FIRST-YEAR GROUP THEORY
EXERCISES FOR SECTION 8

1. Working from right to left, write

\[(1, 2, 4, 5)(3, 5, 7)(1, 4, 7, 6)(4, 6, 5)(1, 5, 2)\]

as a product of disjoint cycles in \(S_7\).

2. Working in \(S_4\) set \(a = (1, 2, 3, 4)\) and \(b = (1, 2)(3, 4)\). Show that \(ab = ba^{-1}\). Deduce that \(S_4\) has a subgroup isomorphic to \(D_4\).

3. Show that \(D_6\) is not isomorphic to \(A_4\).

4. Show that if \(D_n \cong S_n\) then \(n = 3\).

5. Show that the identity element and the three elements \((1, 2)(3, 4)\), \((1, 3)(2, 4)\), \((1, 4)(2, 3)\) form a subgroup \(V\) of \(S_4\) and that \(V\) is the Klein 4-group.

6. Show that \(A_4\) is not Abelian.

7. Working in \(S_7\):

   (1) What is the order of \((1, 2)(3, 4)(5, 6, 7)\)?
   (2) Write down an element of order 10.