FIRST-YEAR GROUP THEORY
EXERCISES FOR SECTION 6

1. Let $n$ be an even positive integer. Show that $D_n \times Z_2$ has no element of order $2n$, and deduce that $D_{2n}$ is not isomorphic to $D_n \times Z_2$.

2. Let $H$ and $K$ be groups. Prove that $H \times K \cong K \times H$.

3. Prove that $(Q, +)$ is not isomorphic to $(Z, +)$.

4. Show that $D_4$ is not isomorphic to the quaternion group.

5. Show that no two of the groups $Z_2 \times Z_2 \times Z_2$, $Z_2 \times Z_4$ and $Z_8$ are isomorphic.

6. Show that $U_{10} \cong Z_4$.

7. Let $G$ be the group of all rational numbers under addition, and let $H$ be the group of positive rational numbers under multiplication. Prove that $G$ is not isomorphic to $H$.

8. Let $G$ be the group of all non-zero rational numbers under multiplication, and let $H$ be the group of positive rational numbers under multiplication. Prove that $G$ is not isomorphic to $H$. 