# Bertrand's paradox

Márton Balázs

University of Bristol

$$\mathbb{P}\{\longrightarrow\}=?$$

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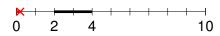
$$\mathbb{P}\{\longrightarrow\}=$$
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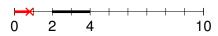


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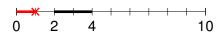
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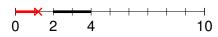
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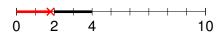


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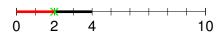


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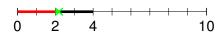
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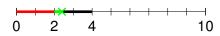
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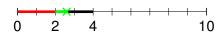
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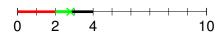
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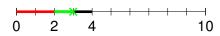
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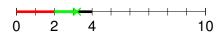
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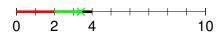
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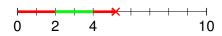
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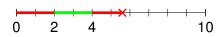
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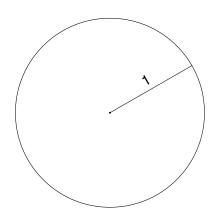
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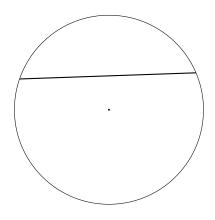


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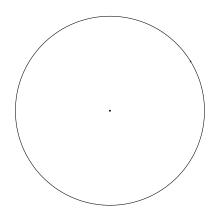


$$\mathbb{P}\{\longleftarrow\} = \frac{1}{10} = \frac{2}{10} = \frac{1}{5}.$$

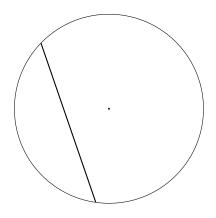




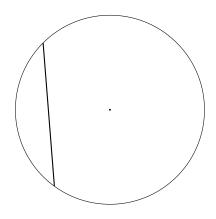
random chord



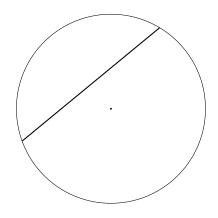
random chord



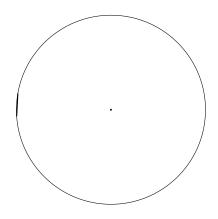
random chord



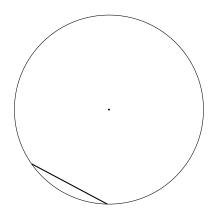
random chord



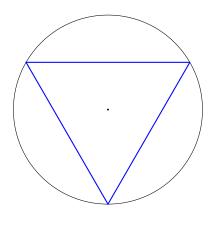
random chord



random chord

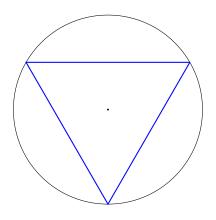


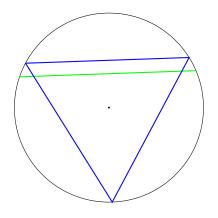
random chord

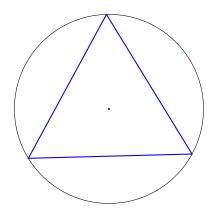


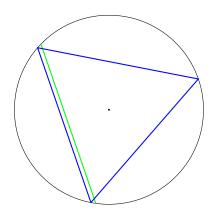
random chord



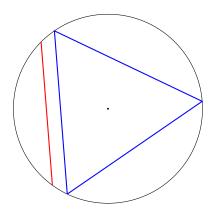


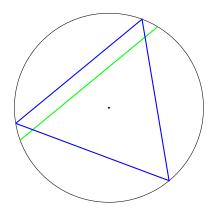




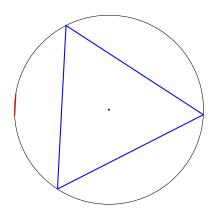


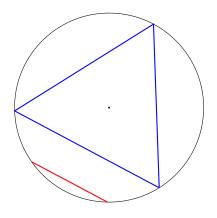
 $\mathbb{P}\{random\ chord\ is\ longer\ than\ one\ side\ of\ the\ \nabla\}$ ?

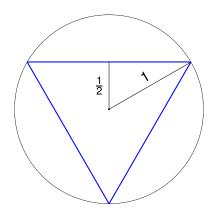


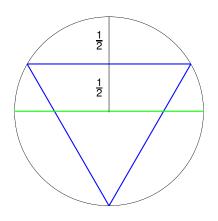


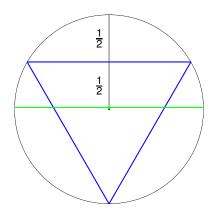
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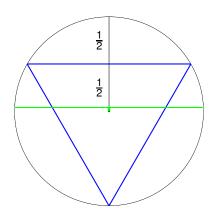


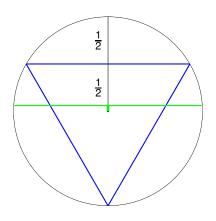


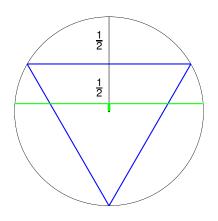


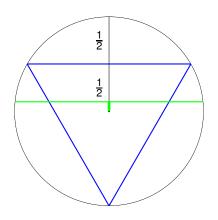


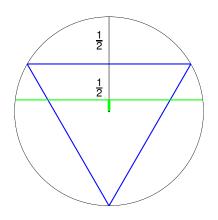


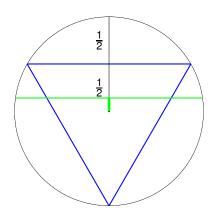


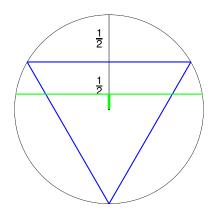


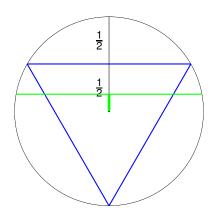


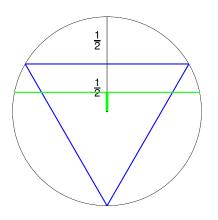


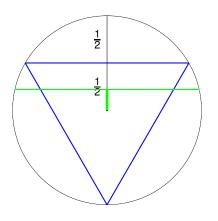


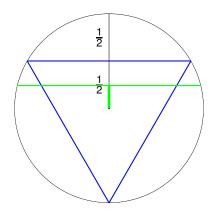


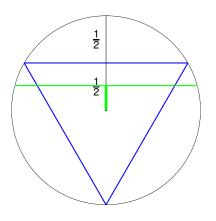


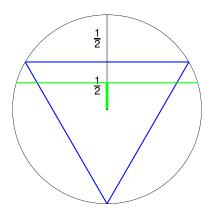


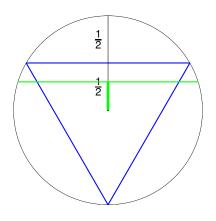


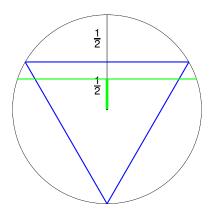


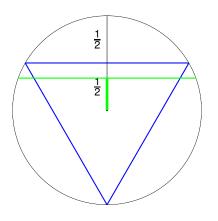


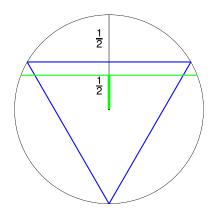


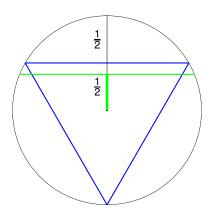


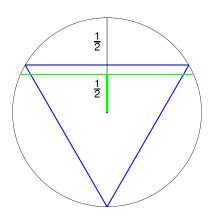


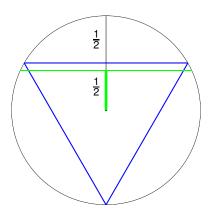


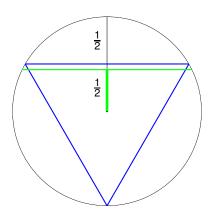


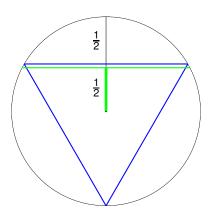


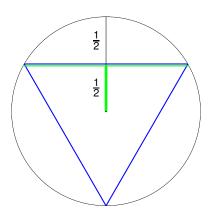


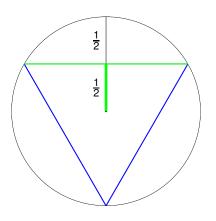


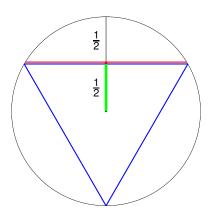


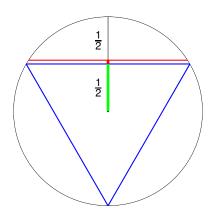


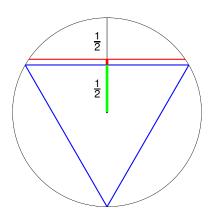


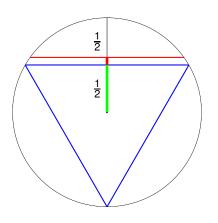


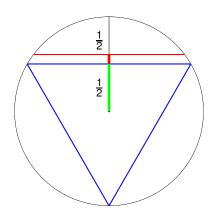


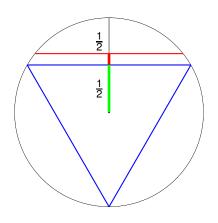


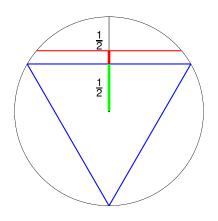


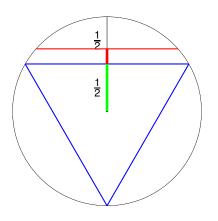


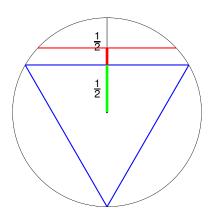


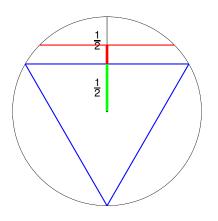


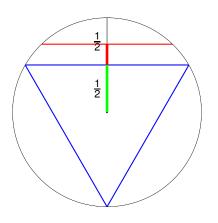


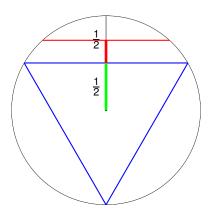


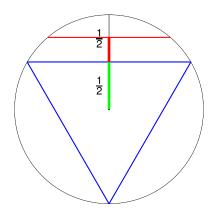


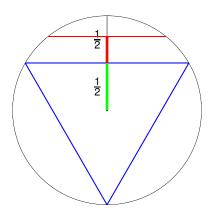


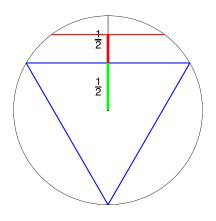


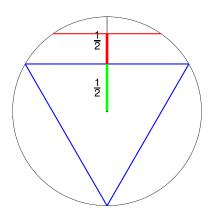


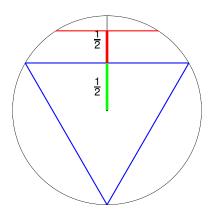


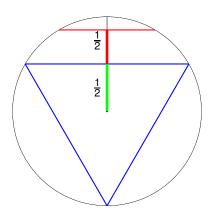


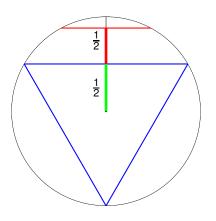


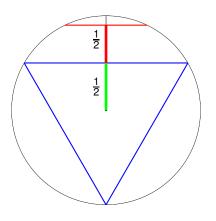


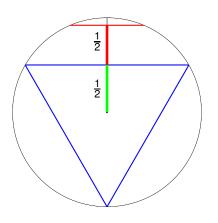


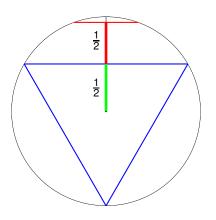


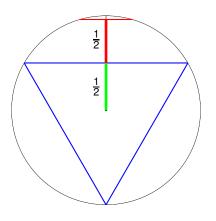


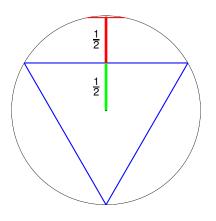


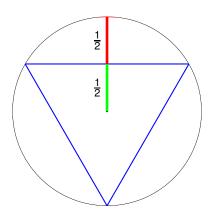




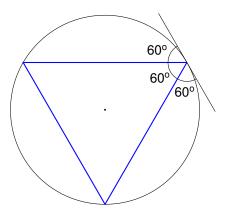


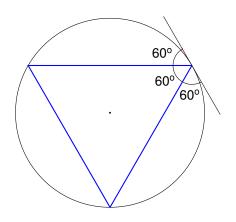


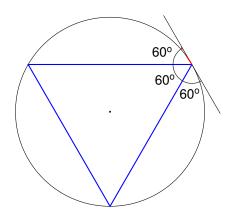


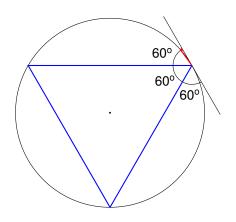


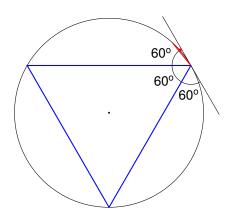
Distance from center:  $\frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} =$ 

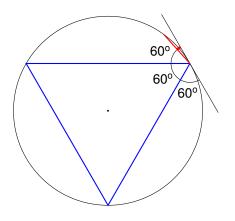


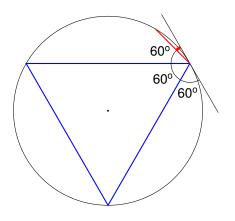


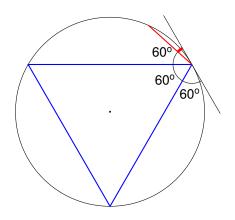


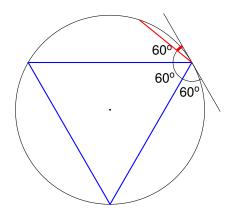


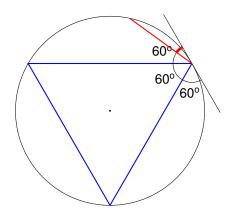


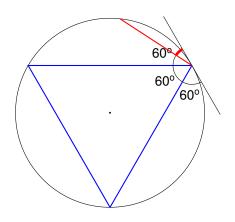


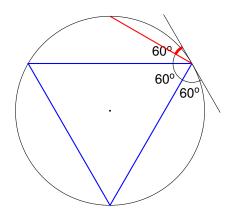


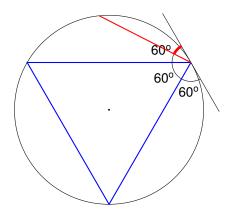


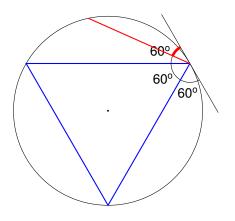


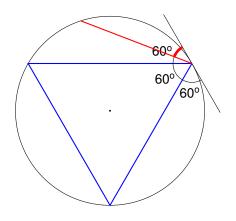


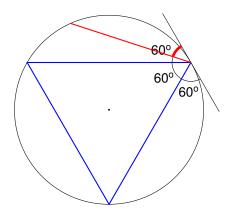


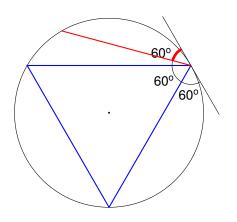


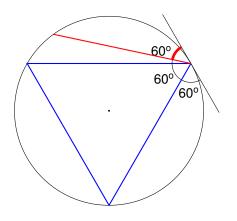


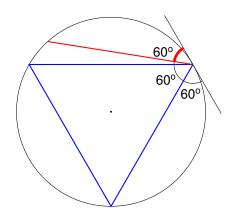


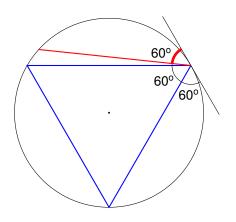


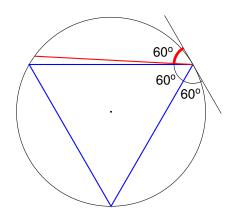


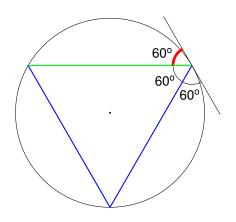


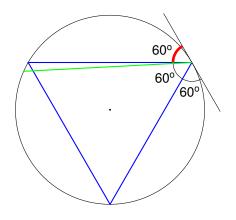


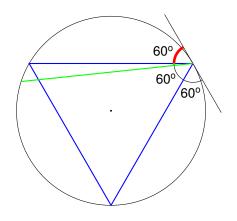


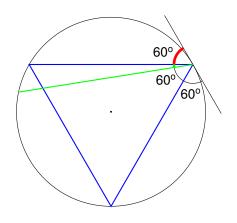


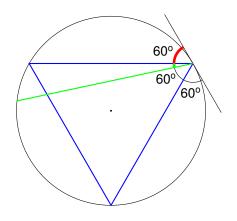


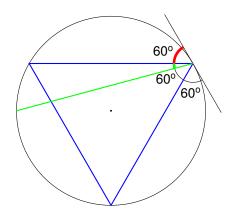


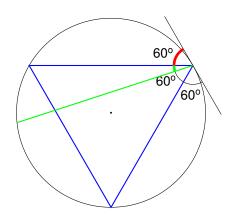


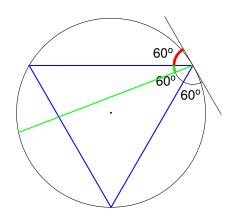


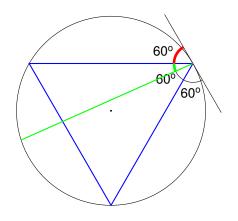


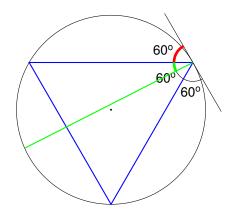


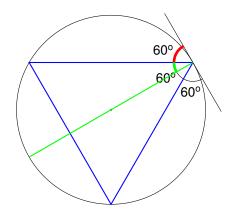


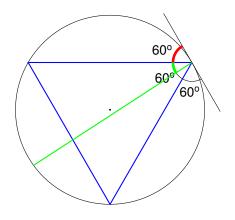


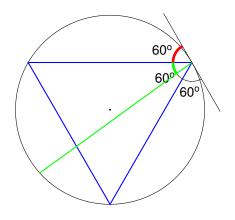


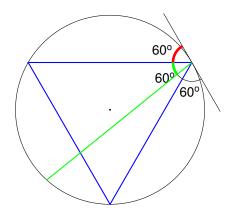


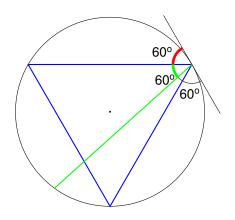


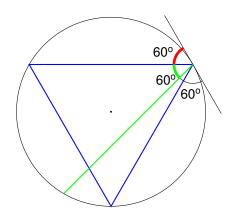


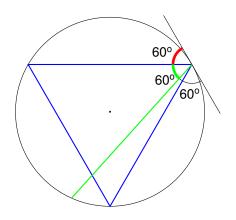


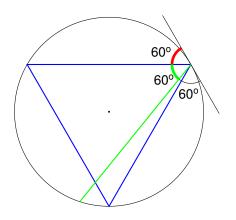


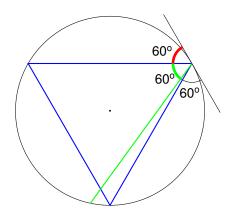


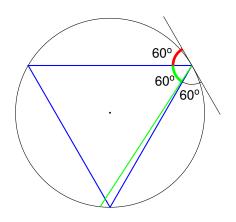


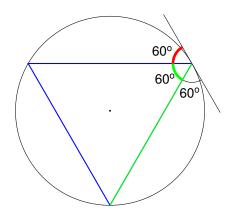


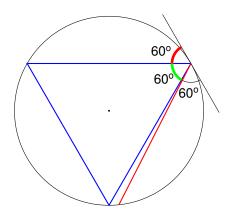


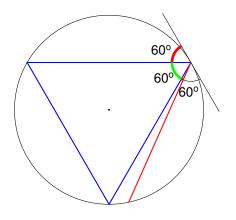


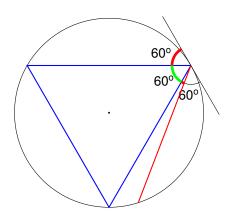


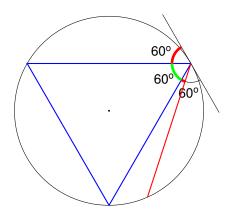


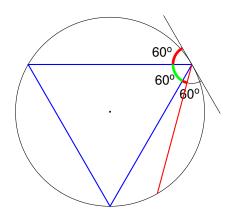


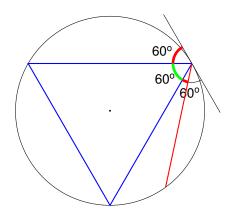


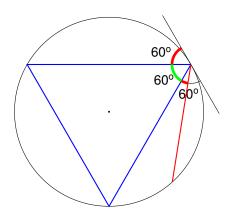


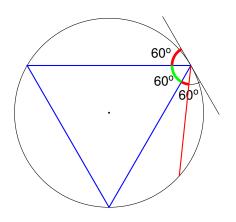


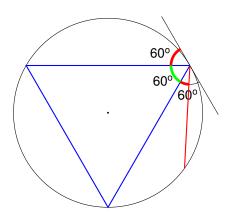


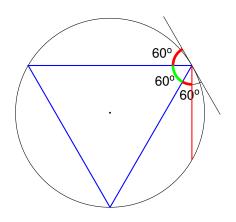


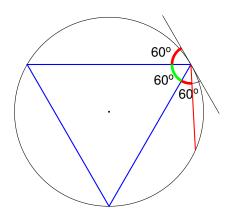


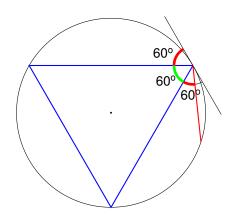


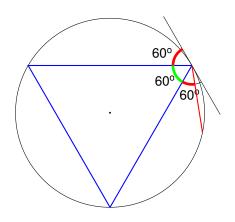


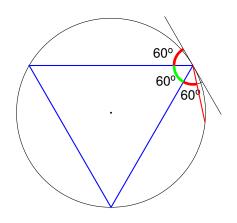


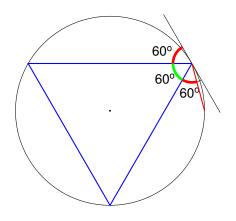


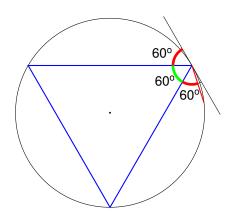


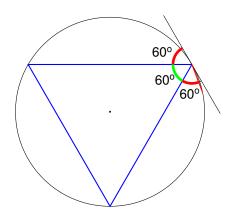


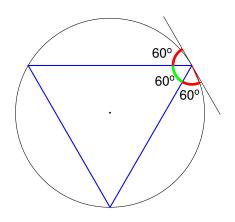


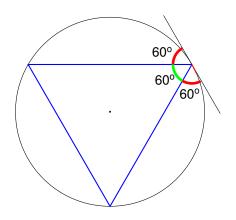


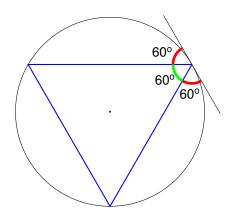




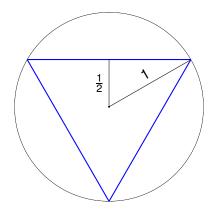


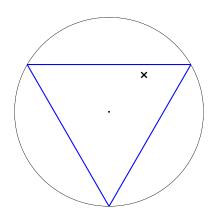


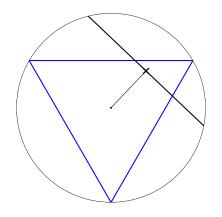


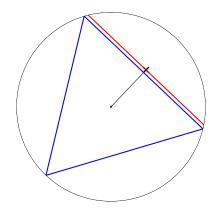


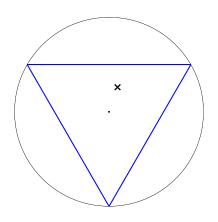
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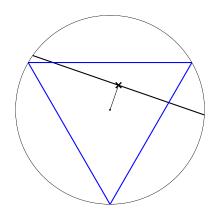


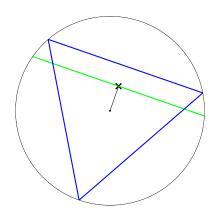


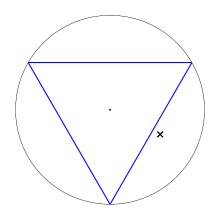


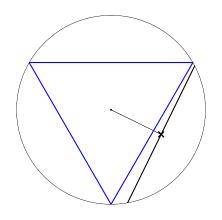


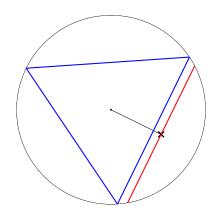


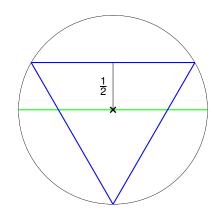


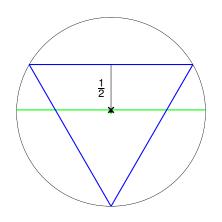


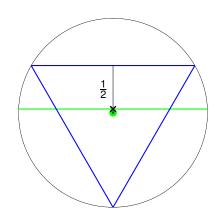


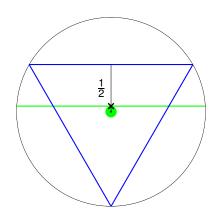


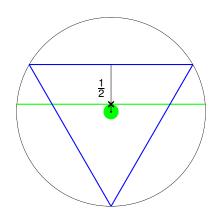


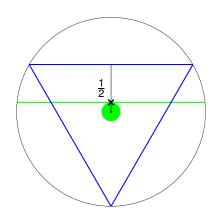


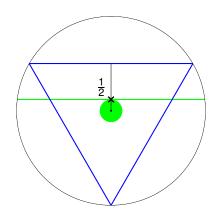


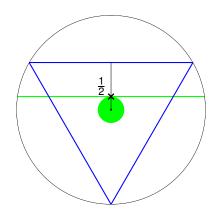


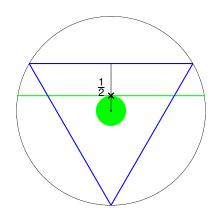


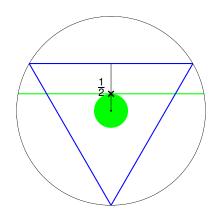


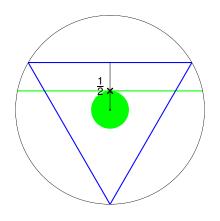


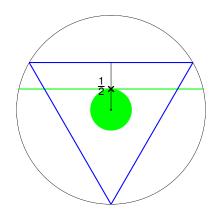


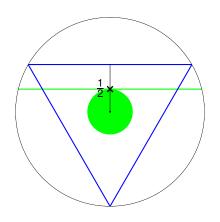


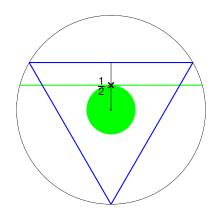


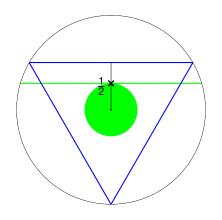


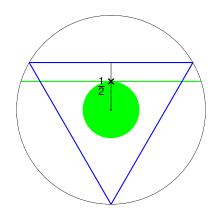


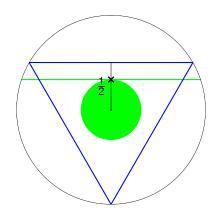


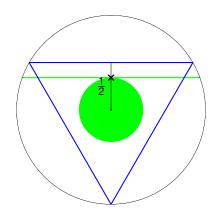


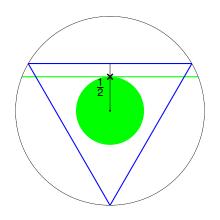


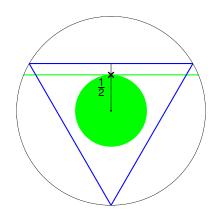


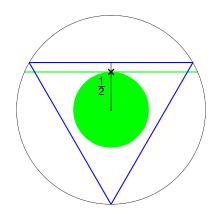


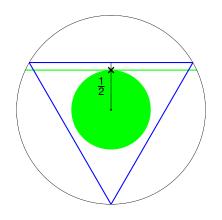


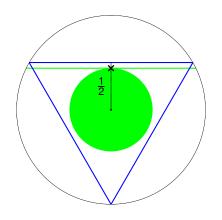


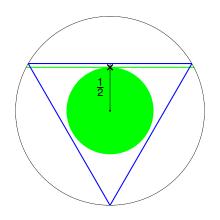


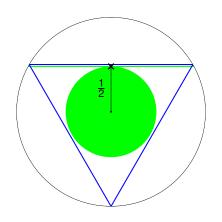


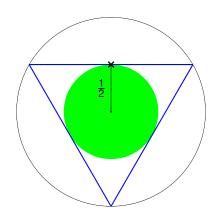


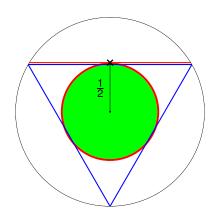


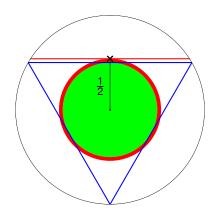


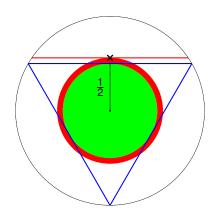


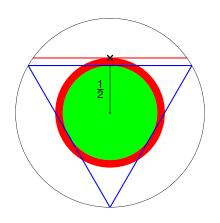


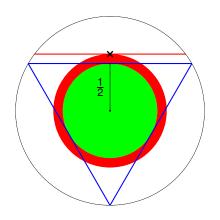


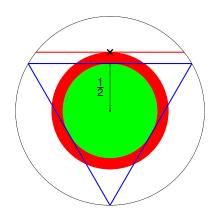


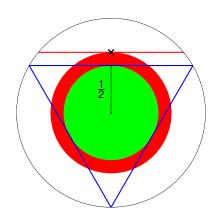


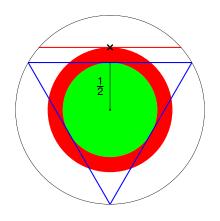


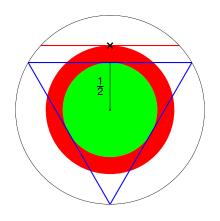


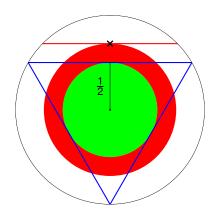


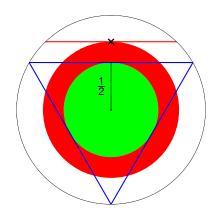


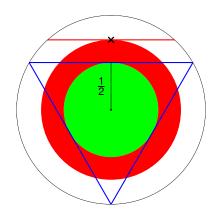


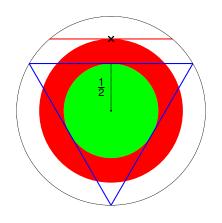


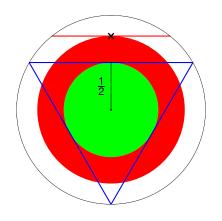


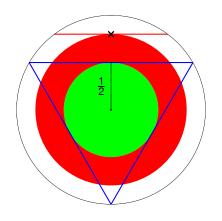


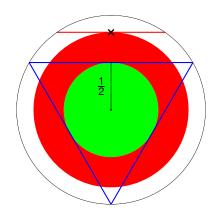


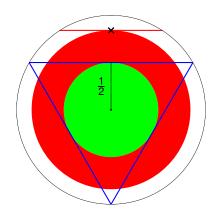


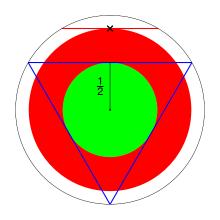


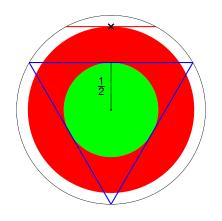


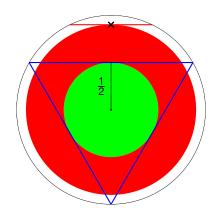


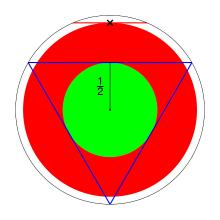


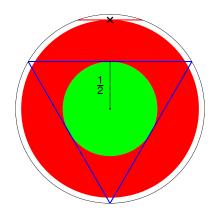


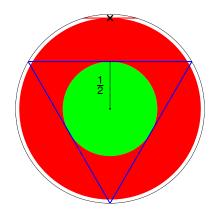


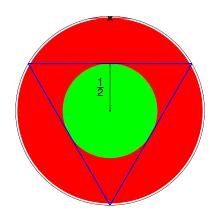


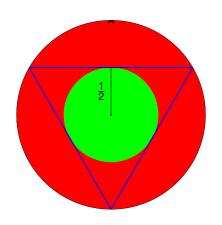












Midpoint: 
$$\frac{\left|\begin{array}{c}\bullet\\ \bullet\end{array}\right|}{\left|\begin{array}{c}\bullet\\ \bullet\end{array}\right|}=\frac{\left(\frac{1}{2}\right)^2\cdot\pi}{1^2\cdot\pi}=\frac{1}{4}.$$

Answer:  $\frac{1}{2}$ ?  $\frac{1}{3}$ ?  $\frac{1}{4}$ ?

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