

Probability 1, Autumn 2016, Problem sheet 1

To be discussed on the week 3 Oct...7 Oct.

Problems marked with "PrCl" are discussed in the large problem class on Friday 30 Sep.

Mandatory HW's are marked with "HW", they are due on the week 10 Oct...14 Oct.¹

Solutions will be available on Blackboard on the 15th Oct.

1.1 How many $\{1, 2, \dots, n\} \rightarrow \{0, 1\}$ functions are there? (That is, functions that assign either 0 or 1 to integers between 1 and n .) And how many $\{0, 1\} \rightarrow \{1, 2, \dots, n\}$ functions?

PrCl 1.2 I have 4 Maths, 3 Chemistry, 2 History and one French language book (no two are identical). In how many ways can I order them on my bookshelf if I want books of the same topics to stay next to each other?

- HW 1.3**
- (a) In how many ways can three girls and three boys sit in a row? (Notice: people are never considered indistinguishable.)
 - (b) In how many ways can three girls and three boys sit in a row if girls sit next to each other and boys sit next to each other?
 - (c) In how many ways can three girls and three boys sit in a row if only boys are required to sit next to each other?
 - (d) In how many ways can three girls and three boys sit in a row if no two children of the same sex are allowed to sit next to each other?

PrCl 1.4 Out of 5 women and 7 men we wish to form a committee of 2 women and 3 men.

- (a) In how many ways can we do that?
- (b) And how many ways if two of the 7 men refuse to serve together?

HW 1.5 Out of 8 women and 6 men we wish to form a committee of 3 women and 3 men.

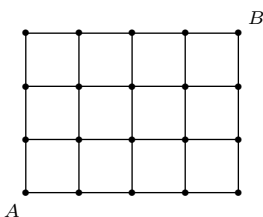
- (a) In how many ways can we do that?
- (b) How many ways if two of the 6 men refuse to serve together?
- (c) How many ways if two of the 8 women refuse to serve together?
- (d) How many ways if one of the women and one of the men refuse to serve together?

1.6 12 ladies and 13 gentlemen attend a dance class. In how many ways can the instructor select 6 ladies, 6 gentlemen, and then pair them?

1.7 A walker starts from 0, and can make a jump of length one in each step in either directions on the integer line \mathbb{Z} . She makes 10 steps in total and then stops.

- (a) How many different trajectories of her motion are possible?
- (b) In how many positions can she end up after 10 steps?
- (c) Suppose she ends up at position 4. In how many ways could she get there in 10 steps?

HW 1.8 On the graph below, how many ways are there from A to B only using steps of length one in the Northern or Eastern direction?



1.9 10 children want to play football.

- (a) How many ways are there to form teams A and B of sizes 5 each?
- (b) How many ways are there to form two teams of sizes 5 each? (!)

¹Details of how to hand in are to be discussed with your tutor.

HW 1.10 Out of n people we want to form a committee of k , one of whom will be the president of the committee.

- (a) By picking the committee first, then its president, argue that there are $\binom{n}{k} k$ ways to do this.
- (b) By choosing the non-president members of the committee first, then picking the president from the remaining people, argue that there are $\binom{n}{k-1} (n - k + 1)$ many ways.
- (c) Pick the president first, then select the non-president members, and argue that the number of ways is $n \binom{n-1}{k-1}$.
- (d) Draw the conclusion:

$$\binom{n}{k} k = \binom{n}{k-1} (n - k + 1) = n \binom{n-1}{k-1}.$$

Prove this also analytically (by writing out all the factorials).