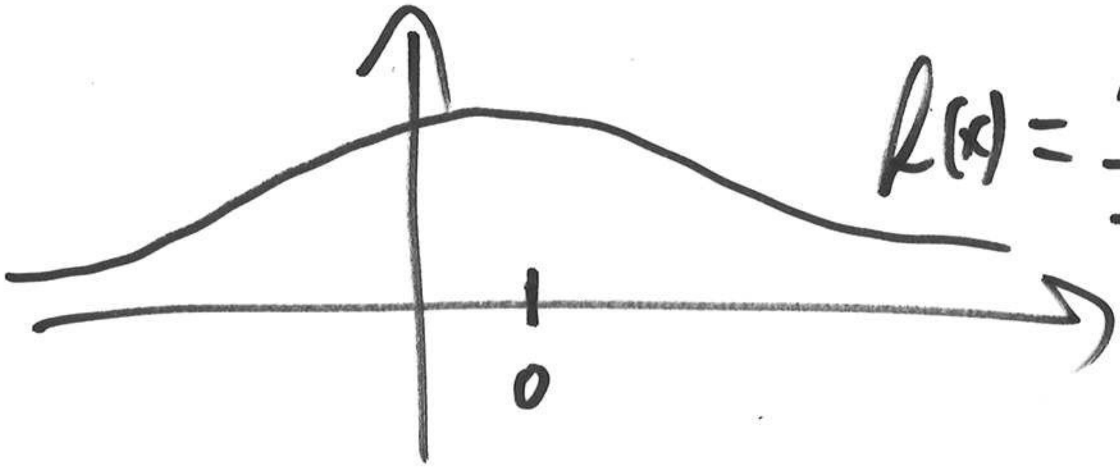
Q&A on $9^{th}$ Jan at 10 am



$$f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$$

iid: $X_1, X_2, \ldots$

$e^{-2}$ ✓

$$\frac{71}{29}$$

# 0. Counting

Experiment 1 : $n$ outcomes

2 $m$ outcomes

combined: $n \cdot m$ outcomes

→ Permutations: orders of $n$ elements

$n!$ many

→ with repetitions: $n_1, n_2, \dots, n_r$

alike alike

→ no. of permutations is $\frac{n!}{n_1! \, n_2! \dots n_r!}$

$n = n_1 + n_2 + \dots + n_r$

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→ $k$-permutations: pick $k$ out of $n$ in order. $\frac{n!}{(n-k)!}$ many ways

→ $k$ - Combinations: pick $k$ out of $n$ without order.

$$\underline{\underline{\binom{n}{k}}} = \frac{n!}{(n-k)!\,k!}$$

Pascal's: $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$

Binom. Thm: $(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$

Multinomial coeff.

$n = n_1 + \dots + n_r$ $$\binom{n}{n_1, n_2, \dots n_r} = \frac{n!}{n_1!\, n_2! \cdots n_r!}$$

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1 Events, $\mathbb{P}$. $\Omega \supseteq E$

$E \cap F$ : BOTH $\bigcap_i E_i$

$E \cup F$ : EITHER $\bigcup_i E_i$

$E^c = \Omega - E$

$(E \cup F)^c = E^c \cap F^c$

$(E \cap F)^c = E^c \cup F^c$ De Morgan

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$E \cap F = \emptyset$ $E, F$ mut. exclusive.

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Axioms: $0 \leq \mathbb{P}(E)$

$\mathbb{P}(\Omega) = 1$

If $E_i$'s are exclusive $\mathbb{P}(\bigcup_i E_i) = \sum_i \mathbb{P}(E_i)$

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$\mathbb{P}(E^c) = 1 - \mathbb{P}(E)$

If $E \subseteq F \Rightarrow \mathbb{P}(E) \leq \mathbb{P}(F)$

$\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$

$\mathbb{P}(\bigcup_{i=1}^n E_i) = \sum_i \mathbb{P}(E_i) - \sum_{i<j} \mathbb{P}(E_i \cap E_j) + \sum_{i<j<k} \mathbb{P}($

$E_i \cap E_j \cdot \mathbb{P}(E_k) - \dots + \dots - \dots$

$$\dots (-1)^{n+1} \mathbb{P}(E_1 \cap \dots \cap E_n)$$

.... equally likely outcomes ...

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② Conditionals.

$$\mathbb{P}(E|F) = \frac{\mathbb{P}(E \cap F)}{\mathbb{P}(F)}$$

$\to F =$ reduced s.p.

$\mathbb{P}(\bullet|F)$ satisfies the 3 axioms.

Law of total prob:

$$\mathbb{P}(E) = \mathbb{P}(E|F)\,\mathbb{P}(F) + \mathbb{P}(E|F^c)\,\mathbb{P}(F^c)$$

$$\mathbb{P}(E) = \sum_i \mathbb{P}(E|F_i)\,\mathbb{P}(F_i)$$

$\hookrightarrow$ complete system partition.

<u>Bayes' Thm</u>:

$$\mathbb{P}(F|E) = \frac{\mathbb{P}(E|F)\,\mathbb{P}(F)}{\mathbb{P}(E|F)\,\mathbb{P}(F) + \mathbb{P}(E|F^c)\,\mathbb{P}(F^c)}$$

Independence: $\mathbb{P}(E \cap F) = \mathbb{P}(E)\,\mathbb{P}(F)$

$E, F, G$

$$\mathbb{P}(E \cap F) = \mathbb{P}(E)\,\mathbb{P}(F)$$
$$\mathbb{P}(E \cap G) = \mathbb{P}(E)\,\mathbb{P}(G)$$
$$\mathbb{P}(F \cap G) = \mathbb{P}(F)\,\mathbb{P}(G)$$
$$\mathbb{P}(E \cap F \cap G) = \mathbb{P}(E) \cdot \mathbb{P}(F) \cdot \mathbb{P}(G)$$

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$$\mathbb{P}(A \mid B) = \mathbb{P}(A \mid BF)\,\mathbb{P}(F \mid B) + \mathbb{P}(A \mid BF^c)\,\mathbb{P}(F^c \mid B)$$

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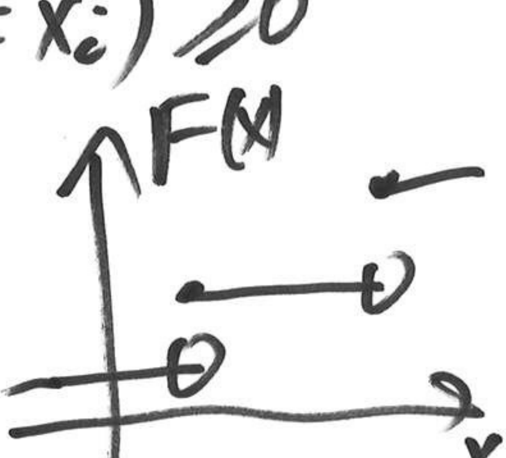
## 3) Discrete r.v.

$$X = x_1, x_2, \dots$$

$$p(x_i) = \mathbb{P}(X = x_i) \geq 0$$

$$\sum_i p(x_i) = 1$$

$$F(x) = \mathbb{P}(X \leq x)$$



$$\mathbb{E}X = \sum_i x_i \, p(x_i)$$

$$\mathbb{E}\, g(X) = \sum_i g(x_i) p(x_i)$$

$$\mathbb{E}X^n = n^{th} \text{ moment}$$

$$\operatorname{Var} X = \mathbb{E}X^2 - (\mathbb{E}X)^2 = \mathbb{E}(X - \mathbb{E}X)^2$$

$$\mathbb{E}(aX + b) = a\,\mathbb{E}X + b$$

$\searrow$ $SD X =$

$= \sqrt{\operatorname{Var} X}$

$$\operatorname{Var}(aX + b) = a^2 \operatorname{Var} X$$

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$$\mathbb{E}(X-a)^+ = \int_{-\infty}^{\infty} (x-a)^+ f(x)\,dx$$

$$g(x) = (x-a)^+$$

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Cont: $\mathbb{P}(X \in B) = \int_B f(x)\,dx$

$\searrow$ density

$$F(a) = \int_{-\infty}^{a} f(x)\,dx$$

$$f(x) = \frac{dF(x)}{dx}$$

$$\mathbb{E} X = \int_{-\infty}^{\infty} x f(x) dx$$

$$\mathbb{E} g(X) = \int_{-\infty}^{\infty} g(x) f(x) dx$$