1. Prove that an $g \text{ isometry of } \mathbb{H}^2$ is elliptic, parabolic, or hyperbolic according as the number of fixed points of $g$ on the boundary $\partial \mathbb{H}^2$ is zero, one, or two. Treat the case of a fixed point at infinity carefully!

2. Find the hyperbolic area of the region bounded by the curves $\{x^2 + (y-2)^2 = 4\}$, $\{x^2 + (y-1)^2 = 1\}$, $\{(x-1)^2 + y^2 = 1\}$, and $\{(x+1)^2 + y^2 = 1\}$.

3. Find the hyperbolic area of the region bounded by the curves $\{x^2 + y^2 = 16\}$, $\{(x-2)^2 + y^2 = 4\}$, and $\{(x+1)^2 + y^2 = 9\}$.

4. Prove that the set
\[
\left\{ \begin{pmatrix} a & 3b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z}, ad - 3bc = 1 \right\}
\]
forms a subgroup of $\text{SL}(2, \mathbb{Z})$.

5. Find a map $g$ that takes the horocycle $\{x + i : x \in \mathbb{R}\}$ to a horocycle of your choice based at 1. What is the Euclidean diameter of your horocycle?

6. Construct a fundamental domain for the action of $\text{SL}(2, \mathbb{Z})$ on $\mathbb{H}^2$ that contains the point $(31/30, 9/10)$. What transformation maps it to the standard fundamental domain?