

Probability 1, Autumn 2016, Problem sheet 3

To be discussed on the week 17 Oct... 21 Oct.

Problems marked with "PrCl" are discussed in the large problem class on Friday 14 Oct. Mandatory HW's are marked with "HW", they are due on the week 24 Oct... 28 Oct, the latest.¹ Solutions will be available on Blackboard on the 29th Oct.

- 3.1** A certain town with a population of 100 000 has three newspapers: I, II, and III. The proportions of townspeople who read these papers are as follows:

I: 26%	I and II: 6%	I and II and III: 2%
II: 18%	I and III: 9%	
III: 22%	II and III: 5%	

(The list tells us, for instance, that 6 000 people read newspapers I and II (and 2 000 of them also read newspaper III)).

- Find the number of people who do not read any of these newspapers.
- Find the number of people who read exactly one newspaper.
- How many people read at least two newspapers?
- If I and III are morning papers and II is an evening paper, how many people read at least one morning paper plus an evening paper?
- How many people read only one morning paper and one evening paper?

- 3.2** On a horse race of 7 horses, let S denote the event that *Star* is among the first three, and M the event that *Magic* has finished in an even position. Assuming each outcome to be equally likely, find the probability of $S \cup M$.

HW 3.3 An urn contains 5 red, 7 green and 8 yellow balls. Drawing 5 without replacement, what is the probability that we have each of the three colors at least once?

PrCl 3.4 We roll a die ten times. What is the probability that each of the results 1, 2, ..., 6 shows up at least once? *HINT: Define the events $A_i := \{\text{number } i \text{ doesn't show up at all during the ten rolls}\}$, $i = 1 \dots 6$. Note that these events are not mutually exclusive.*

- 3.5 The matching problem.** n gentlemen go out for dinner, and they leave their hats in the cloakroom. After the dinner (and several glasses of wine) they pick their hats completely randomly. What is the probability that none of the gentlemen took their own hats? What happens as $n \rightarrow \infty$?

HW 3.6 We roll three fair dice. Assuming no two dice land on the same number, what is the probability that at least one of the three dice shows a six?

- 3.7** We roll a red, a blue and a yellow die (each fair). Denote the numbers they show by R , B and Y , respectively.

- What is the probability that the three numbers are all different?
- Given that the three numbers are all different, what is the probability that $R < B < Y$?
- Determine $\mathbf{P}\{R < B < Y\}$.

HW 3.8 We repeatedly roll two four-sided (tetrahedron) dice at the same time, and only stop when at least one of them shows a four. What is the probability that the other one also shows a four? *HINT: it is not $\frac{1}{4}$.*

- PrCl 3.9** (a) I come from a family of two children. What is the probability that the other child is my sister?
(b) The king comes from a family of two children. What is the probability that the other child is his sister?

Explain your answer.

HW 3.10 Each of three balls, independently, was painted gold or black with probability $\frac{1}{2} - \frac{1}{2}$. The balls were then put in an urn.

- Suppose we see that the black paint was used, that is, at least one of the three balls is black. Then what is the probability that each of the other balls is also painted black?
- Instead, suppose now that the urn tilts, one of the three balls rolls out, and we see it is black. Now what is the probability that each of the other balls is also painted black?

Explain your answer.

¹Details of how to hand in are to be discussed with your tutor.