

Probability 1, Autumn 2015, Problem sheet 1

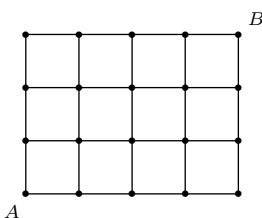
To be discussed on the week 5 Oct...9 Oct.

Problems marked with "PrCl" are discussed in the large problem class on Thursday 1 Oct.

Mandatory HW's are marked with "HW", they are due on the week 12 Oct...16 Oct.¹

Solutions will be available on Blackboard on the 17th Oct.

- 1.1** How many $\{1, 2, \dots, n\} \rightarrow \{0, 1\}$ functions are there? (That is, functions that assign either 0 or 1 to integers between 1 and n .) And how many $\{0, 1\} \rightarrow \{1, 2, \dots, n\}$ functions?
- PrCl 1.2** I have 4 Maths, 3 Chemistry, 2 History and one French language book (no two are identical). In how many ways can I order them on my bookshelf if I want books of the same topics to stay next to each other?
- HW 1.3**
- In how many ways can three girls and three boys sit in a row? (Notice: people are never considered indistinguishable.)
 - In how many ways can three girls and three boys sit in a row if girls sit next to each other and boys sit next to each other?
 - In how many ways can three girls and three boys sit in a row if only boys are required to sit next to each other?
 - In how many ways can three girls and three boys sit in a row if no two children of the same sex are allowed to sit next to each other?
- PrCl 1.4** Out of 5 women and 7 men we wish to form a committee of 2 women and 3 men.
- In how many ways can we do that?
 - And how many ways if two of the 7 men refuse to serve together?
- HW 1.5** Out of 8 women and 6 men we wish to form a committee of 3 women and 3 men.
- In how many ways can we do that?
 - How many ways if two of the 6 men refuse to serve together?
 - How many ways if two of the 8 women refuse to serve together?
 - How many ways if one of the women and one of the men refuse to serve together?
- 1.6** 12 ladies and 13 gentlemen attend a dance class. In how many ways can the instructor select 6 ladies, 6 gentlemen, and then pair them?
- HW 1.7** On the graph below, how many ways are there from A to B only using steps of length one in the Northern or Eastern direction?



- 1.8** 10 children want to play football.
- How many ways are there to form teams A and B of sizes 5 each?
 - How many ways are there to form two teams of sizes 5 each? (!)
- HW 1.9** Out of n people we want to form a committee of k , one of whom will be the president of the committee.
- By picking the committee first, then its president, argue that there are $\binom{n}{k} k$ ways to do this.
 - By choosing the non-president members of the committee first, then picking the president from the remaining people, argue that there are $\binom{n}{k-1} (n-k+1)$ many ways.
 - Pick the president first, then select the non-president members, and argue that the number of ways is $n \binom{n-1}{k-1}$.
 - Draw the conclusion:

$$\binom{n}{k} k = \binom{n}{k-1} (n-k+1) = n \binom{n-1}{k-1}.$$

Prove this also analytically (by writing out all the factorials).

¹Details of how to hand in are to be discussed with your tutor.