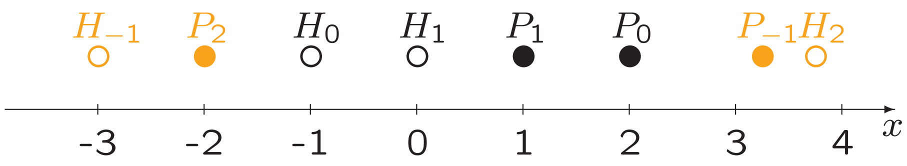

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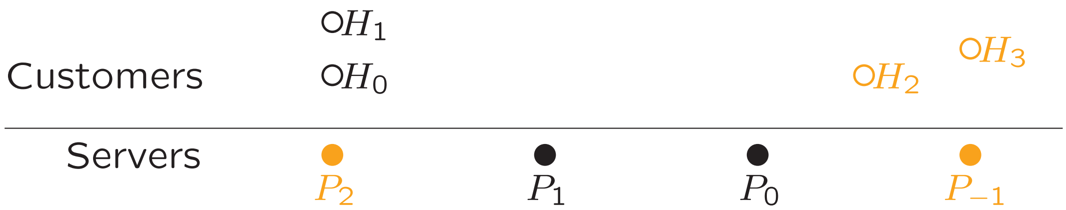

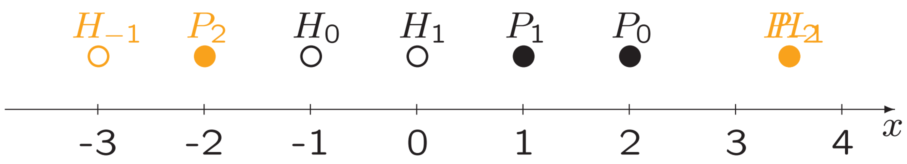

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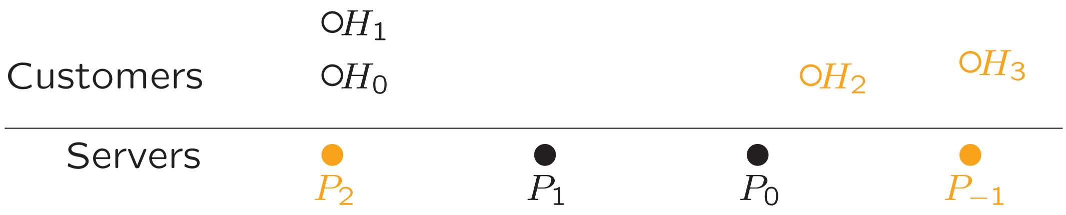

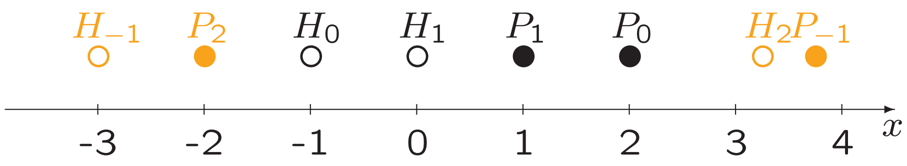

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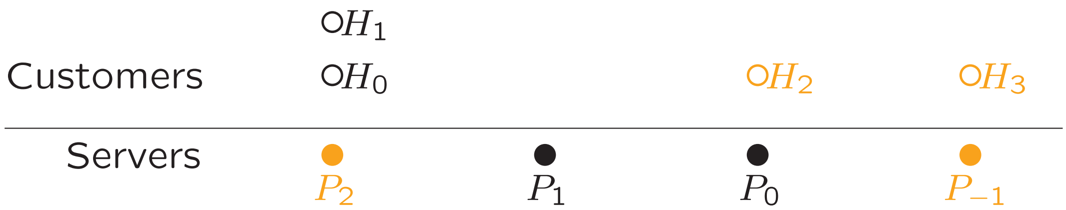

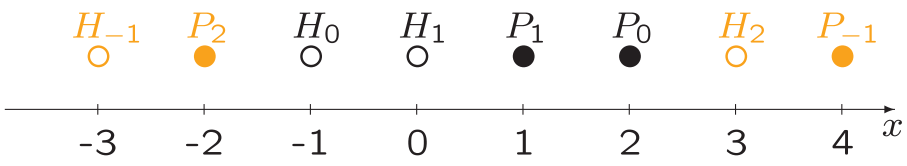

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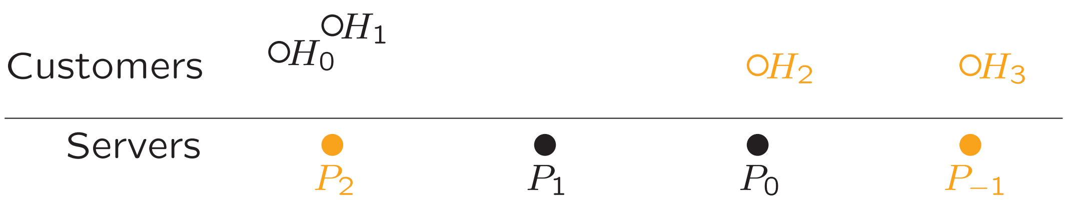

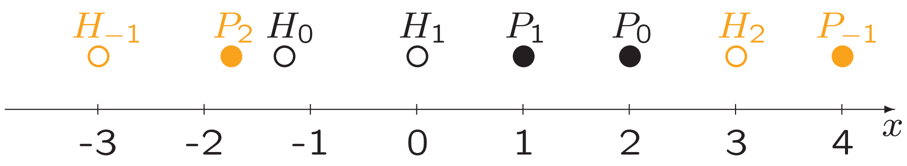

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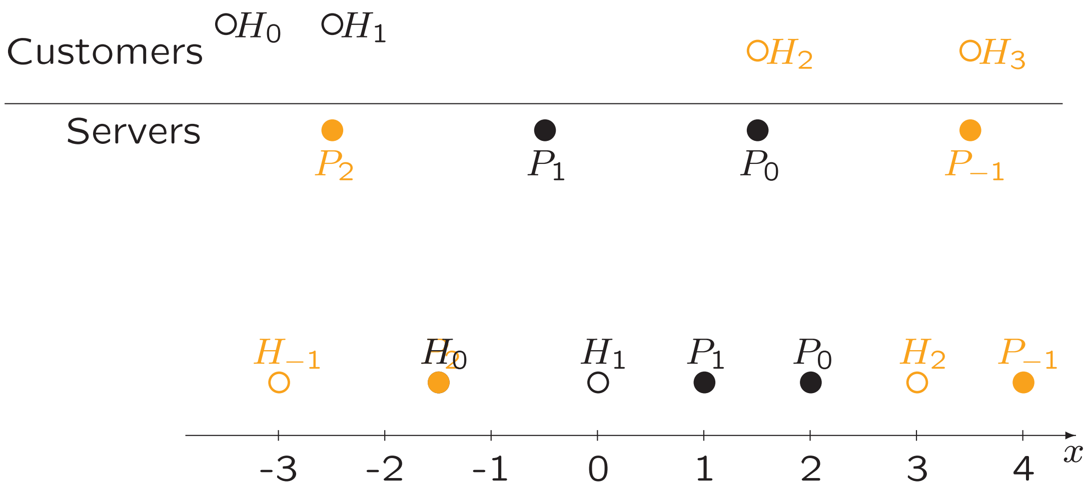

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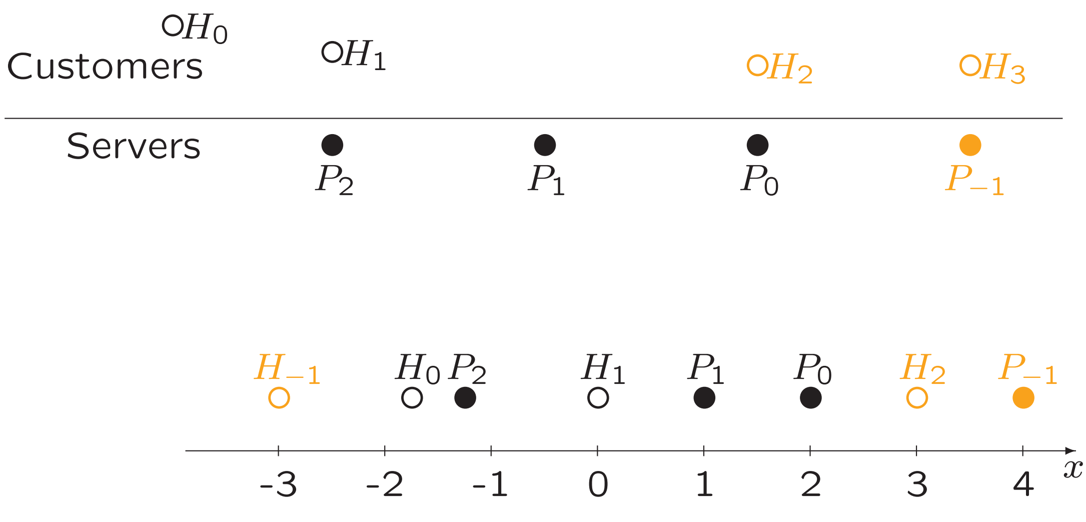

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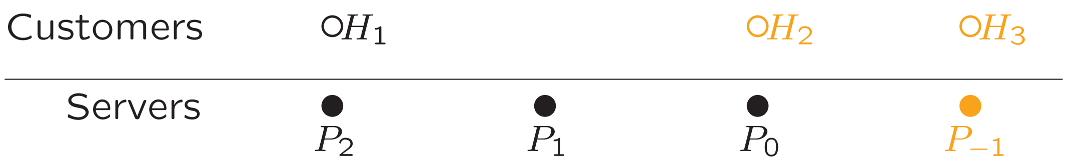

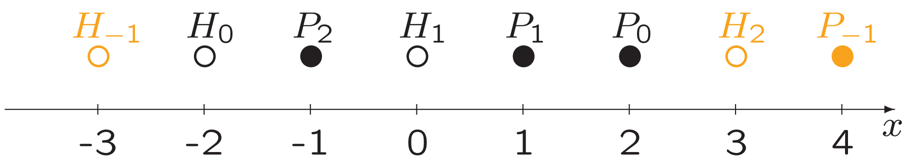

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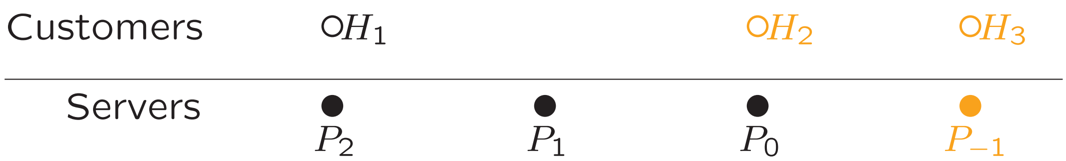

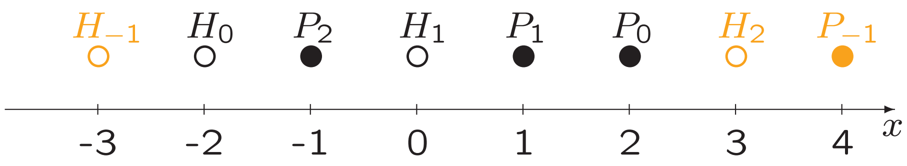

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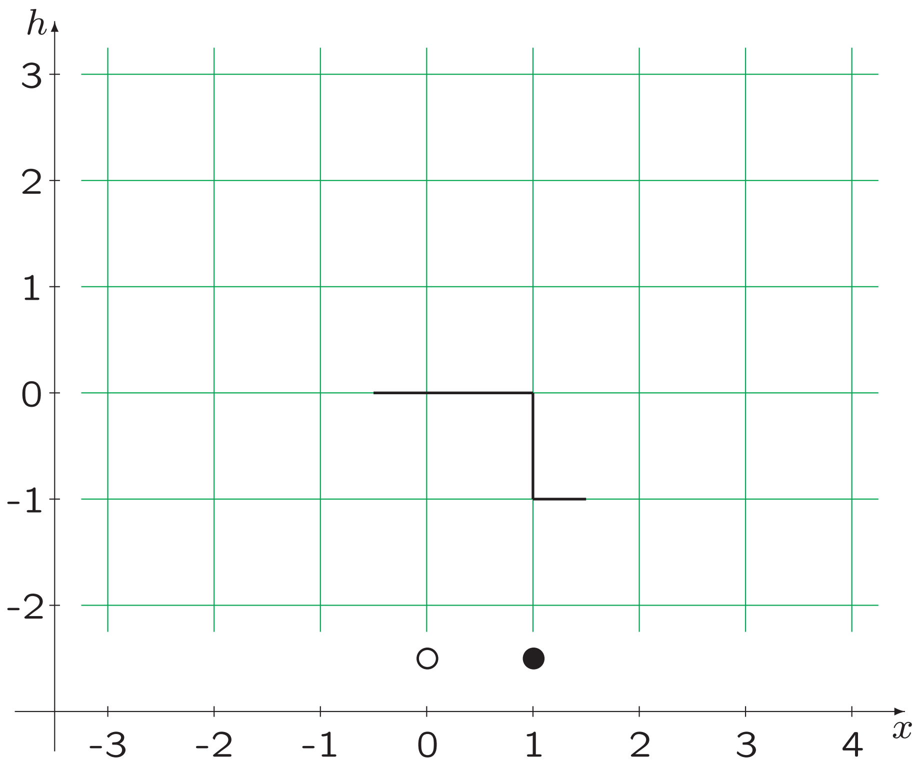
⇝ Equilibrium system of queues as seen right after H_0's jump.

⇝ Burke's Theorem (Kesten 1970): P_0 and H_0 jump as Poisson($1 - \varrho$) and Poisson(ϱ) processes, respectively, and they are independent.

Outfit 4: Last passage percolation



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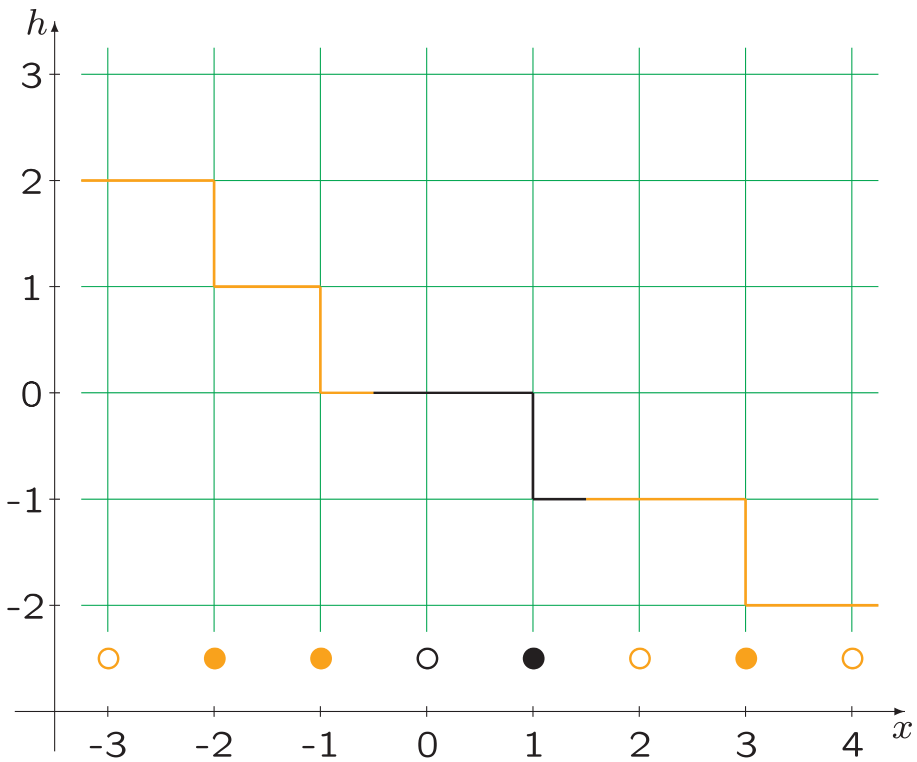

Bernoulli(ϱ) distribution

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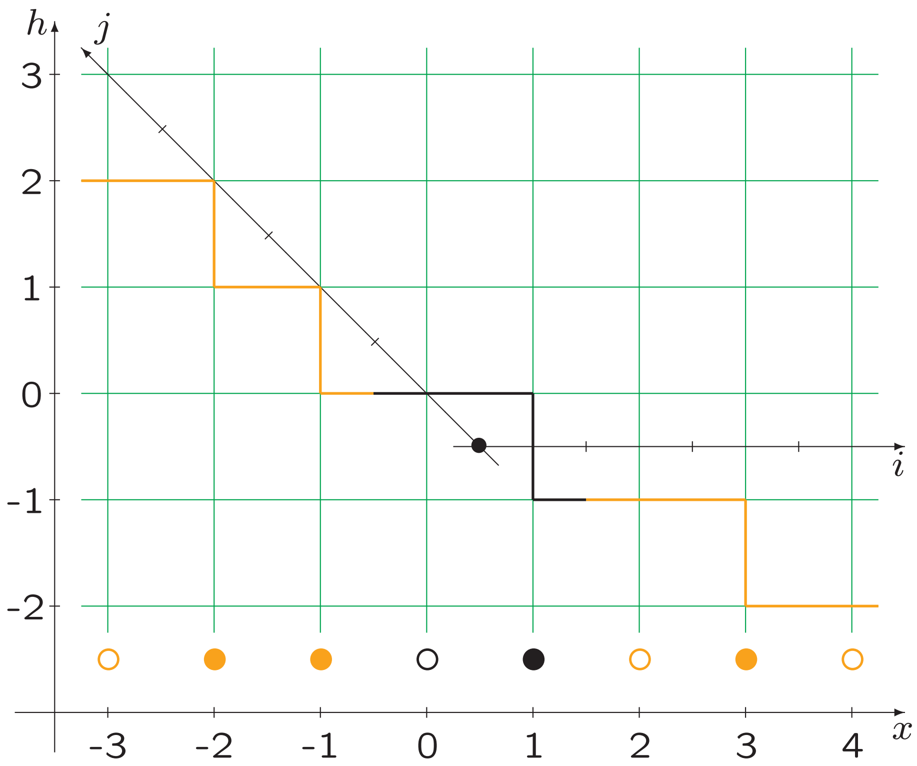

Bernoulli(ϱ) distribution

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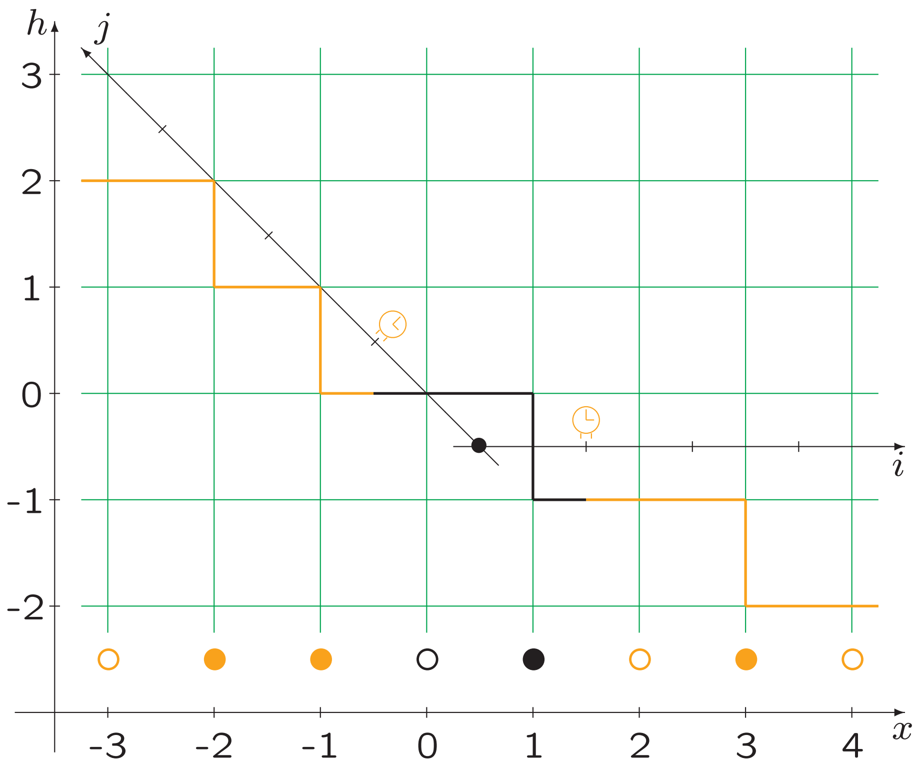

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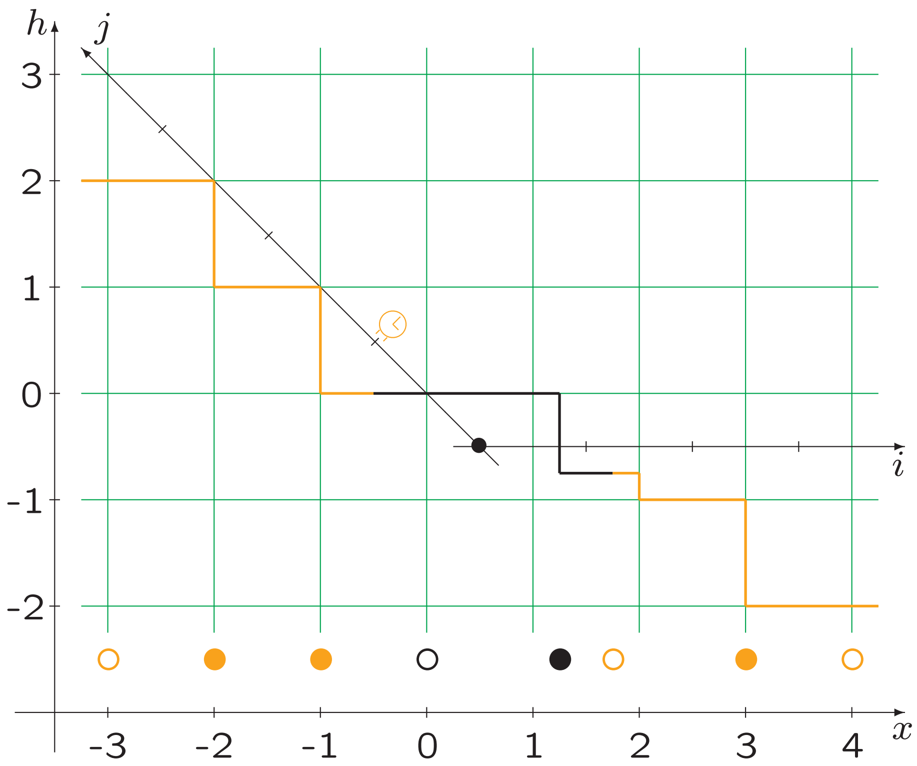

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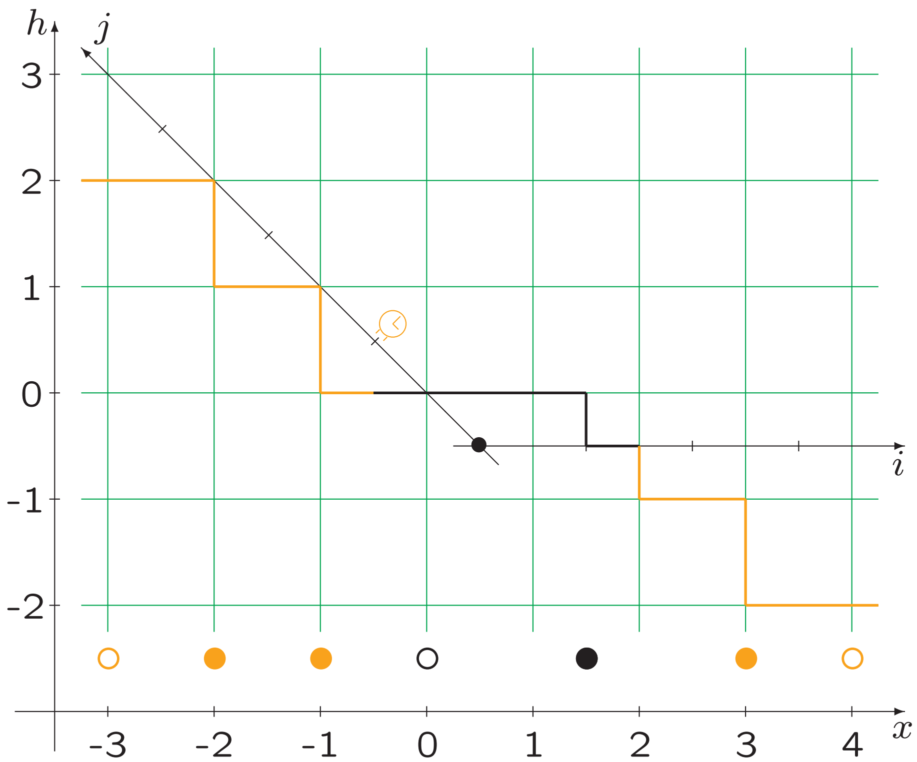

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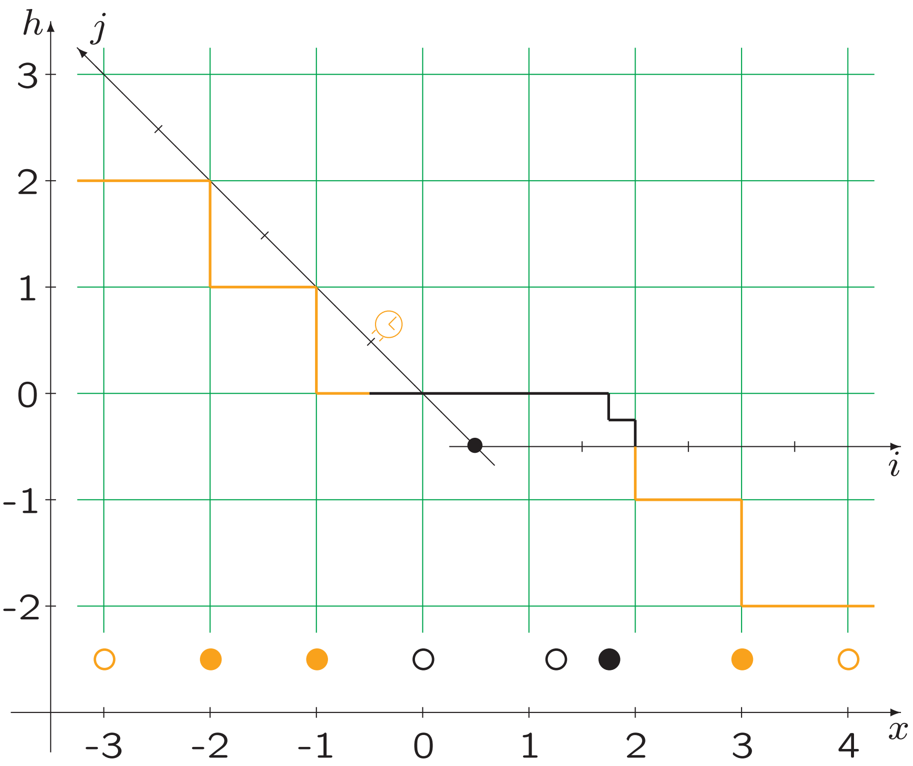

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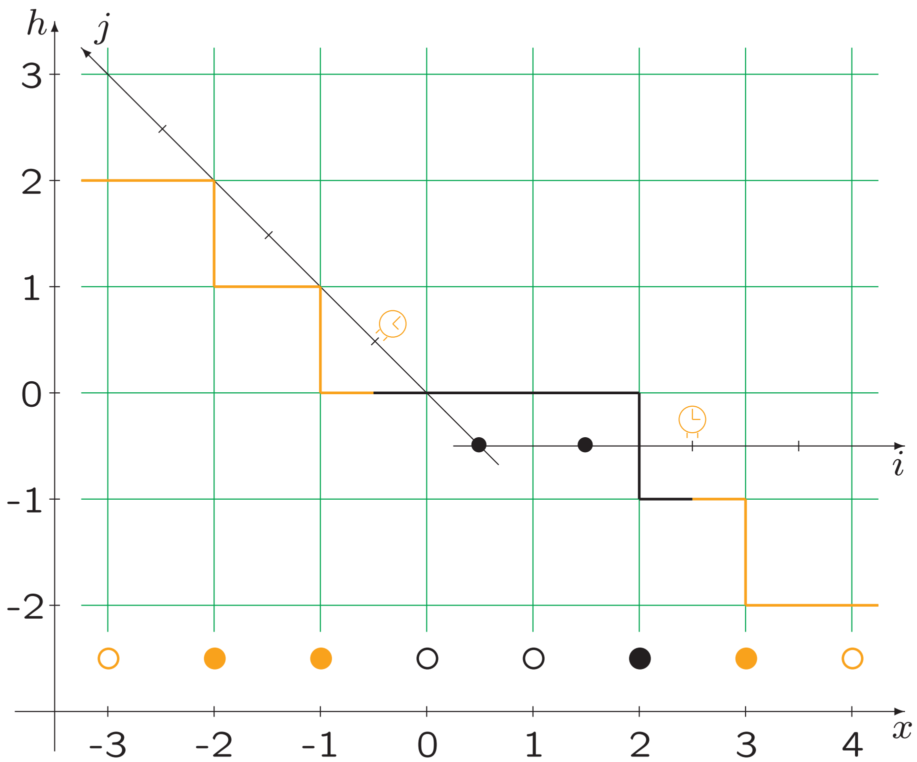

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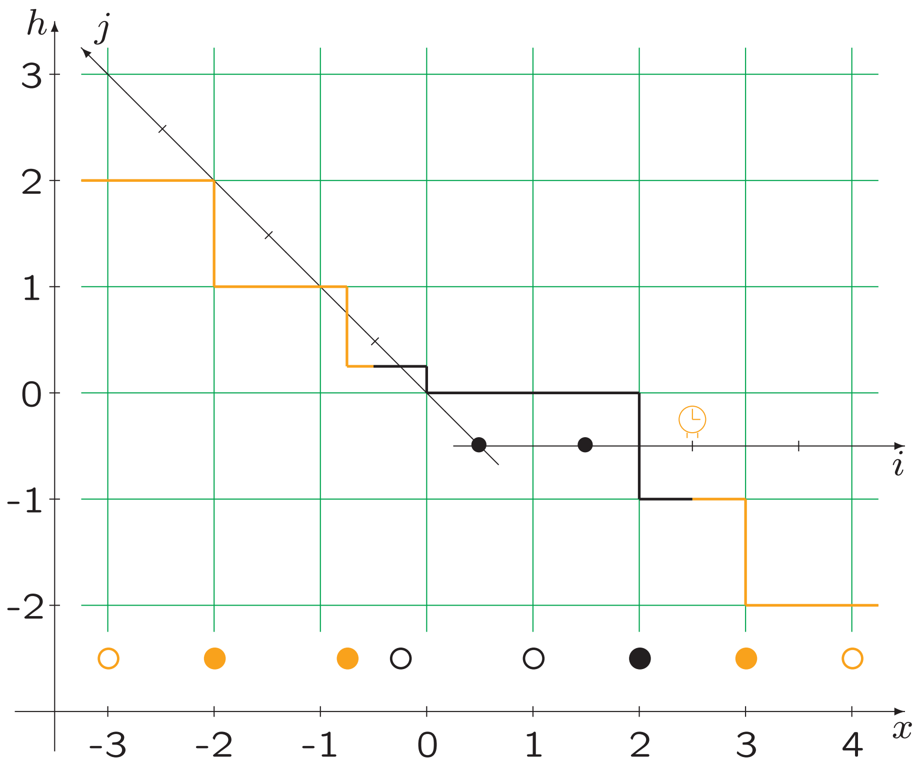

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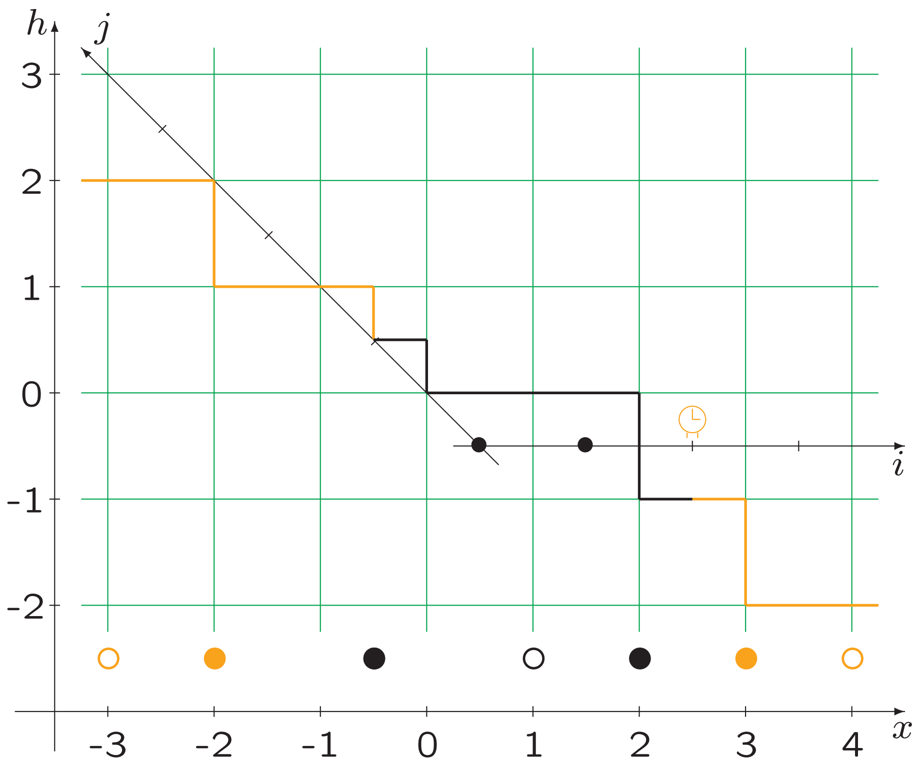

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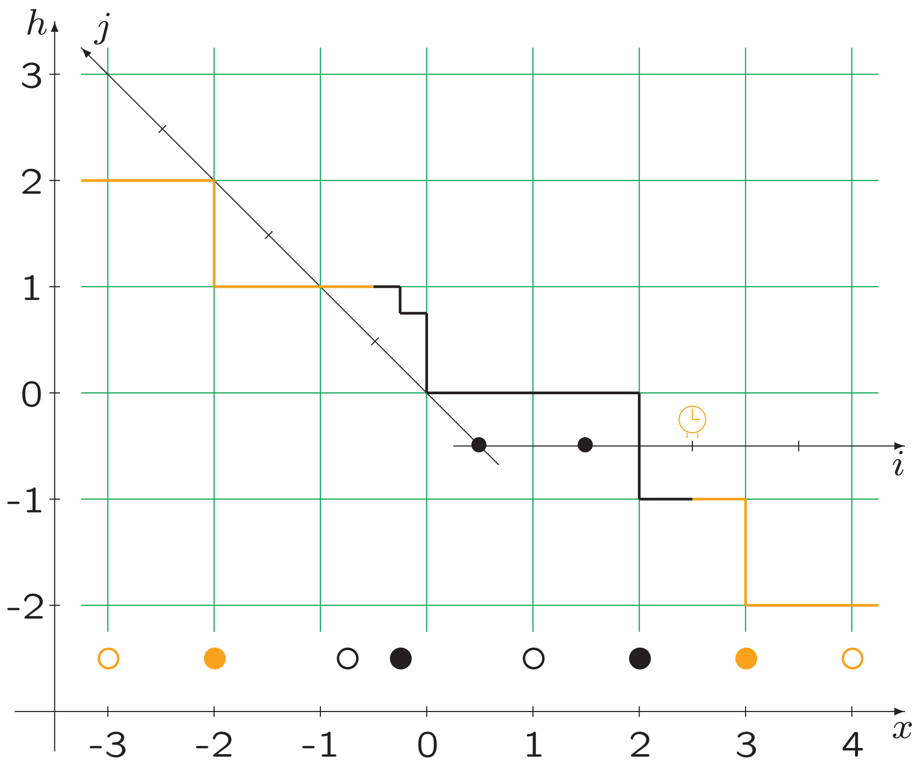

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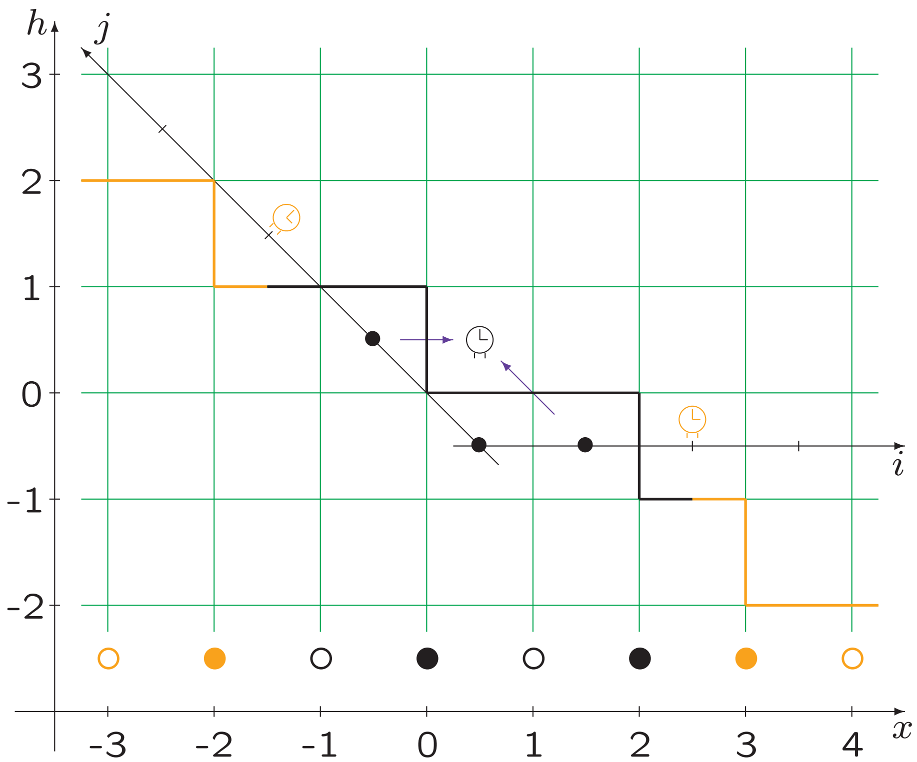

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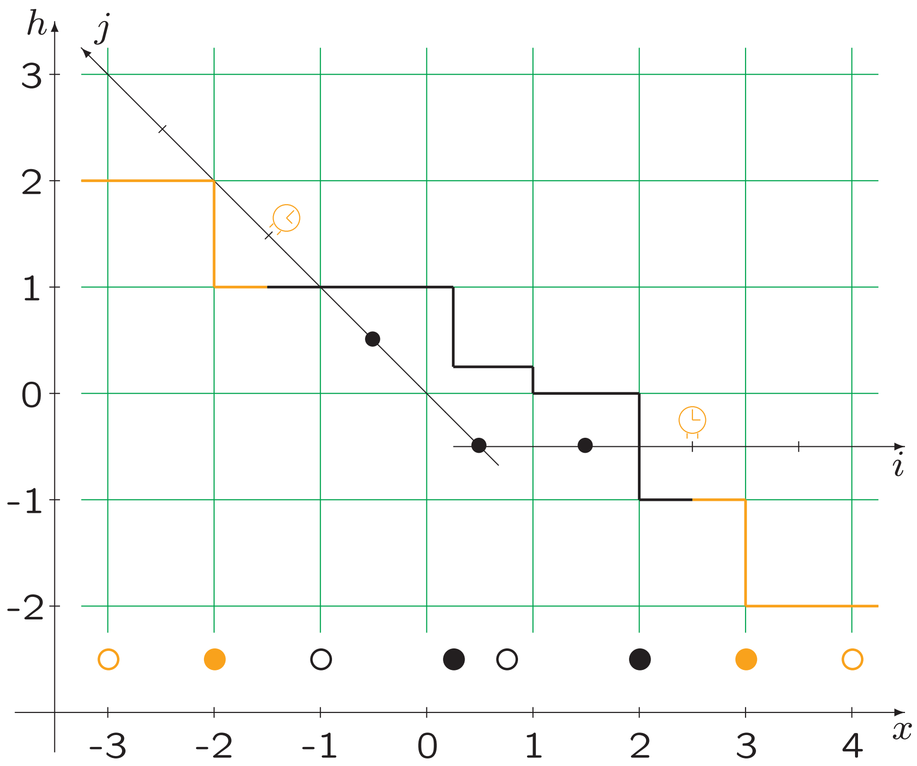

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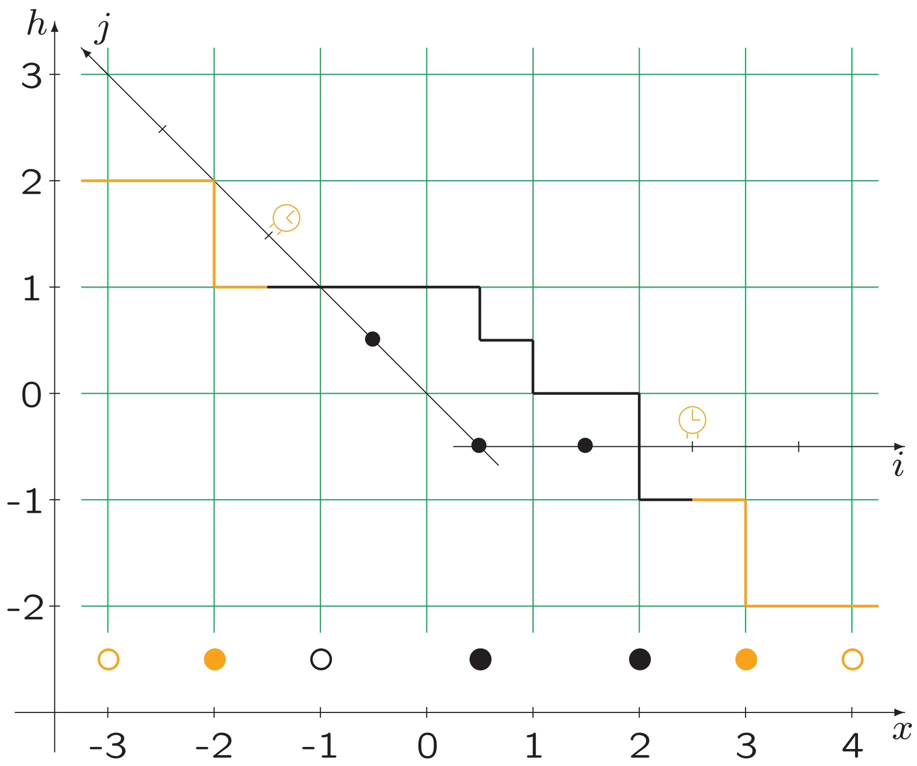

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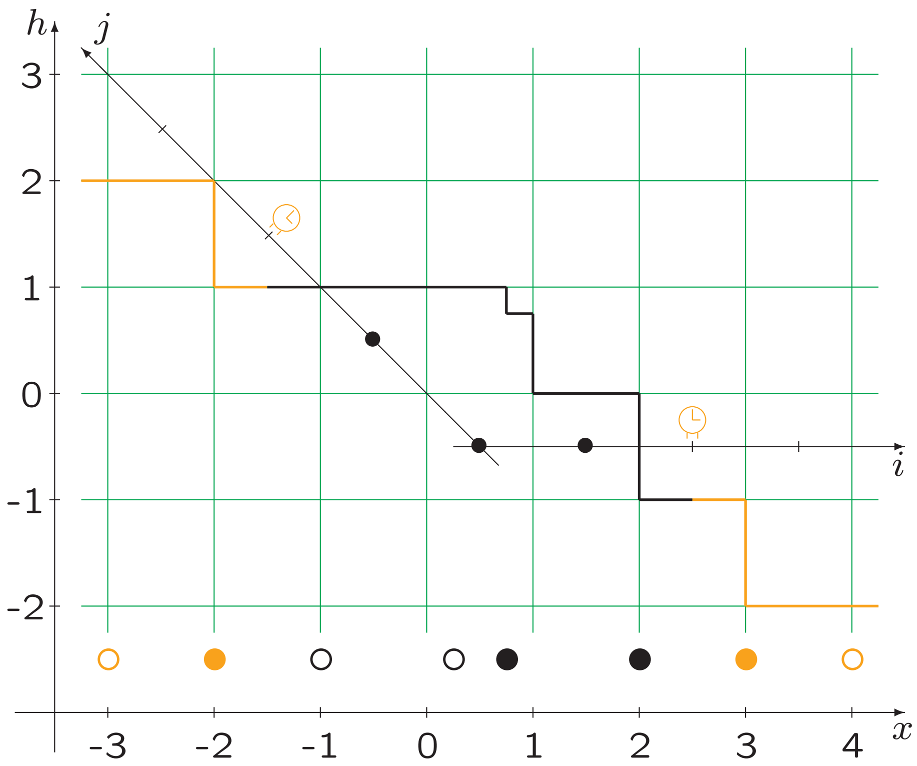

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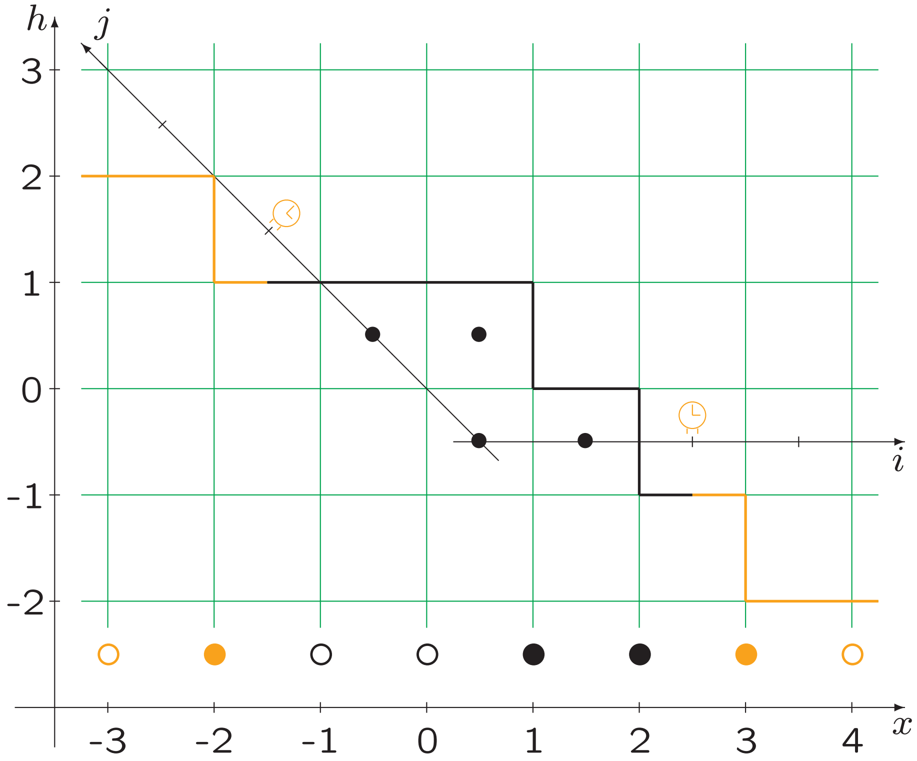

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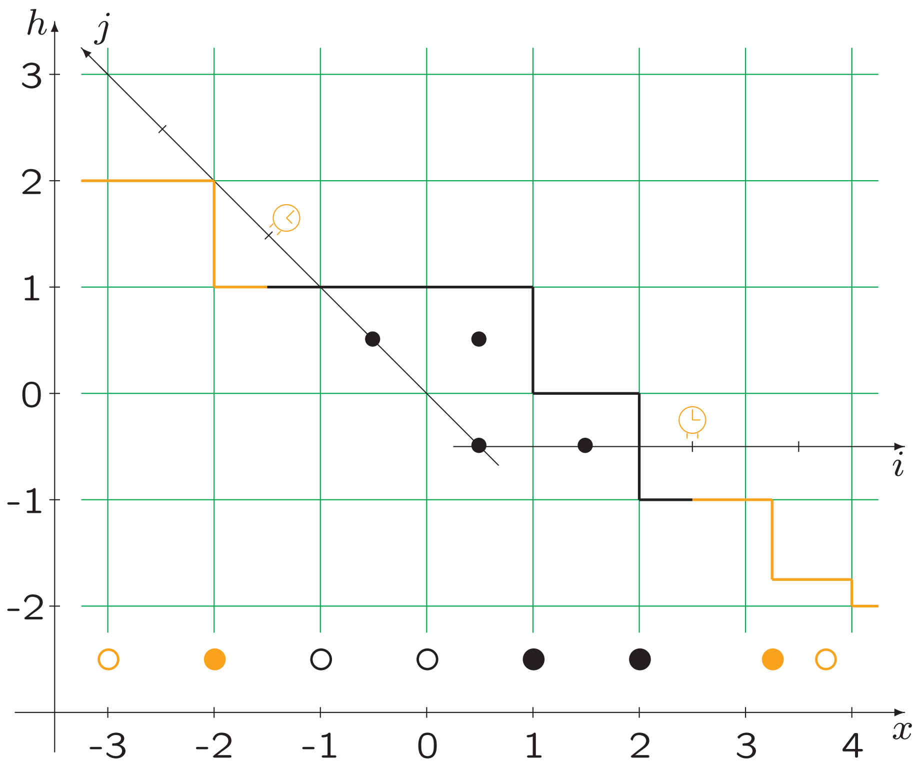

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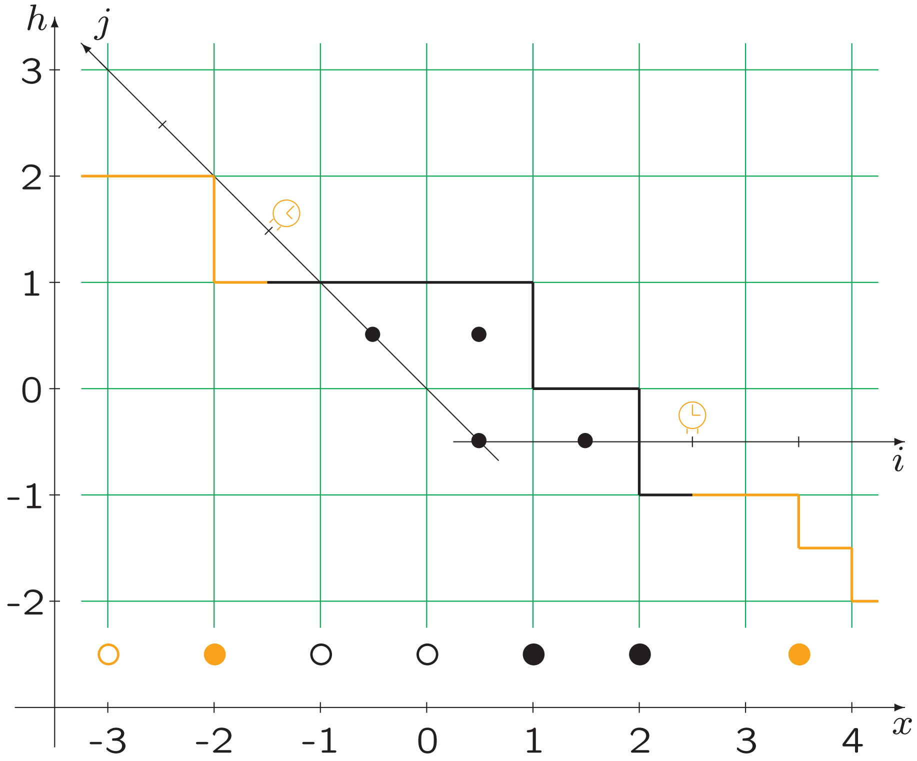

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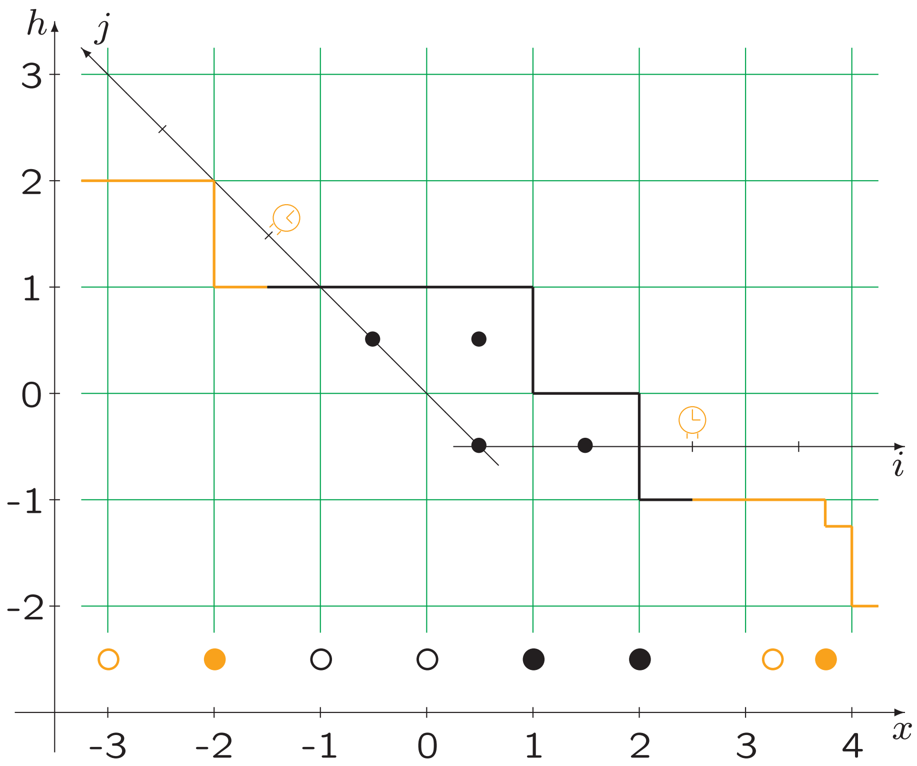

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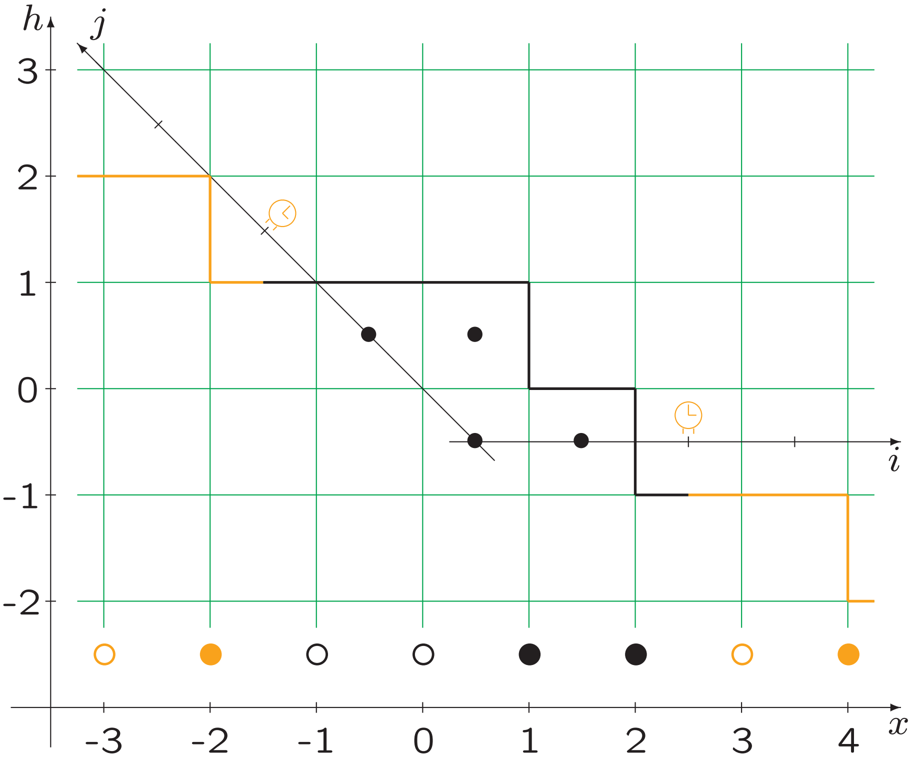

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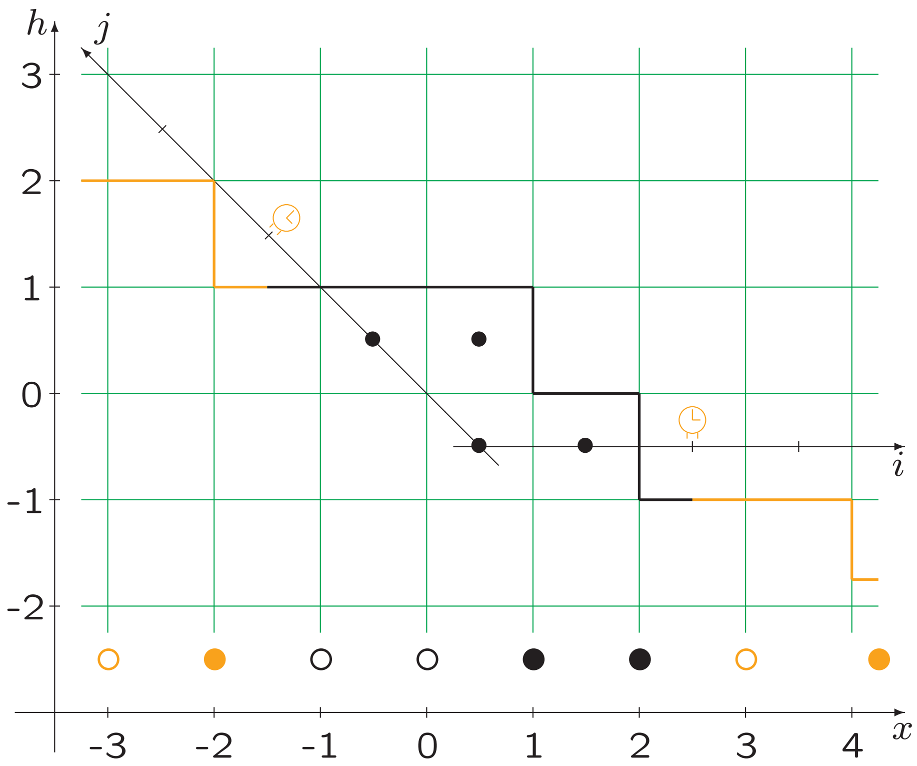

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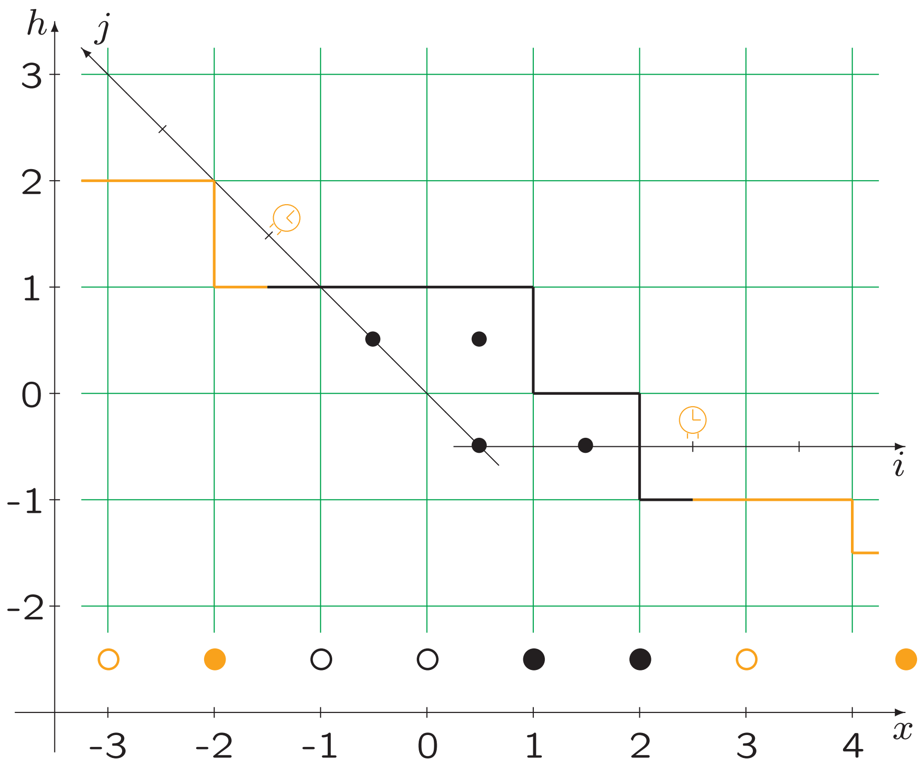

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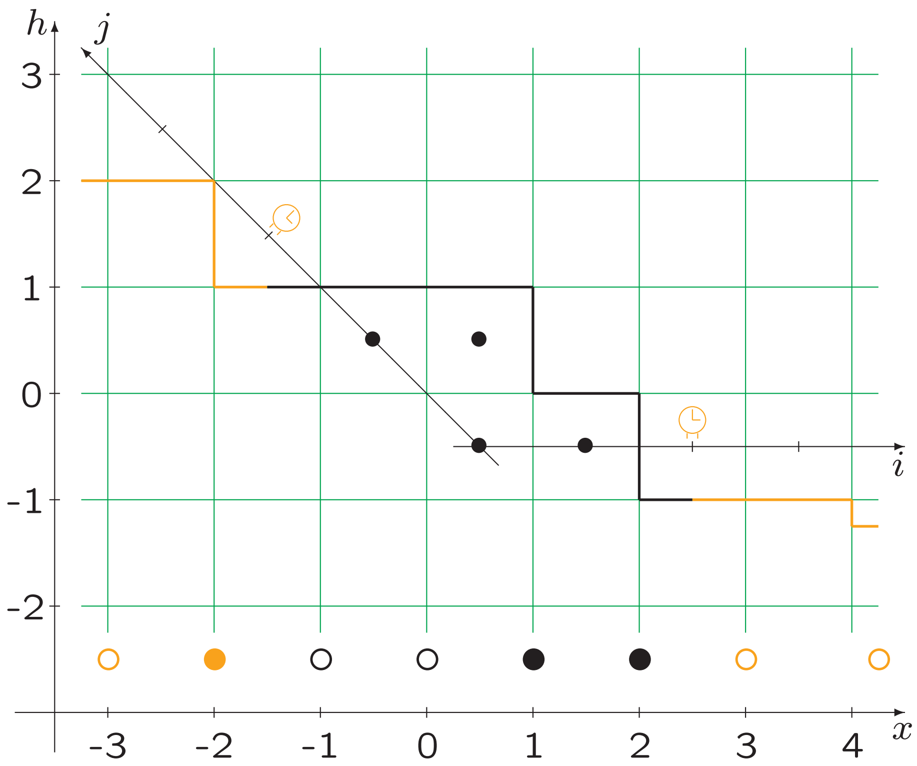

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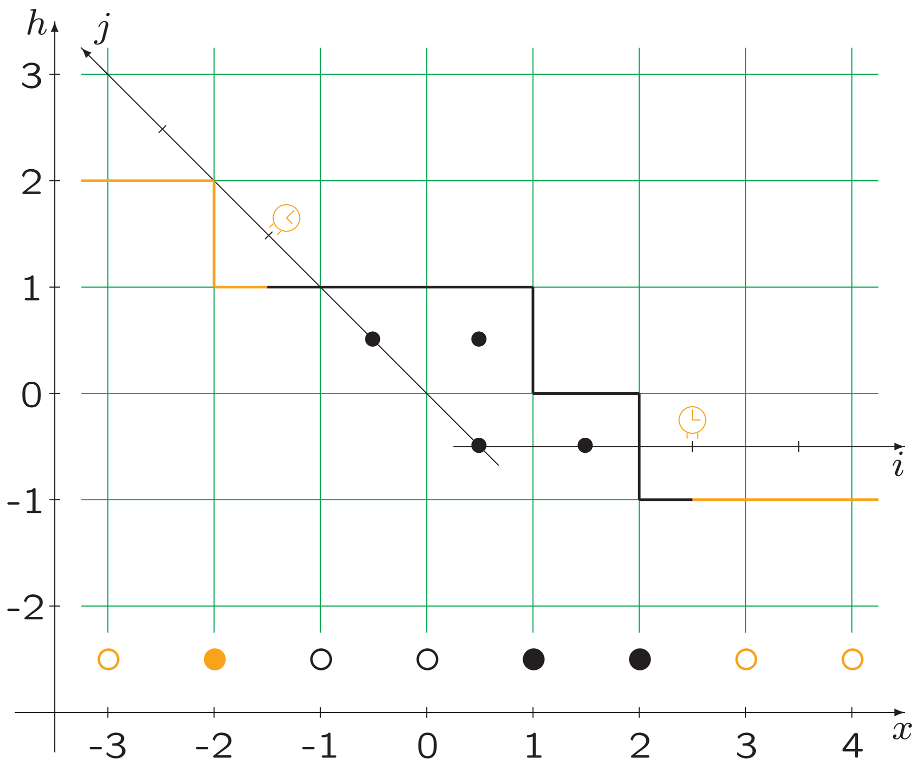

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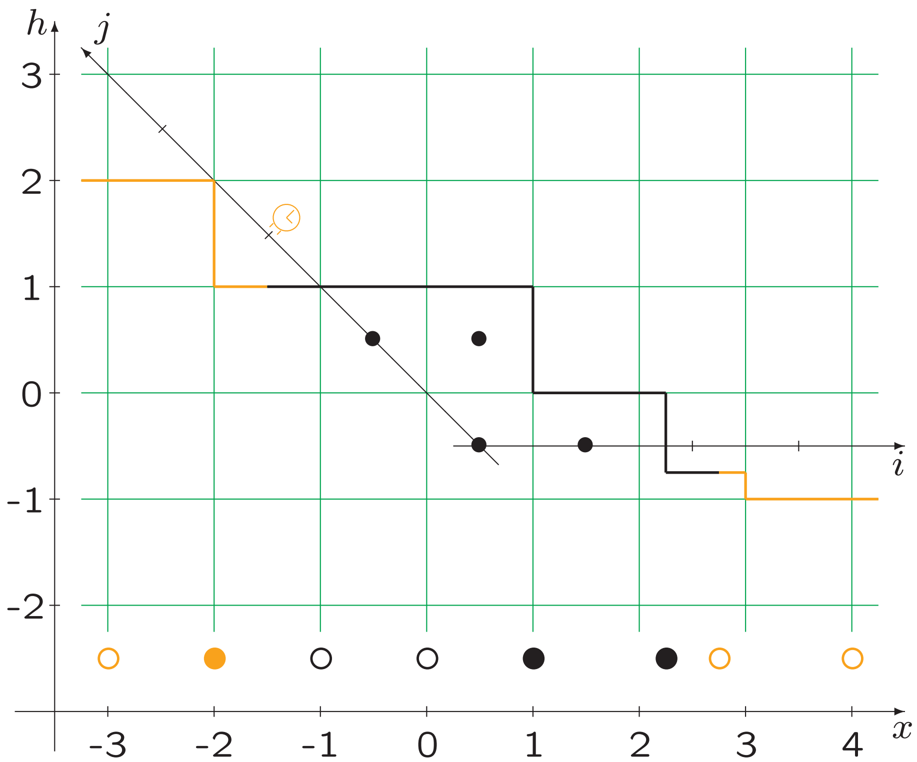

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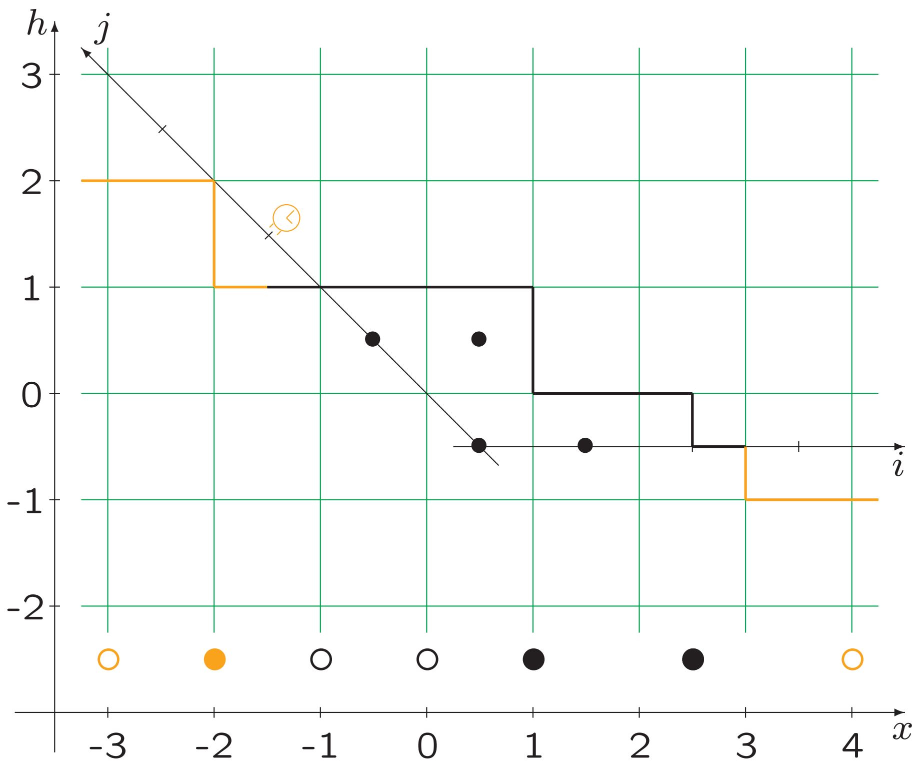

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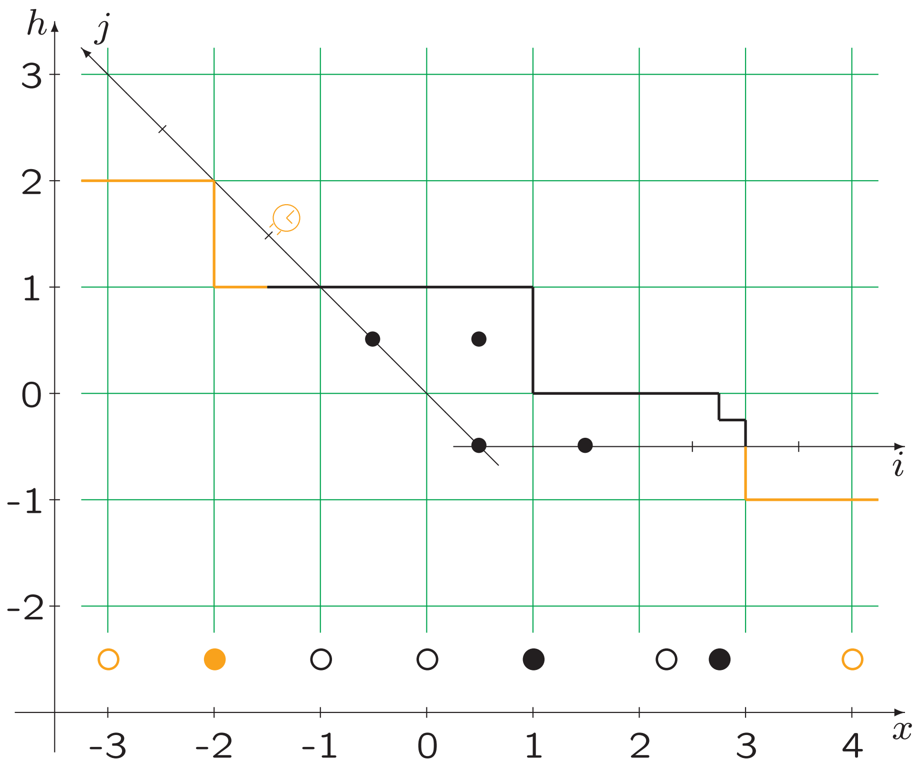

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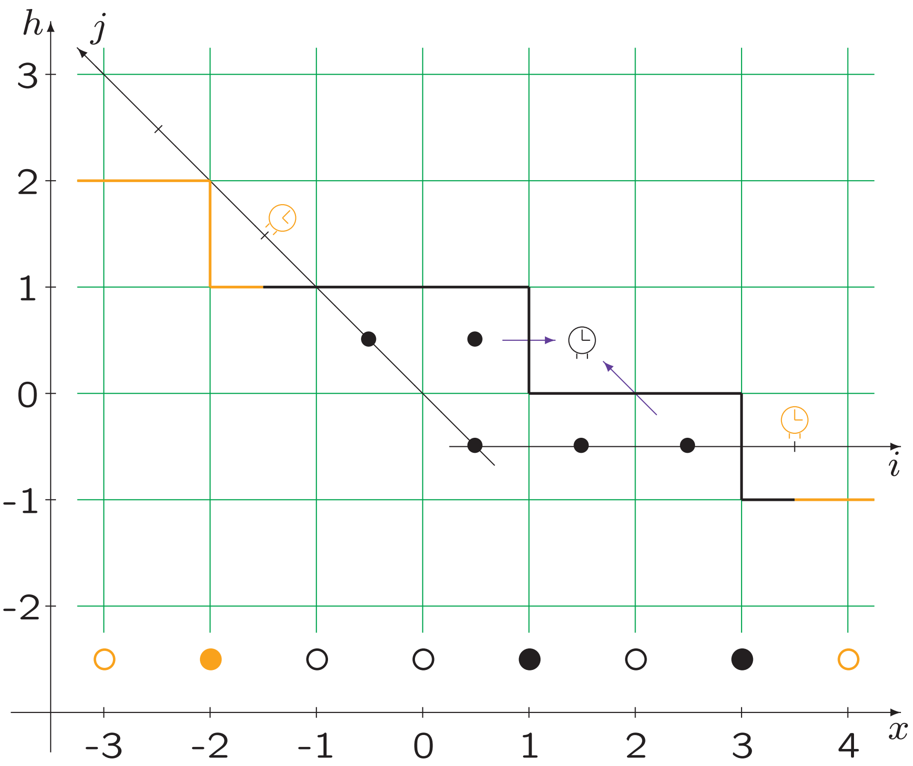

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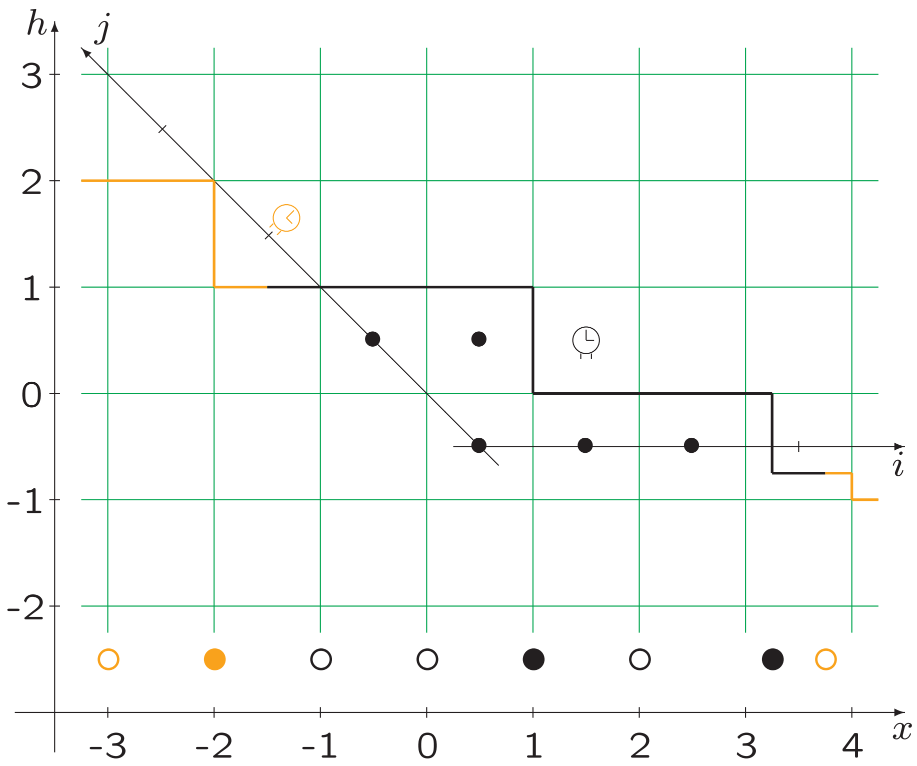

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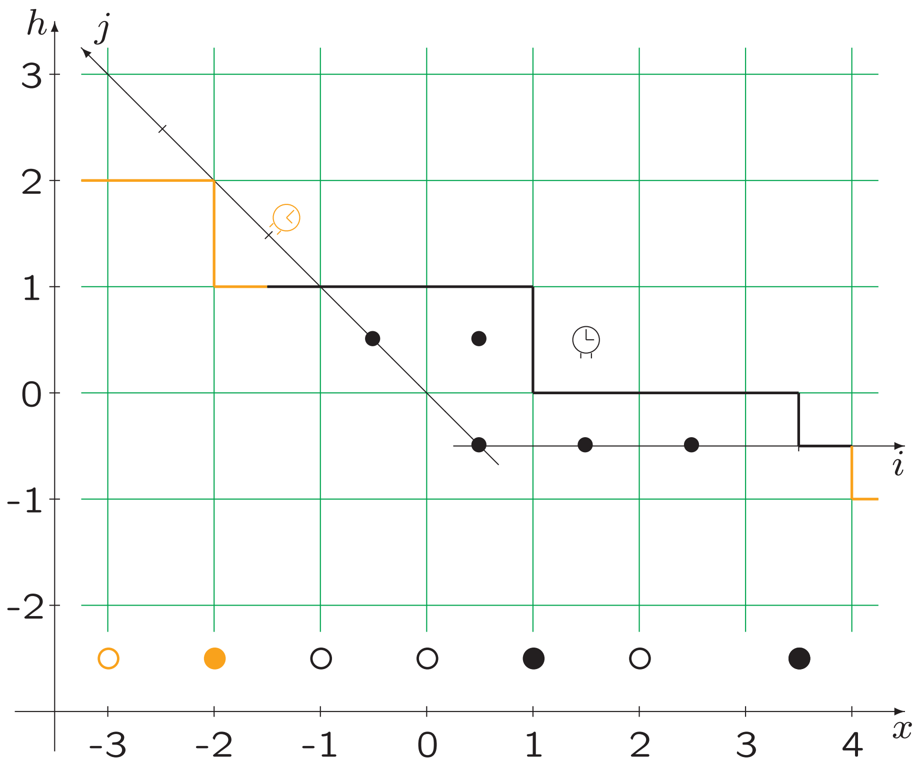

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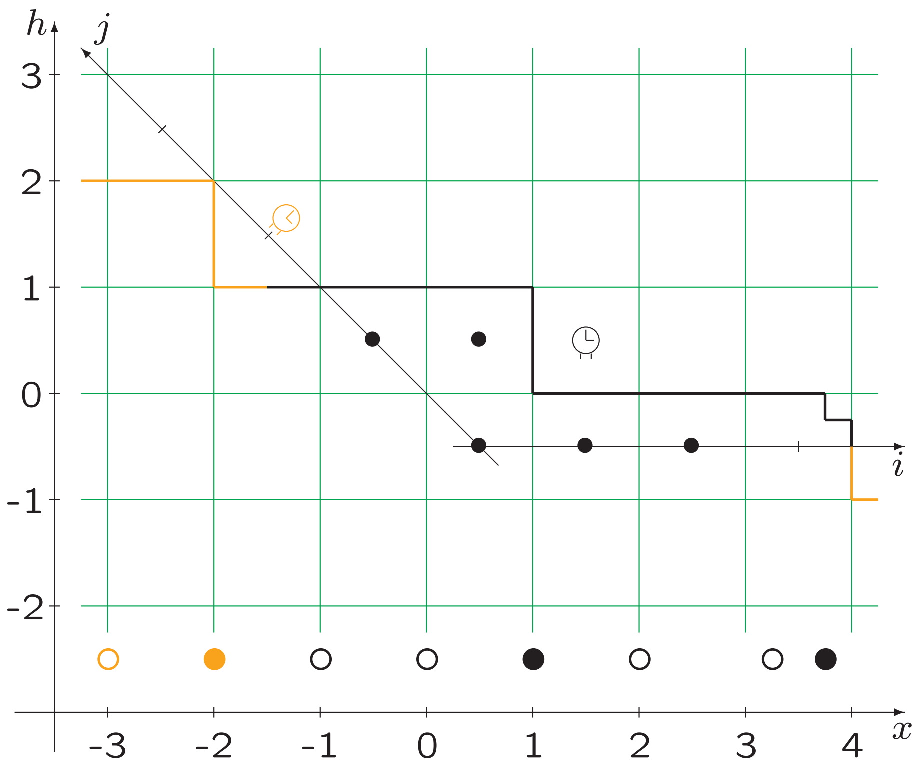

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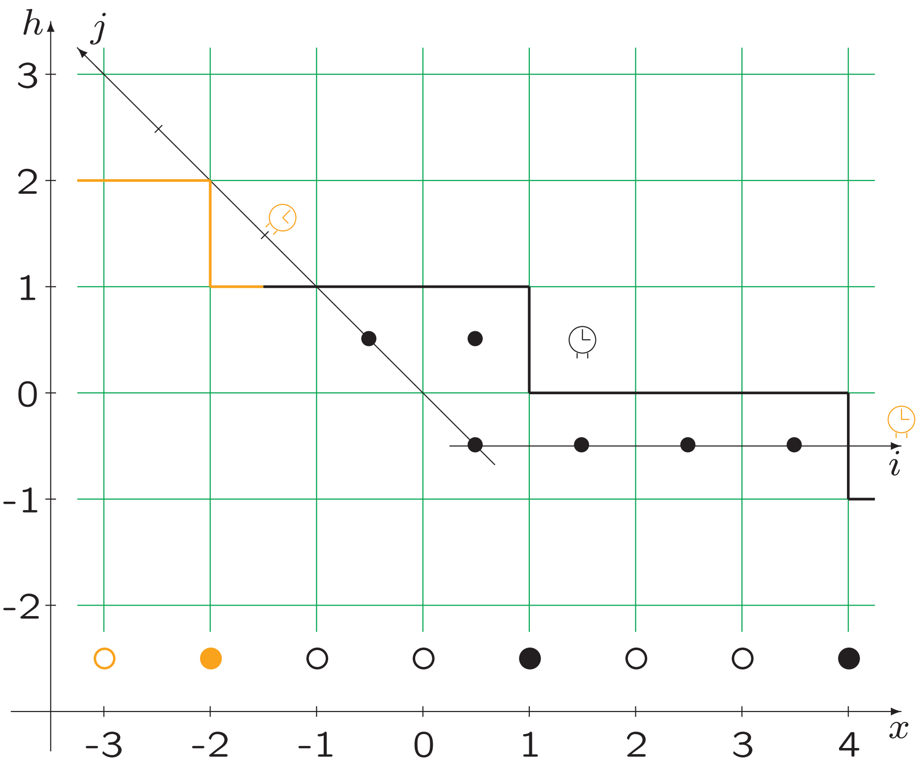

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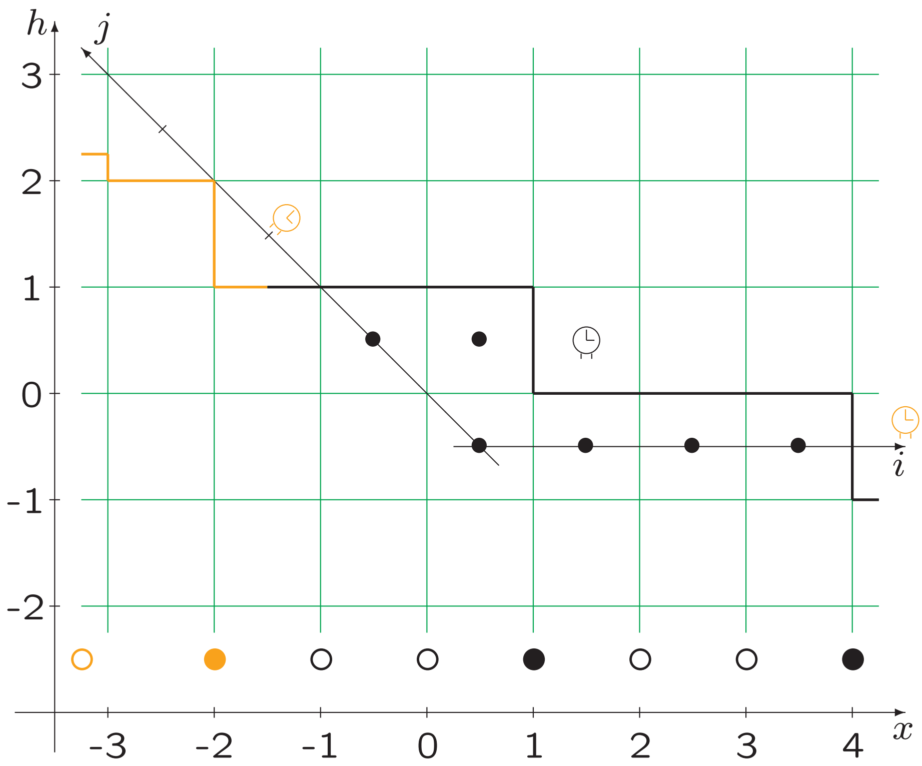

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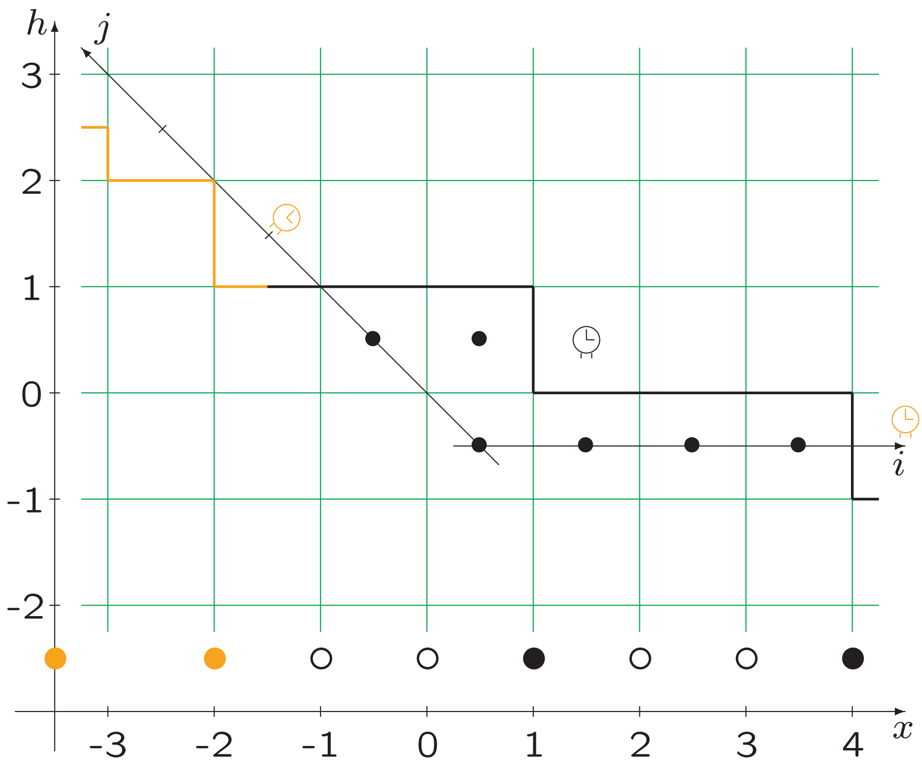

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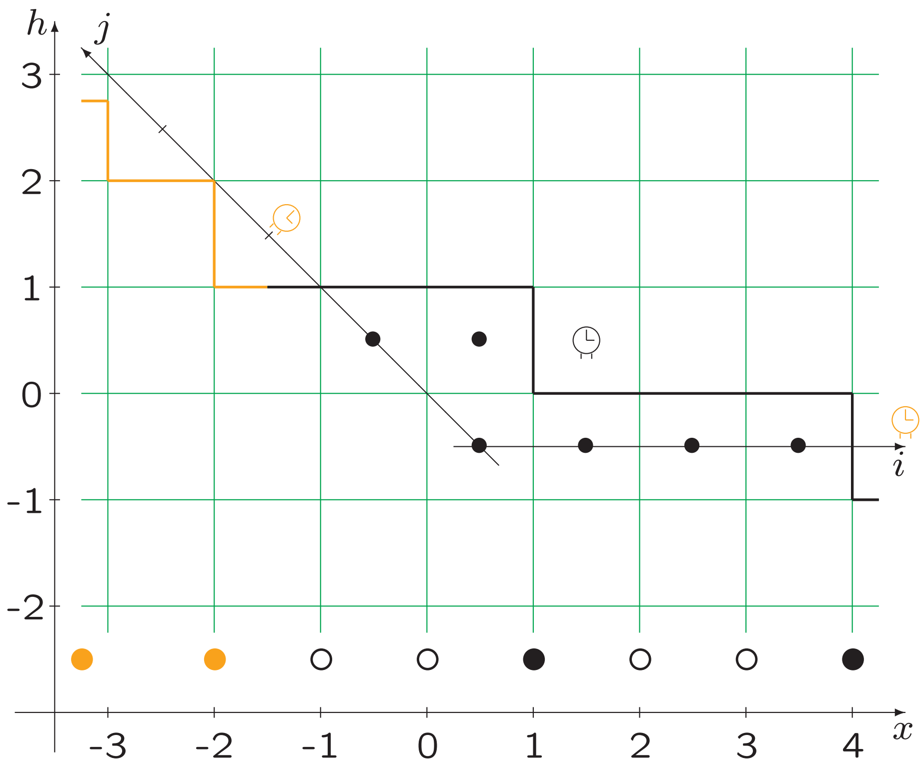

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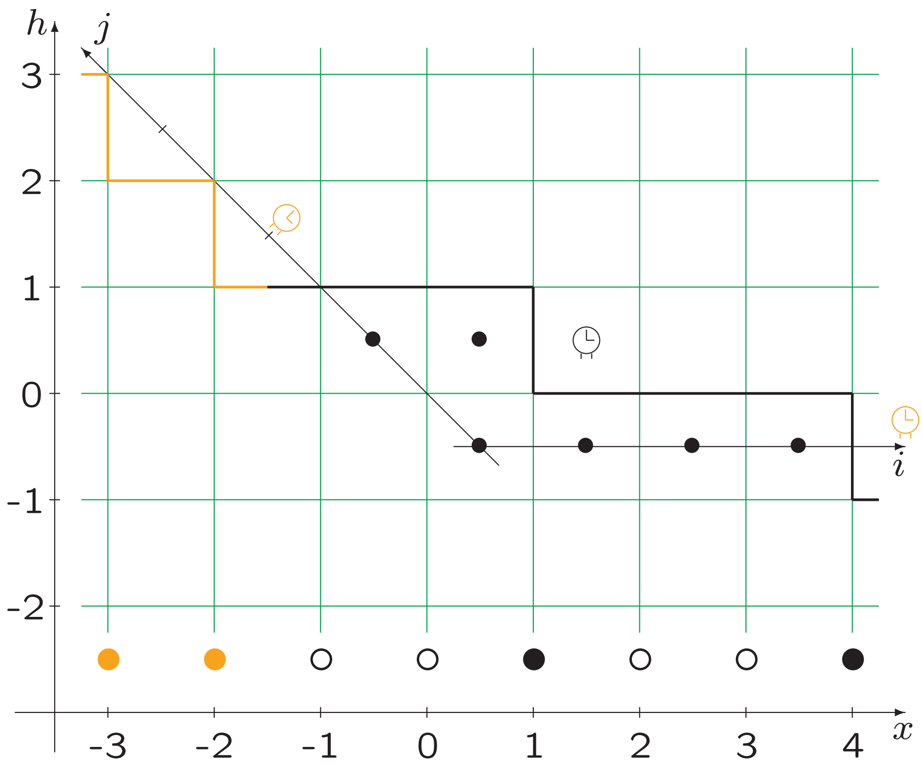

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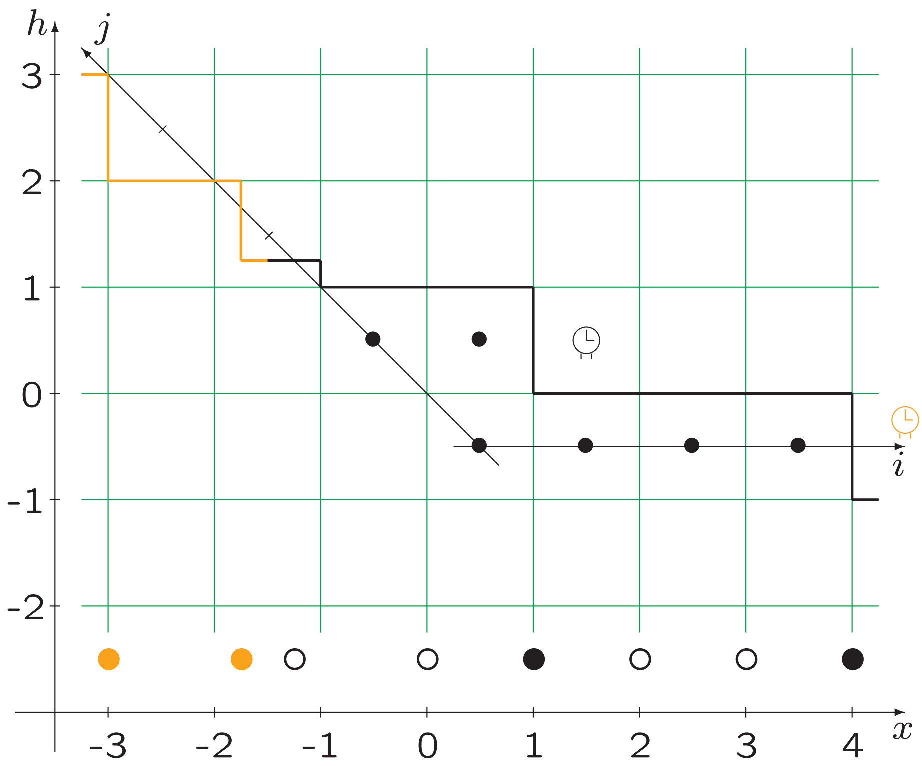

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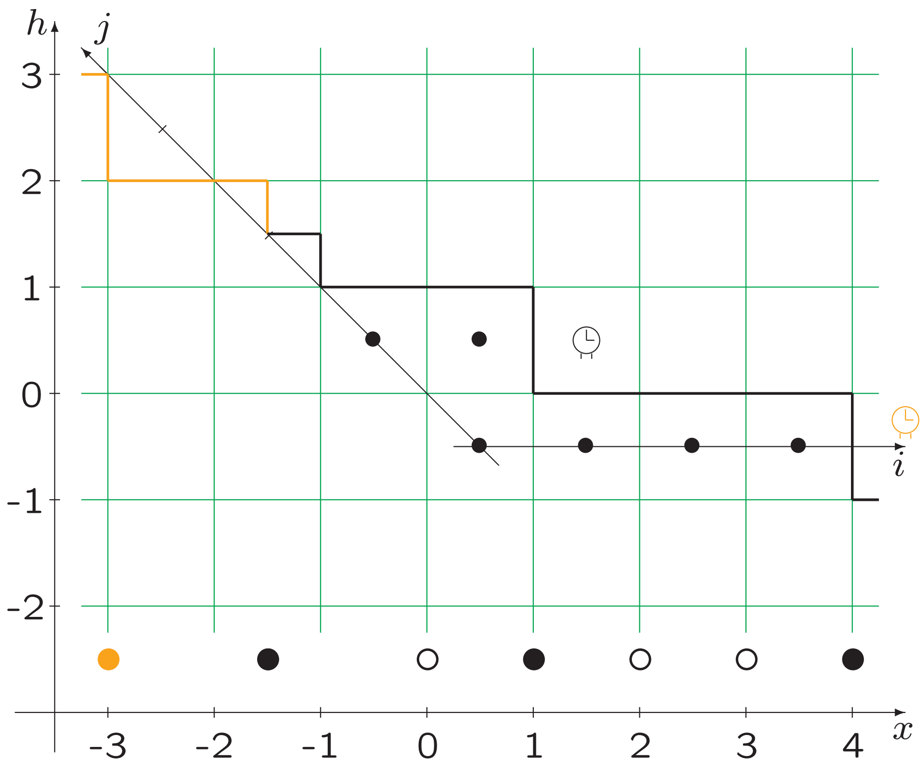

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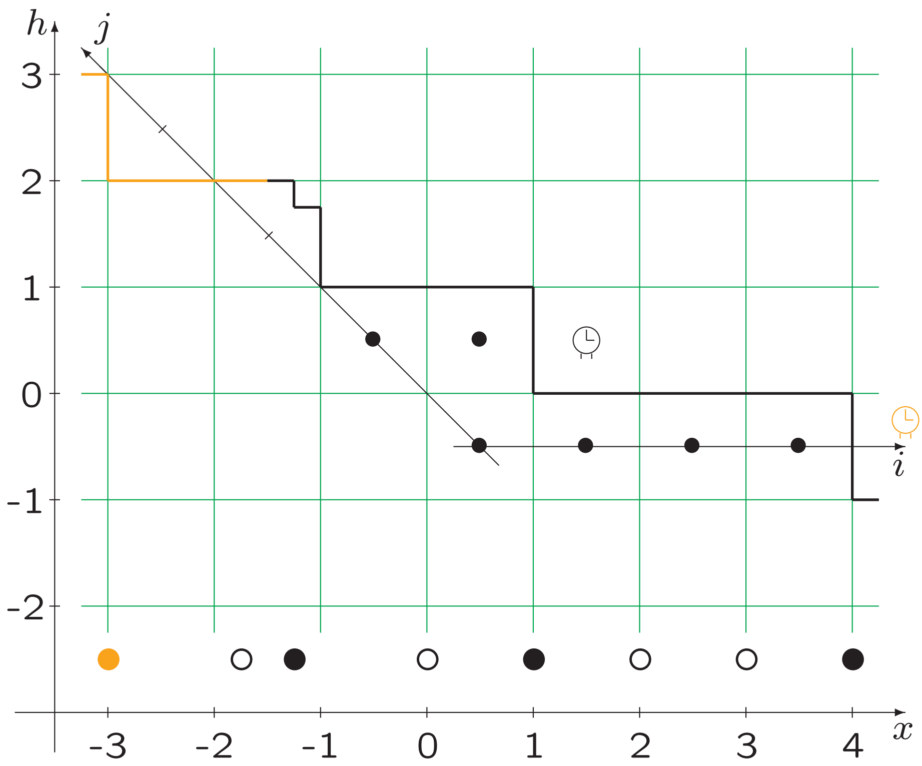

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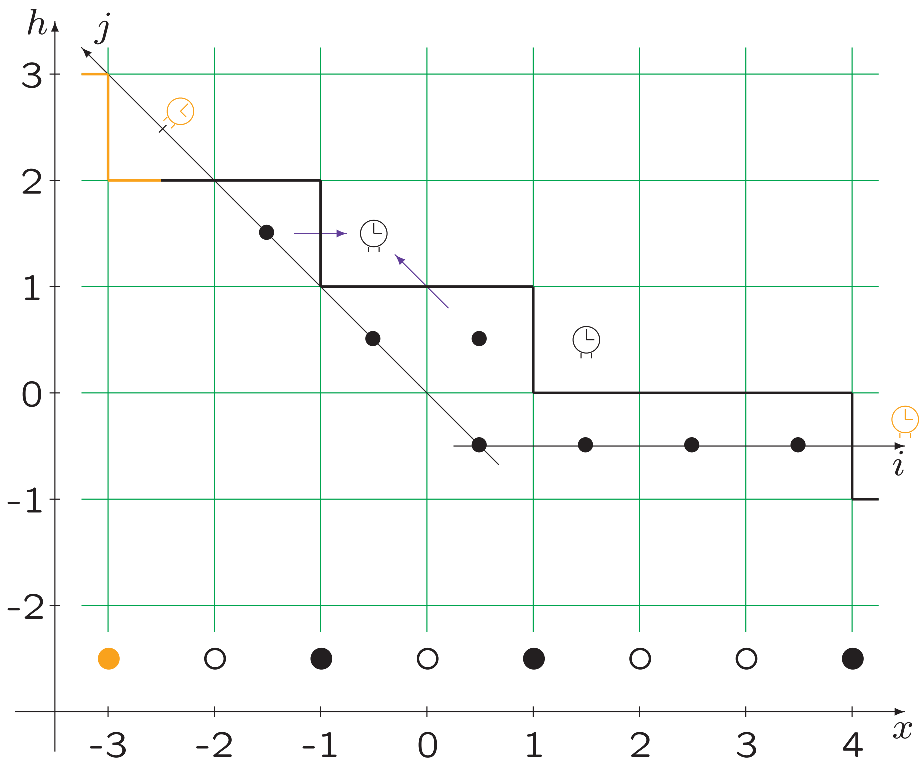

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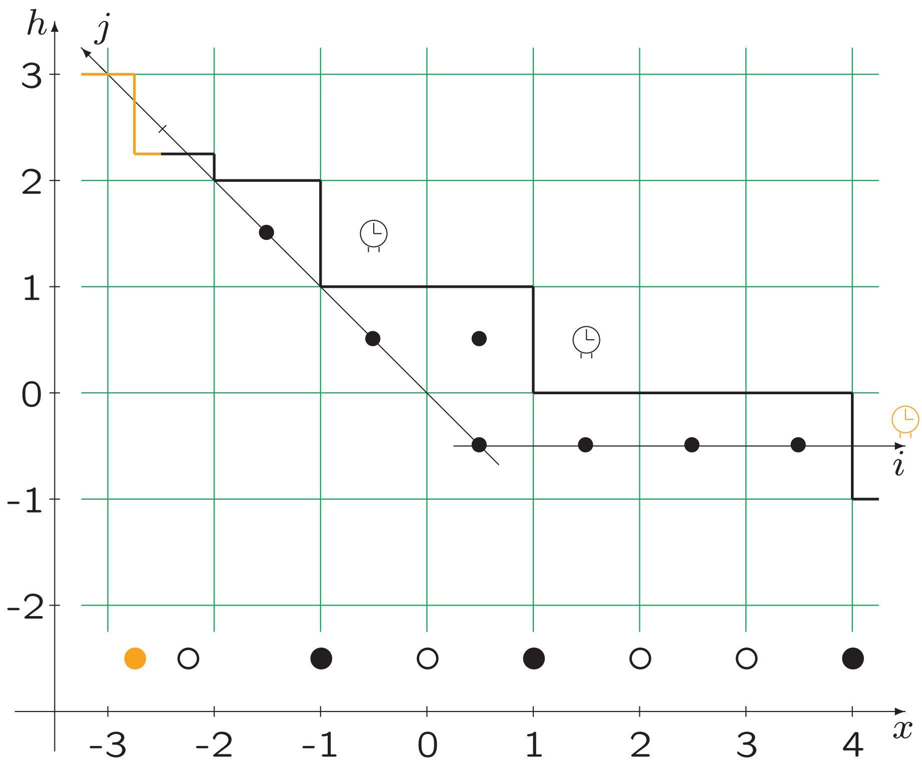

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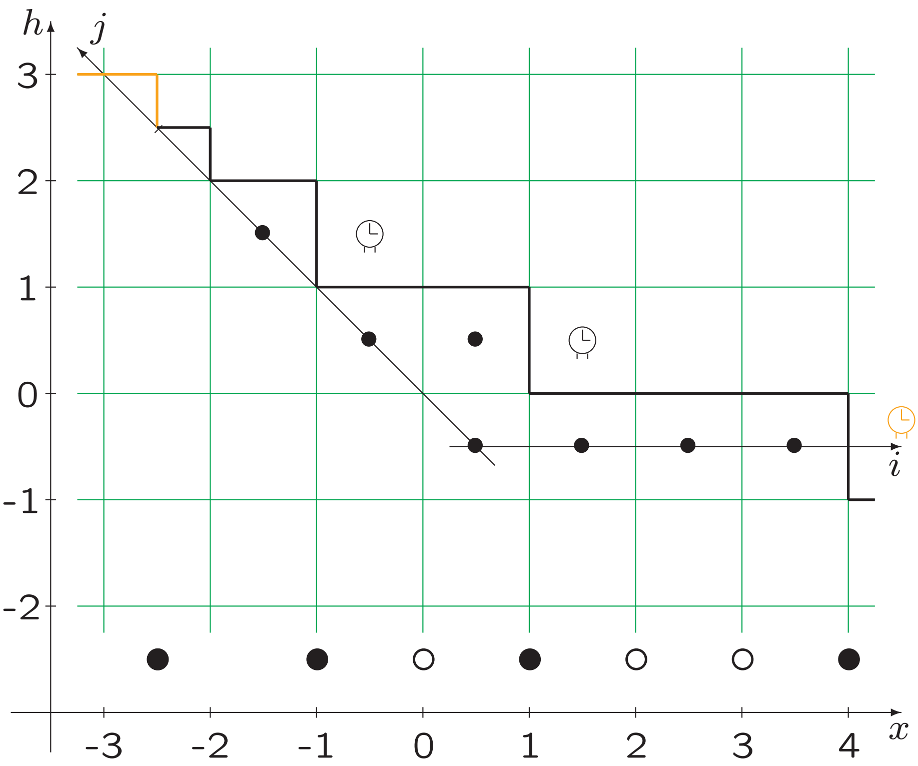

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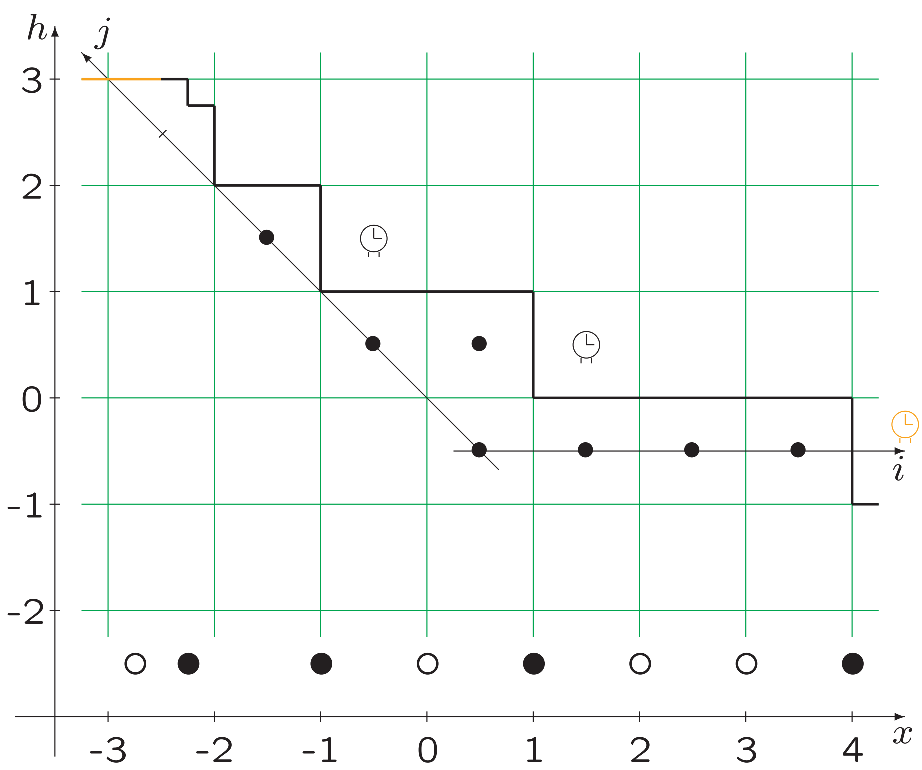

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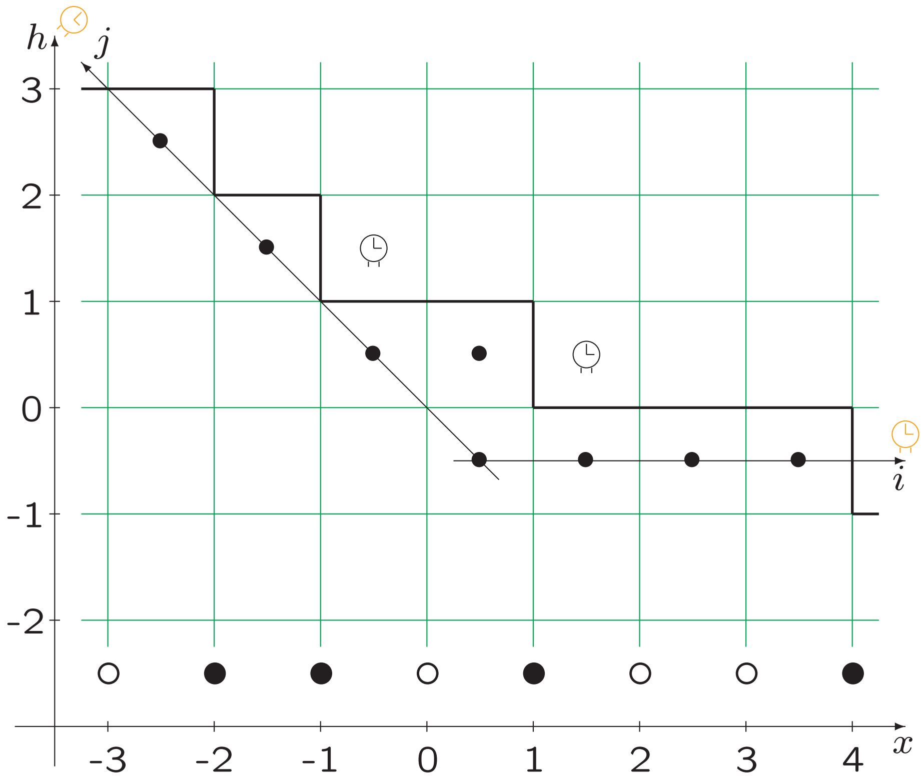

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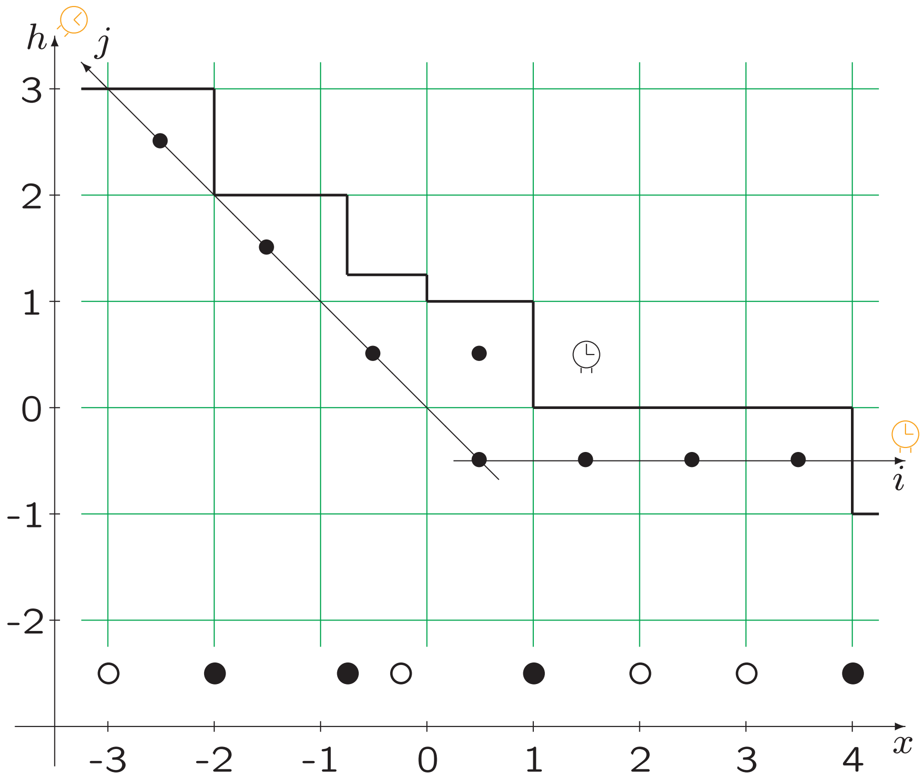
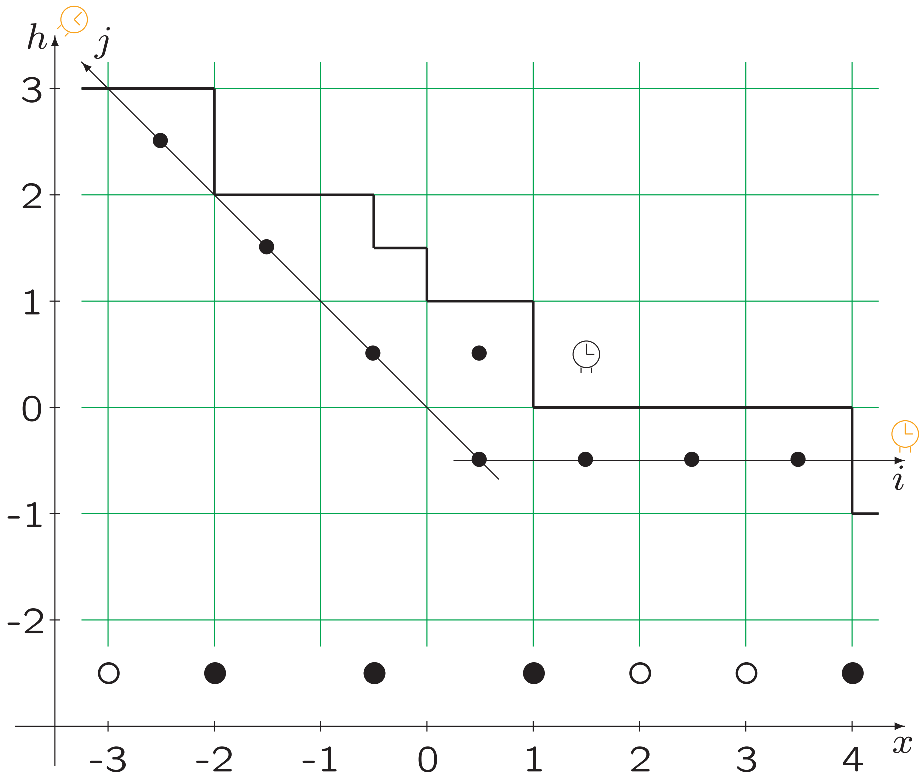
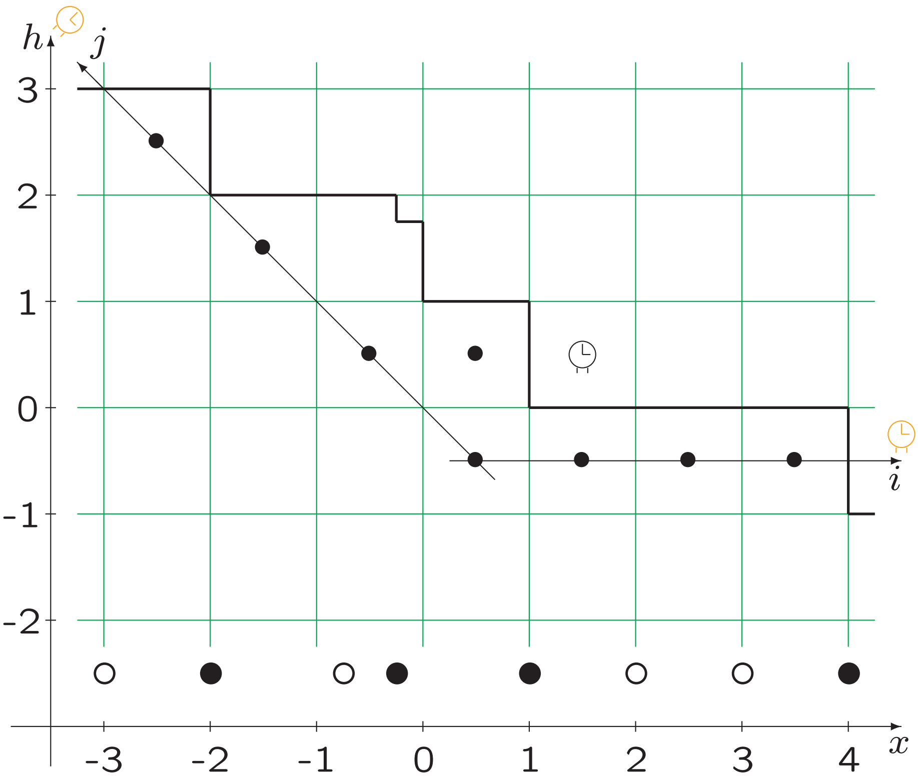


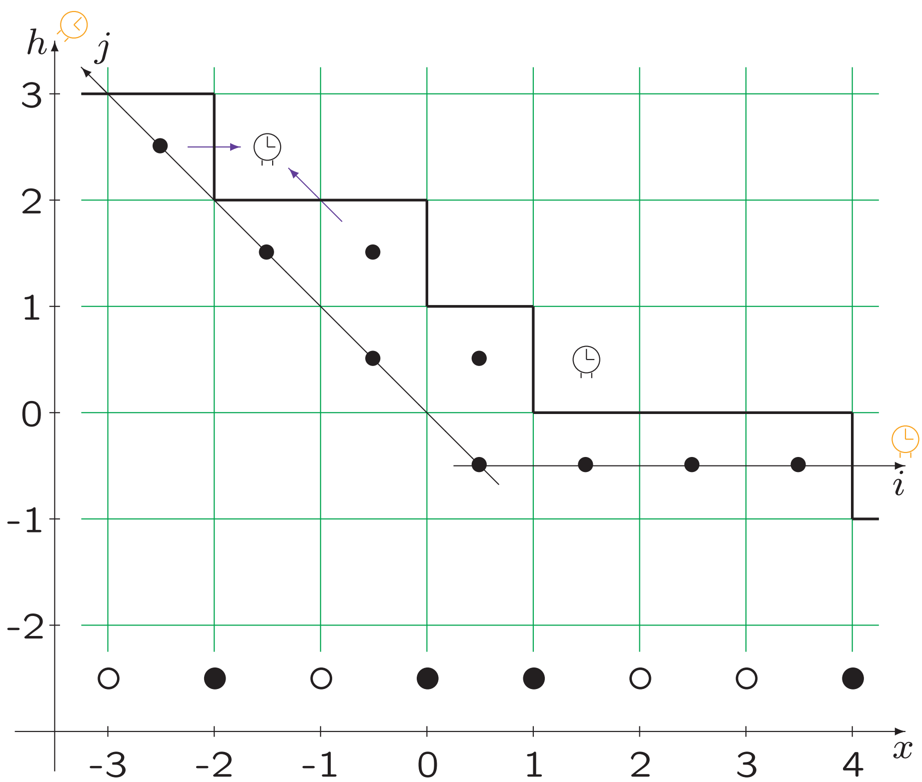
Outfit 4: Last passage percolation

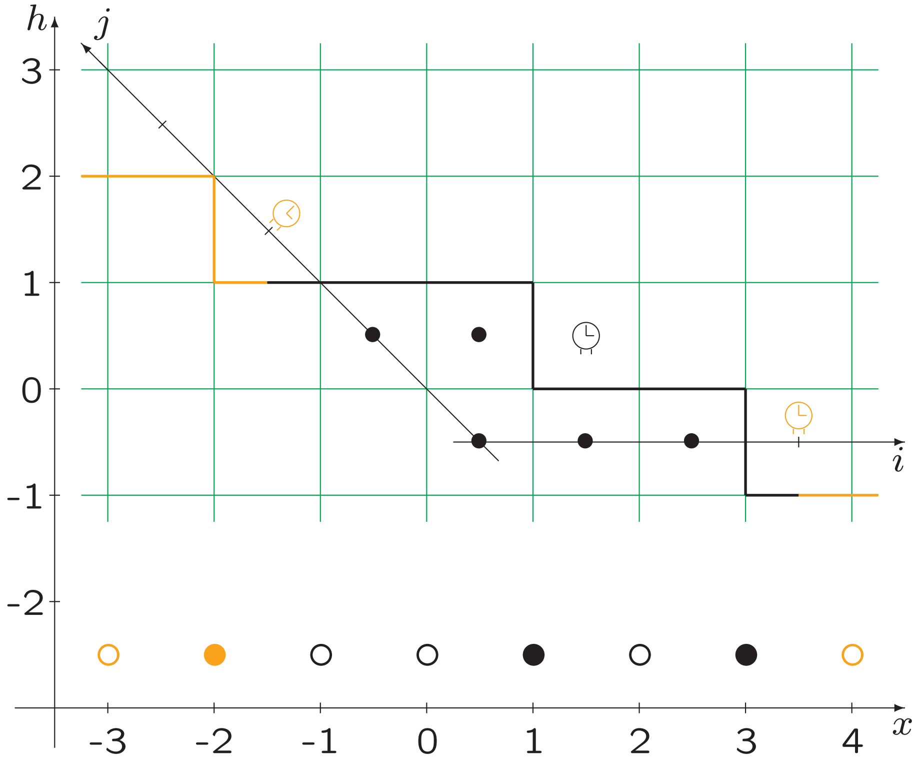


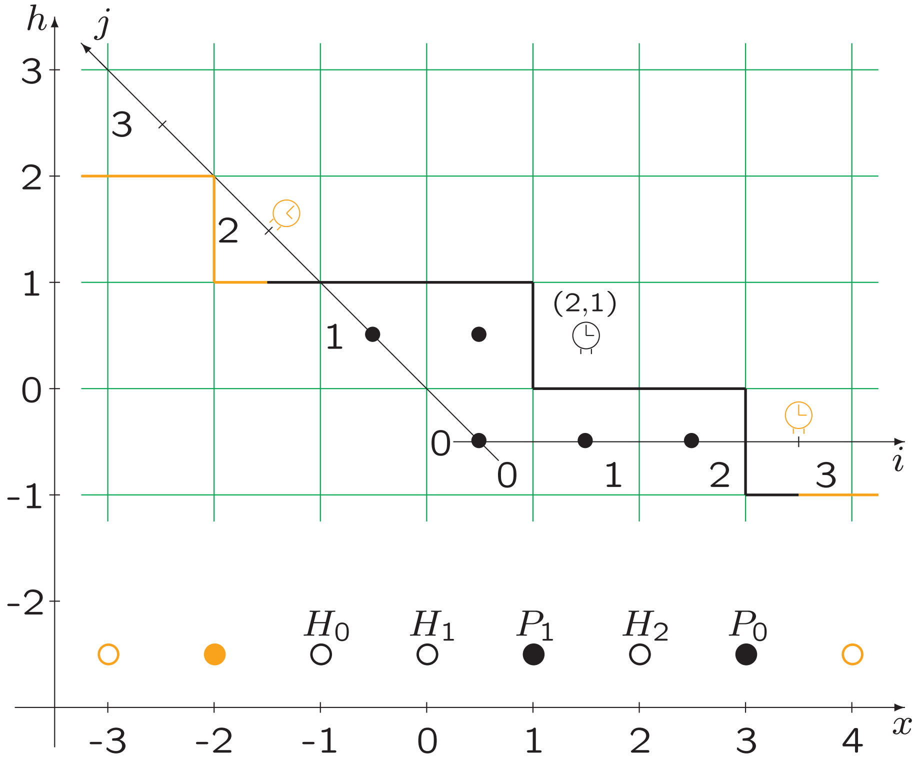
Outfit 4: Last passage percolation

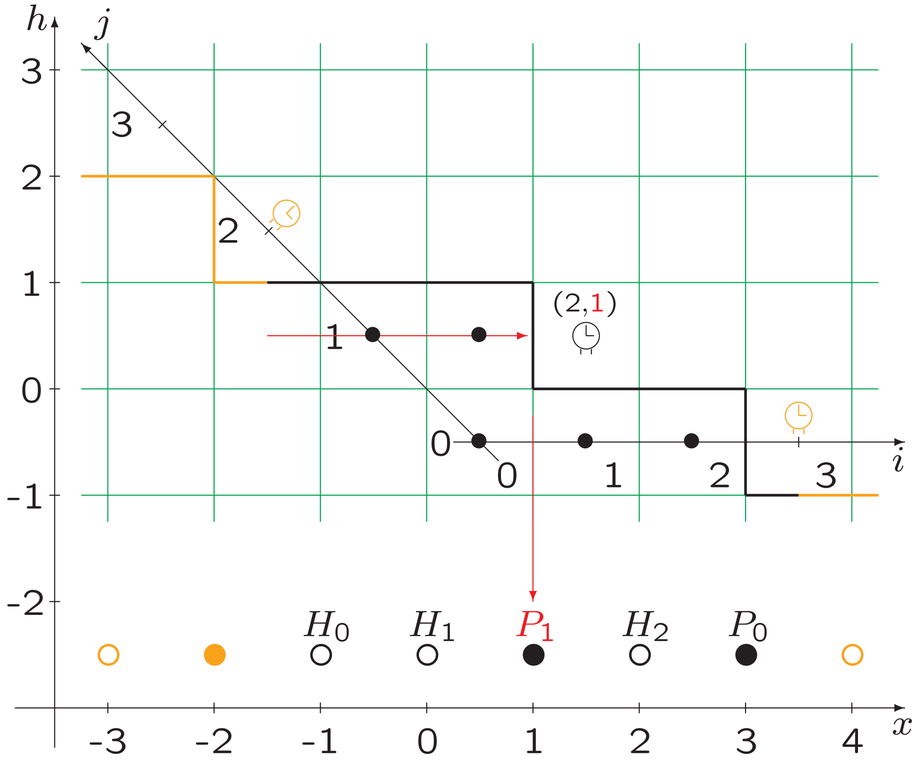


Outfit 4: Last passage percolation

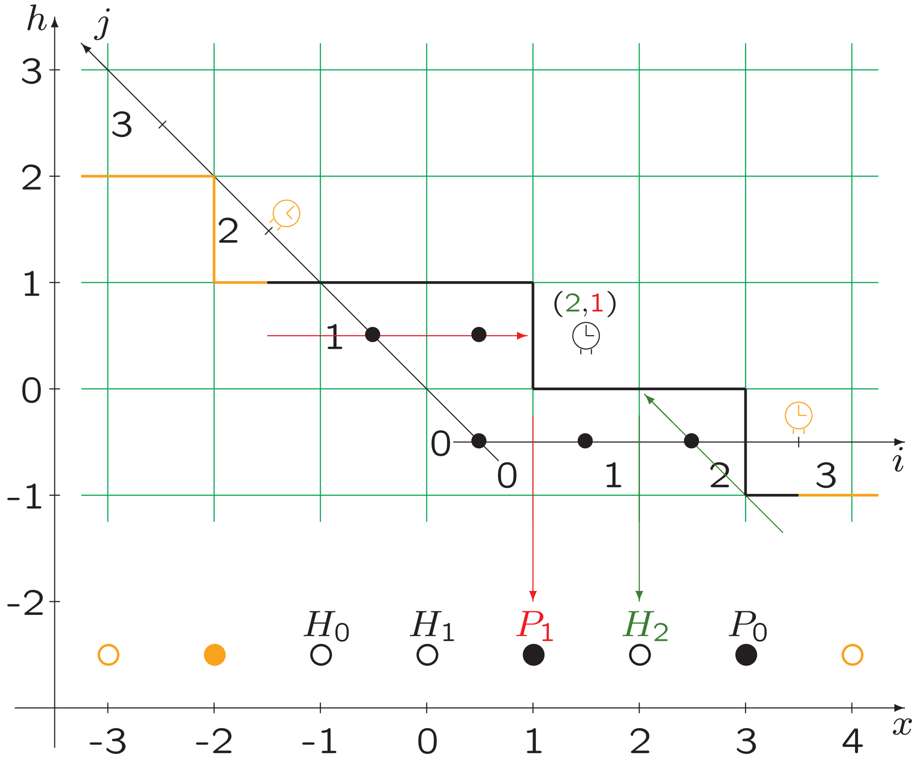



h
j
3
2
1
0
-1
-2
i
x
-3
-2
-1
0
1
2
3
4

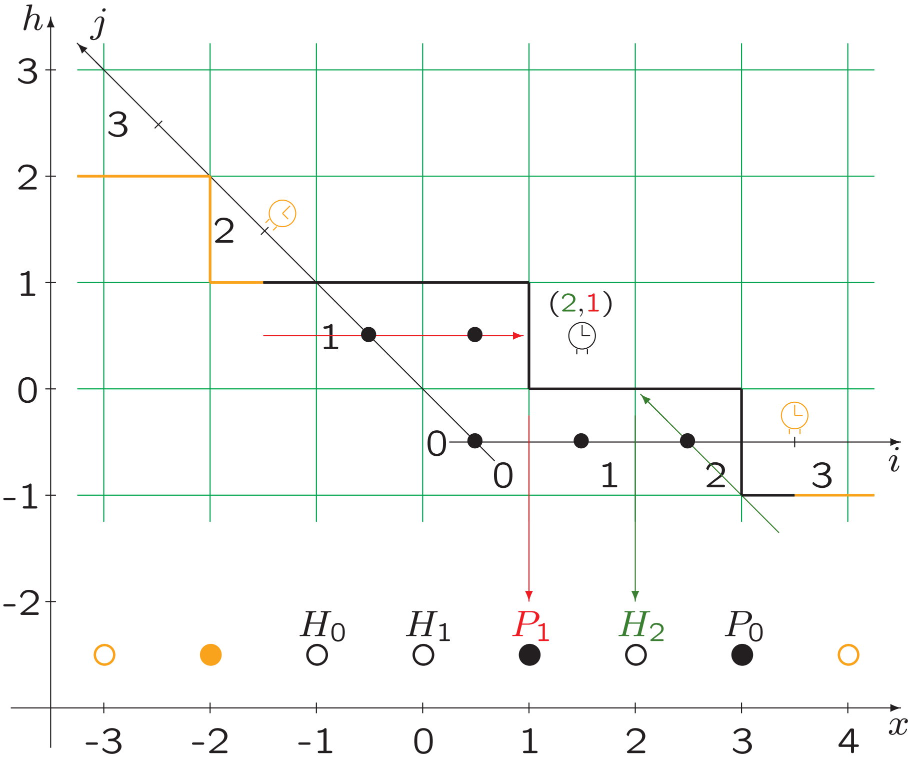

h
j
3
2
1
0
-1
-2
3
2
1
0
0
1
2
3
i
(2,1)
H0
H1
P1
H2
P0
-3
-2
-1
0
1
2
3
4
x



h
j
i
x
3
2
1
0
-1
-2
(2,1)
H_0
H_1
P_1
H_2
P_0
-3
-2
-1
0
1
2
3
4

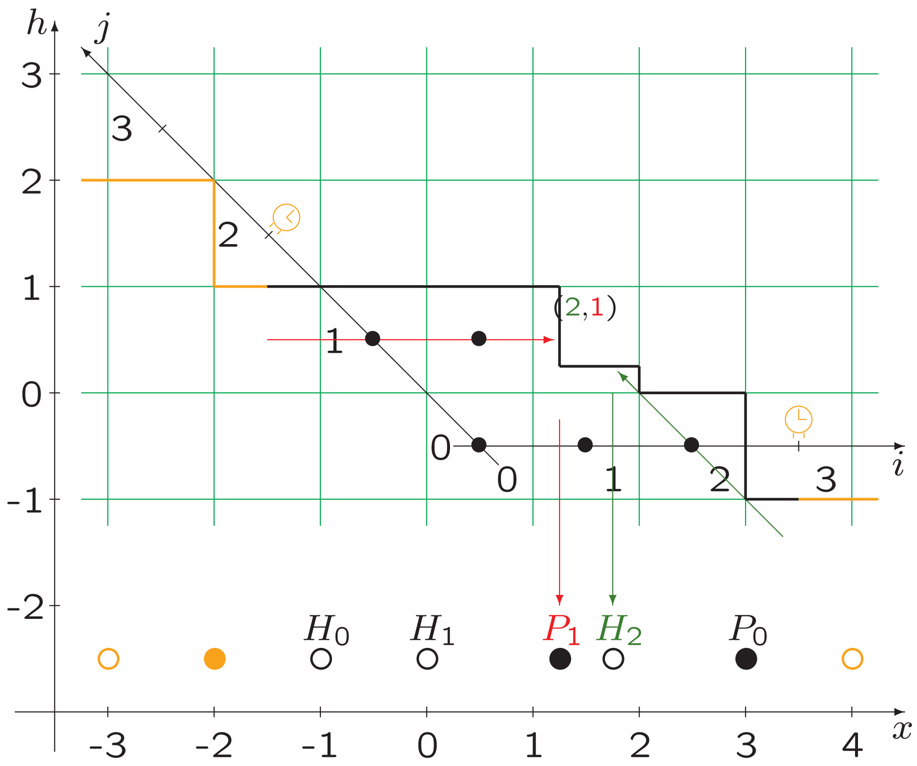


h
j
i
x
(2,1)
H0
H1
P1
H2
P0



Occupation of (i, j) = jump of P_j over H_i.

Occupation of $(2, 1)$ = jump of P_1 over H_2.



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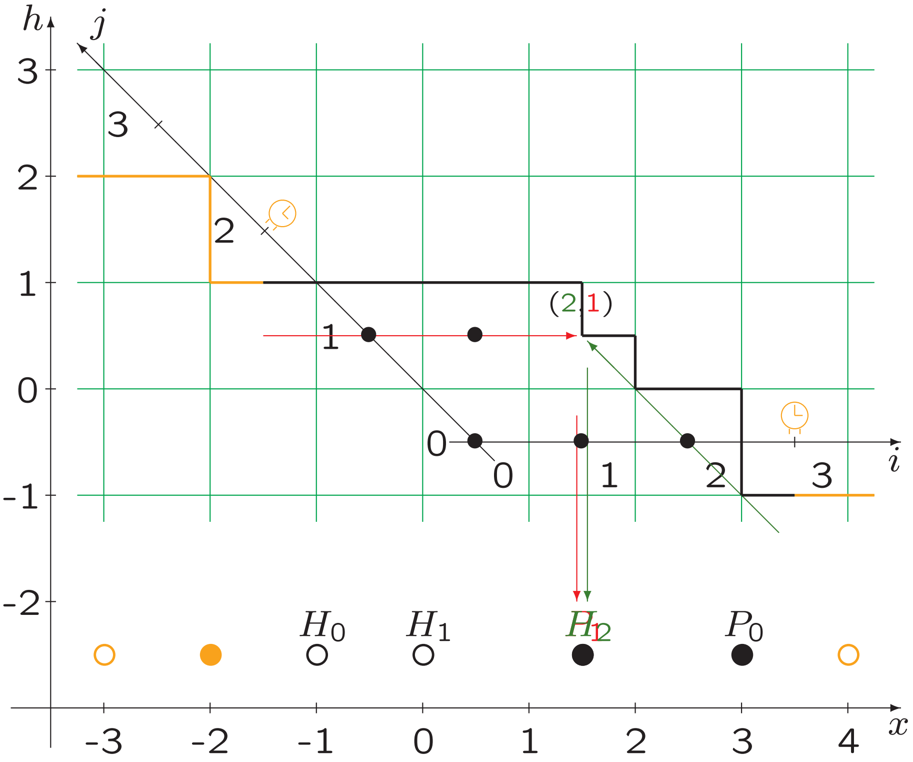

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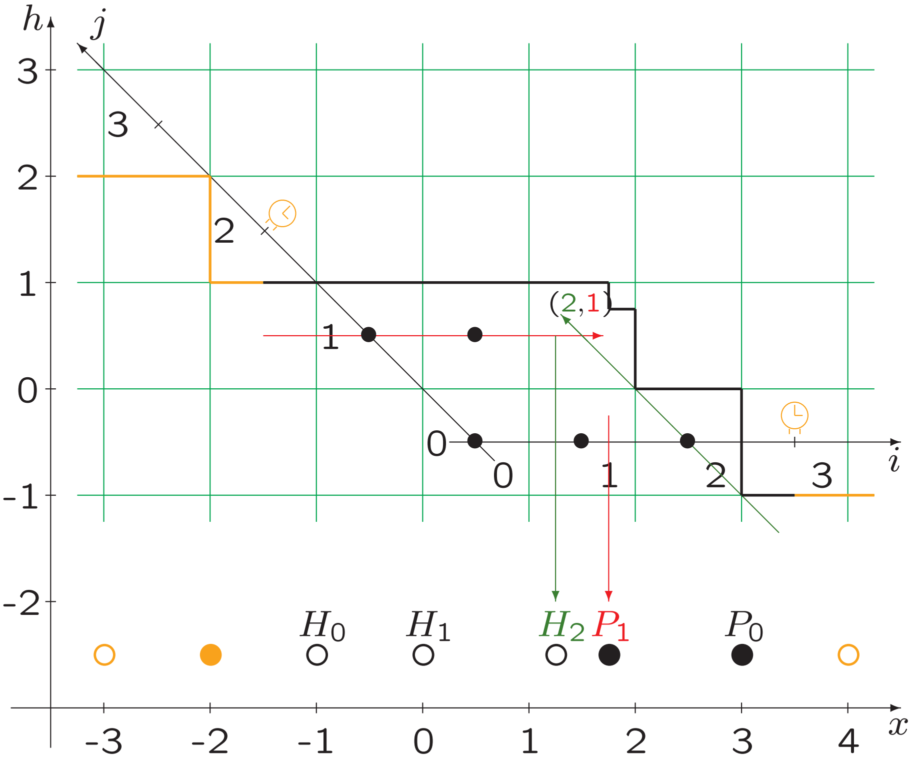
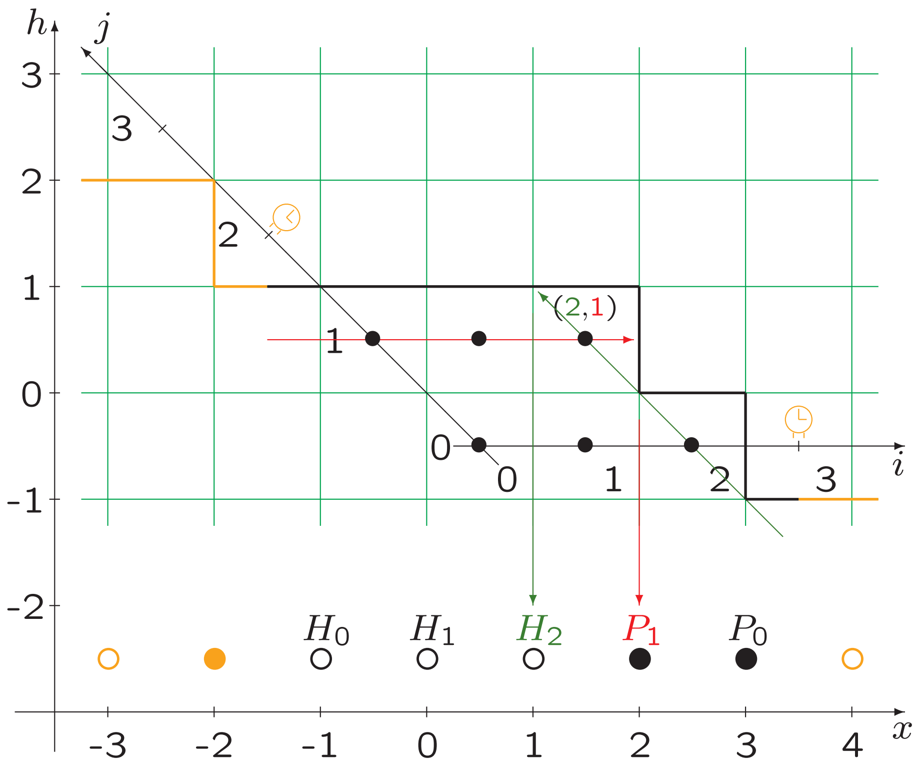


Occupation of (i, j) = jump of P_j over H_i.

Occupation of $(2, 1)$ = jump of P_1 over H_2.



Occupation of (i,j) = jump of P_j over H_i.

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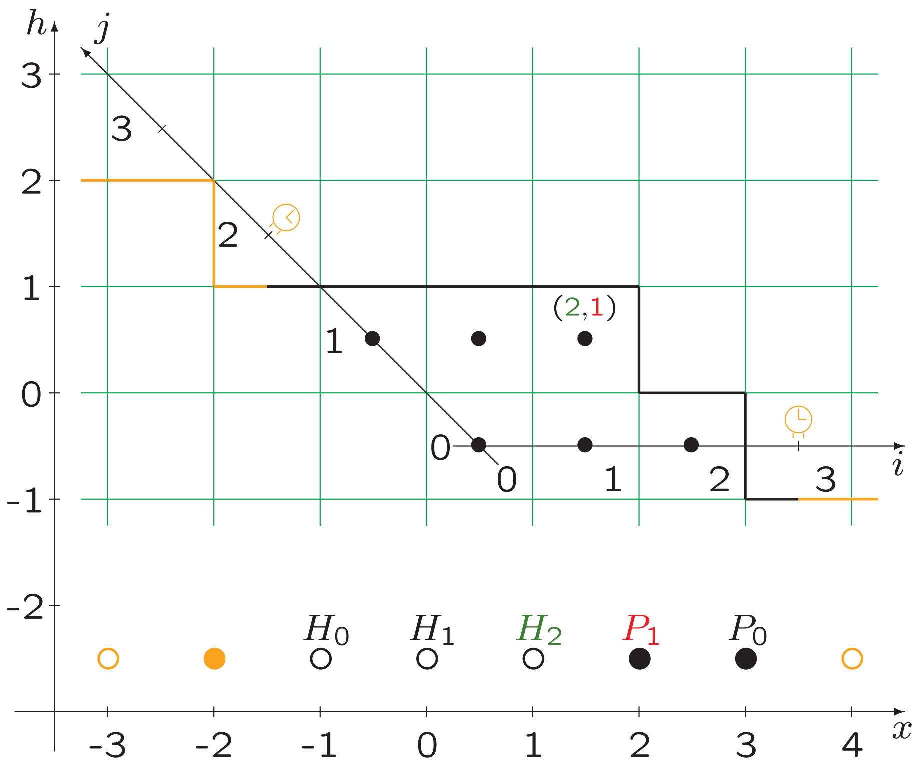
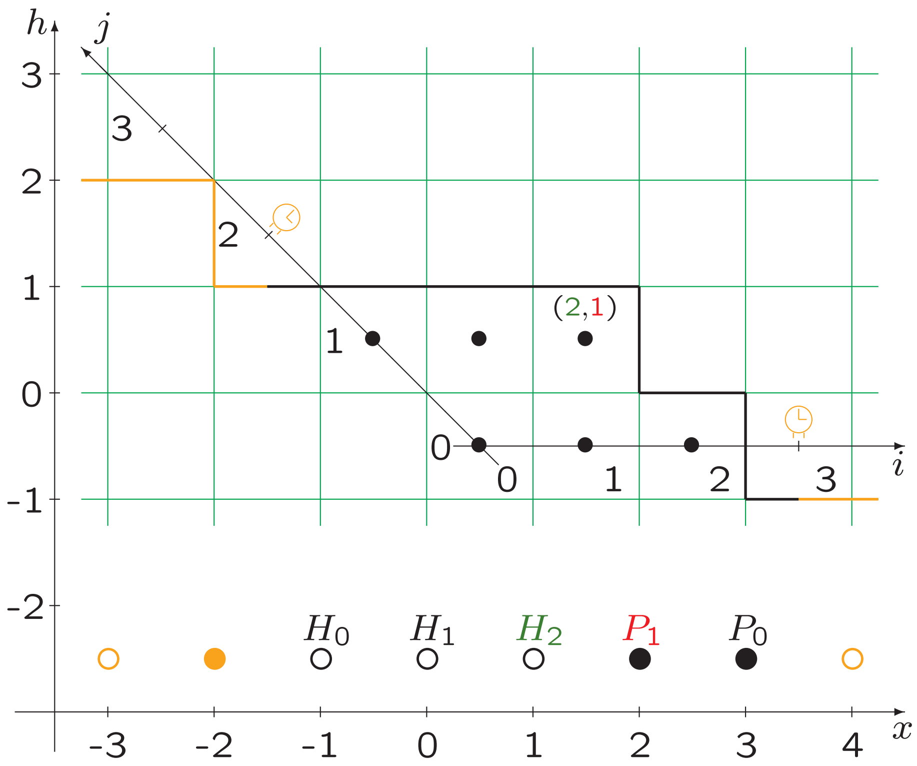


Occupation of (i, j) = jump of P_j over H_i.

Occupation of $(2, 1)$ = jump of P_1 over H_2.

The time when this happens $=: G_{ij}$.



Occupation of (i, j) = jump of P_j over H_i.

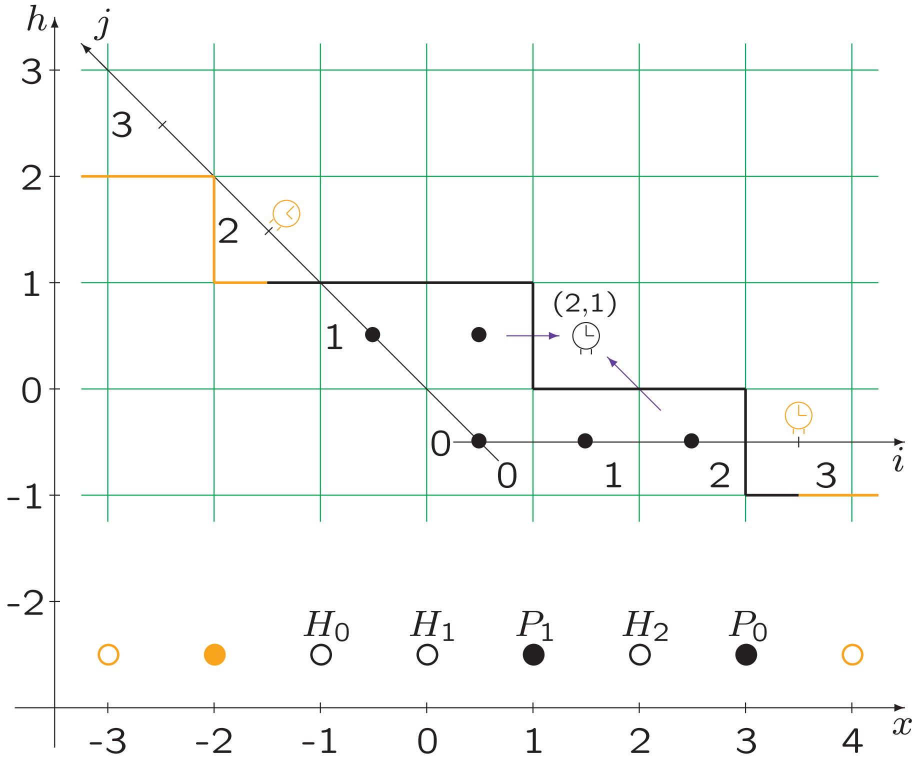
Occupation of $(2, 1)$ = jump of P_1 over H_2.

The time when this happens $=: G_{ij}$.

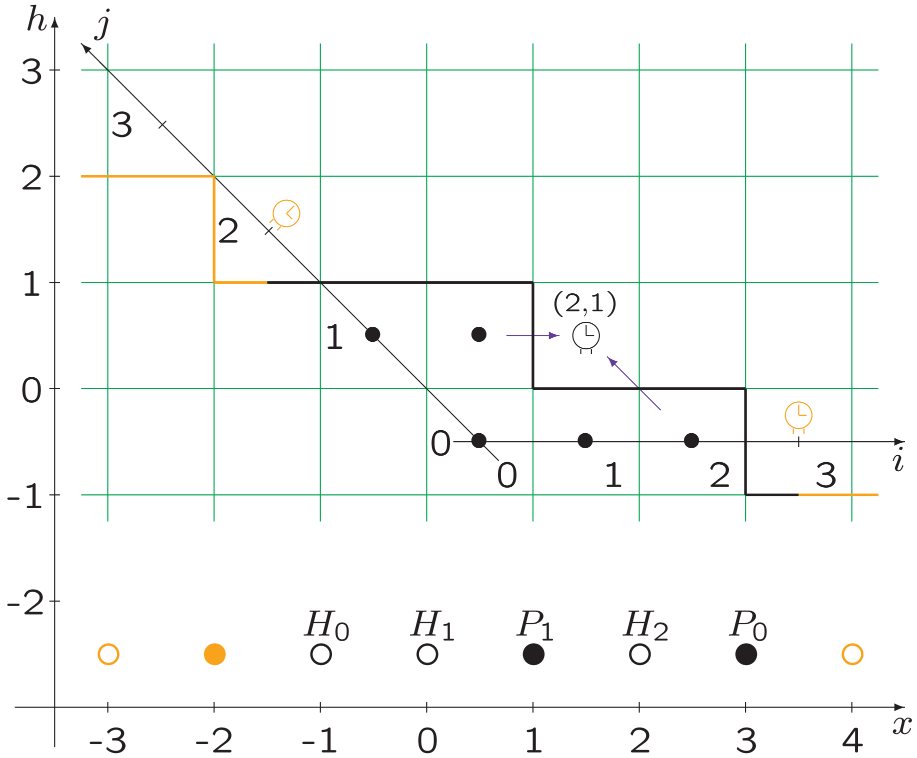
The characteristic speed $V = C(\varrho)$ translates to

$$m := (1 - \varrho)^2 t \text{ and } n := \varrho^2 t.$$

Will present results on G_{mn}.



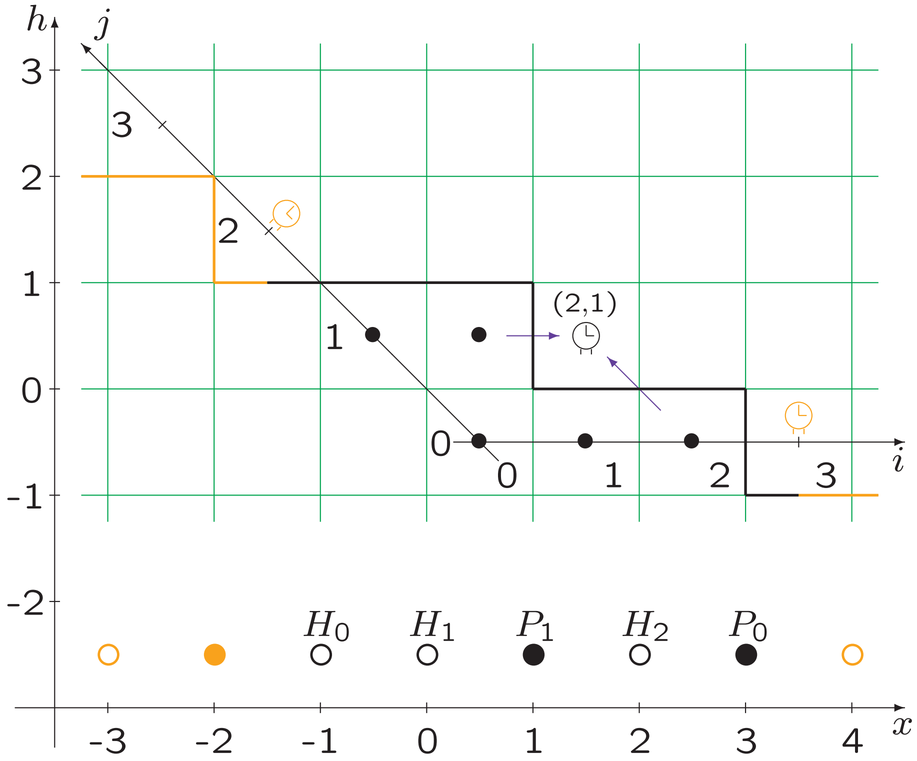
h
j
i
x
(2,1)
H_0
H_1
P_1
H_2
P_0



Burke's Theorem:

P_0 jumps according to a Poisson$(1-\varrho)$ process,

governed by the right orange part



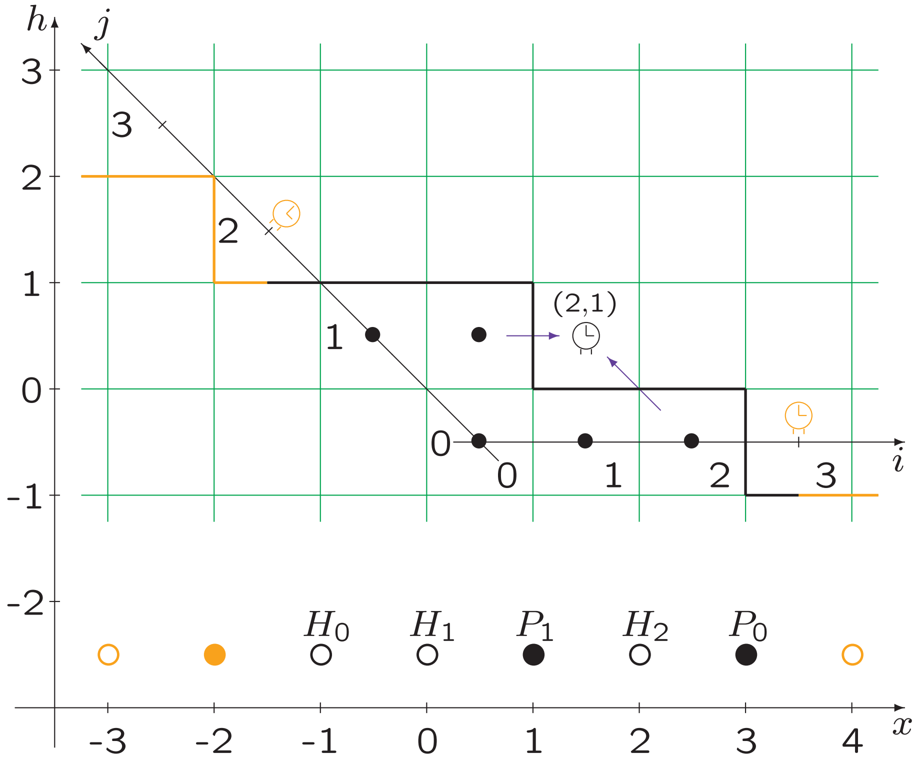
Burke's Theorem:

P_0 jumps according to a Poisson($1-\varrho$) process,

governed by the right orange part

H_0 jumps according to a Poisson(ϱ) process,

governed by the left orange part



Burke's Theorem:

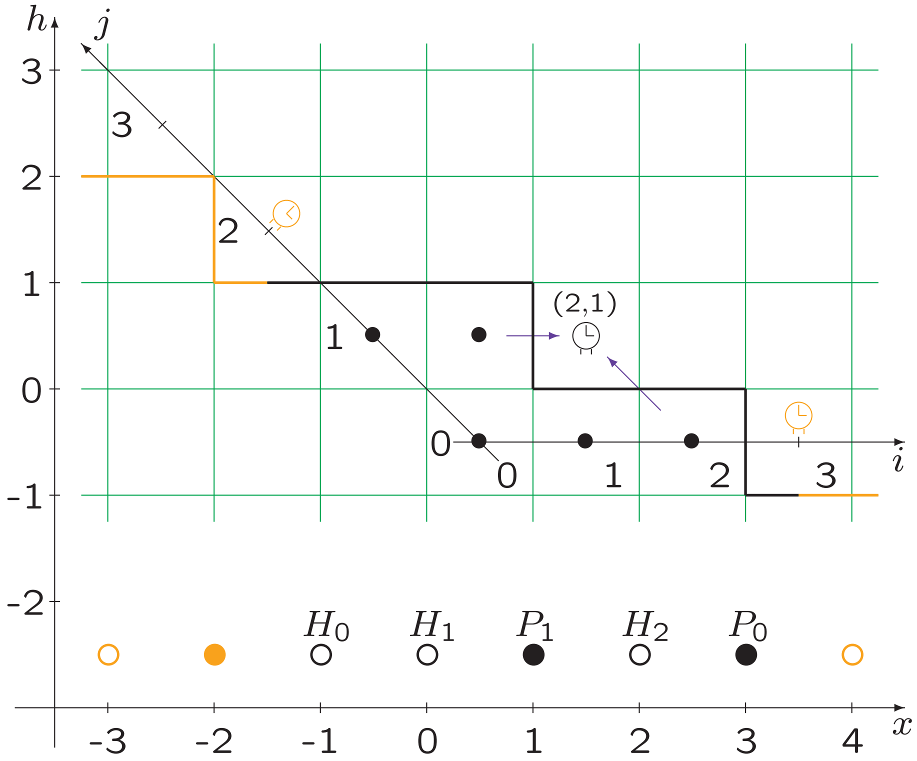
P_0 jumps according to a Poisson$(1-\varrho)$ process,

governed by the right orange part

H_0 jumps according to a Poisson(ϱ) process,

governed by the left orange part

independently of the ⏲'s.



Burke's Theorem:

P_0 jumps according to a Poisson$(1-\varrho)$ process,

governed by the right orange part

H_0 jumps according to a Poisson(ϱ) process,

governed by the left orange part

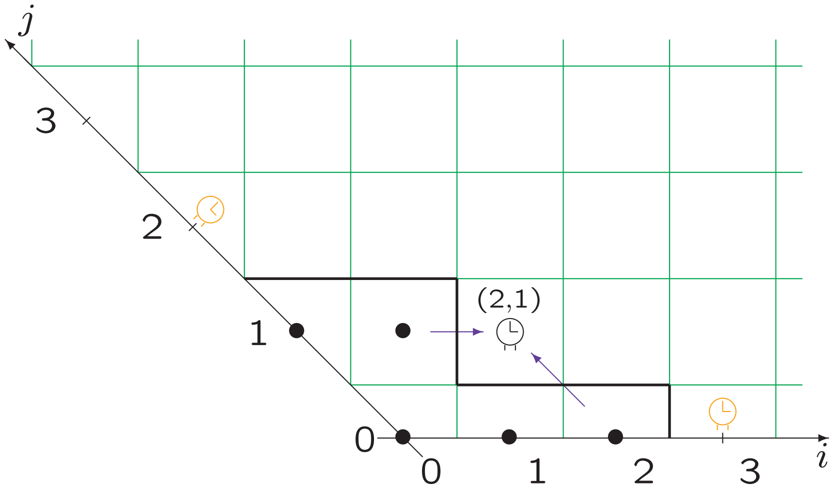
independently of the ◷'s.

Therefore:

◷ ∼ Exponential$(1-\varrho)$

◷ ∼ Exponential(ϱ) } independently

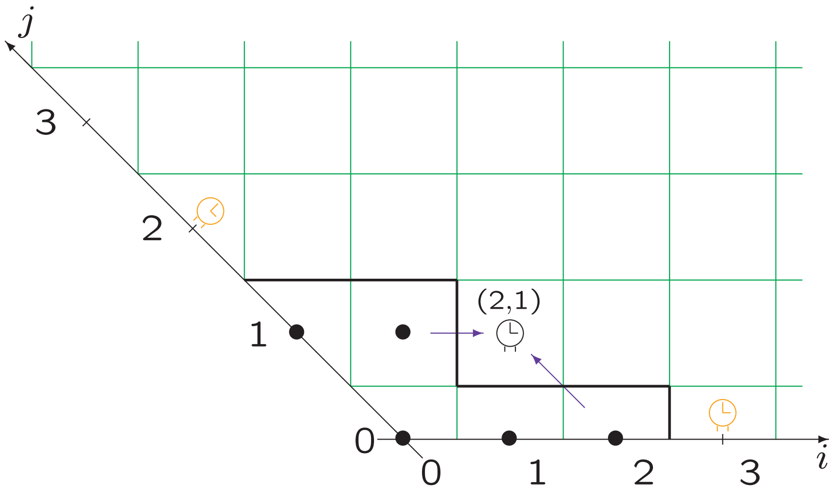
◷ ∼ Exponential(1)



 ∼ Exponential$(1-\varrho)$
 ∼ Exponential(ϱ)
 ∼ Exponential(1)

} independently

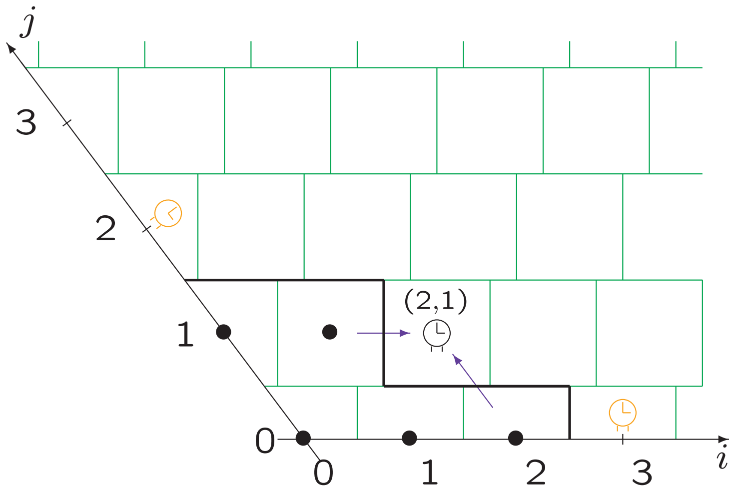
The last passage model



$\sim$ Exponential$(1-\varrho)$
$\sim$ Exponential(ϱ)
$\sim$ Exponential(1)

} independently

The last passage model



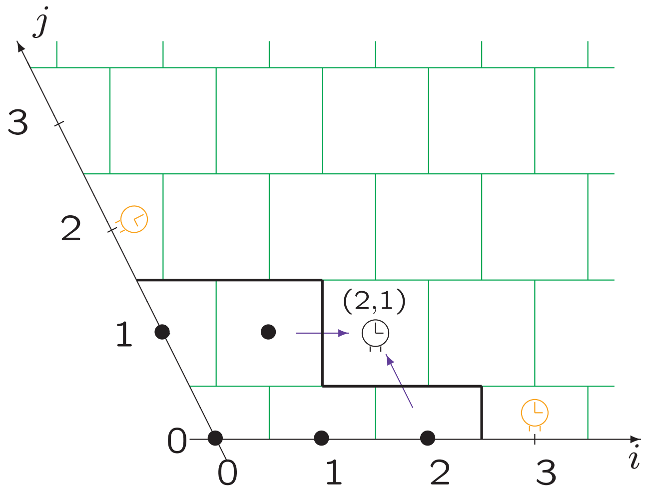
 $\sim$ Exponential$(1-\varrho)$

 $\sim$ Exponential(ϱ)

 $\sim$ Exponential(1)

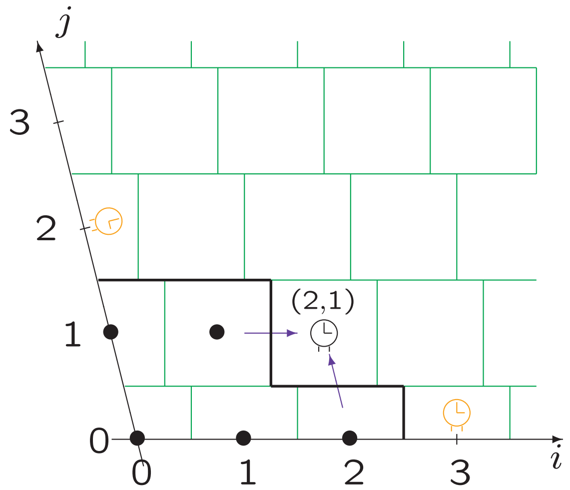
} independently

The last passage model



clock $\sim$ Exponential$(1-\varrho)$
clock $\sim$ Exponential(ϱ) } independently
clock $\sim$ Exponential(1)

The last passage model

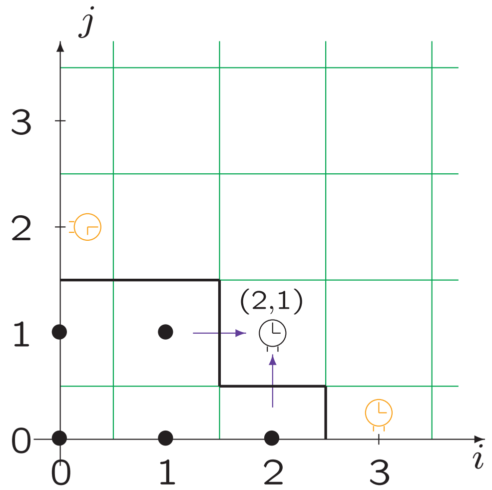


$\sim$ Exponential$(1-\varrho)$

$\sim$ Exponential(ϱ) } independently

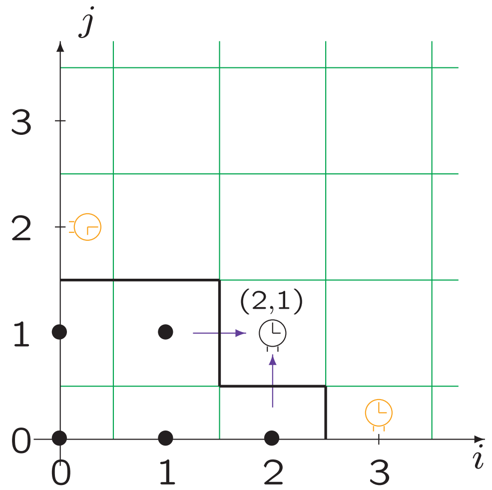
$\sim$ Exponential(1)

The last passage model



$$\left.\begin{array}{l} \text{(orange clock)} \sim \text{Exponential}(1-\varrho) \\ \text{(orange clock)} \sim \text{Exponential}(\varrho) \\ \text{(clock)} \sim \text{Exponential}(1) \end{array}\right\} \text{independently}$$

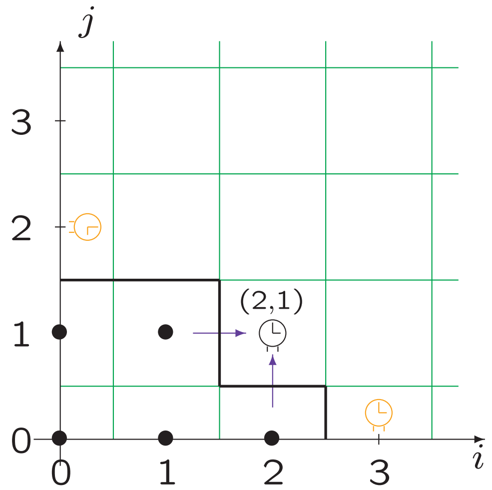
The last passage model



$$\left.\begin{array}{l} \sim \text{Exponential}(1-\varrho) \\ \sim \text{Exponential}(\varrho) \\ \sim \text{Exponential}(1) \end{array}\right\} \text{independently}$$

starts ticking when its west neighbor becomes occupied

The last passage model



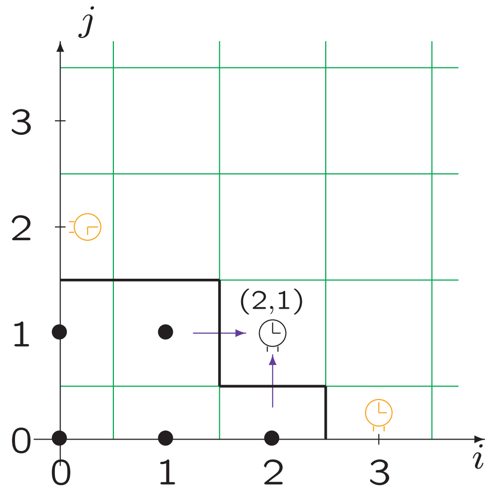
$$\sim \text{Exponential}(1-\varrho)$$
$$\sim \text{Exponential}(\varrho)$$
$$\sim \text{Exponential}(1)$$

independently

starts ticking when its west neighbor becomes occupied

starts ticking when its south neighbor becomes occupied

The last passage model



$\sim$ Exponential($1-\varrho$)

$\sim$ Exponential(ϱ) independently

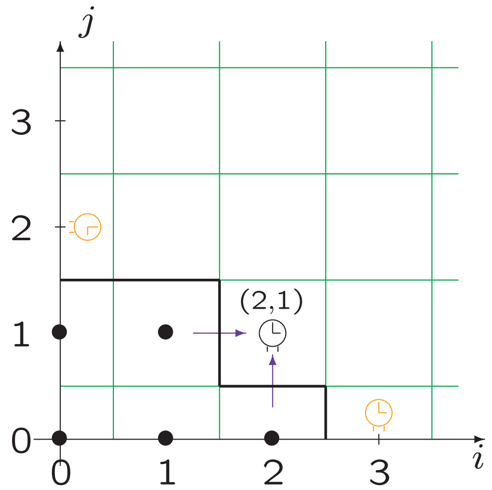
$\sim$ Exponential(1)

starts ticking when its west neighbor becomes occupied

starts ticking when its south neighbor becomes occupied

starts ticking when both its west and south neighbors become occupied

The last passage model



M. Prähofer and H. Spohn 2002

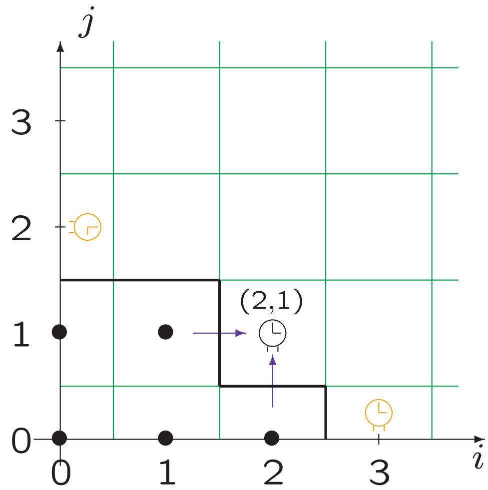
$\sim$ Exponential($1-\varrho$)
$\sim$ Exponential(ϱ)
$\sim$ Exponential(1)
independently

starts ticking when its west neighbor becomes occupied

starts ticking when its south neighbor becomes occupied

starts ticking when both its west and south neighbors become occupied

The last passage model



M. Prähofer and H. Spohn 2002

$$\left.\begin{array}{l} \text{(clock)} \sim \text{Exponential}(1-\varrho) \\ \text{(clock)} \sim \text{Exponential}(\varrho) \\ \text{(clock)} \sim \text{Exponential}(1) \end{array}\right\} \text{independently}$$

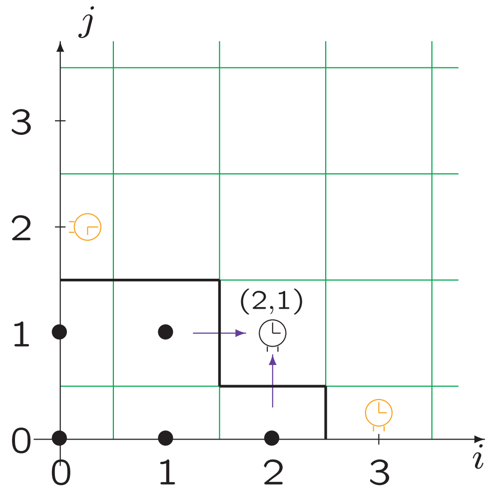
(clock) starts ticking when its west neighbor becomes occupied

(clock) starts ticking when its south neighbor becomes occupied

(clock) starts ticking when both its west and south neighbors become occupied

G_{ij} = the occupation time of (i, j)

The last passage model



M. Prähofer and H. Spohn 2002

$\sim$ Exponential$(1-\varrho)$
$\sim$ Exponential(ϱ)
$\sim$ Exponential(1)
} independently

starts ticking when its west neighbor becomes occupied

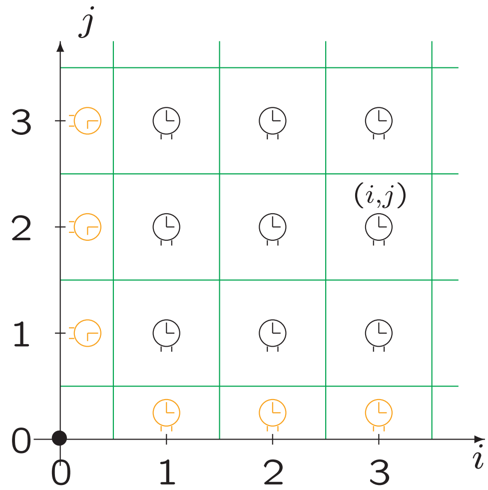
starts ticking when its south neighbor becomes occupied

starts ticking when both its west and south neighbors become occupied

G_{ij} = the occupation time of (i,j)

G_{ij} = the maximum weight collected by a north-east path from $(0,0)$ to (i,j).

The last passage model



M. Prähofer and H. Spohn 2002

(orange clock, west boundary) $\sim$ Exponential($1-\varrho$)
(orange clock, south boundary) $\sim$ Exponential(ϱ)
(clock) $\sim$ Exponential(1)
} independently

(orange clock) starts ticking when its west neighbor becomes occupied

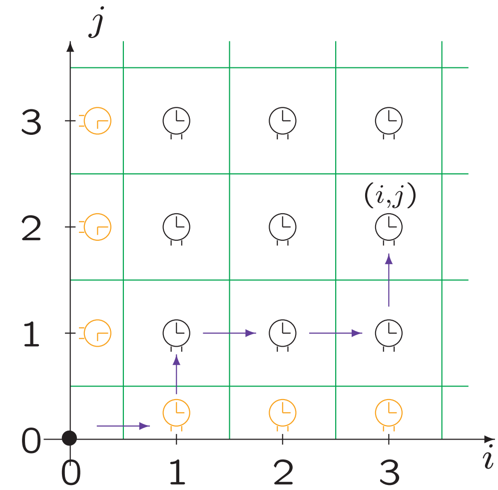
(orange clock) starts ticking when its south neighbor becomes occupied

(clock) starts ticking when both its west and south neighbors become occupied

G_{ij} = the occupation time of (i, j)

G_{ij} = the maximum weight collected by a north-east path from $(0, 0)$ to (i, j).

The last passage model



M. Prähofer and H. Spohn 2002

$\sim$ Exponential$(1-\varrho)$
$\sim$ Exponential(ϱ)
$\sim$ Exponential(1)
independently

starts ticking when its west neighbor becomes occupied

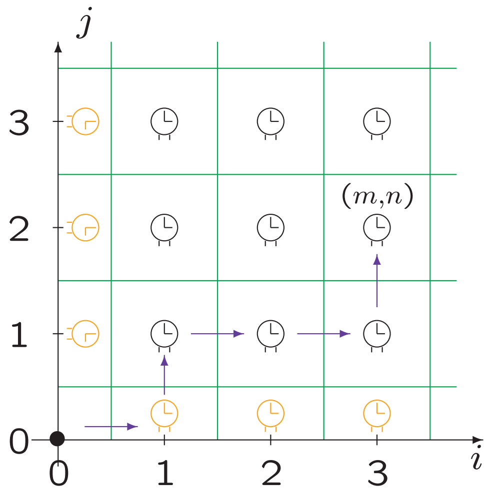
starts ticking when its south neighbor becomes occupied

starts ticking when both its west and south neighbors become occupied

G_{ij} = the occupation time of (i,j)

G_{ij} = the maximum weight collected by a north-east path from $(0,0)$ to (i,j).

5. Results



On the characteristics

$$m := (1-\varrho)^2 t \text{ and } n := \varrho^2 t,$$

Theorem:

$$0 < \liminf_{t\to\infty} \frac{\mathbf{Var}(G_{mn})}{t^{2/3}} \le \limsup_{t\to\infty} \frac{\mathbf{Var}(G_{mn})}{t^{2/3}} < \infty.$$

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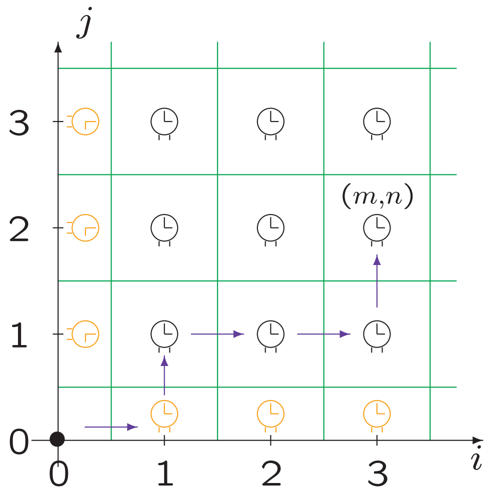

On the characteristics

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Johansson (2000) identifies the limiting distribution of $\widetilde{h}_{Vt}(t)/t^{1/3}$ in terms of Tracy-Widom GUE distributions, when  and  $\sim$ Exponential(1) (rarefaction fan).

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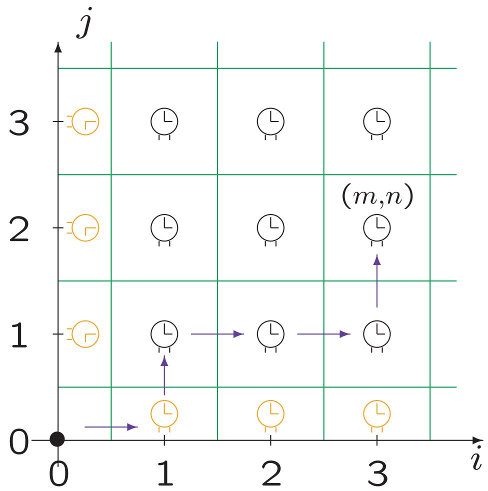

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P. L. Ferrari and H. Spohn (2005) identify the limiting distribution of $h_x(s) - \mathbf{E}[h_{C(\varrho)t}(t)]$ when x and s are off characteristics by $t^{2/3}$ and $t^{1/3}$, respectively.

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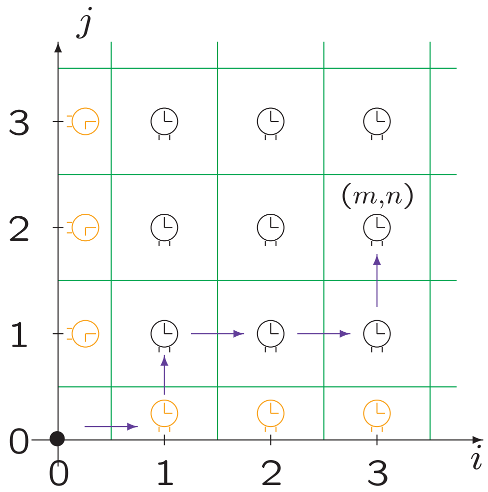

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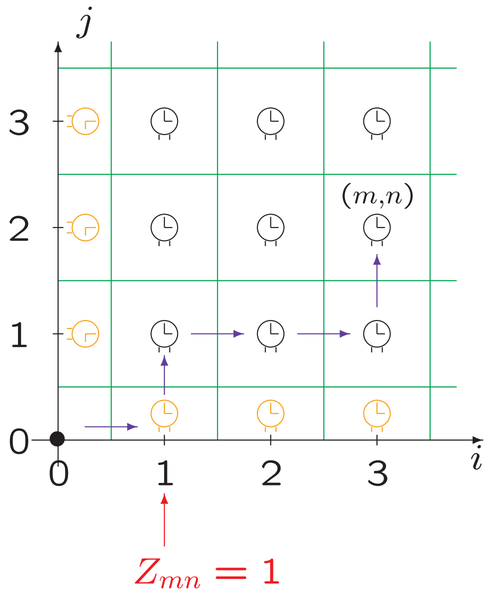
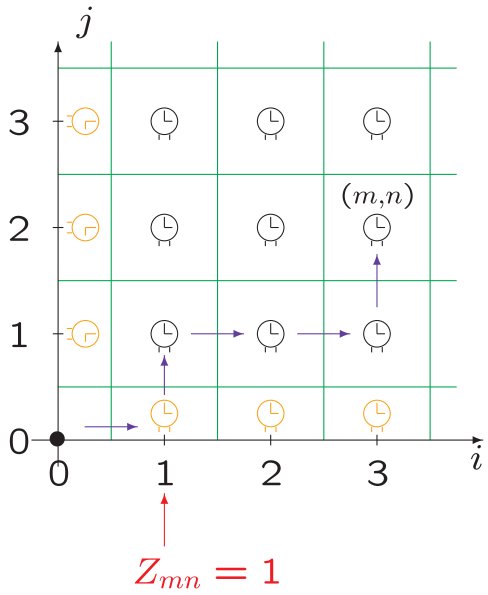
P. L. Ferrari and H. Spohn (2005) identify the limiting distribution of $h_x(s) - \mathbf{E}[h_{C(\varrho)t}(t)]$ when x and s are off characteristics by $t^{2/3}$ and $t^{1/3}$, respectively.

Their method: RSK correspondence, random matrices.



Z_{mn} is the exit point of the longest path to

$$(m,\ n) = ((1-\varrho)^2 t,\ \varrho^2 t).$$



Z_{mn} is the exit point of the longest path to

$$(m,\ n) = ((1-\varrho)^2 t,\ \varrho^2 t).$$

Theorem:

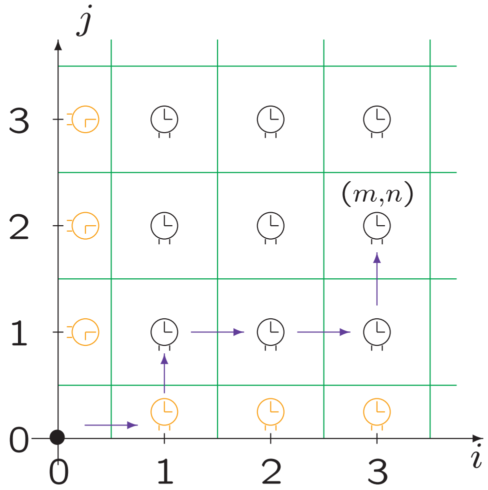
For all large t and all $a > 0$,

$$\mathbf{P}\{Z_{mn} \geq a t^{2/3}\} \leq C a^{-3}.$$

Given $\varepsilon > 0$, there is a $\delta > 0$ such that

$$\mathbf{P}\{1 \leq Z_{mn} \leq \delta t^{2/3}\} \leq \varepsilon$$

for all large t.



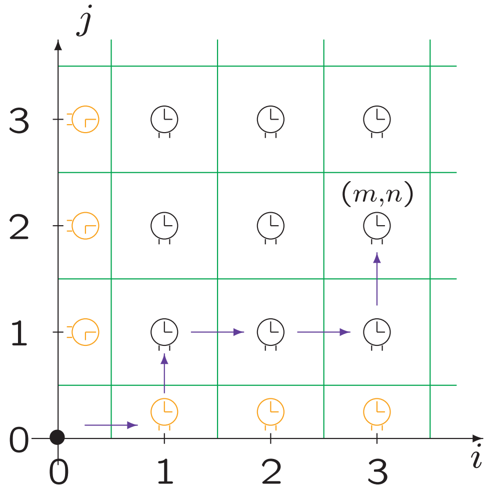
<u>Equilibrium</u>:

 $\sim$ Exponential$(1-\varrho)$

 $\sim$ Exponential(ϱ)

 $\sim$ Exponential(1)

} independently

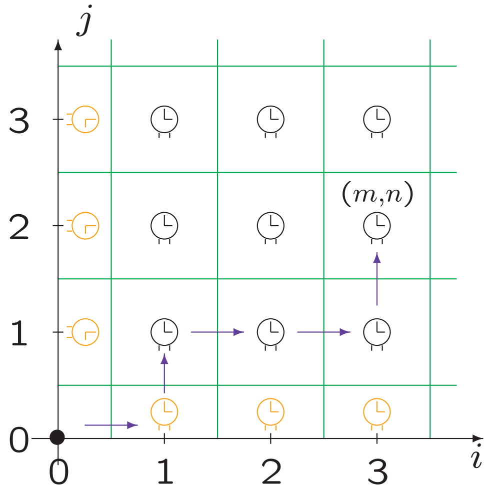


Equilibrium:

$$\left.\begin{array}{l} \sim \text{Exponential}(1-\varrho) \\ \sim \text{Exponential}(\varrho) \\ \sim \text{Exponential}(1) \end{array}\right\} \text{independently}$$

Rarefaction fan:

$$\left.\begin{array}{l} < \text{Exponential}(1-\varrho) \\ < \text{Exponential}(\varrho) \\ \sim \text{Exponential}(1) \end{array}\right\} \text{independently}$$



Equilibrium:

$\sim$ Exponential($1-\varrho$)
$\sim$ Exponential(ϱ)
$\sim$ Exponential(1)
} independently

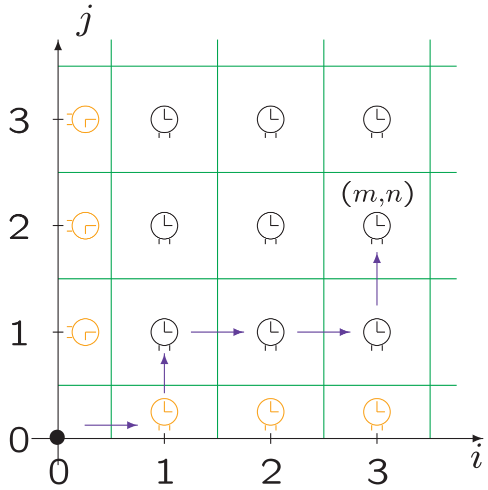
Rarefaction fan:

$<$ Exponential($1-\varrho$)
$<$ Exponential(ϱ)
$\sim$ Exponential(1)
} independently

Theorem:

For $0 < \alpha < 1$ and all $t > 1$,

$$\mathbf{P}\{|G_{mn} - t| > at^{1/3}\} \le Ca^{-3\alpha/2}.$$



Equilibrium:

 $\sim$ Exponential$(1 - \varrho)$

 $\sim$ Exponential(ϱ) independently

 $\sim$ Exponential(1)

Rarefaction fan:

 $<$ Exponential$(1 - \varrho)$

 $<$ Exponential(ϱ) independently

 $\sim$ Exponential(1)

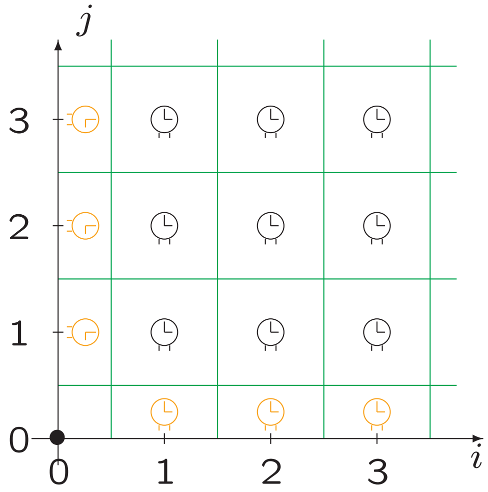
Theorem:

For $0 < \alpha < 1$ and all $t > 1$,

$$\mathbf{P}\{|G_{mn} - t| > at^{1/3}\} \leq Ca^{-3\alpha/2}.$$

Also transversal $t^{2/3}$-deviations of the longest path.

6. Last passage equilibrium



Equilibrium:

 $\sim$ Exponential($1-\varrho$)

 $\sim$ Exponential(ϱ)

 $\sim$ Exponential(1)

} independently

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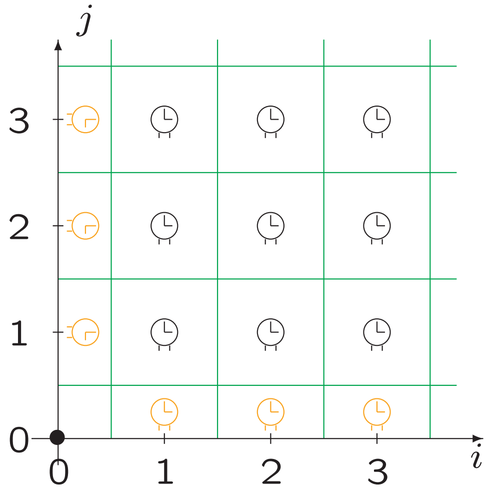

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G-increments:

$$I_{ij} := G_{ij} - G_{\{i-1\}j} \quad \text{for } i \geq 1,\ j \geq 0, \qquad \text{and}$$

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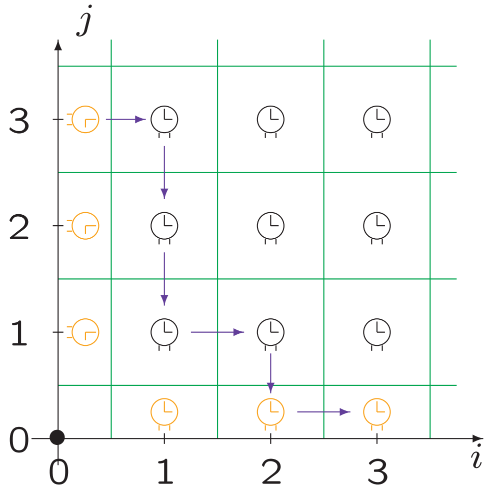

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$\rightsquigarrow$ Any fixed southeast path meets *independent* increments

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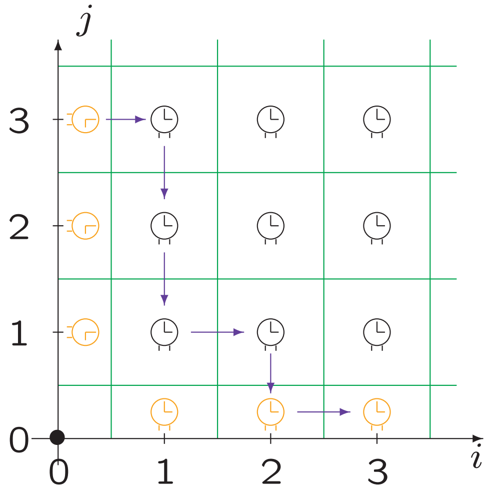

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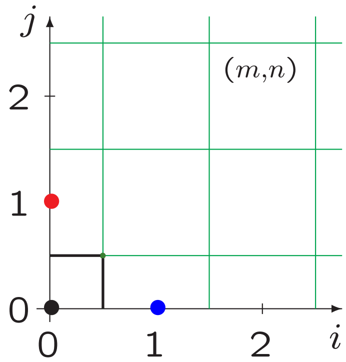
Of course, this doesn't help directly with G_{mn}.

7. The competition interface



Ferrari, Martin, Pimentel (2005)

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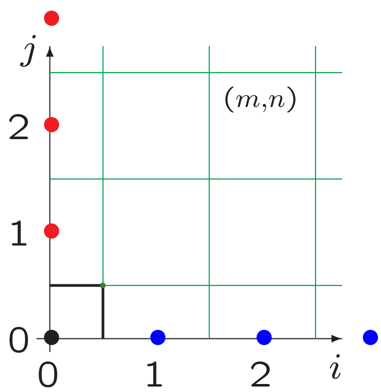

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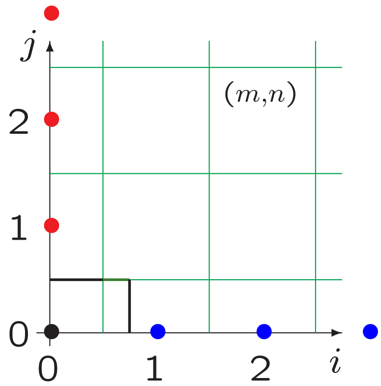

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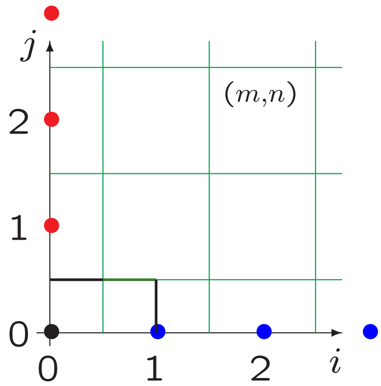

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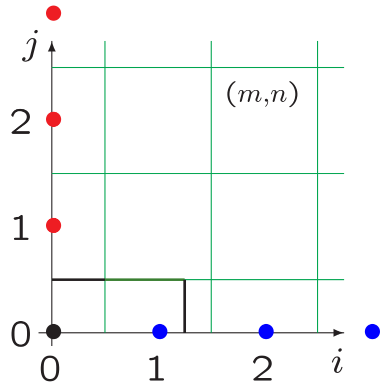

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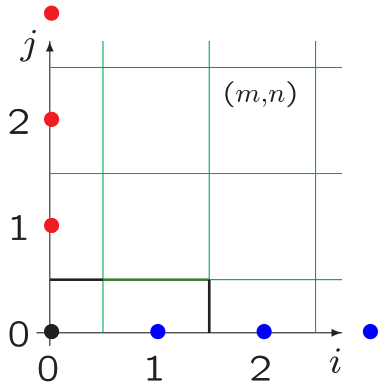

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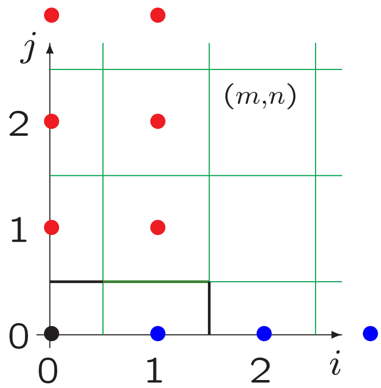

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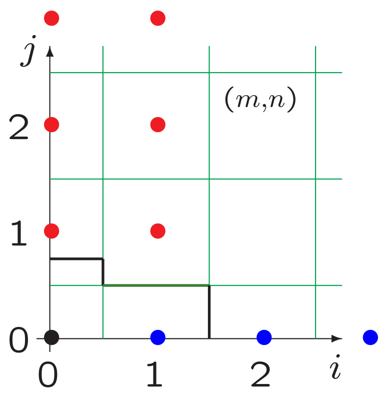

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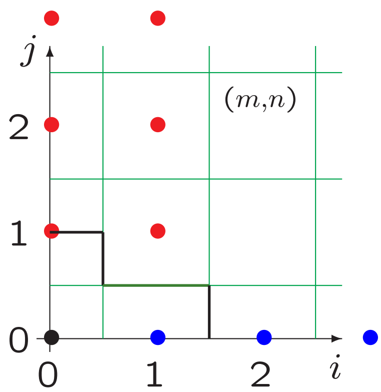

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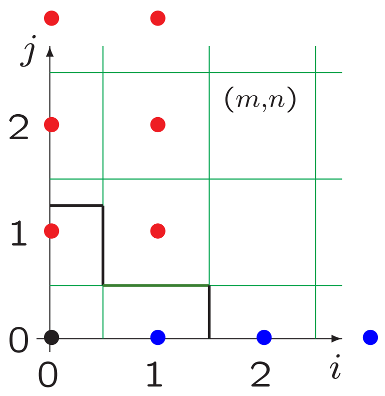

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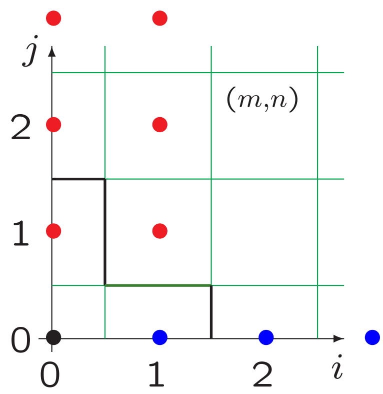

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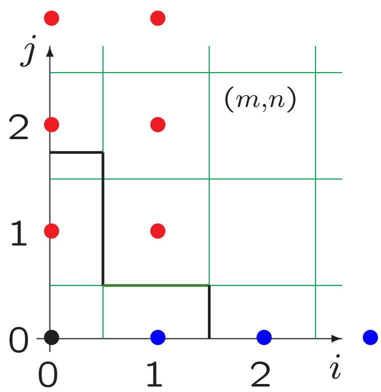

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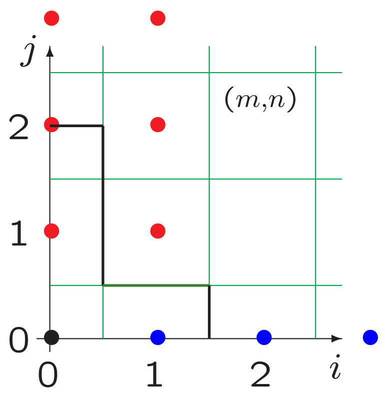

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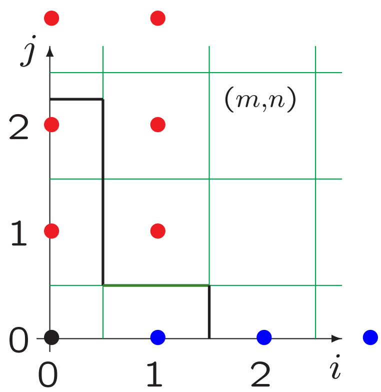

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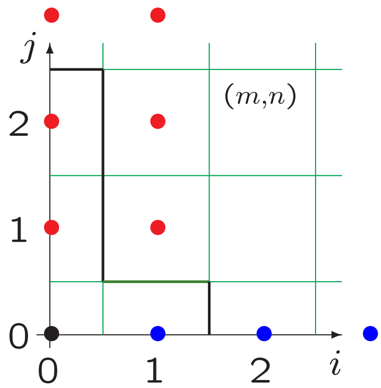

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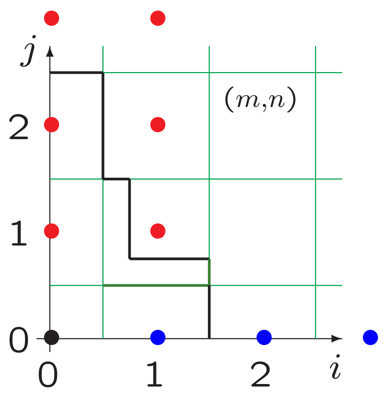

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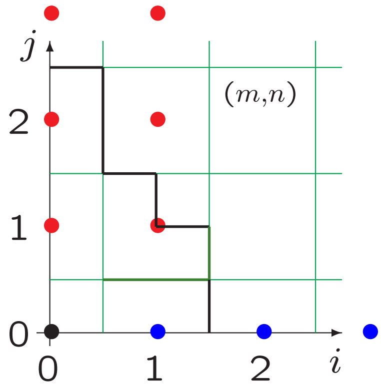

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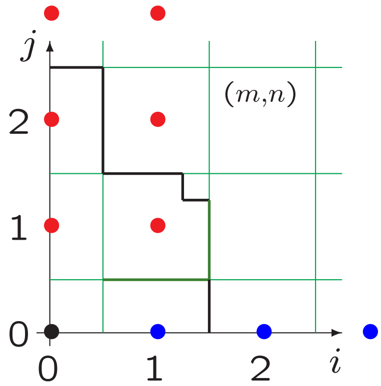

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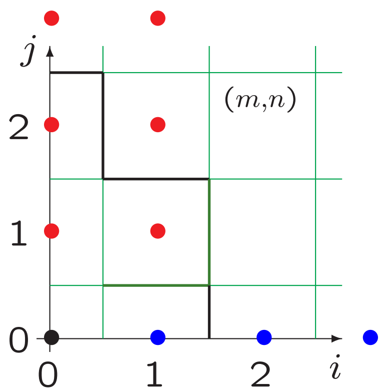

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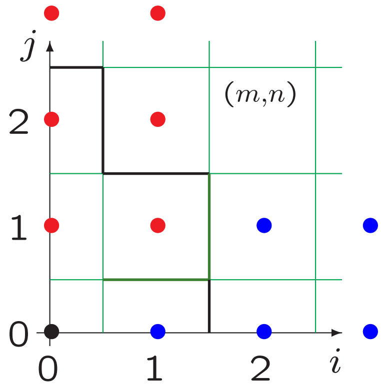

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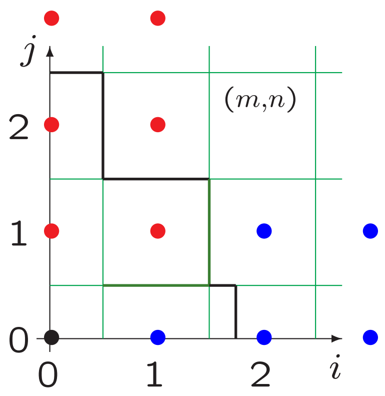

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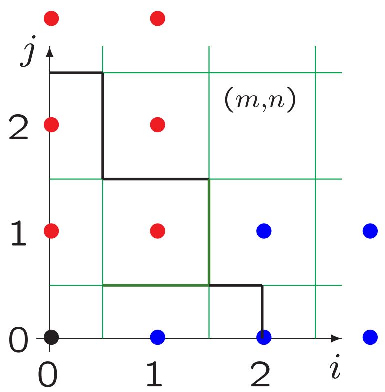

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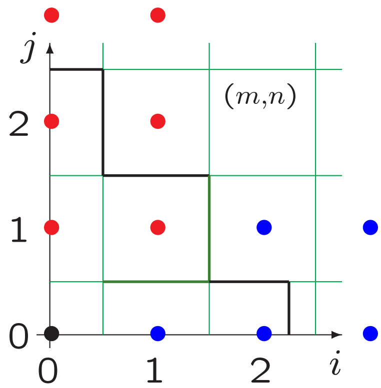

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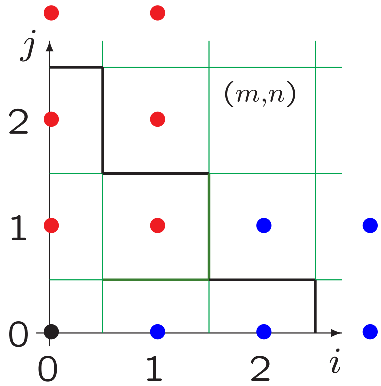

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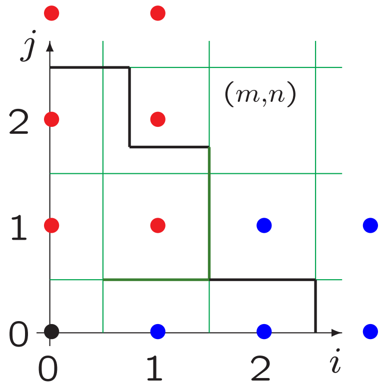

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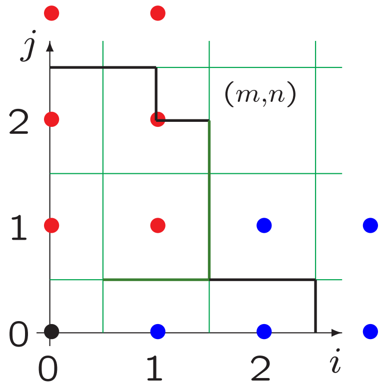

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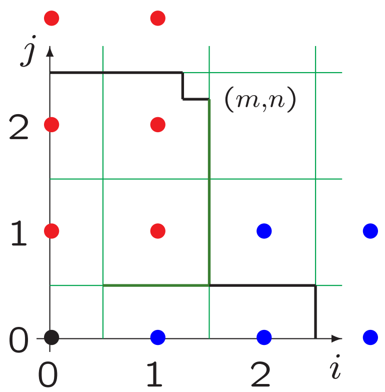

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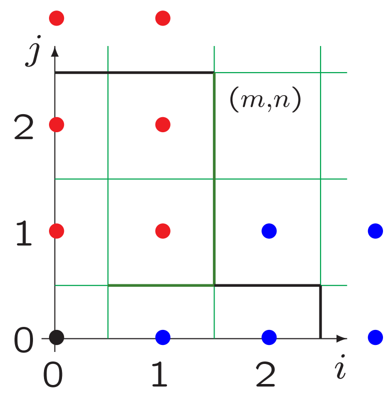

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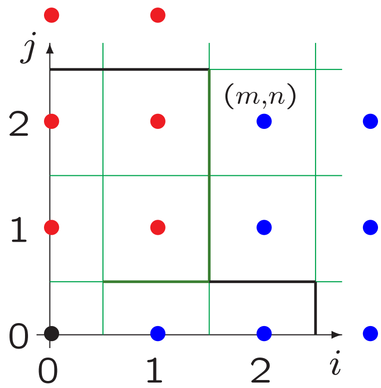

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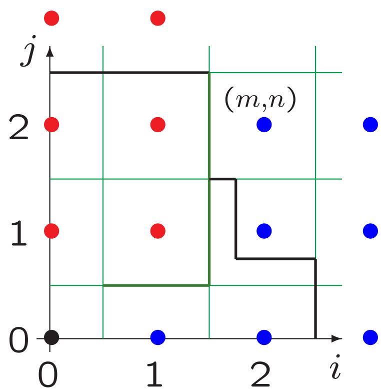

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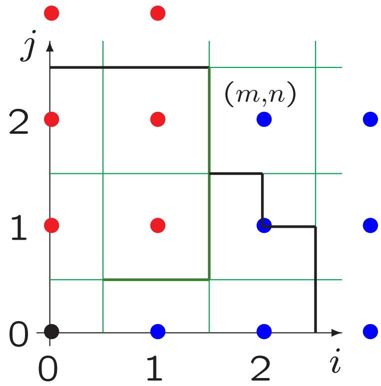

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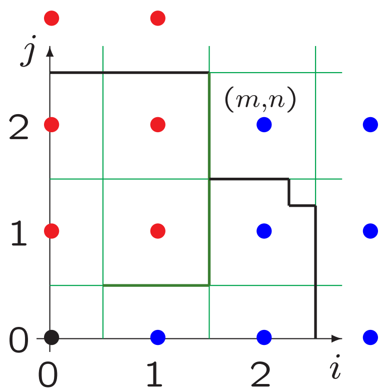

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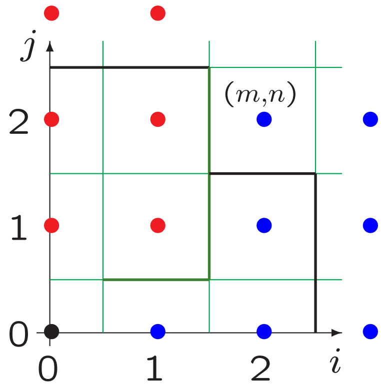

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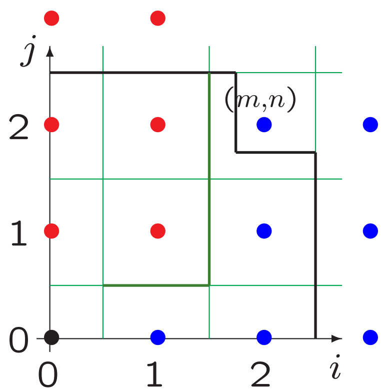

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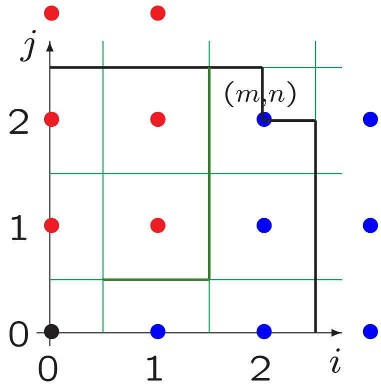

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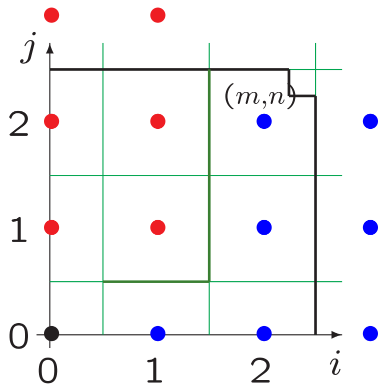

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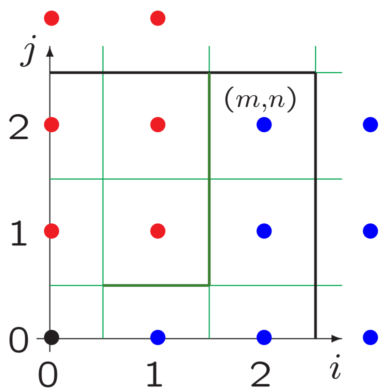

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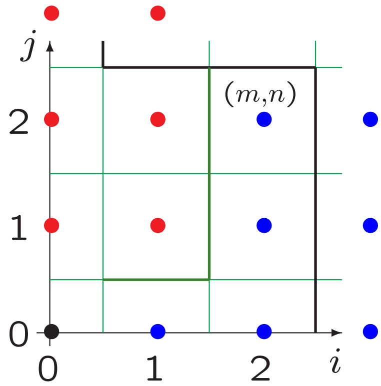

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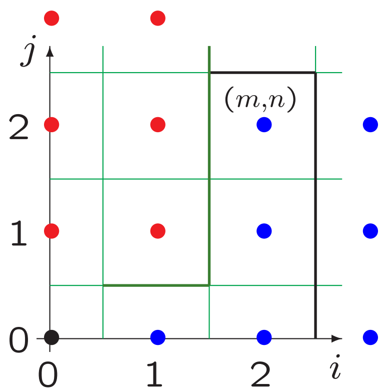

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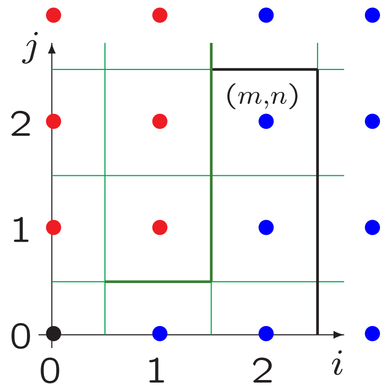

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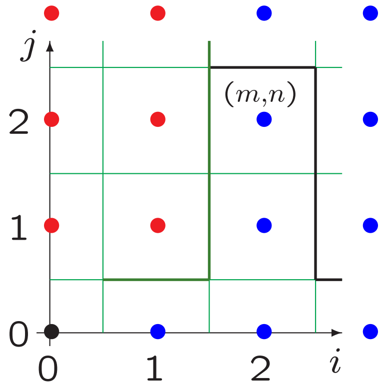

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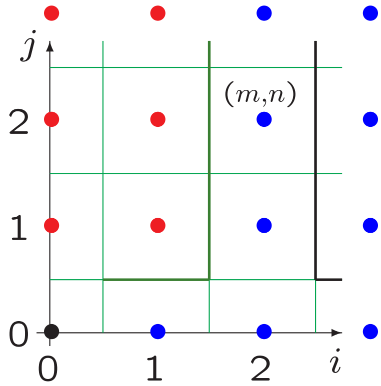

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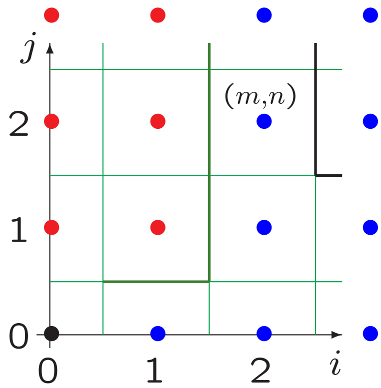

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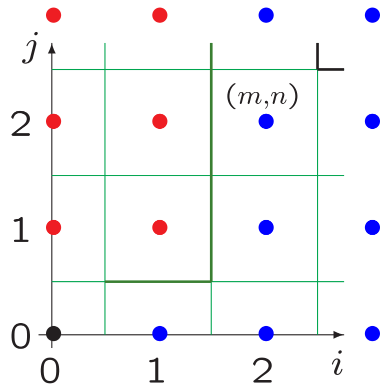

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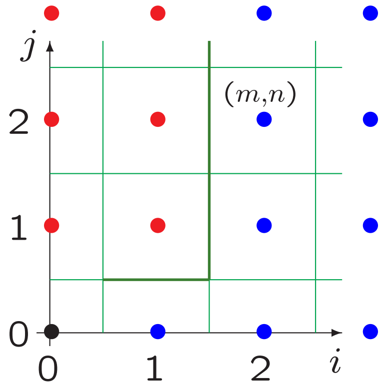

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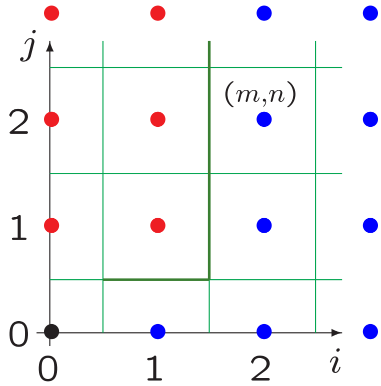

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The competition interface follows the same rules as the *second class particle* of simple exclusion.

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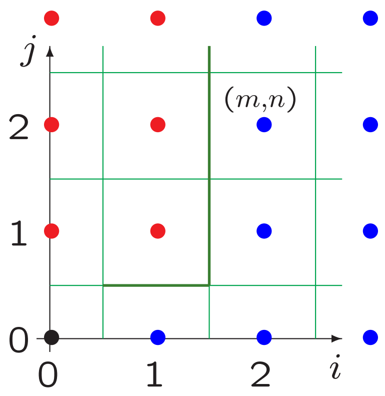

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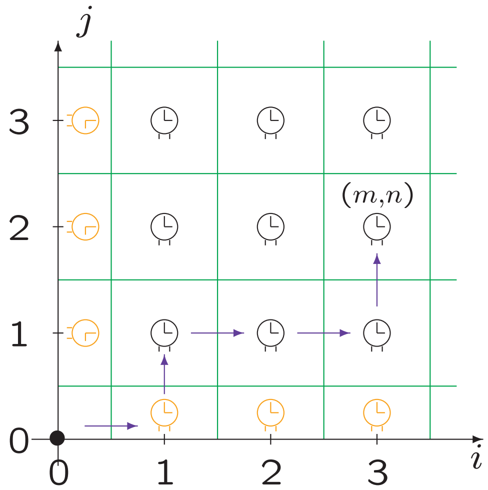
The competition interface follows the same rules as the *second class particle* of simple exclusion.

If it passes left of (m,n), then G_{mn} is not sensitive to decreasing the weights on the j-axis.

If it passes below (m,n), then G_{mn} is not sensitive to decreasing the weights on the i-axis.

8. Upper bound (E. Cator and P. Groeneboom)



G^{ϱ}: weight collected by the longest path.

Z^{ϱ}: exit point of the longest path.

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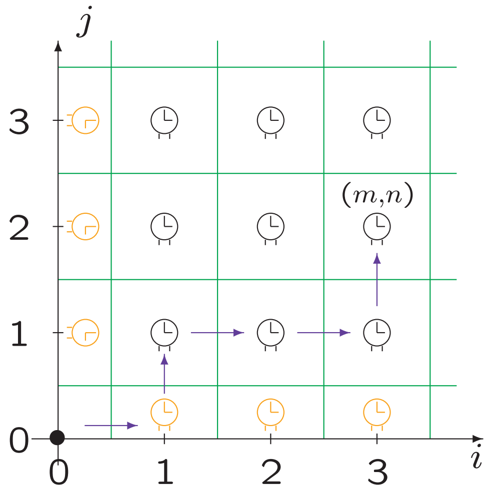

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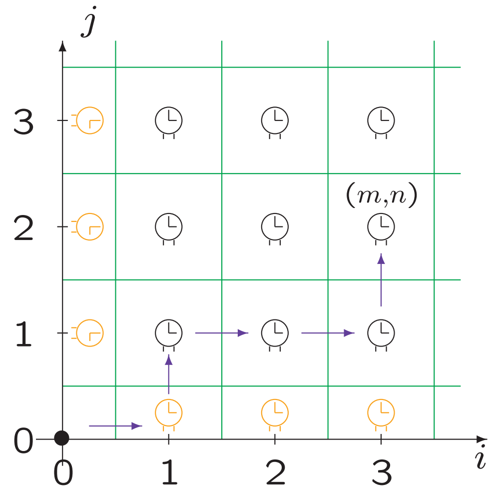

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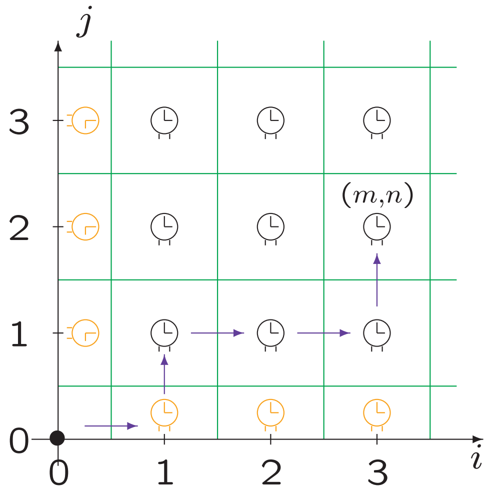

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for any z, any $0 < \lambda < 1$.

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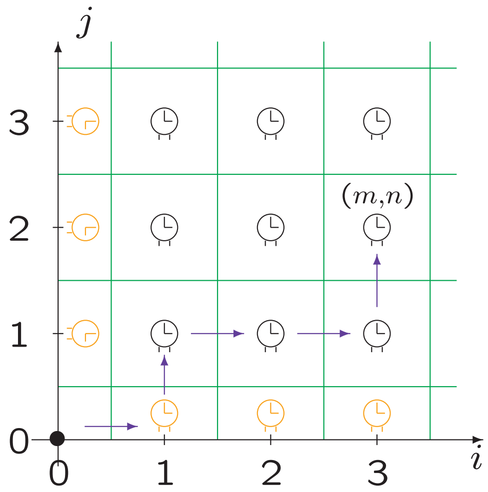

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$$\mathbf{P}\{Z^\varrho > u\} = \mathbf{P}\{\exists z > u \,:\, U_z^\varrho + A_z(t) = G^\varrho\}$$

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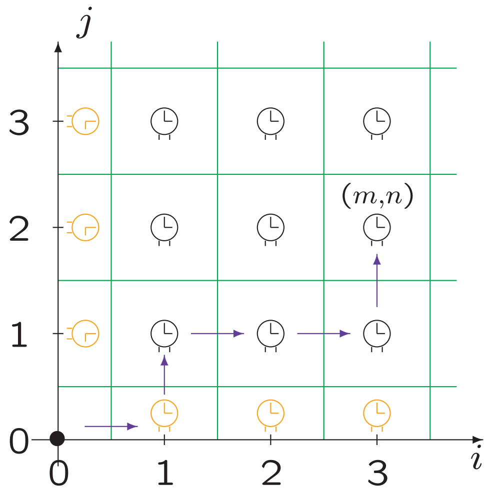

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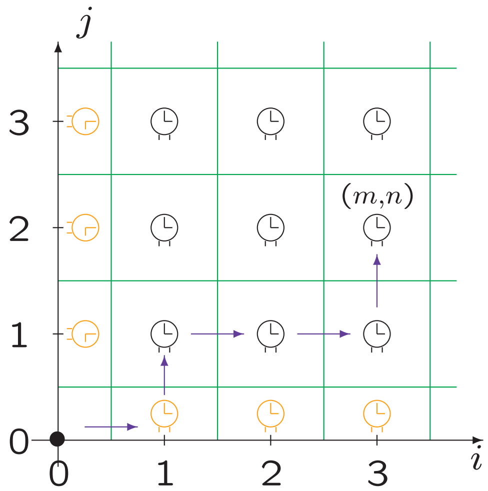

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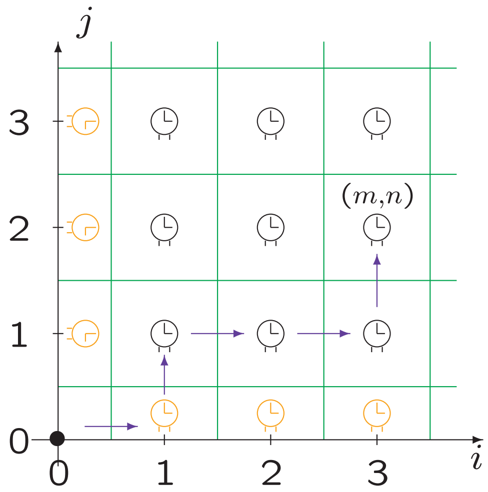

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Prove, by a perturbation argument, that **Var**(G) is related to $\mathbf{E}(U_{Z+})$.

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A large deviation estimate connects $\mathbf{P}\{Z^{\varrho} > y\}$ and $\mathbf{P}\{U^{\varrho}_{Z^{\varrho}+} > y\}$.

$$\rightsquigarrow \mathbf{P}\{U^{\varrho}_{Z+} > y\} \le C\Big(\frac{t^2}{y^4} \cdot \mathbf{E}(U^{\varrho}_{Z^{\varrho}+}) + \frac{t^2}{y^3}\Big)$$

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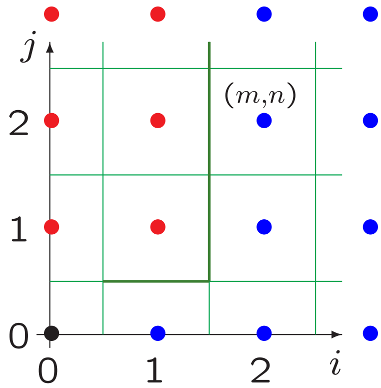
Conclude

$$\limsup_{t\to\infty} \frac{\mathbf{E}(U^\varrho_{Z^\varrho+})}{t^{2/3}} < \infty, \qquad \limsup_{t\to\infty} \frac{\mathbf{Var}(G^\varrho)}{t^{2/3}} < \infty.$$

9. Time-reversal and the lower bound

(E. Cator and P. Groeneboom)



⇝ Z-probabilities are connected to competition interface-probabilities.

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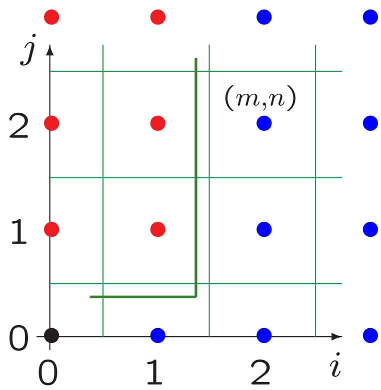

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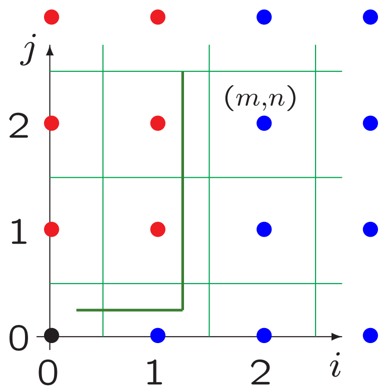

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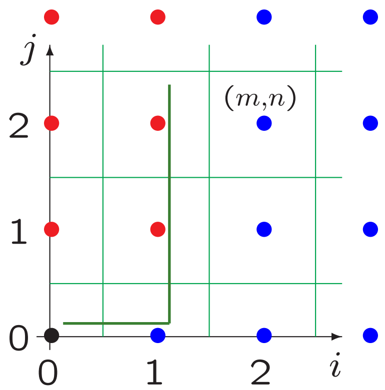

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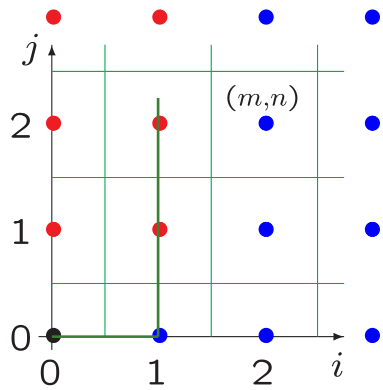

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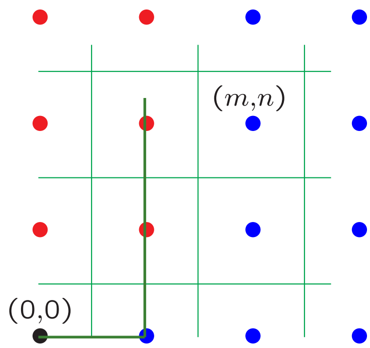

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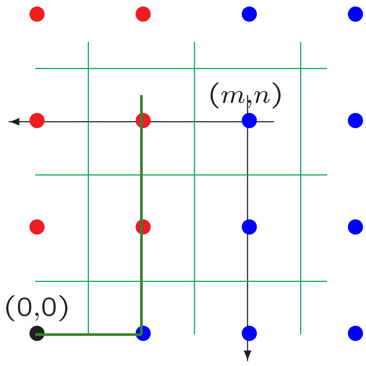

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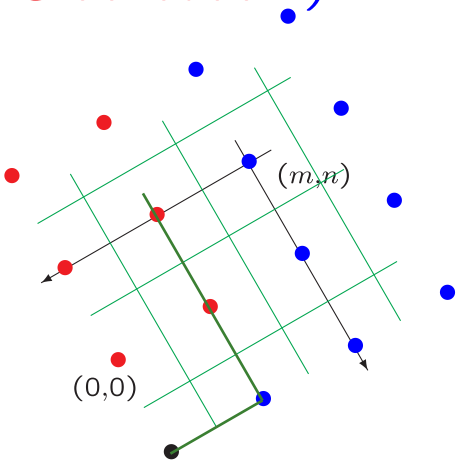

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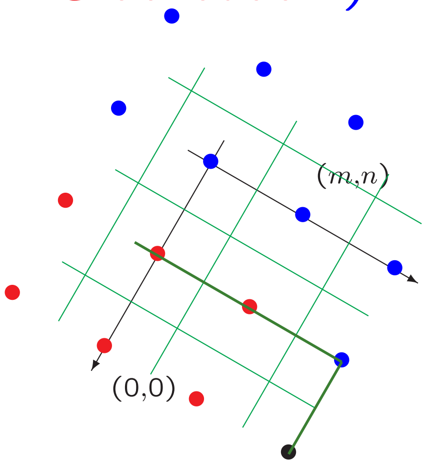

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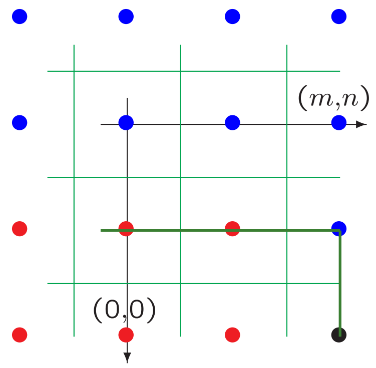

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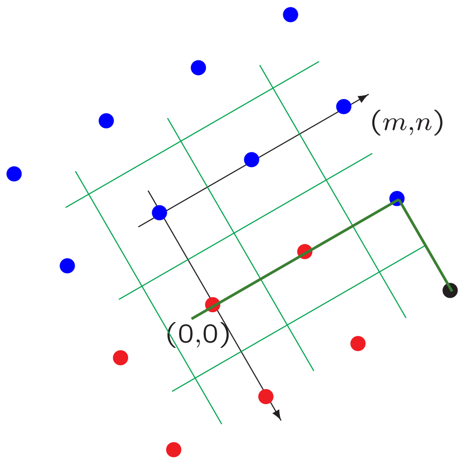

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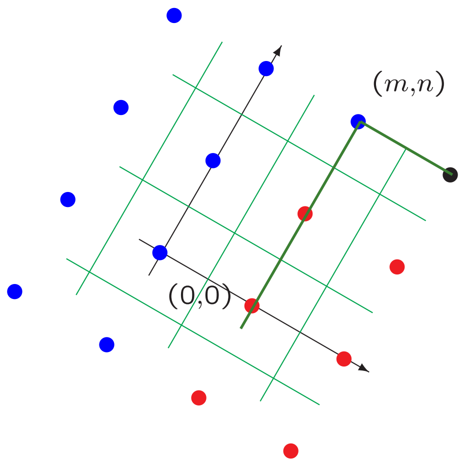

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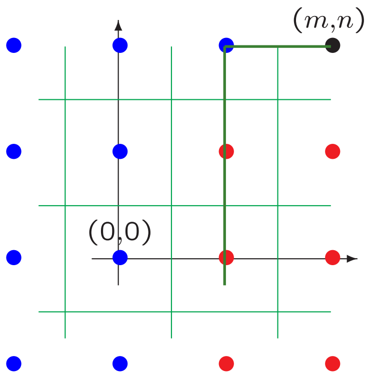

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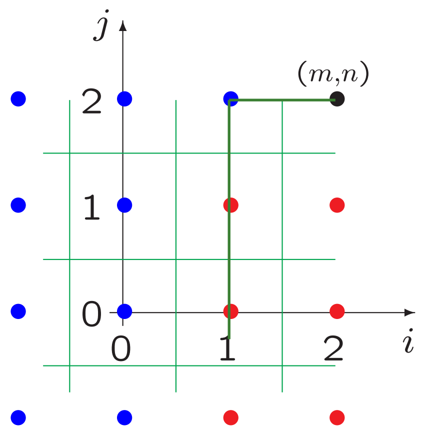

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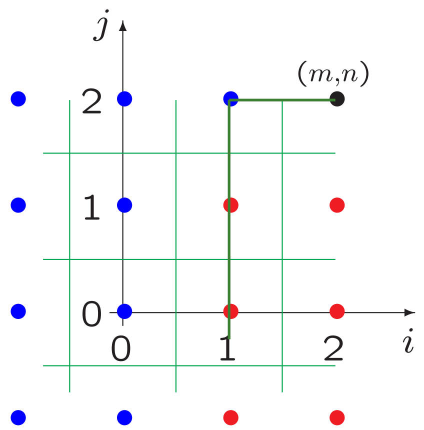

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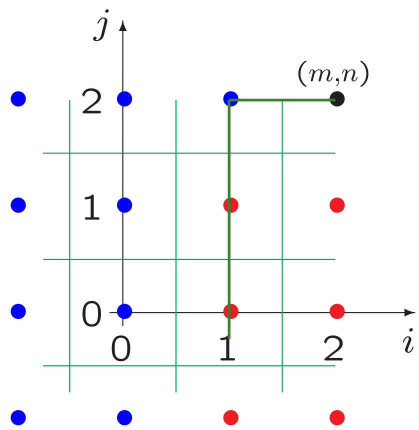

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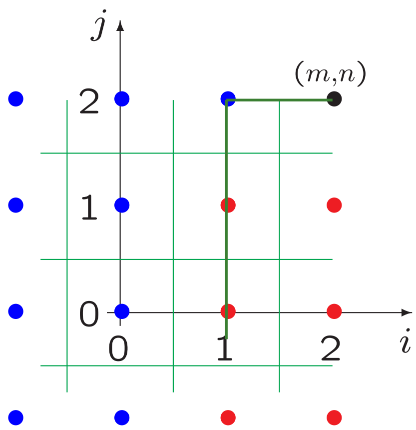

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Conclude

$$\liminf_{t\to\infty} \frac{\mathbf{E}(U^{\varrho}_{Z^{\varrho}+})}{t^{2/3}} > 0, \qquad \liminf_{t\to\infty} \frac{\mathbf{Var}(G^{\varrho})}{t^{2/3}} > 0.$$

10. Further directions

$\longrightarrow$ We only have deviation probability results for the case of the rarefaction fan. How about **Var**(G) in this case?

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$\longrightarrow$ Generalize even more: drop the last-passage picture. These methods have the potential to extend to other particle systems directly (zero range, bricklayers', ...?).

Thank you.