

Problem Sheet 10

Remember: when online, you can access the Statistics 1 data sets from an **R** console by typing

```
load(url("http://www.stats.bris.ac.uk/%7Emapjg/Teach/Stats1/stats1.RData"))
```

- *1. A tyre company claims its tyres have a mean useful lifetime of 42,000 miles. A consumer association bought one of the company's tyres from each of 10 randomly chosen outlets and tested them on a test rig that simulated normal road conditions. The observed lifetimes (in thousands of miles) were

42 36 46 43 41 35 43 45 40 39.

Thinking carefully about the context, what is an appropriate alternative hypothesis H_1 to use in testing the manufacturer's claim? Use the data to test whether or not there is sufficient evidence to reject the manufacturer's claim, using a test procedure with significance level $\alpha = 0.05$. The data are contained in the Statistics 1 data set `tyre.lifetimes`.

[If you do this question by hand calculation, it may help to know that `pt(0.8808, 9)` gives 0.7993325 and `qt(0.95, 9)` gives 1.833113.]

[Your answer should include a statement of any model assumptions, a brief description of your working at each stage of the test procedure including the p -value and the critical region for the test, and a summary of your conclusions. You may find it helpful to check your numerical calculations using the `t.test()` function in R.]

2. In a study of blood glucose level, measurements were made on a sample of 52 pregnant women in their third trimester of pregnancy. Their values (in milligrams/100 millilitres of blood) were found to have a sample mean of 70.12. Healthy women who are not pregnant are known to have a mean value of 80 (mg/100ml), with a standard deviation of 10 (mg/100ml), and we will assume that the standard deviation for pregnant women is also 10 (mg/100ml).
- Use the data to test the hypothesis that pregnant women have a lower glucose level than women who are not pregnant. [Your answer should include a statement of any model assumptions, a brief description of your working at each stage of the test procedure including the p -value, and a summary of your conclusions. You will probably need to use **R** to calculate the actual p -value.]
 - Now assume the test was carried out using a procedure with significance level $\alpha = 0.01$. Calculate the probability of type II error under the alternative hypothesis that the true mean value of glucose level in pregnant women is 79 (mg/100 ml). Hence calculate the power of the test to discriminate between the null hypothesis and this alternative.
3. (*This question reminds you that sometimes the null hypothesis is true but due to chance the test statistic is such that the null hypothesis is rejected. In particular, under a test procedure with significance level α , about $100\alpha\%$ of samples will result in the null hypothesis being rejected even when H_0 is true.*).

Choose valid numerical values of μ_0 and σ^2 for normally distributed data and choose a sample size n . Then draw a random sample of size n using the **R** command:

```
> mysamp <- rnorm( n, mean=mu0, sd=sigma)
```

substituting in for your chosen values of n , μ_0 and σ . Examine your sample in `mysamp` by typing its name:

```
> mysamp
```

Now perform the one-sample t -test of $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$ using the command:

```
> t.test(mysamp, mu=mu0)
```

again substituting in your value for `mu0`. Is the test statistic significant, i.e. is H_0 rejected? (you can tell either from the p -value – if it is less than α then you can reject at the α significance level – or by seeing whether the confidence interval contains μ_0).

The two stages of sampling and testing can be replaced by one line:

```
> t.test( rnorm(n, mean=mu0, sd=sigma), mu=mu0)
```

Repeat the whole procedure until you have performed 20 tests in all (use the up arrow key and ENTER to repeat the command). Out of the 20 repeats of the sampling and testing how many times was the test significant? On average, how many times should it be significant?

4. The table below shows a data set with five pairs of values, (x_i, y_i) $i = 1, \dots, 5$. Assume that the data satisfy the simple Normal linear regression model $Y_i = \alpha + \beta x_i + e_i$, where the e_i are i.i.d. $N(0, \sigma^2)$. This data set was introduced in Question 2 on Problem Sheet 5.

x	1	3	4	6	7
y	0	1	2	5	4

- (a) Test the null hypothesis $H_0: \alpha = 0$ against the alternative $H_1: \alpha \neq 0$ using a test procedure with significance level 0.05.
- (b) Test the null hypothesis $H_0: \beta = 0$ against the alternative $H_1: \beta \neq 0$ using a test procedure with significance level 0.05.

In each case calculate the p -value of the observed test statistic.

- *5. A study was conducted to examine the dependence of metabolic rate on body mass for 7 dogs, yielding data given in the table below.

Body mass (kg)	31.20	24.00	19.80	18.20	9.61	6.50	3.19
Metabolic rate (kcal/day)	1113.2	981.8	908.2	840.8	626.2	429.5	280.9

It was decided to analyse these data on a log scale; defining x as log body mass and y as log metabolic rate, we fit a Normal linear regression model $Y_i = \alpha + \beta x_i + e_i$. Summary statistics calculated from the data are

$$\sum x_i = 17.800 \quad \sum y_i = 45.590 \quad \sum x_i^2 = 49.239 \quad \sum y_i^2 = 298.433 \quad \sum y_i x_i = 118.365.$$

Calculate the end points of 99% confidence intervals for α and β .

[In **R**, I found `qt(c(0.95, 0.975, 0.99, 0.995), 5)` gives

[1] 2.015048 2.570582 3.364930 4.032143; some of these values may be useful.]

6. The table below shows the rainfall (in ins) for the spring and the following autumn for each of ten consecutive years. Let x_i denotes the observed spring rainfall in the i th year and y_i denotes the corresponding observed autumn rainfall. Assume that the data satisfy the simple Normal linear regression model $Y_i = \alpha + \beta x_i + e_i$, where the e_i are i.i.d. $N(0, \sigma^2)$. The data are contained in the Statistics 1 data set `rain`.

Spring rainfall (x)	1.6	5.3	2.8	9.6	6.7	1.5	5.4	8.5	4.1	3.9
Autumn rainfall (y)	4.6	6.0	2.9	11.1	8.2	1.3	9.1	10.2	5.2	8.3

- (a) Find a 90% confidence interval for β .
- (b) Test the null hypothesis $H_0: \beta = 0$ against the alternative $H_1: \beta \neq 0$, using a test procedure with significance level equal to 0.10.
- (c) Show that in general a test of the null hypothesis $H_0: \beta = 0$ against the alternative $H_1: \beta \neq 0$ with significance level (say) γ will accept the null hypothesis if and only if the corresponding $100(1 - \gamma)\%$ confidence interval for β contains the value $\beta = 0$.
- *7. The table below shows the average weight (in kg) of piglets in a litter, for seven litters of varying size. The data are contained in the Statistics 1 data set `pig` and were introduced in Question 3 on Problem Sheet 5.

Litter size (x)	1	3	5	8	8	9	10
Average weight (y)	1.6	1.5	1.5	1.3	1.4	1.2	1.1

Summary statistics are:

$$\sum x_i = 44, \sum y_i = 9.6, \sum x_i^2 = 344, \sum y_i^2 = 13.36, \sum y_i x_i = 57.$$

When a simple linear regression of average weight on litter size is performed in **R** using the command `piglets <- lm(wt ~ littersize, data=pig)`, the output of the command `summary(piglets)` includes the following lines:

Coefficients:

```

              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.683051   0.064520  26.086 1.55e-06 ***
littersize   -0.049576   0.009204  -5.387 0.00297 **
---
```

```

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.07558 on 5 degrees of freedom
```

- (a) Show clearly how each of the numerical values on the three lines beginning `(Intercept)`, `littersize` and `Residual standard error` were calculated, and explain the interpretation of the asterisks `***` and `**` at the end on the lines.
- (b) What conclusion would you reach from the output for testing the null hypothesis $H_0: \beta = 0$ against the alternative $H_1: \beta \neq 0$?

Note: for each of the following questions your answer should include a statement of any model assumptions, a brief description of your working at each stage of the test procedure including the approximate p -value and the critical region for the test, and a summary of your conclusions. You may find it helpful to check your numerical calculations using the `t.test()` function in R.

- *8. Eight athletes ran a 400 metre race at sea level and at a later meeting ran a 400 metre race at high altitude. Their times in seconds are shown in the table below. Test at the null hypothesis that race times are unaffected by altitude, against the alternative that race times are greater at high altitude using a test procedure with significance level $\alpha = 0.05$. The data are contained in the Statistics 1 data set `runner`.

Runner	1	2	3	4	5	6	7	8
Sea Level	48.3	47.6	49.2	50.3	48.8	51.1	49.0	48.1
High Altitude	50.4	47.3	50.8	52.3	47.7	54.5	48.9	49.9

[These values may be useful: `pt(2.1958, 7)` is `0.9679363` and `qt(0.95, 7)` is `1.894579`.]

[P.S. Do you think times should be faster or slower at altitude? Think about this, then do some research - see, e.g.

<http://news.bbc.co.uk/sport1/hi/athletics/2259347.stm>.]

9. To investigate the relative size of secretarial starting salaries in the public and private sector, 9 private sector posts and 10 public sector posts were chosen at random from jobs advertised on the web. The table below shows the advertised starting salaries (in £1,000; this data set is quite old!). You may assume that the population variances are the same in the private and public sectors. Use the data to test at the 0.05-level the hypothesis that starting salaries are the same in the two sectors against the alternative that private sector starting salaries are higher. The data are contained in the Statistics 1 data set `secretaries`.

Private sector	12.1	13.4	11.3	10.6	9.7	12.5	9.6	13.6	11.2	
Public sector	9.3	8.5	8.2	13.1	8.8	11.9	10.1	9.8	12.2	10.4

10. A study was conducted of 10 households to see if alerting them to high usage rates of electricity reduced their actual consumption. A small monitor was installed in each household, which activated a red flashing light whenever the current rate of usage exceeded a pre-set threshold. The monthly usage (in kilowatt-hours) before and after installation of the monitor is given below. Test at the 0.05-level whether the monitor is effective at reducing electrical consumption. The data are contained in the Statistics 1 data set `kwh`.

Household	1	2	3	4	5	6	7	8	9	10
Before	940	1370	1030	2030	1540	2300	1800	910	640	1200
After	900	1230	1060	2100	1250	2200	1820	900	630	1110

11. In a study to examine whether increasing the amount of calcium in the diet reduced blood pressure, a group of 10 men were given a calcium supplement in their diet for 12 weeks, and a control group of 11 men received a placebo (a pill that appeared identical, but contained no active substance). The table below shows the relative change (in mm of mercury) in blood pressure over the 12 week period (before - after) for each subject. What do you conclude from the results shown? The data are contained in the Statistics 1 data set `bp`.

Calcium group	-7	4	-18	-17	3	5	-1	-10	-11	2	
Placebo group	1	-12	1	3	-3	5	-5	-2	11	1	3